Selecting Sampling Interval in the Improved Spectral Domain Method to Simulate Microstrip Circuits

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Abstract **─** In this paper, we investigate the spectral method that calculates the S-parameters for microstrip circuits and antennas. A sampling interval selection criterion is proposed. The scheme for low-loss cases is specially discussed. Several examples are employed to demonstrate the engineering applications of the spectral domain method and its results are compared with previously published and measured results.

Index Terms **-** Microstrip circuits, sampling interval, and spectral domain method.

I. INTRODUCTION

The time domain Prony's method [1, 2] and its improved version [3] have been proposed to derive complex resonant frequencies of a scatterer from its transient response. The optimum sampling technique for the original time domain Prony's method has been presented in [4] and the issue of choosing the sampling interval for the improved time domain Prony's method has been addressed in [5]. A combination of FDTD and time domain Prony's methods [6] have been employed for the analysis of microwave integrated circuits and to obtain their scattering parameters. The concept of spectral domain Prony's method has been introduced in [7], and has been applied to the problem of analyzing planar microstrip circuits. It employs the least squares procedure to estimate the complex scattering parameters. It does this by extracting the magnitude and phase of the incident and reflection waves from the sampled voltage data, and it does not require the knowledge of the characteristic impedance of the microstrip feed line. However, if the dimensions of microstrip lines at the feed and load ends are relatively small, then the distance between the sampling points to which the Prony's method is applied is small. Under these circumstances, the conventional spectral domain Prony's method suffers from the problem of ill-conditioned equations whose solution has considerable errors, especially at low frequency. We have introduced its improved version in [8] to circumvent this problem. The improved version has higher accuracy and smaller computational domain compared with the conventional spectral domain Prony's method, and is in good agreement with analytic formula as well as measured data. In this work we propose the sampling interval selection criterion based on error analysis and numerical experiments. Further, this method cannot get physical attenuation for low-loss cases, but it can get good results for S-parameters. We will demonstrate this point in the following sections and propose a special scheme for lossy cases.

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II. SPECTRAL-DOMAIN PRONY METHOD

The sampled values of the complex voltage at the feed end of the microwave circuit can be given by [7]

$$V_n = A_1 z_1^n + A_2 z_2^n, \quad n = 0, 1, \cdots, N-1, \quad (1)$$

where both A_1 and A_2 are complex unknowns, as yet undetermined, and

$$z_1 = e^{-\gamma d}, \ \ z_2 = e^{\gamma d}, \quad (2)$$

where γ (γ=α+jβ) is the complex wave number, and d is the sampling interval along the microstrip transmission line. In equation (1), V_n are obtained from the time domain data at N equally spaced nodes via Fourier transformation.

It is assumed that z_1 and z_2 are the roots of the algebraic equation

$$B_2 + B_1 z - z^2 = 0. \quad (3)$$

From equations (1) and (3), one can obtain

$$\begin{bmatrix} V_1 & V_0 \\ V_2 & V_1 \\ \vdots & \vdots \\ V_{N-1} & V_{N-2} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} V_2 \\ V_3 \\ \vdots \\ V_N \end{bmatrix}. \quad (4)$$

Using the least squares procedure, the original spectral domain Prony's method solves equations (4), (3) and (1) to get A_1 and A_2 i.e., the incident and reflection voltages, and knowledge of which one can calculate all of the entries of the scattering matrix.

An improved method has been presented in [8]. From equations (2), (3) and (4), we have

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_{N-1} \end{bmatrix} \left[e^{\gamma d} + e^{-\gamma d} \right] = \begin{bmatrix} V_2 + V_0 \\ V_3 + V_1 \\ \vdots \\ V_N + V_{N-2} \end{bmatrix}. \quad (5)$$

Numerical simulations show that solving the above matrix cannot get physical attenuation constant for small loss cases. Considering that loss is generally low and sampling interval is very small (generally 0.01 mm ~ 2 mm), even if we set α = 0, we can get good results of scattering parameters. When it is very lossy, we still use the above matrix.

For a lossless line, the matrix shown in equation (5) reduces to

$$\begin{bmatrix} Re\begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_{N-1} \end{pmatrix} \\ Im\begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_{N-1} \end{pmatrix} \end{bmatrix} 2\cos(\beta d) = \begin{bmatrix} Re\begin{pmatrix} V_2 + V_0 \\ V_3 + V_1 \\ \vdots \\ V_N + V_{N-2} \end{pmatrix} \\ Im\begin{pmatrix} V_2 + V_0 \\ V_3 + V_1 \\ \vdots \\ V_N + V_{N-2} \end{pmatrix} \end{bmatrix}, \quad (6)$$

where *Re* (•) and *Im* (•) imply the real and the imaginary parts, respectively. We derive β from these above equations. It is evident that the values of thus obtained, have numerical artifacts and, hence, we fit the computed values with a straight line passing through the origin. This is due to the fact that β is proportional to the frequency f

$$\beta = \frac{2\pi\sqrt{\varepsilon_e}}{c} f, \quad (7)$$

where ε_e is the effective dielectric constant and c is the speed of light in free space. Then, the improved method leads to S-parameters that are much more reasonable and physically acceptable than the original results. A more accurate approach for solving α and β is to introduce analytic solution of α and β. For a microstrip line, the phase constant can be written as shown in equation (7), and the attenuations due to dielectric loss and due to conductor loss can be given by [9],

$$\alpha_d = \frac{k_0 \varepsilon_r (\varepsilon_e - 1) \tan\psi}{2\sqrt{\varepsilon_e}(\varepsilon_r - 1)} \quad (8)$$

$$\alpha_c = \frac{R_s}{Z_0 W}, \quad (9)$$

where k_0 is the free space wave number, $\tan\psi$ is the loss tangent of the dielectric, R_s is the surface resistivity of the conductor, W is the width of the microstrip line, Z_0 is the characteristic impedance of the feeding transmission line whose expression is also given in [9]. For coplanar waveguides, its attenuation has been analyzed in [10-14]. In fact, for these cases as well as other complex problems [15-17], it is very complicated to calculate their phase and attenuation constant. So, we solve equation (5) for very lossy cases and equation (6) for lossless and low loss cases. We subsequently solve equation (1) to get incident and reflection voltages and then calculate the S-parameters.

III. SAMPLING INTERVAL

There are several popular excitation techniques in FDTD simulations: lumped port, wave port, mode port, aperture field, and plane wave source [18]. The difference of the wave port from the lumped port is that the wave port is used to excite a matched port and the lumped port is used to excite an open port. Since the feed line is terminated at the domain boundary, it is perfectly matched at the domain boundary. We then use wave port excitation, not lumped port excitation. Another suitable excitation source for uniform transmission lines is a mode port [18, 19]. For solving and storing modal field, it needs more memory and computational time than wave port.

Theoretically, we choose d to be as large as possible to improve the property of equation (1). Unfortunately, if d is too large, the computational cost will greatly rise. Consequently, there is an optimal choice for the sampling interval that balances computational cost in the FDTD method and numerical error in the spectral domain method. To derive the selecting criterion of sampling interval, we firstly define a function $y = f(x)$ plotted in Fig. 1. As shown in this figure, the same error $\pm \Delta y$ has different confidence intervals, and with large gradient comes small interval, in which it has higher probability to obtain the exact solution. From equation (6), we approximately have

$$d \sin(\beta d) \Delta \beta = \delta, \tag{10}$$

where δ is the relative numerical error of the sample data. The relative error depends on a number of factors such as the cell-size in the FDTD method, and the structure of microstrip circuit. If we want to get a reasonable β, the sampling interval d must satisfy

$$(\beta d)^2 > \delta \ . \tag{11}$$

Numerical simulations for the band pass filter with wave port cases indicate that the suitable sampling interval approximately satisfies

$$d > \frac{0.01}{\beta_{\min}} \ . \tag{12}$$

One can also get good results if selecting such sampling interval for simple microstrip circuits as the patch antenna and low pass filter given in [20]. Although being an approximation, the above expression provides a selecting criterion that has not been presented before. Note that the sampling interval should be larger than the minimum cell size of the FDTD simulation.

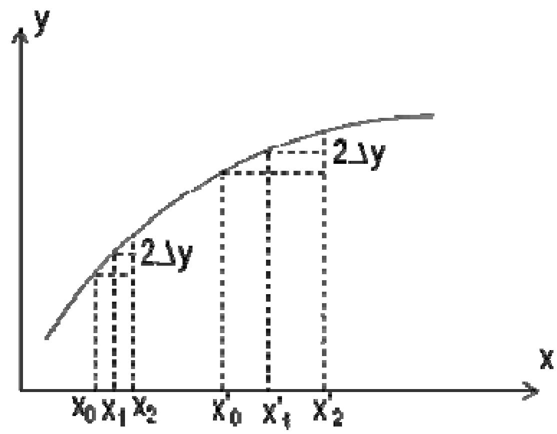


Fig. 1. Relationship between interval and gradient with the same error.

IV. EXAMPLE

We now use the spectral domain method to simulate a microstrip patch antenna, whose actual dimensions are shown in Fig. 2. The relative permittivity of the substrate is 2.2 and the conductivity of the patch and feed line is 5.8×10^8 S/m. We get the conductor loss by the use of equation (9), which is plotted in Fig. 3 (a). The computational domain for this problem is 25 mm × 50 mm × 20 mm and the sampling interval d is equal to 0.5 mm. Since the conductance of the conductor is finite, several cells are needed to simulate the conductors in the z-direction. It is very important to ensure that the FDTD mesh coincides with the edges of the feed line and the patch. We take the minimum cell sizes to be 0.05, 0.05, and 0.01 mm, in the x-, y-, and z-directions, respectively. Thus a non uniform mesh of (225 × 320 × 261 cells) is generated including six-layers of PML. A wave port is employed as an excitation source. To calculate the S-parameters, we use these three approaches as follows:

i. using equation (5) and set $\alpha = 0$, then fitting β,
ii. using equation (6) and fitting β,
iii. using analytic α and β from equations (8) and (9).

The numerical results of the attenuation constant are shown in Fig. 3 (b). We then plot its magnitude in Fig. 4. Moreover, the results of the phase constants are shown in Fig. 5. From the example, we conclude that it is difficult to solve the attenuation constant for the present spectral domain, although the method can get good results for the scattering parameters. As mentioned above, if the sampling distance is large enough, the current method can get good result of attenuation constant, but it will significantly add computational domain and require more memory. Using the current method, we can get good results for the S-parameters even though we cannot get physical attenuation for low-loss cases in the case of small sampling interval.

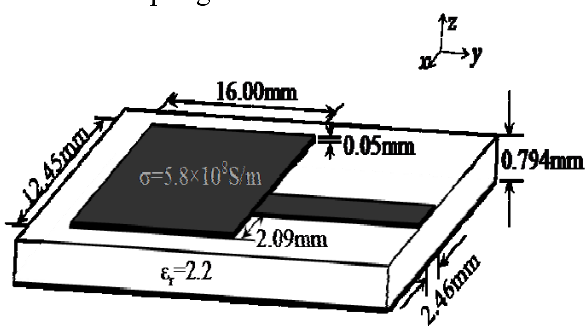
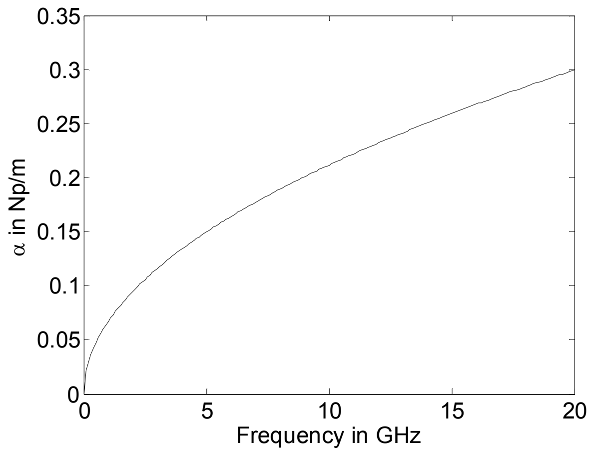
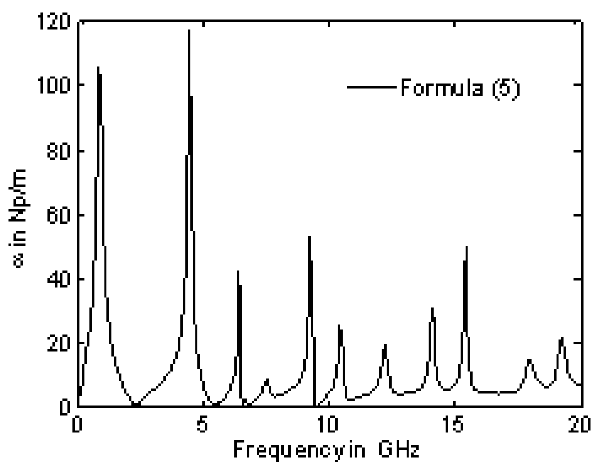


Fig. 2. Microstrip patch antenna.

Finally, we study a microstrip branch line coupler with four ports. The substrate relative permittivity is 2.2. The dimensions of the branch line coupler are given in [19] and repeated here in Fig. 6. The computation domain is 50 mm × 20 mm × 3 mm, in the x-, y-, and z-directions, respectively. The minimum spaced steps used are $\Delta x = 0.5$ mm, $\Delta y = 0.38675$ mm, and $\Delta z = 0.2$ mm and the non-uniform mesh is 85 × 47 × 13 cells, which is relatively small for a single PC. The corresponding time step is approximately 0.532 ps. The sampling interval is 0.5 mm and a wave port is employed to excite port 1. The computation time for this circuit is approximately only one minute on a single PC with two cores and 6 GB memory. It is a lossless case, so we use equation (6) to get the phase constant and plot it and its fitting curve shown in Fig. 7. The scattering coefficient results, as shown in Fig. 8, indicate good agreement in the location of the response nulls and crossover point. The desired branch line coupler performance is seen in the sharp S_{11} and S_{21} nulls, which occur at approximately at the same frequency as the crossover in S_{11} and S_{21}. At this point S_{31} and S_{41} are both approximately -3 dB, indicating that almost all of the power from port 1 is equally divided and transmitted through the device to ports 3 and 4.



(a)



(b)

Fig. 3. Attenuation constant versus frequency for (a) analytic solution and (b) using equation (5).

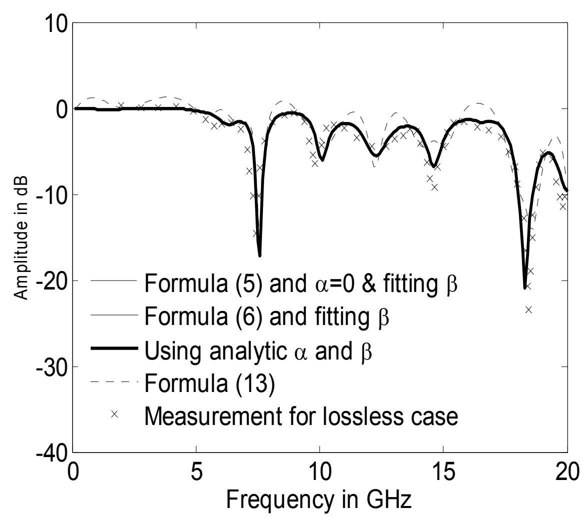


Fig. 4. Magnitude of S_{11} versus frequency.

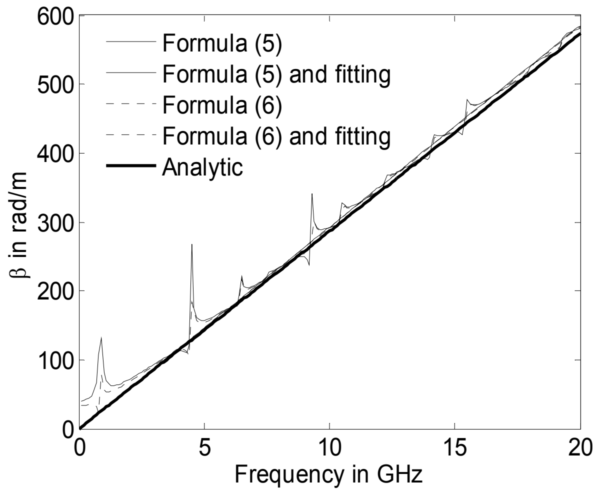


Fig. 5. Phase constant versus frequency.

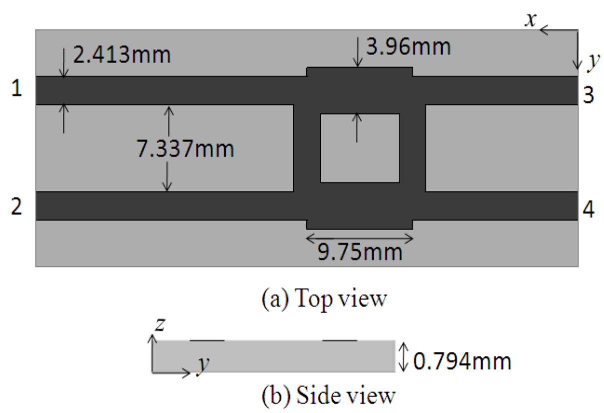


Fig. 6. Dimensions of the microstrip branch line coupler.

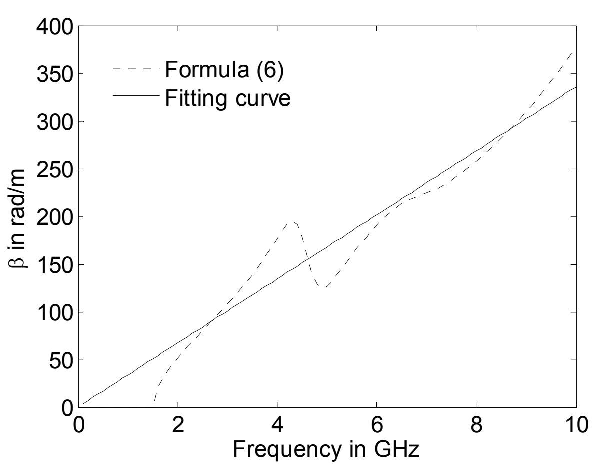


Fig. 7. The computational results of the phase constant and its fitting curve.

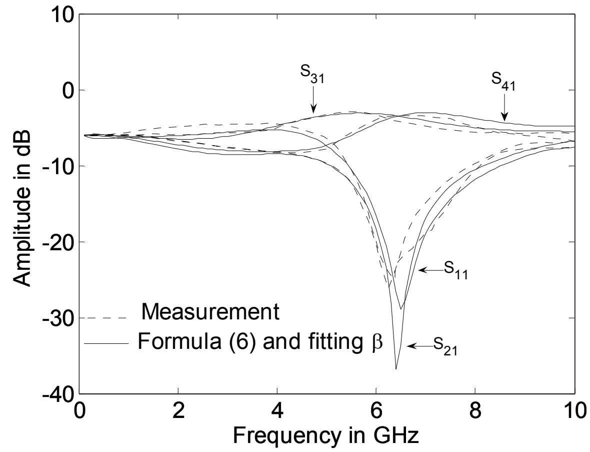


Fig. 8. Scattering parameters of the branch line coupler.

V. CONCLUSION

We have investigated the improved spectral domain method for the computation of scattering parameters of microstrip circuits. An empirical formula has been proposed for selecting a suitable sampling distance of the total voltages along the feed line. Moreover, we have also presented three approaches to analyze the lossy cases in spectral domain, and found that lossy problems can be processed as lossless cases in the present method. Using the current method, we can still get good results for S-parameters even though we cannot get physical attenuation for low-loss cases in the case of small sampling interval.

REFERENCES

[1] M. L. Van Blaricum and R. Mittra, "A technique for extracting the poles and residues of a system directly from its transient response," *IEEE Trans. Antennas Propagat.*, vol. Ap-23, pp. 777-781, Nov. 1975.

[2] J. A. Pereda, L. A. Vielva, A. Vegas, and A. Prieto, "Computation of resonant frequencies and quality factors of open dielectric resonators by a combination of the finite-difference time-domain (FDTD) and Prony's methods," *IEEE Microwave and Guided wave Letters*, vol. 2, pp. 431-433, Nov. 1992.

[3] A. J. Mackay and A. McCowen, "An improved pencil-of-function method and comparisons with traditional methods of pole extraction," *IEEE Trans. Antennas Propagat.*, vol. Ap-35, pp. 435-441, April 1987.

[4] R. W. Kulp, "An optimum sampling procedure for use with the Prony method," *IEEE Trans.*

Electromag. Compat. vol. EMC-23, pp. 67-71, May 1981.

[5] J. Lee and H. Kim, "Selecting sampling interval of transient response for the improve Prony method," *IEEE Trans. Antennas Propagat.*, vol. Ap-51, pp. 74-77, Jan. 2003.

[6] W. L. Ko and R. Mittra, "A combination of FDTD and Prony's methods for analyzing microwave integrated circuits," *IEEE Trans. Microwave Theory and Techniques*, vol. 39, pp. 2176-2181, Dec. 1991.

[7] K. Naishadham and X. P. Lin, "Application of spectral domain Prony's method to the FDTD analysis of planar microstrip circuits," *IEEE Trans. Microwave Theory and Techniques*, vol. 42, pp. 2391-2398, Dec. 1994.

[8] Z. Gong, W. Yu, and X. Yang, "Improvements to spectral domain Prony's method for analyzing microstrip circuits," *Electronics Letters*, vol. 47, no. 5, pp. 330-331, March 2011.

[9] D. M. Pozar, *Microwave Engineering, 2nd Ed.*, John Wiley & Sons Inc., New Jersey, 1998.

[10] R. N. Simons, *Coplanar Waveguide Circuits, Components, And Systems*, John Wiley & Sons Inc., New Jersey, 2001.

[11] R. E. Collin, *Foundations For Microwave Engineering, 2nd Ed.*, Wiley-IEEE Press, New Jersey, Dec. 2000.

[12] K. C. Gupta, R. Garg, I. Bahl, and P. Bhartia, *Microstrip Lines and Slotlines, 2nd Ed.*, Artech House Inc., Norwood, MA, 1996.

[13] F. D. Paolo, *Networks And Devices Using Planar Transmission Lines*, CRC Press LLC, Boca Raton, FL, 2000.

[14] M. Fujii and S. Kobayashi, "Accurate analysis of losses in waveguide structures by compact two-dimensional FDTD method combined with autoregressive signal analysis," *IEEE Trans. Microwave Theory and Techniques*, vol. 44, pp. 970-975, June 1996.

[15] H. Chen, Y.-H. Wu, Y.-M. Yang, and Y.-X. Zhang, "A novel and compact bandstop filter with folded microstrip/CPW hybrid structure," *J. of Electromagn. Waves and Appl.*, vol. 24, pp. 103-112, 2010.

[16] A. Hussain and Q. A. Naqvi, "Fractional rectangular impedance waveguide," *Progress In Electromagnetics Research,* PIER 96, pp. 101-116, 2009.

[17] T. Vaupel, "A fast spectral domain solver for the characterization of larger microwave structures in multilayered environments," *Appl. Comp. Electro. Soc. (ACES) Journal*, vol. 24, no. 5, pp. 493-503, Oct. 2009.

[18] W. Yu, X. Yang, Y. Liu, and R. Mittra, *Electromagnetic Simulation Techniques Based on the FDTD Method*, John Wiley & Sons Inc., New Jersey, 2009.

[19] W. Yu, R. Mittra, Y. Liu, and X. Yang, *Parallel Finite-Difference Time-Domain Method*, Artech House Inc., Norwood, MA, 2006.

[20] D. M. Sheen, S. M. Ali, M. D. Abouzahra, and J. A. Kong, "Application of the three-dimensional finite-difference time-domain method to the analysis of planar microstrip circuits," *IEEE Trans. Microwave Theory and Techniques*, vol. 38, pp. 849-857, July 1990.