FINITE ELEMENT ANALYSIS AND EVALUATION OF ELECTROMAGNETIC FIELDS GENERATED BY ATMOSPHERIC DISCHARGES

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Abstract - A lightning return stroke channel is modeled by a high loss transmission line. The differential equations that represent its dynamic behavior are solved by the application of the one-dimensional finite element method (FEM). By the combination of FEM results with the use of Maxwell equations applied to the dipole method, electric and magnetic (EM) fields are evaluated at various positions in space.

I. INTRODUCTION

The great increase in the number of components in new electronic equipment constitutes an everpresent concern for engineers dealing with problems of electromagnetic compatibility. As a consequence of the phenomenon, inaccurate readings may occur and system regulatory and control functions may be improperly activated. In a worst case scenario, this may lead to the destruction of the equipment. In general, the components operate at low voltage levels and thus remain quite susceptible to electromagnetic disturbances. The disturbances may originate from various sources, but the ones caused by lightning are among those principally responsible for the most serious occurrences [1].

During the last decade, much progress has been made in solving electromagnetic transient problems on digital computers. Ultimately, the usefulness of computer simulations must be proved by comparing the results with measurements obtained from field tests [2]. This will check not only the correctness of the algorithms but also the adequacy of the models. A careful error analysis of the measurements is often essential if differences have to be explained.

To evaluate the electromagnetic field generated by lightning return strokes more accurately, it is of fundamental importance to choose a model which can adequately represent the evolution of surge along the channel. An adequate, representative model can be defined by using a physical or mathematical construction which behaves as, or approximates closely, the natural phenomenon in question. The Bruce-Golde (BG) model, the Transmission Line (TL) model, the Traveling Current Source (TCS) model, and others [1]-[5] are among some of the commonly employed models.

The work presented here is based on the determination of EM fields generated by lightning return strokes, using a high loss transmission line model of the channel. The equations that govern the dynamics of this problem are solved through finite analysis, with results then applied to calculate EM fields at diverse points in space [2],[6]-[8].

II. THE TRANSMISSION LINE MODEL FOR STROKE

Consider a lossy transmission line where L', C', G' and R' are the inductance, capacitance, conductance and resistance per unit length, respectively [3]. At a point x along the line, voltage and currentare related by

$$\begin{cases} -\dfrac{\partial v(x,t)}{\partial x} = R'i + L'\dfrac{\partial i(x,t)}{\partial t} \\ -\dfrac{\partial i(x,t)}{\partial x} = G'v + C'\dfrac{\partial v(x,t)}{\partial t} \end{cases} \tag{1}$$

The general solution of (1) was obtained by Heaviside and Poincaré. Using the Heaviside operator

$$\begin{cases} -\dfrac{\partial v(x,t)}{\partial x} = (R'+L'p)\, i(x,t) \\ -\dfrac{\partial i(x,t)}{\partial x} = (G'+C'p)\, v(x,t) \end{cases} \tag{2}$$

$$\begin{cases} v(x,t) = e^{\gamma x} f_1(t) + e^{-\gamma x} f_2(t) \\ i(x,t) = -\sqrt{\dfrac{G'+C'p}{R'+L'p}}\left(e^{\gamma x} f_1(t) - e^{-\gamma x} f_2(t)\right) \end{cases} \tag{3}$$

With appropriate algebra the solution of (3) for an infinite line excited by a unit step function can be written as

$$\begin{cases} v(x,t) = e^{\frac{-\alpha x}{v}} + \beta \dfrac{x}{v}\displaystyle\int_{\frac{x}{v}}^{t} \dfrac{e^{-\alpha t} I_1(\beta k)}{k} dt \\ i(x,t) = \sqrt{\dfrac{C'}{L'}}\left[e^{-\alpha t} I_0(\beta k) + (\alpha - \beta)\displaystyle\int_{\frac{x}{v}}^{t} e^{-\alpha t} I_0(\beta k) dt \right] \end{cases} \tag{4}$$

for $t > x/v$, where $k = \sqrt{t^2 - x^2/v^2}$, I_0 and I_1 are are zeroeth-order and first-order Bessel functions of imaginary arguments, respectively [5], and

$$v = \frac{1}{\sqrt{L'C'}} \quad \text{speed of propagation,} \quad (5)$$

$$\alpha = \frac{1}{2}\left(\frac{R'}{L'} + \frac{G'}{C'}\right) \quad \text{attenuation constant,} \quad (6)$$

$$\beta = \frac{1}{2}\left(\frac{R'}{L'} - \frac{G'}{C'}\right) \quad \text{propagation constant.,} \quad (7)$$

The determination of the waveform at any point x on the line from (4) involves a numerical integration that is prohibitively time-consuming for an infinite line. A simplified solution can be obtained from the lossless transmission line equations [4], where the losses are represented by resistances at the ends and center of the transmission line; This is a good approximation for R<< Z.

III. POINT-MATCHED FINITE ELEMENT METHOD

This method [4,8-12] requires the line to be subdivided into a one-dimensional finite number of subregions called elements. Each element has several points called interpolation nodes. This allows "v" and "i" to be written in the form:

$$\begin{cases} v(x,t) = \sum_{i=1}^{M} \phi_i(x)\, V_i(t) \\ i(x,t) = \sum_{j=1}^{N} \psi_j(x)\, I_j(t) \end{cases} \quad (8)$$

where "M" and "N" represent the number of nodes of the "V" and "I" finite element segments, respectively, and ϕ_i and ψ_j are basis functions that interpolate the voltage and current within each element using the values at the nodes as interpolation coefficients.

This approach is referred to as the point-matched time domain finite element method (TDFE) because $\phi_i(x)$ and $\psi_j(x)$ are defined to be

$$\begin{cases} \phi_i(x) = \begin{cases} 1 \text{ at } x = x_i \\ 0 \text{ at other nodes} \end{cases} \\ \psi_j(x) = \begin{cases} 1 \text{ at } x = x_j \\ 0 \text{ at other nodes} \end{cases} \end{cases} \quad (9)$$

In the leap-frog scheme, the time derivatives are represented by the forward Euler difference given by

$$\begin{cases} \dfrac{\partial V_i^{n+\frac{1}{2}}}{\partial t} \approx \dfrac{V_i^{n+1} - V_i^n}{\Delta t} \\ \dfrac{\partial I_j^n}{\partial t} \approx \dfrac{I_j^{n+\frac{1}{2}} - I_j^{n-\frac{1}{2}}}{\Delta t} \end{cases} \quad (10)$$

where I_j^n is the current at x_j at time $n\Delta t$ and $V_i^{n+\frac{1}{2}}$ is the voltage at x_i at time $(n+1/2)\Delta t$.

Using appropriate interpolation functions, the following final equations are obtained

$$\begin{cases} V_i^{n+1} = \dfrac{2C' - G'\Delta t}{2C' + G'\Delta t} V_i^n - \dfrac{2\Delta t}{(2C' + G'\Delta t)\,\Delta x}\left(I_{j+1}^{n+\frac{1}{2}} - I_j^{n+\frac{1}{2}}\right) \\ I_j^{n+\frac{1}{2}} = \dfrac{2L' - R'\Delta t}{2L' + R'\Delta t} I_j^{n-\frac{1}{2}} - \dfrac{2\Delta t}{(2L' + R'\Delta t)\,\Delta x}\left(V_{i+1}^n - V_i^n\right) \end{cases} \quad (11)$$

Note that the solution to the Finite Element Method represented by (11) is stable [8] if

$$v \le \frac{\Delta x}{\Delta t} \quad (12)$$

where "v" is the wave propagation speed. This implies that the wave must not propagate more than one subdivision in space during one time step. To obtain the solution [8] set

$$\Delta x = v\Delta t. \quad (13)$$

IV. ANALYTICAL FORMULATION FOR DISTANT FIELDS

The lightning return stroke channel is modeled by a transmission line which is approximately represented as a thin wire vertical antenna attached to ground. Then, evaluations of the far EM fields generated by the channel are made.

The particular case of a vertical antenna of negligible thickness and height "H" above a perfectly conducting ground plane, and carrying a conduction current i(z,t) is illustrated in Fig.1; The boundary conditions at the ground plane are satisfied by constructing a mirror image of the antenna as shown in Fig.1 [7].

Starting with Maxwell's equations, one can determine the electromagnetic fields in terms of retarded potentials:

$$\begin{cases} \vec{E} = -\vec{\nabla}\phi - \dfrac{\partial \vec{A}}{\partial t} \\ \vec{B} = \vec{\nabla} \times \vec{A} \end{cases} \tag{14}$$

subject to the Lorentz gauge given by

$$\vec{\nabla} \cdot \vec{A} + \frac{1}{c^2}\frac{\partial \phi}{\partial t} = 0 \tag{15}$$

By manipulating (14) and (15) in the case of a rising current pulse in the z-direction of the channel, one obtains the following set of differential equations for the EM fields:

$$d\vec{E}_{real} = \frac{\mu_0 c^2}{4\pi}(pz-z)px\left[\frac{3i}{ra^4 c} + \frac{3}{ra^5}\int_0^t i d\tau + \frac{1}{ra^3 c^2}\frac{\partial i}{\partial t}\right]\hat{a}_x dz$$
$$+\frac{\mu_0 c^2}{4\pi}(pz-z)py\left[\frac{3i}{ra^4 c} + \frac{3}{ra^5}\int_0^t i d\tau + \frac{1}{ra^3 c^2}\frac{\partial i}{\partial t}\right]\hat{a}_y dz$$
$$+\frac{\mu_0 c^2}{4\pi}\left[(-px^2 - py^2 + 2(pz-z)^2)\left(\frac{i}{ra^4 c} + \frac{1}{ra^5}\int_0^t i d\tau\right)\right.$$
$$\left. -(px^2+py^2)\frac{1}{ra^3 c^2}\frac{\partial i}{\partial t}\right]\hat{a}_z dz \tag{16}$$

$$d\vec{B}_{real} = -\frac{\mu_0}{4\pi}py\left[\frac{i}{ra^3} + \frac{1}{ra^2 c}\frac{\partial i}{\partial t}\right]\hat{a}_x dz +$$
$$+\frac{\mu_0}{4\pi}px\left[\frac{i}{ra^3} + \frac{1}{ra^2 c}\frac{\partial i}{\partial t}\right]\hat{a}_y dz \tag{17}$$

where $i = i(z,t') = i(z, t - ra/c)$, and t' is the retardation time.

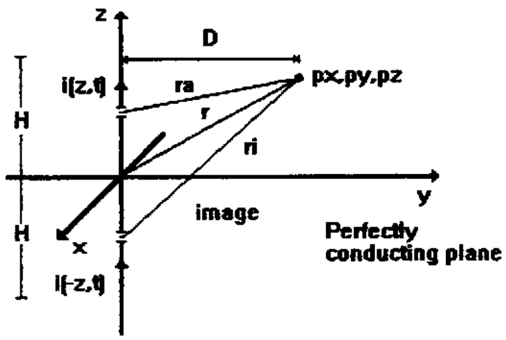

Fig.1: Vertical antenna of height "H" above a perfectly conducting ground plane

With regard to the contributions of the image antenna, it should be noted that they are calculated in a manner analogous to a real antenna by substituting "ra" for "ri" in Fig.1. In this way, the resultant EM fields can be obtained by integration along these two antennas in the z-direction, with the values of i(z,t) and i(-z,t) obtained from the solution of the transmission line equations (1) through the use of the finite element method.

V. RESULTS

For the lightning return stroke simulations that follow, typical channel parameters encountered in the relevant literature were used [1,4,7]. Channel height was assumed to be 4 km, and the following additional parameters were used: C'=3.5 pF/m, $v = 1/\sqrt{L'C'}$ = 80 m/μs, G'=0 S/m. The losses (R') vary from 0 to 1 Ω/m. A triangular current function (1.125 x 25 μs) with a 10 kA peak value placed at ground level was used as the source of the channel model. Simulations were also carried out by using the formulation proposed by Uman [7] for a lossless line; The results are presented and compared with the solutions obtained from the formulation given in this paper, viz. equations (11). Finally, by the introduction of a finite value of resistance in the channel, EM fields are estimated for specific points in space.

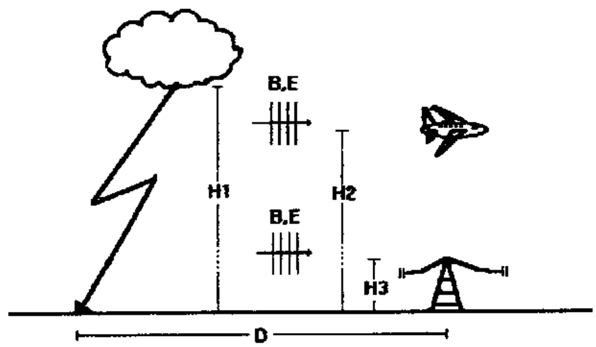

Fig.2: Cloud to ground lightning discharge and the resulting electromagnetic interference.

The results obtained with the finite element method show considerable accuracy in comparison with similar results obtained with Uman formulation [7] (see Figures 3 through 6). The oscillations observed in Figures 4, 6, 8 and 9 are due to problems caused by the influence of the retardation time and by the definition of the time step used to calculate the current and EM fields.

Introduction of resistance in the channel yields interesting results which can be observed in the EM waveforms (Figures 7 through 9). Some attenuation and distortion in B was expected and occured once the introduction of resistance attenuated and distorted the current waveform in the channel.

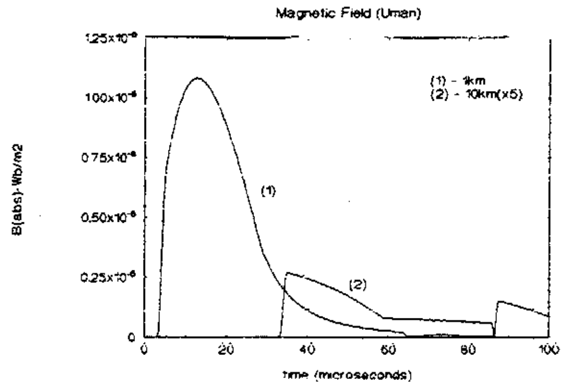

Fig.3: Magnetic field at distances of 1.0 and 10.0 km from the lightning return stroke, in the xy-plane (Uman) [7].

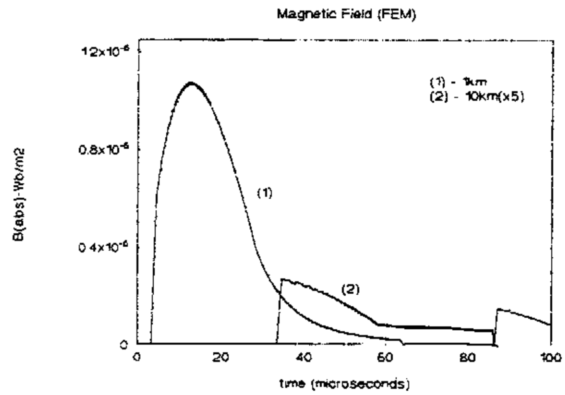

Fig.4: Magnetic field at distances of 1.0 and 10.0 km from the lightning return stroke, in the xy-plane, with R'=0 Ω/m (FEM).

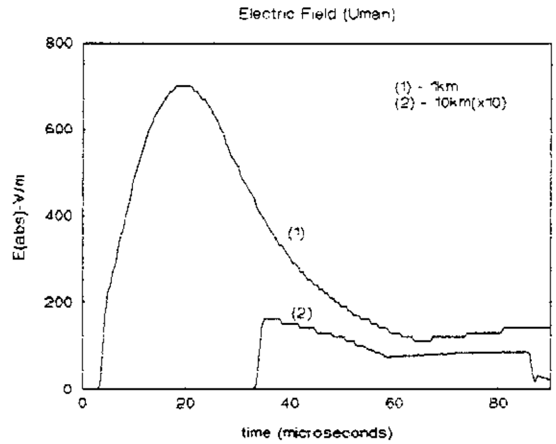

Fig.5: Electric field at distances of 1.0 and 10.0 km from the lightning return stroke, in the xy-plane (Uman) [7].

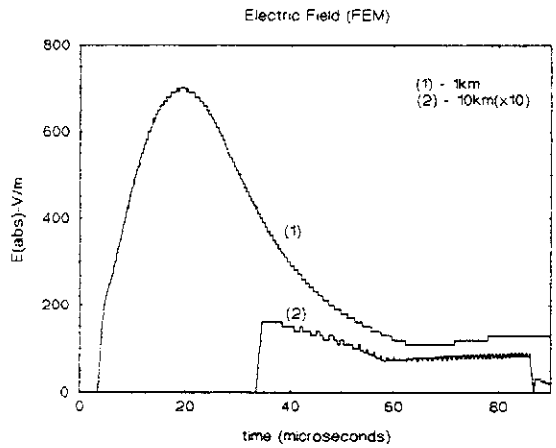

Fig.6: Electric field at distances of 1.0 and 10.0 km from the lightning return stroke, in the xy-plane, with R'=0 Ω/m (FEM).

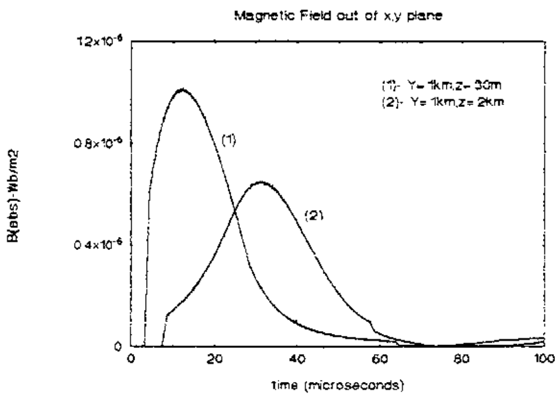

Fig.7: Magnetic field at z=30 m, y=1 km and z=2 km, y=1 km with R=1 Ω/m (FEM).

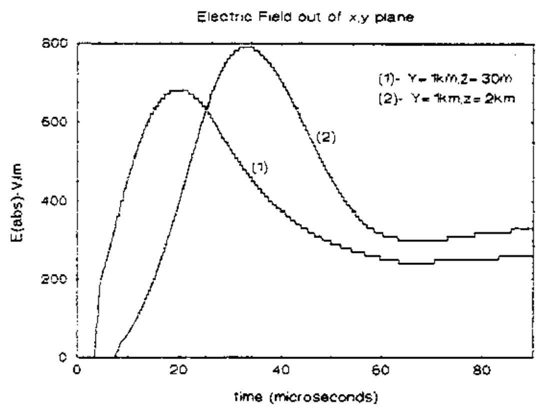

Fig.8: Electric field at z=30 m, y=1 km and z=2 km, y=1 km with R = 1 Ω/m (FEM).

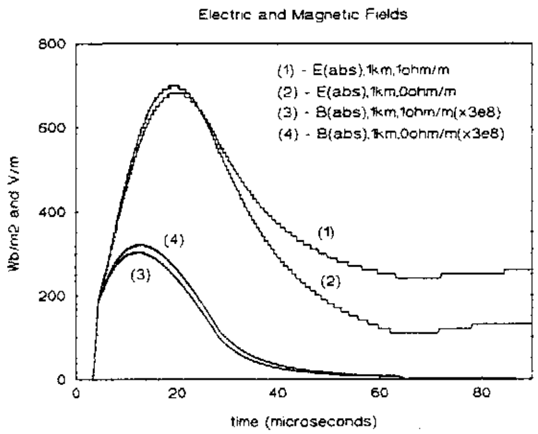

Fig.9: EM fields at a distance of 1 km from the lightning return stroke, in the xy-plane, with R=0 Ω/m and R=1 Ω/m (FEM).

Regarding the electric field E, the following observation can be made: its curve increased and decreased more slowly, suggesting a channel charge and discharge time alteration caused by the introduction of finite values of resistance R' in the channel (equation (11)).

Finally, in Figures 7 and 8, EM field waveforms were obtained at points with $pz \neq 0$, suggesting situations such as those shown in Fig.2.

It is important to note that the following simplifications were made in the construction of the model: (a) the channel was considered uniform and vertical, (b) a perfectly conducting ground plane was used, and (c) the absence of subsequent discharges was assumed.

VI. CONCLUSIONS

A consistent mathematical model of a lightning return stroke channel was presented, with the objective to evaluate the EM fields generated by the natural phenomenon of lightning at specific points in space. Different simulations were carried out, and the results compared favorably with those in the referenced literature.

The elevated losses introduced in the lightning return stroke channel, as suggested in the referenced literature [10,12], cause numerical problems in most mathematical models of representation. On this point, a great contribution in flexibility by the use of the finite element method was observed, which properly treats the phenomenon of propagation in transmission lines with elevated losses. It is worth emphasizing that the utilization of the finite element method in the various problems goes far beyond others. Its flexibility can be properly applied to other lightning return stroke characteristics, such as corona effect, nonuniform parameters in lightning channels, and so on.

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