Wire Grid Moment Method (NEC) Models of a Patch Antenna

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Abstract: Wire grid models of an air-loaded resonant circular patch antenna are used to calculate several observable quantities. The resonant frequency, input reactance, and bandwidth are accurately calculated, but the input resistance is underestimated by the grid models due to the existence of separate currents on both the top and bottom of the measured solid patch. The Lawrence Livermore (NEC3) Method of Moments computer program was used to generate the grid models. Average wire lengths of about .03 λ are required for accurate results. A comparison with other studies suggests that the .03 λ segmentation may be required because the patch is close to the ground plane (.0175 λ), and/or because errors in the near field calculation are compounded by the large fraction of energy stored in the near field since the patch is a high Q resonator.

Introduction

The electrical behavior of a conducting surface such as a patch antenna can be approximated by a wire grid. The wire grid consists of numerous short and interconnected segments of wire arranged such that the grid conforms to the surface of the patch. Wire grid computer models of a patch were used to calculate the resonant frequency, input impedance, and impedance bandwidth of the patch, and the calculated results are compared with measurements. The measured results were obtained from the open literature [1]. The near and far fields of the patch were not measured and so are not available for comparison with calculations.

The measured patch was a single isolated planar circular disc on a polystyrofoam substrate. Figure 1 shows the measured patch cross-sectional dimensions. It is about 1/2 wavelengths in diameter and is 1/57 wavelengths (.0175 λ) over a ground plane. The large patch size and low resonant frequency (f_m=549MHz) used for these measurements improved the accuracy of the fabrication and measurement. Both the measured and calculated results are scalable to a higher frequency and smaller patch since the dielectic constant of the substrate is about 1.0.

The computer program used to calculate the input impedance of the wire grids was the Numerical Electromagnetics Code 3 - Method of Moments (NEC3-MOM) developed at Lawrence Livermore National Laboratories. The details of the equations, solution, and computer program are fully described in the excellent manual available with the NEC3-MOM code [2], and so will not be repeated here.

Wire Grid Computer Models

A major limitation to the modeling of patch antennas using the approach described in this paper is that the NEC3-MOM code currently available cannot model a dielectic slab over a ground plane. Therefore, measurements for a patch over a very low dielectric constant (ϵ_r = 1.014) foam substrate were used in order to provide a comparison with air loaded (ϵ_r = 1.0) wire grid computer models.

Figures 2 through 6 show the wire grid configurations used. The diameter of all the grids was the same as the patch: .2908 m which is .53 λ at f_m where f_m is the measured resonant frequency of the patch (549 MHz). The measured patch was driven by a coax center conductor extending up through a hole in the ground plane and connecting to the patch. It is modeled in the grids as an ideal voltage source placed on a feed wire segment connecting the ground plane to the grid. The feed wire is perpendicular to the page in figures 2 through 6, and its position is indicated by a small circle. The feed wire was included in the grid using the Numerical Green's Function option of NEC3 so that rotational symmetry could still be used for the rest of the grid to reduce computer time. The thickness and conductivity of the patch was not known. The grid models assumed infinite conductivity.

Some details for each grid, and a summary of the results, are listed in table I: Column 2 lists the average wire length (in wavelengths at f_m) between junctions for each grid. Some wires between two junctions were broken into several segments but the average total wire length between the junctions is listed in column 2. Column 3 gives the total number of wire segments in the complete grid. Column 4 is the total surface area of the grid relative to twice the surface area of <u>one side</u> of the planar patch. Column 5 shows the resulting calculated grid resonant input resistance R_c of the feed wire, normalized by the measured input resistance R_m at resonance of the patch. Column 6 lists the calculated grid resonant frequency f_c normalized by the measured patch resonant frequency f_m. The second to last column is the impedance bandwidth of the grid as a percent of the resonant frequency of the grid. The impedance bandwidth was defined as the frequency band over which the input resistance was greater than .707 of its resonance value [3][4]. The measured bandwidth (from the curve in figure 7) is 1.9%. The resistance values and bandwidths for some of the cases are estimated from a curve such as shown in figure 7 with a limited number of data points. The last column is the smallest ratio of segment length to segment radius of any segment in the grid.

<u>Resonant Frequency Results</u>

Figure 7 plots the results for case i prior to normalization. All the calculated resonant frequencies (after normalization by f_m) are listed in column 6 of table I. For cases h and i the resonant frequency of the patch is calculated to within 1% of the measured resonant frequency. This is less than the anticipated numerical error in the NEC3 code [2] since some of the shortest segments had length/radius ratios less than 8 (listed in the last column of table I).

Table I. Summary of Grid Models and Calculated Results

Case	Average Wire Length	Number of Segments	Surface Area	Estimated Rc/Rm	fc/fm	Estimated Band-width	Minimum Segment Len/Rad
a	0.096	56	0.078	2.65	0.849	3.0%	104.00
b	0.058	177	0.620		0.855		3.13
c	0.058	177	1.000	1.06	0.886	2.8%	1.93
d	0.039	609	0.576	1.17	0.961	2.6%	3.65
e	0.039	609	1.151	0.80	0.965	2.3%	1.82
f	0.039	609	2.303	0.51	0.952	2.6%	0.91
g	0.035	1105	1.000	0.88	0.955	2.6%	1.89
h	0.028	737	0.974	0.81	0.996	3.1%	3.06
i	.02 est.	>1000	2.0	0.88	0.996	1.9%	5.00

Figure 8 plots the normalized resonant frequency vs the average wire length between junctions for the 9 cases (plots column 6 vs column 2 of table I). That figure shows that the calculated frequency converges to the measured value when the wire length is reduced to .03 λ. The grid can therefore provide a very accurate prediction of the patch resonant frequency, but the .03 λ wire length requirement is more stringent than the .05 to .20 λ usually required for accurate wire grid modeling [2][5].

A small source of error in the resonant frequency is due to the 1.4% difference in substrate dielectric constant between the measured patch substrate and calculated grid model. This should result in a calculated resonant frequency about 1.4% lower than the measured resonant frequency, based on a resonant cavity model for the patch. The calculated frequency for cases h and i was only 0.4% lower than the measured frequency, so the calculated resonant frequency is actually about 1% higher than expected. A small source of error in resonant frequency may also occur because radially directed currents are required to go to zero at the edge of the patch, but in the grid some current may continue a small distance on the circumferential rim wires before going to zero. However, as can be seen by a comparison of cases g and h and figures 5 and 6, this source of error does not appear to be very great.

Input Impedance Results

The value of the input resistance at resonance is calculated to within 12% for case i. This error is probably due to the following difference between a wire grid and a planar surface: The planar patch has currents on both the top and the bottom of the patch, while a grid combines these two currents into a single current on the grid. The feed is connected to the underside of the patch, so the measured input impedance is due to the current on only the underside of the planar patch. The current on the top of the planar patch does not contribute to the measured input impedance (except indirectly inasmuch as it couples to and changes the current on the underside). The grid input impedance on the other hand will be calculated using the combined total current on the grid at the feed point. The resulting current flowing from the feed wire onto the grid may therefore be larger or smaller than the actual current flowing onto the underside of the patch. A larger feed wire current would result in a lower input impedance since $Z = V/I$. This result is seen in figure 8 which compares the measured and calculated impedances. The fact that the calculated grid impedance is only 12% lower than the measured impedance for case i at resonance suggests that the currents on the top of the measured planar patch are small compared to those on the underside. This is also to be expected since a patch resembles a resonant cavity [6], with the field mostly reflected at the edge of the patch and confined between the patch and ground plane. The larger currents would therefore be found on the underside of the planar patch since the fields there are much greater than over the top of the patch. The input reactance was not explicity measured, but at resonance it goes to zero and this is correctly predicted by all the grid models, and is plotted for case i in figure 7.

Surface Area and Wire Radii of Grids

Calculated errors may be due to the reactance of the grid surface, which is sensitive to surface area [5][7]. The literature on wire grid models of solid surfaces recommends that the total surface area of all the wires in the grid be twice the surface area of <u>one</u> side of a closed surface, or twice the surface area of <u>both</u> sides of a surface that is not closed [5][7][8]. The surface area for the grids is obtained by adjusting the wire radius. The patch is not a closed surface physically, yet electromagnetically it resembles a resonant cavity [6], which is a closed surface. Column 4 of table I lists the grid surface area divided by twice the surface area of <u>one</u> side of the patch. The grid surface area has a significant effect on the input resistance and bandwidth, but an insignificant effect on resonant frequency, as can be seen for cases d,e, and f. This suggests that a resonant patch should be considered as a closed surface for the purposes of surface area equivalence, since case e yielded better results than case f. However case i produced the best results when the patch was considered as an open surface (twice both sides).

The surface area requirement posed two problems for case a (and to a lesser extent for case b also): First, the wire radius which would be required is a large fraction of the 0.953 cm height of the patch over the ground plane. The resulting proximity of the ground plane to the bottom of the wires would be a significant alteration of the model. Secondly, enlarging the wire radius to provide enough surface area could cause numerical errors, since the NEC3 manual recommends an 8/1 or greater segment length/radius ratio [2] (there are no straight two wire junctions in the grids, so the EK option allowing a 2/1 segment length/radius ratio could not be used). Several cases were tried using the same grid as case a, but with wire radii of .4 cm or .8 cm (providing twice the surface area of one or both sides of the patch respectively). These large radii produced input resistances in the thousands of ohms (Rc/Rm>10), or caused other obvious errors in the results. A modification was then tried to the 56 segment model: The grids with large radii were raised higher over the ground plane so that the spacing between the ground plane and bottom of the wires was equal to the measured patch spacing of 0.953 cm. This reduced the extremely large input resistances to values around that obtained for case a. The resonant frequency was not significantly affected by changing the grid height over the ground plane. A second modification also tried for the 56 segment grid was to scale its dimensions based on the assumption that the radius of the wires on the periphery increases the effective diameter of the grid, although the NEC3 algorithm does not indicate that this should make very much difference [9]. Up to π wire radii were added to the diameter of the grid. This size scaling resulted in higher resonant frequencies but had little effect on the value of the input resistance.

Wire Grid Segmentation Requirements

Ludwig [5] modeled an infinite cylinder using a wire grid. When using the optimal surface area for the grid (twice the area of the cylinder), he accurately calculated the far field and currents on the grid. To obtain accurate far field results when the cylinder was in a cavity resonant mode, he needed .2 λ grid spacing, whereas in a non-resonant mode he needed .33 λ spacing. Results for the cylinder in a resonant mode, however, were much more sensitive to errors in the grid surface area than were results for the non-resonant cylinder. He also accurately calculated the near field at some distance from the grid, but found that the near field right on the grid, between the grid wires, is not accurately calculated using the grid model, but that this error is reduced when more wires are used. The diameter of the infinite cylinder used by Ludwig was much greater than the diameter or spacing over the ground plane of the patch described in this paper. Due to the small size of the patch, and its very close proximity to its image in the ground plane, the results for the patch may have been much more sensitive to calculated errors of the near field close to the grid, and therefore required smaller wire lengths between junctions.

Another previous study [10][11] used a wire grid to model wide planar strip dipoles each 0.5 λ long, 0.12 λ wide, and 0.25 λ above a ground plane. The dipoles were therefore somewhat smaller than the patch, but much higher above the ground plane. Wire lengths between junctions of about 0.1 λ yielded accurate results for the input impedance and far field patterns of the dipoles near resonance. Longer wire lengths were not tried since the width of the dipoles was .12 λ. Those results also suggest that it may be the close proximity of the patch to the ground plane that requires a finer grid segmentation.

The dipoles were modeled at resonance, but they do not resonate in a resonant cavity mode. The resonating strip dipoles have an impedance bandwidth that is very much wider than the bandwidth of the patch, so their Q is much lower than that of a patch since the Q of a resonator is a measure of the sharpness of the bandwidth [3]:

$$Q = 2\pi f_m / \text{Bandwidth}$$

The Q of a resonant structure is also given by:

$$Q = 2\pi\ E_s\ /\ E_l$$

Where f_m is the resonant frequency, E_s is energy stored in the reactive near field, and E_l energy lost per cycle. Therefore, the higher the Q of a resonator, the larger the fraction of energy being stored in the reactive near field.

Errors in the computed near field should have a greater effect on the calculated results when a large fraction of the energy is stored in the near field (i.e. for a high Q resonator). This provides another possible explanation for the short wire length (0.03 λ) required to model the patch.

<u>Conclusions</u>

The results of this study indicate that a wire grid using NEC3 is a valid model for the resonant frequency, bandwidth, and input reactance of an air loaded patch antenna. The input resistance to the patch is underestimated by the grid due to the presence of currents on both the top and bottom of the solid planar patch. Average wire lengths of about .03 λ between junctions are required for accurate results. A comparison with grid models of other structures described the literature suggests that this short .03 λ segmentation requirement for the patch may be due to one or both of the following reasons: the very close proximity of the patch to the ground plane, or due to errors in the near field calculation compounded by the large fraction of energy stored in the near field since the patch is a high Q resonator.

References

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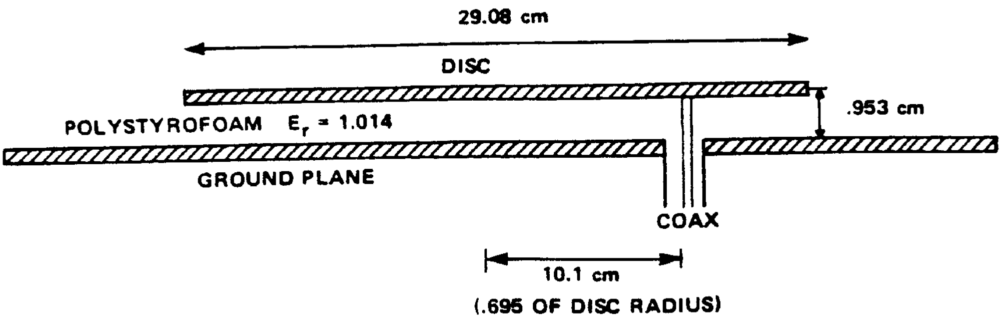

Figure 1. Cross Section of Circular Patch

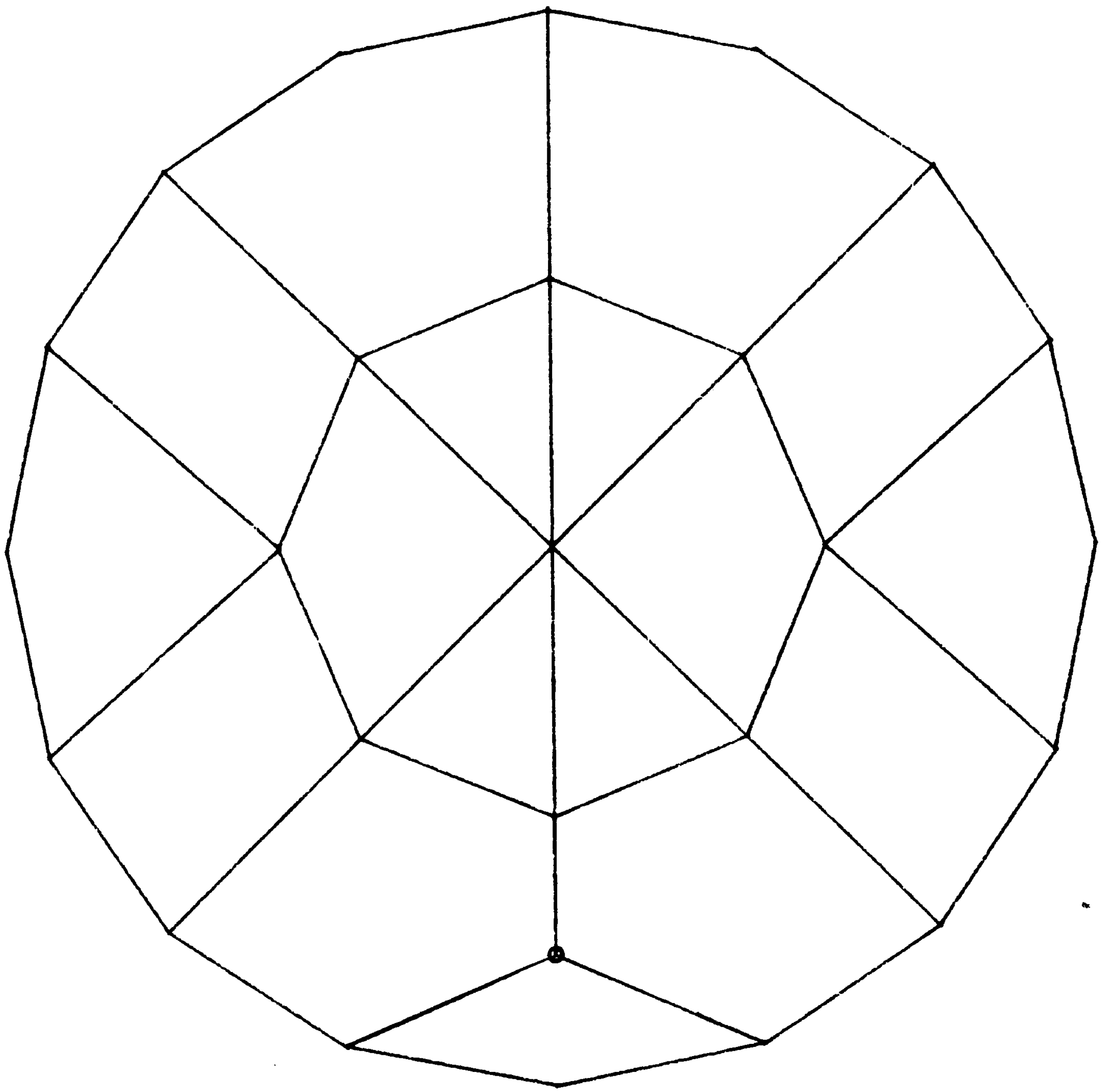

Figure 2. Wire Grid with .096 λ Average Wire Length.

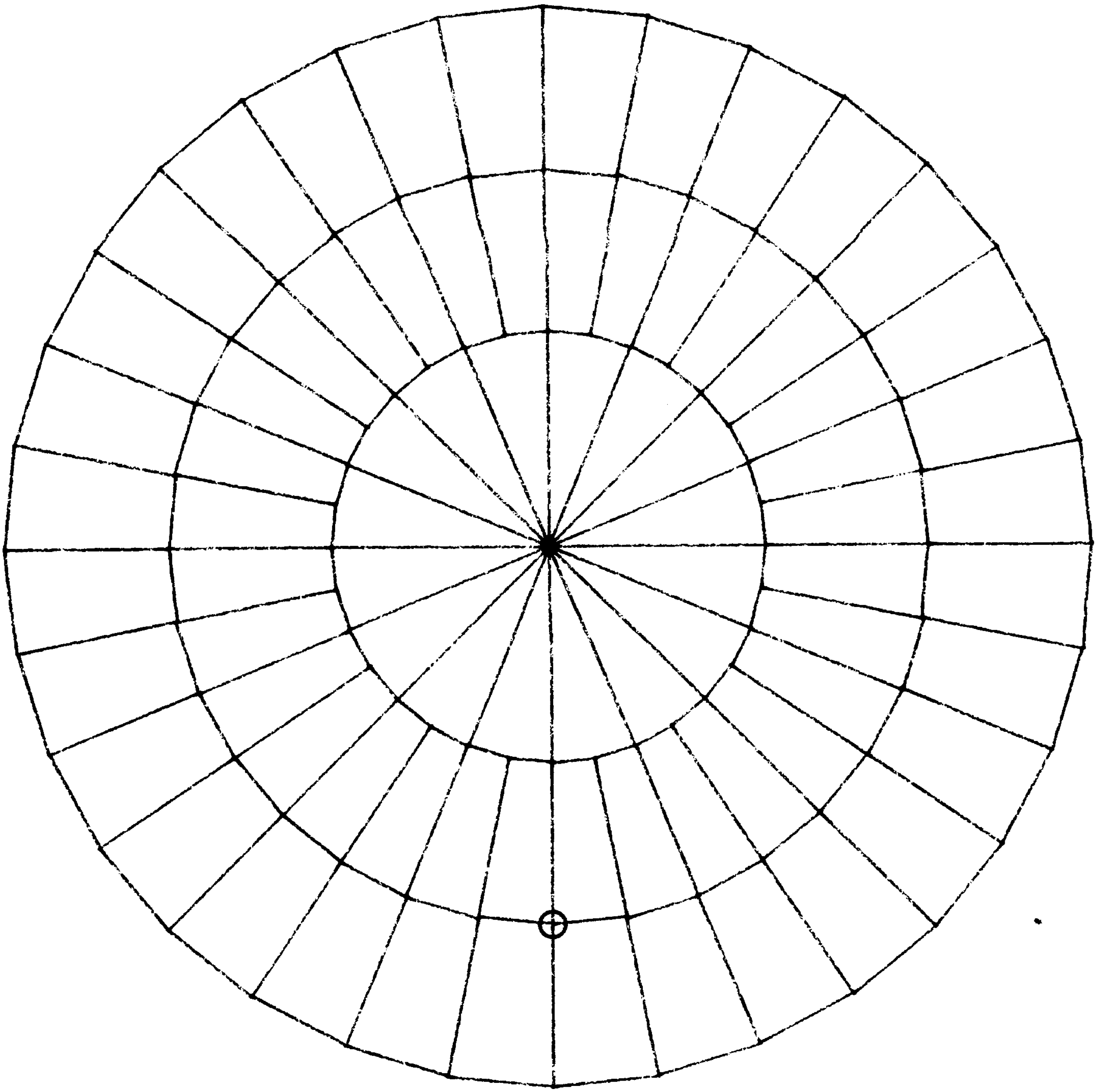

Figure 3. Wire Grid with .058 λ Average Wire Length.

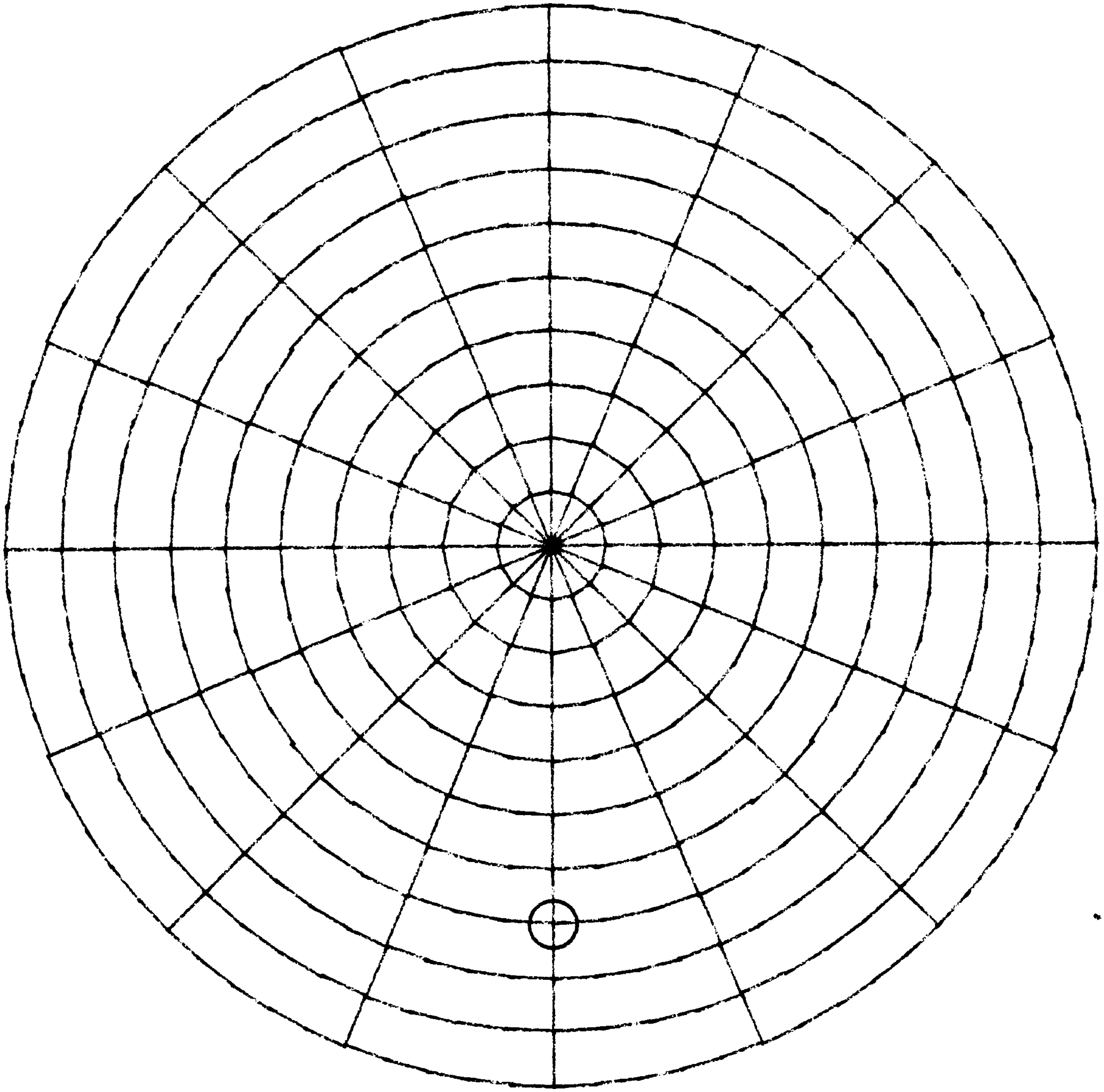

Figure 4. Wire Grid with .039 λ Average Wire Length.

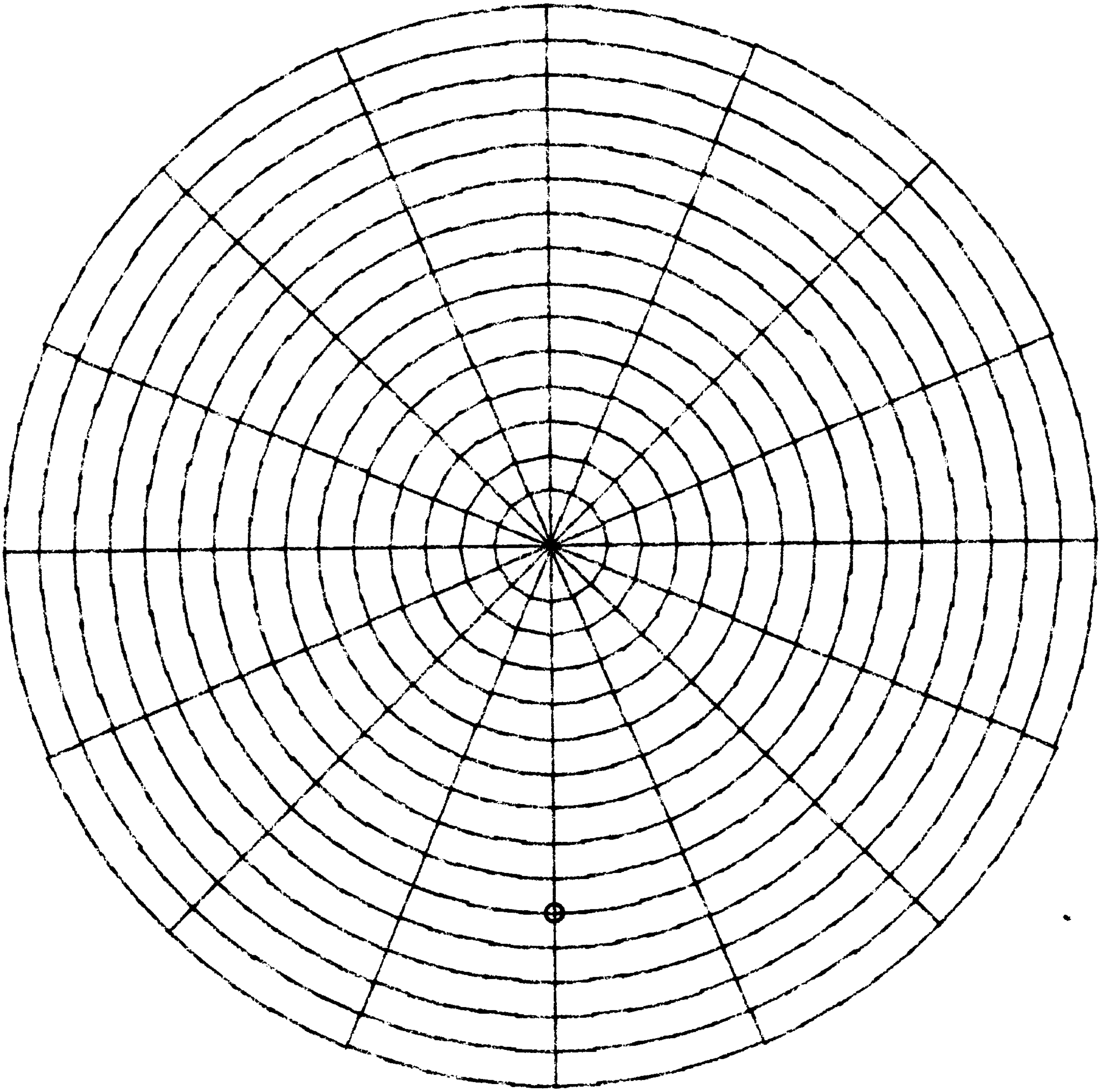

Figure 5. Wire Grid with .035 λ Average Wire Length.

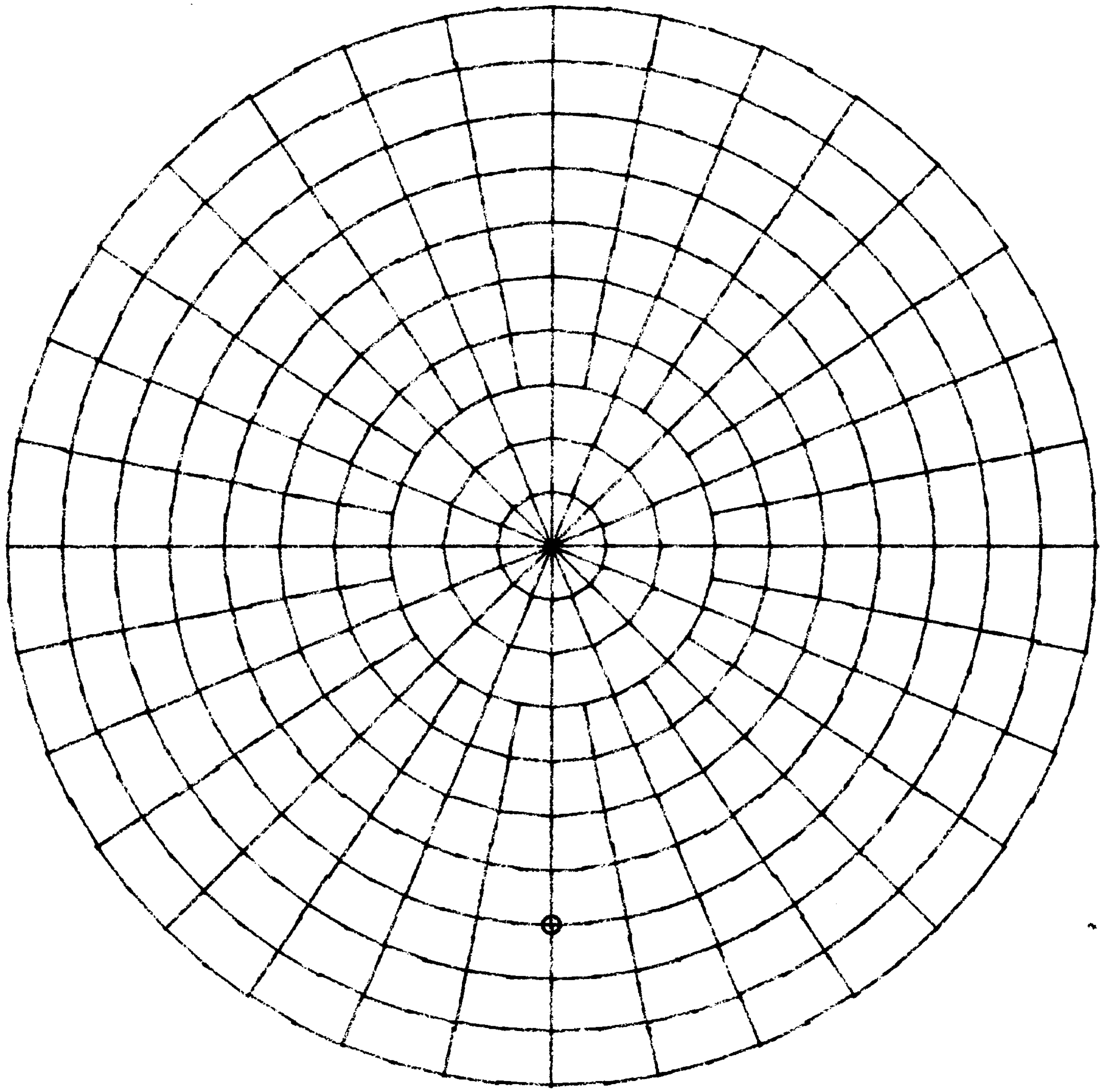

Figure 6. Wire Grid with .028 λ Average Wire Length.

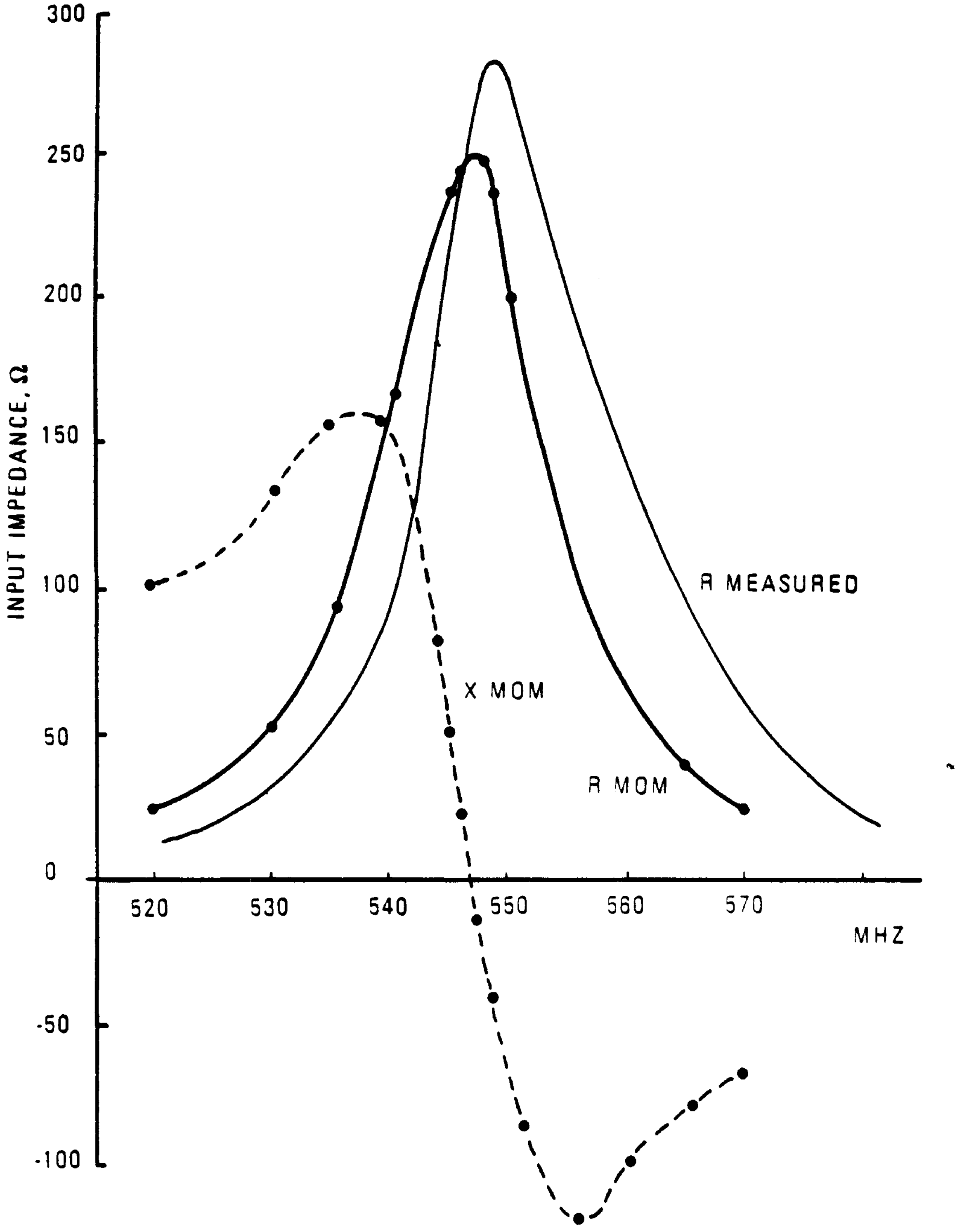

Figure 7. Measured and Calculated Input Resistance. Calculated Reactance also Shown. Calculations are for Case i of Table I.

Figure 8. Calculated Resonant Frequency of Grids vs. Average Wire Length Between Grid Junctions.