Implementation of Generalized Transmission-Line Equations to Transmission Line Parameter Extraction

Y. W. Liu, D. S. Zhao, and K. K. Mei

Dept. of Electronic Engineering, City University of Hong Kong
E-mail: eeliuy@cityu.edu.hk, Phone: (852) 2788-9861, Fax: (852)2788-7791

Abstract: It is usually ignored that the transmission line parameters extracted by conventional transmission line equations are variable with the internal impedance of excitation source and the loading of the line. To obtain the correct parameters, the match for both excitation and loading has to be applied. Problem is that exact matches for some extreme cases are not easy to realize. To solve this problem, in this paper, generalized transmission-line equations are implemented to the transmission-line parameter extraction with the cooperation of full-wave solver Zeland IE3D. The parameters extracted by the generalized transmission line equations are invariant to both excitation and loading. Except for this, the local radiation parameters generated from the transmission line discontinuities can also be found.

Keywords: generalized transmission line equation, parameters extraction, and invariance.

I. INTRODUCTION

For a simply shaped uniform transmission-line, its per-unit-length parameters are easily found by analytical formula and the transmission-line equations can be used to solve the line's problem with arbitrary excitation and load. For a complicatedly shaped non-uniform transmission-line, however, its per-unit-length parameters are hard to be determined by analytical formula, and have to be extracted by numerical methods. When the conventional transmission-line equations are used to extract line's per-unit-length parameters, what we find is that the extracted parameters vary with the excitation internal impedance and the line load. The reason for this is that the derivation of the conventional transmission-line equations is based on the assumption of an infinite line or non-reflection that is equivalent to the matching conditions for both excitation and load. In other words, the extracted parameters are incorrect if the matching condition for both excitation and load is not satisfied. It should be emphasized that the matched condition has been used in finding correct equation coefficients only. After the parameters (equation coefficients) are determined, the equations can be used to solve the line's problem with arbitrary excitation and load. Unfortunately, the exact matches for some extreme cases, such as a dipole antenna, are hard to be obtained. To solve this problem, in this paper, generalized transmission-line equations are implemented to the transmission-line parameter extraction. Since the derivation of the generalized transmission-line equations is based on the assumption of the finite rather than infinite line, the parameters extracted by the generalized transmission-line equations are invariance to both excitation and load. In comparison with the derivation of the conventional transmission-line equations, the generalized transmission-line equations have introduced two new terms, the dependent voltage source αV and dependent current source βI , which are interpreted as the local radiation sources. Here, V and I stand for the voltage and current at each discrete segment of the transmission-line, while α and β are the coefficients of V and I , respectively. In order to extract the line parameters by using the generalized transmission line equations, two pairs of voltage and current solutions along the line have to be used. By means of the MoM commercial software like Zeland IE3D, Sonnet, and Ensemble, the two pairs of solutions for two different loads can be obtained. Thus, the parameters can be found by substituting these two pairs of solutions into the generalized transmission line equations. The parameters extracted by the generalized equations are invariant to both internal source impedance and loads. To show the invariance of the extracted parameters to the excitation and load, in this paper, two numerical examples of microstrip line structures are given. Except for this, the local radiation effects from the discontinuities can also be obtained when the generalized equations are applied to non-uniform transmission line structures.

II. IMPLEMENTATION OF GENERALIZED EQUATIONS INTO PARAMETER EXTRACTION

As is well known, the following conventional transmission line equations have been derived from Kirchhoff's laws on a basis of an infinite-length line and TEM mode assumption [2]

$$\begin{aligned} \frac{dV(z)}{dz} &= -Z(z)\cdot I(z) \\ \frac{dI(z)}{dz} &= -Y(z)\cdot V(z) \end{aligned} \qquad (1)$$

where $Z = j\omega L + R$ and $Y = j\omega C + G$ are per-unit-length series impedance and shunt admittance, respectively. To let the equations be used, the per-unit-length line parameters L , R , C and G (i.e., coefficients of the equations) must be found by using either analytical formula or numerically extracted technique. In the past years, almost all literatures resorted to directly solving Maxwell's equations to find the parameters under static and quasi-static field assumption [2] – [5]. However, when equations (1) are used in the parameter extraction, one of things, which is easily ignored, is that the extracted parameters are directly dependant on internal impedance and line's load impedance. In other words, to obtain correct parameters, both excitation and loading matches have to be imposed so that the assumption of the infinite line can be satisfied. The problem is that exact matches at both ends of the line are not easy to realize because the characteristic impedance of the line is unknown. To solve this problem, the following generalized transmission line equations derived from a finite line [6]

$$\begin{aligned}\frac{dV}{dl} &= -Z(l)I(l) + \alpha(l)V(l) \\ \frac{dI}{dl} &= -Y(l)V(l) + \beta(l)I(l)\end{aligned} \qquad (2)$$

are implemented. For a lossless line, the distributed parameters $Z(l)$ and $Y(l)$ can be written as $j\omega L(l)$ and $j\omega C(l)$, and the corresponding definitions are exactly like those in the conventional transmission-line equations.

Comparing the conventional transmission-line equations (1), the generalized equations introduce two new parameters $\alpha(l)$ and $\beta(l)$, which can be interpreted as the coefficients of dependent voltage source and dependent current source in circuit theory, or be interpreted as the coefficients of local radiations between discontinuities of neighboring segments in field theory. When the transmission-line is a uniform structure, the values of $\alpha(l)$ and $\beta(l)$ are to be zero so that the generalized equations have the same formulations as the conventional uniform line equations. At first glance, the generalized equations seem to be more complex than the conventional equations because there are four parameters to be extracted at each discrete segment. In fact, as stated above, the two additional parameters make the extracted parameters be invariant to line's excitation and load so that the correct line's parameters can easily extracted. Since there are four line parameters $Z(l)$, $Y(l)$, $\alpha(l)$ and $\beta(l)$ at each discrete segment in generalized equations (2), we need to use two pairs of the distributed voltage and current solutions along the transmission-line to determine the line four parameters. Now most full-wave solver tools like Zeland IE3D, Sonnet, and Ensemble can be used for this purpose.

III. NUMERICAL EXAMPLES

To show how to implement the generalized equations to the line parameter extraction, the following two examples are presented. One is a finite-length uniform microstrip line. Another is a microstrip bend with two arms. For these two structures, the relative dielectric constant ε_r of the substrate is 9.8, the height *h* between the metal strip and ground is 0.635mm, the thickness *t* of the metal strip is 2μm, and the width *w* of the metal strip is 0.6mm. For simplicity, the metal are supposed to be lossless. The full-wave simulation tool, Zeland IE3D Software [7], is used to compute the distributed voltages and currents along the microstrip structures.

For the first example, the line parameters *L*(*l*), *C*(*l*), $\alpha(l)$ and $\beta(l)$ for three cases are extracted by the generalized transmission line equations. The first case is for short and open loads (A1). The second case is for the loads with complex numbers of 20+20j and 100+100j (B1). The third case is for the complex number internal impedance of 20+20j and 100+100j for excitation source (C1). The internal source impedances for the first two cases are 50Ω, and terminated load for the last case is 50Ω. All of the parameters *L*(*l*), *C*(*l*), $\alpha(l)$ and $\beta(l)$ along the uniform microstrip line found by the equations (2) are shown in Figures 1-4, respectively. It can be seen that the extracted parameters are almost the same for the three cases. This implies that the line parameters extracted by the generalized equations are invariant to

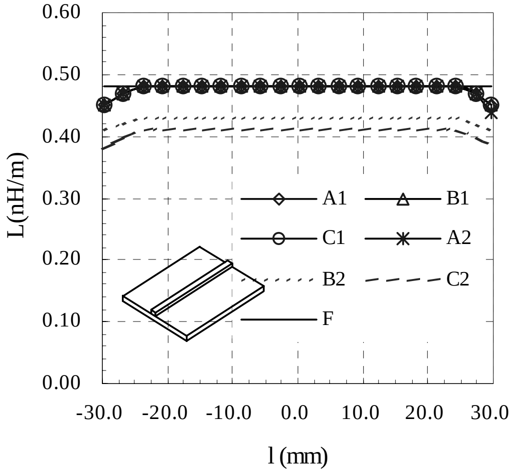

Fig. 1 Comparison of inductance obtained from generalized equations and traditional equations at 1GHz.

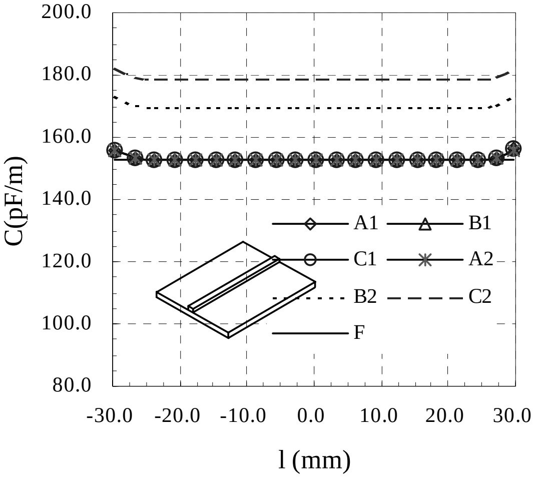

Fig. 2 Comparison of capacitance obtained from generalized equations and traditional equations at 1GHz.

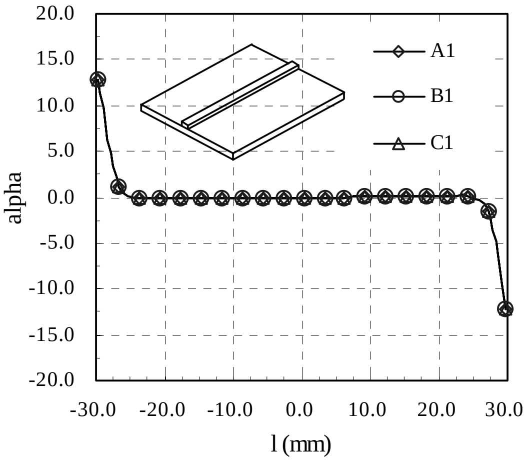

Fig. 3 Distributions of alpha obtained from generalized equations at 1GHz.

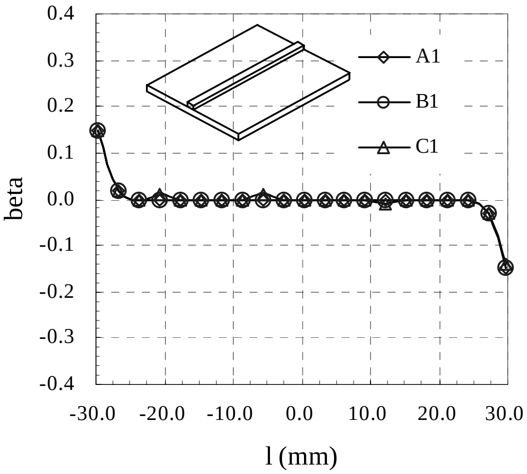

Fig. 4 Distributions of beta obtained from generalized equations at 1GHz.

both internal source impedance and termination loads. While the same cases are applied to the conventional equations (1) (corresponding to cases A2, B2 and C2), the extracted $L(l)$ and $C(l)$ are variant with both excitation and load except for the excitation and load matching conditions (case F), as show in Figs. 1 and 2. In other words, the values of $L(l)$ and $C(l)$ extracted by the conventional equations (1) at matching conditions (case F) are of a good agreement with those extracted by generalized equations (2) and calculated by analytical formulae [3], but the values of $L(l)$ and $C(l)$ extracted by the conventional equations (1) at mismatching conditions are at a great difference from those extracted with generalized equations (2). In addition, we find that the local radiation coefficients $\alpha(l)$ and $\beta(l)$, as shown in Fig. 3 and Fig. 4, are near to zero except for in the vicinity of both line ends. The non-zero $\alpha(l)$ and $\beta(l)$ in the vicinity of both line ends are reasonable because the line ends for a finite-length transmission line itself just are discontinuous places of the line. For second example, the microstrip bend is not uniform structure, so the conventional transmission line equations could not be directly applied to it. However, the generalized equations can be directly used. Two cases are calculated for this bend structure. Both cases have the excitation source of $50\,\Omega$. The first case is for short and open loads (A1). The second case is for 20+20j and 100+100j loads (B1).

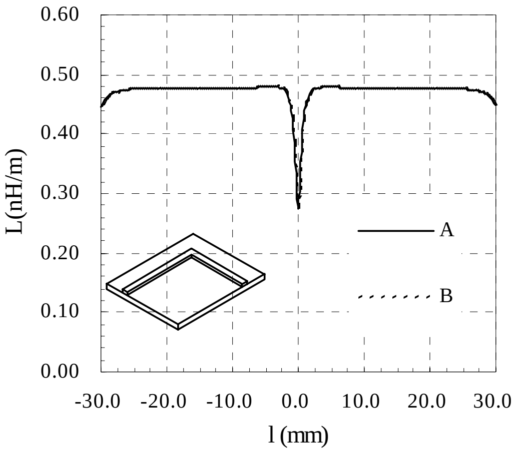

Fig. 5 Comparison of inductance obtained from generalized equations with different loads at 1GHz.

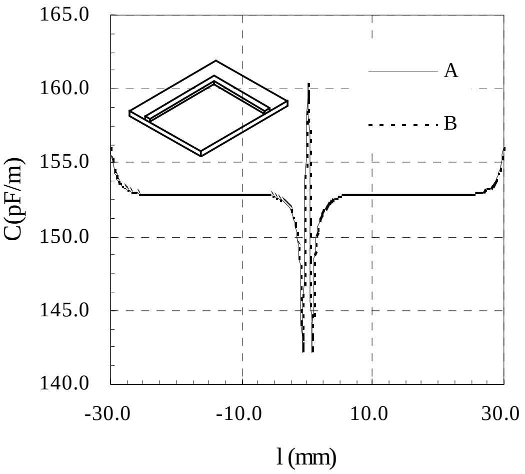

Fig. 6 Comparison of capacitance obtained from generalized equations with different loads at 1GHz.

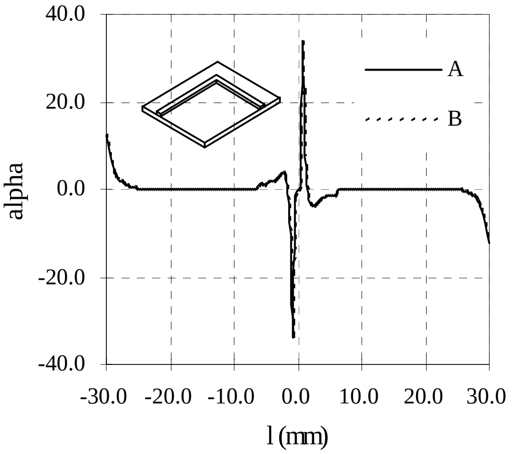

Fig. 7 Comparison of alpha obtained from generalized equations with different loads at 1GHz.

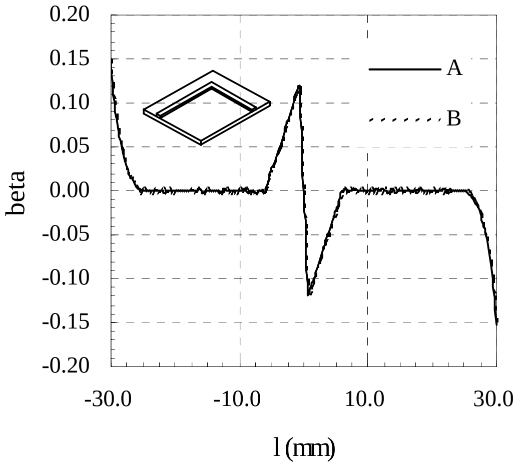

Fig. 8 Comparison of beta obtained from generalized equations with different loads at 1GHz.

The line parameters $L(l)$, $C(l)$, $\alpha(l)$ and $\beta(l)$ are extracted by the generalized equations (2). Figures 5-8 show the results of $L(l)$, $C(l)$, $\alpha(l)$ and $\beta(l)$ along the whole bend structure. For the second case, the line characteristic impedance is far away from that of traveling waves. However, like the first example, the two cases of parameters are also invariant to the port conditions. In addition, the characteristics of the bend discontinuity can be also obtained, as shown in Fig. 7 and Fig. 8, where the larger values of $\alpha(l)$ and $\beta(l)$ appear around the corner of bend while near-zero values occur at the flat part of two arms.

IV. CONCLUSION

In this paper, the generalized transmission-line equations are implemented to extract the transmission line parameters by means of numerical simulation tools like Zeland IE3D Software. The distinguished property for the general equations is that the extracted line parameters are invariant to both internal source impedance and terminated loads. Besides, the local radiation effects from line discontinuities can also be obtained during the process of the parameter extraction.

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Yaowu Liu received the M.S. and Ph.D. degrees from the University of Electronic Science and Technology of China, in 1981 and 1984, respectively. He was a research associate with the Department of Electrical Engineering, University of Utah, Salt Lake City, from April to November in 1986 and a research Scientist with the Department of Electrical Engineering, State University of New York at Binghamton, from May to August in 1993, respectively. From 1986 to 1995, he was a research Scientist with the Department of Electrical Engineering and Computer Science, University of California at Berkeley. Since March 1995, he has jointed in the Department of Electronic Engineering, City University of Hong Kong and now is an Assistant Professor. His research interests include electromagnetic theory and numerical computational methods to solve electromagnetic problems such as scattering, antennas, microwave circuits, and very large-scale integration interconnects.

Deshuang Zhao was born in China in 1974. He received the B.S.E.E. degree from Hennan Normal University in 1998 and the M.S.E.E degree from Electronic Science and Technology of China (UESTC) in 2001. He is now a Research Assistant in Department of Electronic Engineering, City University of Hong Kong. His research interests include electromagnetic theory, microwave circuits, and numerical method.

Kenneth K. Mei received the B.S.E.E., M.S., and Ph.D. degrees in electronic engineering from the University of Wisconsin, Madison, in1959, 1960, and 1962, respectively. He joined the faculty of the Department of Electrical Engineering and Computer Science of the University of California, Berkeley in 1962 where he was a Professor from 1972 to 1994. Since 1994 he has been a Professor in the Department of Electronic Engineering, City University of Hong Kong and now is an Adjunct Professor in Shanghai University. He was a Director of Wireless Communications Research Center, City University of Hong Kong from 1998 to 2002. He began his research in electromagnetics in the area of computation, including the method of moments, finite element/difference, hybrid methods time-domain methods and most recently, the measured equation of invariance and Maxwellian circuits.

Dr. Mei received the Best Paper Award and Honourable Mention of the Best Paper Award in 1967 and 1974, respectively, from the IEEE Antennas and Propagation Society. He is a member of RSI/USNC.