A Fast MEI Scheme for the Computation of Scattering by Very Large Cylinders

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Abstract: A fast scheme for measured equation of invariance (MEI) method is presented in this paper. The scheme combines a strategic technique of the interpolation and extrapolation of MEI coefficients with a special algorithm of cyclic block band matrix to fast solve the scattering problems of very large conducting cylinders. The circumferential dimension of scattering objects could exceed 10,000 wavelength. Computational speed could be 2-3 order faster than conventional MEI method. The fast scheme is especially applicable to scattering problems of very large conducting objects in which other numerical methods may fail.

1. Introduction

One of the advantages of the measured equation of invariance (MEI) method [1-4] is that the MEI can be used to solve scattering problems of normal size objects very fast. However when the target is very large, most numerical methods have difficulties in obtaining correct results. Even the MEI is no exception. In the case of the MEI, the reason is that the integration required to generate the MEI coefficients is very time consuming, and the results of the integration could be inaccurate which causes excessive errors in the resulting MEI coefficients and subsequent failure of the computation. In this paper, the interpolation and extrapolation technique for the MEI coefficients is used to overcome this problem. Such a technique allows us to extrapolate the MEI coefficients at several low frequencies to higher frequencies of interest. Thus the execution time for finding the MEI coefficients at the high frequencies can be greatly reduced, and the accuracy of the MEI coefficients can be greatly increased. The details of the interpolation and extrapolation technique for the

This work was supported in part by the City University of Hong Kong Strategic Grant 7000665 and Small Scale Research Grant 9030545.

MEI coefficients can be found in reference [5]. Compared with the total execution time for solving the large object scattering problems, after the utilization of the interpolation and extrapolation technique, finding the MEI coefficients becomes a minor part of the total computation time, and the solution of the MEI sparse matrix becomes the dominant part. In order to accelerate the whole computational process, the solution of the MEI matrix should be speeded up. To do so, one option is choosing parallel computation. Another is choosing special algorithms. For the parallel computation, a PVM (Parallel Virtual Machine) based parallel sparse matrix solver is investigated. A cluster of Pentium PC's is chosen as the hardware platform. Ethernet is used in connecting Pentiums as a network. When partition number of the MEI matrix is 4 - 16, a speedup of 7 for the MEI matrix can be achieved [6]. At the same time, a special algorithm, which is applicable for the structure of the MEI matrix, is developed. This special algorithm will be described in this paper. In 2-D scattering cases, the MEI matrix is such a sparse structure, in which all the elements are in a diagonal band area except a few off-diagonal elements at the beginning and the end of the mesh. This is a typical cyclic block tridiagonal structure. For such a special structure, we modify the Thomas algorithm [7], which directly solves block tridiagonal matrix, to meet our need. The computational time of our MEI matrix solver is proportional to N, the dimension of the matrix. For the scattering of a conducting circular cylinder with diameter 4,000 wavelength, the MEI solver is about 30 times faster than the Berkeley's sparse solver [8], and is about 4 times faster than the above PVM-based parallel sparse matrix algorithm. Thus using the fast MEI scheme in this paper, which combines the interpolation and extrapolation technique of the MEI coefficients with algorithm of the cyclic block band matrix, the scattering problems of very large cylinders can be solved much faster than with the conventional MEI or MoM algorithms.

2. The MEI Sparse Matrix

For the MEI computation mesh, suppose N_S be total grid points per one layer, and N_L be the total number of layers. Five points difference-difference formula is used for the nodes on the inner layers of the mesh,

$$\sum_{i=0}^{i=4} c_i \phi_i = 0 \qquad (1)$$

where $\{c_i\}$ are the finite-difference coefficients, ϕ_i is the field at node i. For the grids on the truncated boundary, the MEI equation is used

$$\sum_{i=0}^{n-1} a_i \phi_i = 0 \tag{2}$$

where $\{a_i\}$ are the MEI coefficients to be determined, and $n-1$ is the number of neighbors of node 0.

After boundary conditions of the conducting object are applied, the MEI-FD matrix is nested in the following form,

$$\begin{bmatrix} B_1 & C_1 & 0 & 0 & \cdots & 0 & A_1 \\ A_2 & B_2 & C_2 & 0 & \cdots & 0 & 0 \\ 0 & A_3 & B_3 & C_3 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & \cdots & 0 \\ \vdots & \vdots & \cdots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & A_{N_s-1} & B_{N_s-1} & C_{N_s-1} \\ C_{N_s} & 0 & \cdots & 0 & 0 & A_{N_s} & B_{N_s} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \vdots \\ \vdots \\ \Phi_{N_s-1} \\ \Phi_{N_s} \end{bmatrix} = \begin{bmatrix} \Phi_1^I \\ \Phi_2^I \\ \Phi_3^I \\ \vdots \\ \vdots \\ \Phi_{N_s-1}^I \\ \Phi_{N_s}^I \end{bmatrix} \tag{3}$$

where $\{A_i\}$ $\{B_i\}$ and $\{C_i\}$ are $(N_L-1)\times(N_L-1)$ matrices, $\{\Phi_i\}$ and $\{\Phi_i^I\}$ are $(N_L-1)\times 1$ column vectors which represent the unknown scattered fields and known incident fields, respectively.

Obviously, the MEI matrix, i. e. eq. (3), is a typical cyclic block band matrix, in which all the elements are in a diagonal band area except a few off-diagonal elements at the beginning and the end of the mesh.

3. Strategy to obtain MEI coefficients

In this section, we will briefly describe how to use the interpolation and extrapolation technique to obtain MEI coefficients at the high frequencies of interest from those calculated at low frequencies being first calculated.

The MEI coefficients have the characteristics of spatial interpolability, frequency interpolability and frequency extrapolability [5] when the ratio of the wavelength and the discretization size stays the same. The MEI coefficients are interpolable between nodes, i.e., if the nodes are rearranged for different frequencies to keep the ratio invariant, we may obtain the MEI equations at the nodes of the new distribution from interpolations of the MEI coefficients at the nodes of the original distribution. Fig. 1 shows the arrangements of the nodes at three different frequencies increased by a factor of two. The mesh size is reduced by a factor of two, at the same time, the spacing between the mesh boundary and object boundary is also reduced by a factor of two.

We expect the MEI coefficients to smoothly approach a limit when the frequency increases to the optical region. Our expectation is based on the similarity of the local geometry of a node as the mesh boundary approaches the object boundary and on the fact that in geometrical optics the same rules of calculation are applied to all frequencies. Numerical examinations in [5] have indicated that the MEI coefficients do approach a limit with the increase in frequency, and the accuracy of the MEI coefficients obtained by the extrapolation is enough for obtaining accurate results. We express the MEI coefficients at node 0 as polynomials of the normalized wavelength with the degree J-1,

$$a_i(\lambda) = \sum_{j=0}^{J-1} \alpha_{i,j} \lambda^j \tag{4}$$

Based on the mesh organization depicted in Fig.1, we calculate the MEI coefficients at few normalized measuring wavelength such as 2, 1, 0.5, 0.25, ... with number J, by using the conventional MEI method [1]. Then the coefficients $\{\alpha_{i,j}\}$ in eq. (4) can be found. Once the coefficients $\{\alpha_{i,j}\}$ are obtained, the MEI coefficients at any normalized wavelength can be determined by using eq.(4). Since we only calculate the MEI coefficients at the low frequencies through the integration, computational time is very small. In addition, the interpolation and extrapolation time for obtaining the MEI coefficients at the high frequency of interest is also small. Compared with direct calculation of the MEI coefficients at the high frequency by the integration, the computation time can be tens to thousands of times smaller depending on how large the scattering objects are.

The technique presented above is not just a way to obtain the MEI coefficients with economical computation time, but a way to obtain the MEI coefficients accurately for the very large electromagnetic boundary value problems.

4. The Fast Solver for the MEI Matrix

We have pointed out in section 3, that the CPU time can be greatly reduced if the interpolation and extrapolation technique is used in determining the MEI coefficients at the high frequency. Thus most of the CPU time will be used for solving the MEI matrix. In order to reduce the execution time even further, a fast MEI matrix solver has been developed.

As seen in section 2, most elements in the MEI matrix are tridiagonal nonzero block elements, and only few elements are off-tridiagonal nonzero block elements on each reverse-diagonal corner. This is a typical cyclic tridiagonal block structure. For such a special structure, the direct algorithm called Thomas algorithm [7], which has been used to get the solutions of tridiagonal block matrix systems, is expanded to solve this kind of cyclic tridiagonal block matrix systems. This is the MEI matrix solver. Unlike the conventional sparse matrix routine, the MEI matrix solver does not need to pivot and reorder the matrix before solving the matrix. All the MEI matrix solver does is solving the block matrix analytically. In followings, we will briefly describe this algorithm.

Using the first $N_S - 1$ equations in eq.(3), the forward substitution can be completed to obtain the coefficient matrices $\{\Delta_i\}, \{P_i\}$ and $\{Q_i\}$ recursively in the following formula,

$$\begin{bmatrix} P_1 & C_1 & 0 & \cdots & 0 \\ 0 & P_2 & C_2 & \cdots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & P_{N_S-2} & C_{N_S-2} \\ 0 & 0 & \cdots & 0 & P_{N_S-1} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_{Ns-2} \\ \Phi_{N_S-1} \end{bmatrix} = \begin{bmatrix} \Delta_1 - Q_1 \Phi_{N_S} \\ \Delta_2 - Q_2 \Phi_{N_S} \\ \vdots \\ \Delta_{N_S-2} - Q_{N_S-2} \Phi_{N_S} \\ \Delta_{N_S-1} - Q_{N_S-1} \Phi_{N_S} \end{bmatrix} \quad (5)$$

where $P_1 = B_1$, $\Delta_1 = \Phi_1^I$, $Q_1 = A_1$, and

$$P_i = B_i - A_i P_{i-1}^{-1} C_{i-1} \quad (6)$$

$$\Delta_i = \Phi^I{}_i - A_i P_{i-1}^{-1} \Delta_{i-1} \quad (7)$$

$$Q_i = -A_i P_{i-1}^{-1} Q_{i-1} \quad (8)$$

$i = 2,3,\ldots,N_s - 1$. And the unknown sub-matrix Φ_{N_S} can be determined by

$$\Delta_{N_S} = \Phi_{N_S}^I - A_{N_S} P_{N_S-1} \Delta_{N_S-1} - C_{N_S} W_1 \quad (9)$$

$$P_{Ns} = B_{N_S} - A_{N_S} P_{N_S-1} \left(C_{N_S-1} + Q_{N_s=1} \right) - C_{N_S} W_2 \quad (10)$$

$$\Phi_{N_S} = P_{N_S}^{-1} \Delta_{N_S} \quad (11)$$

where the matrices W_1 and W_2 can be determined during the above forward substitution recursively. Then, using backward substitution, all the unknown sub-matrices can be recursively calculated by,

$$\Phi_i = P_i^{-1}(\Delta_i - C_i\Phi_{i+1} - Q_i\Phi_{N_S}) \tag{12}$$

where $i = N_s - 1,\ldots,1$.

In general, if a large number of mesh buffer layers is utilized for the finite difference, all the nonzero sub-matrices in the MEI matrix are still very sparse too. The operators between these sub-matrices can be carried out by general sparse matrix solvers to speed up the computation. However, for 2-D scattering problems, only 2 - 3 mesh layers are needed to obtain robust results in the MEI method [1]. In these cases, the number of zero elements in the sub-matrices is small. Thus the zero elements in the sub-matrices are directly treated as nonzero elements, and the additional matrix fill for these zero elements is small. For example, for three layer mesh with a 6 node MEI's sub-mesh scheme, the total number of nonzero elements is $11N_S$, while the total elements to be filled in eq.(3) is $14N_S$. Another advantage is that since the size of the sub-matrices is small, all the operators between the sub-matrices can be given analytically, thus the calculation for the zero elements is avoided.

The CPU time of the MEI matrix solver is nearly proportional to N_S, the number of nodes per one layer, rather than proportional to N_S^{α} ($\alpha > 1$) as is the case with the general sparse matrix solver [5]. As the dimension of the MEI matrix increases, the time-saving of the MEI matrix solver will increase.

5. Statistics of CPU Time

The CPU time comparison of the MEI matrix solver with the Berkeley's sparse package [8] is presented in Fig.2, which shows that when N_S is small, the CPU time of the MEI matrix solver and the Berkeley's sparse solver are almost identical. However, when the size of the MEI matrix increases, the CPU time of the MEI matrix solver is much less than the CPU time of the Berkeley's sparse solver. For solving the scattering of a cylinder at above 10^5 wavelength circumferential dimension, the MEI matrix solver is about 30 times faster than the Berkeley's general sparse solver. The CPU time for obtaining MEI coefficients using the interpolation and extrapolation technique is also plotted in Fig.2. It is found that without using the interpolation and extrapolation technique, the time consumed by the direct integration to obtain MEI coefficients for a geometry at 2,000 wavelength circumferential dimension will exceed 10hrs, but the interpolation and extrapolation technique needs a few minutes only.

6. Numerical Results

The above scheme allows us to fast solve the scattering problems of large 2-D conducting convex objects. For illustration purpose, we first calculate TM scattering surface current density of a conducting circular cylinder with diameter d = 4,000λ (wavelength) under 0^0 plane wave incidence. The surface current densities compared to the result of geometrical optics are shown in Fig.3. To solve this problem, the MEI coefficients at the lower frequencies are calculated at normalized wavelength 0.8, 0.4, 0.2, 0.1 for a cylinder of diameter d = 10λ. The normalized wavelength of the high frequency is 0.0025. The CPU time to find the MEI coefficients is about 2 minutes. The CPU time to solve the MEI matrix solver is about 124 seconds. But the Berkeley's sparse solver should take the CPU time of 72 minutes.

The second example is the scattering of a square conducting cylinder of 64 meter in circumerferial dimension. The normalized wavelength 2, 1, 0.5, 0.25 of the low frequencies is extrapolated to the wavelength of interest 0.0625, for which the ratio of the circumerferial dimension to the wavelength is 1,024. Fig.4 gives the surface current densities by a 30^0 TE incidence wave illumination. Fig.5 shows the remarkable agreement between the fast MEI scheme and the GTD on the RCS when an incidence wave with angle 45^0 illuminates with the same configuration of Fig.4.

7. Conclusion

In this paper, we present a fast numerical scheme for the MEI method to solve the scattering problems of the very large conducting cylinders in which circumferential dimension exceeds 10,000 wavelength, in a common workstation. Since the scheme combines the extrapolation and interpolation technique of the MEI coefficients with the special algorithm of cyclic block band matrix, the whole computation can be speeded up to 2-3 orders with reasonable numerical accuracy.

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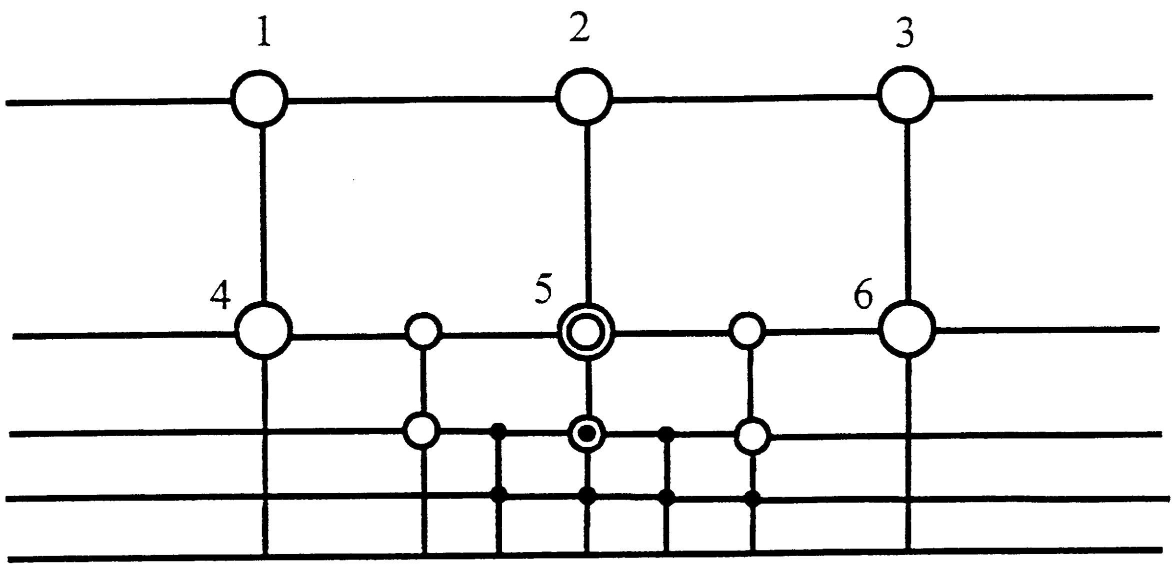

Fig. 1 Mesh strategy with the mesh size reduced by a factor of two.

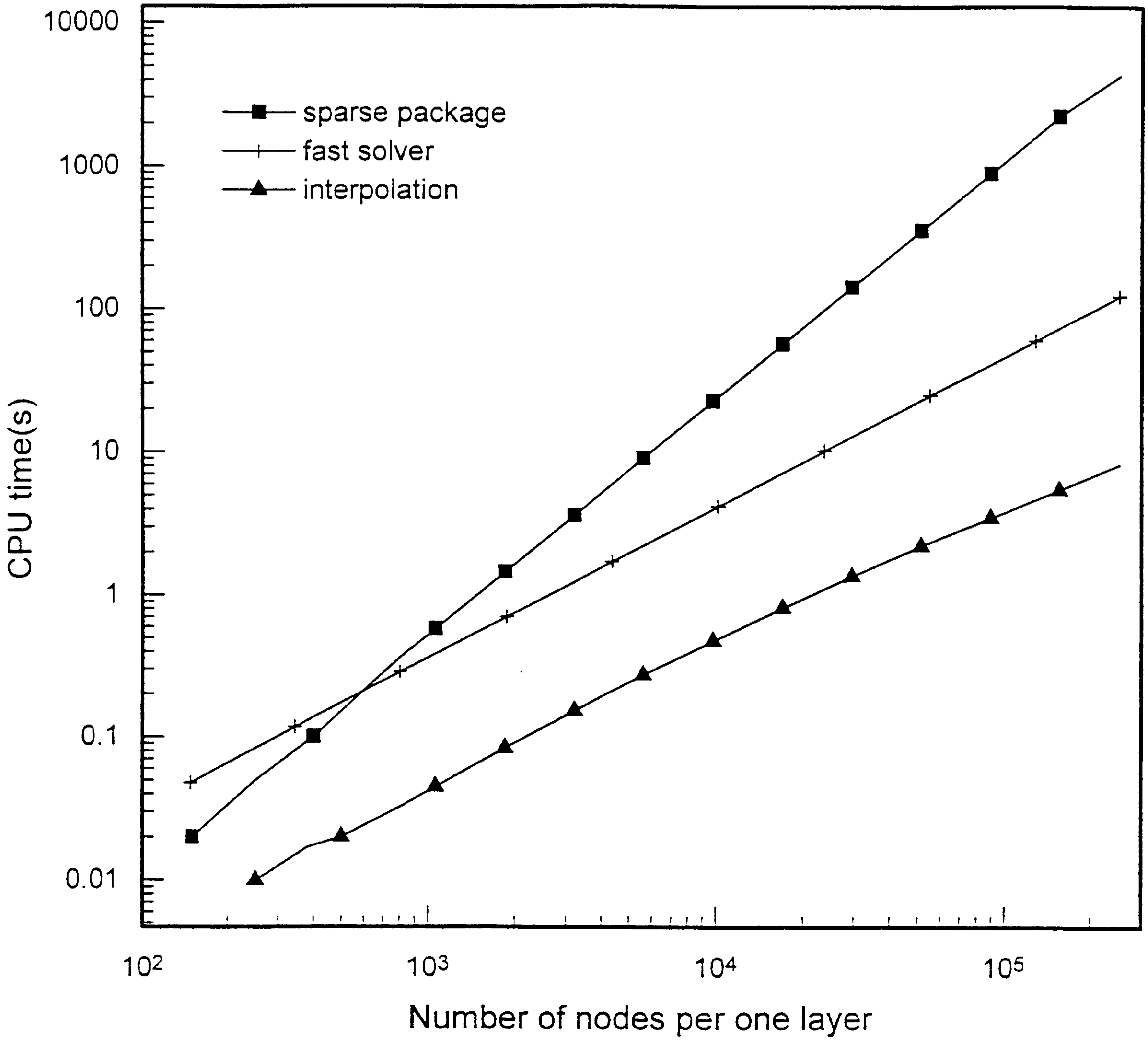

Fig. 2 CPU time vs. number of nodes per one layer on a Sun-Workstation.

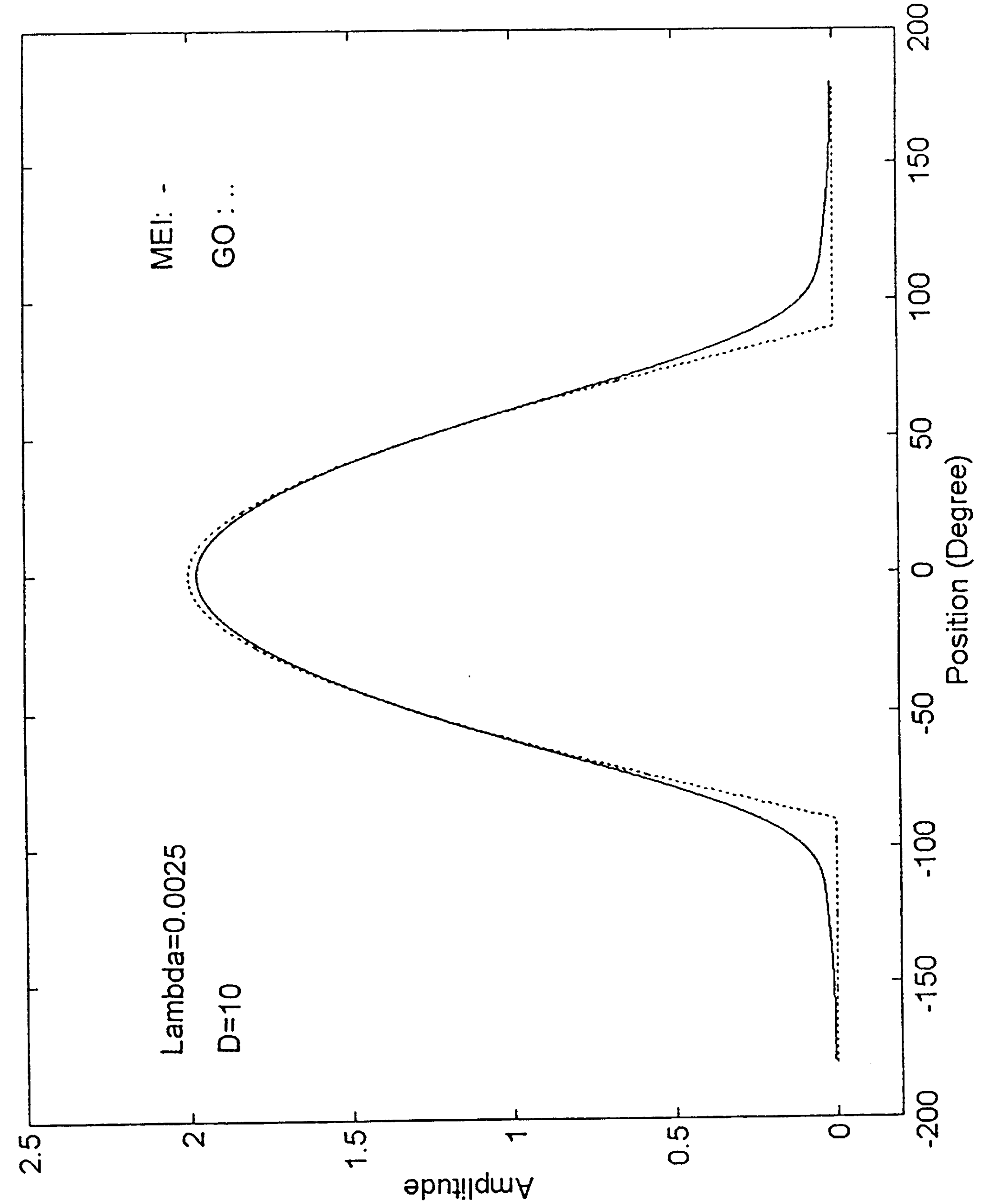

Fig. 3 Surface current distribution on a circular cylinder illuminated by a TM wave with d = 4,000λ.

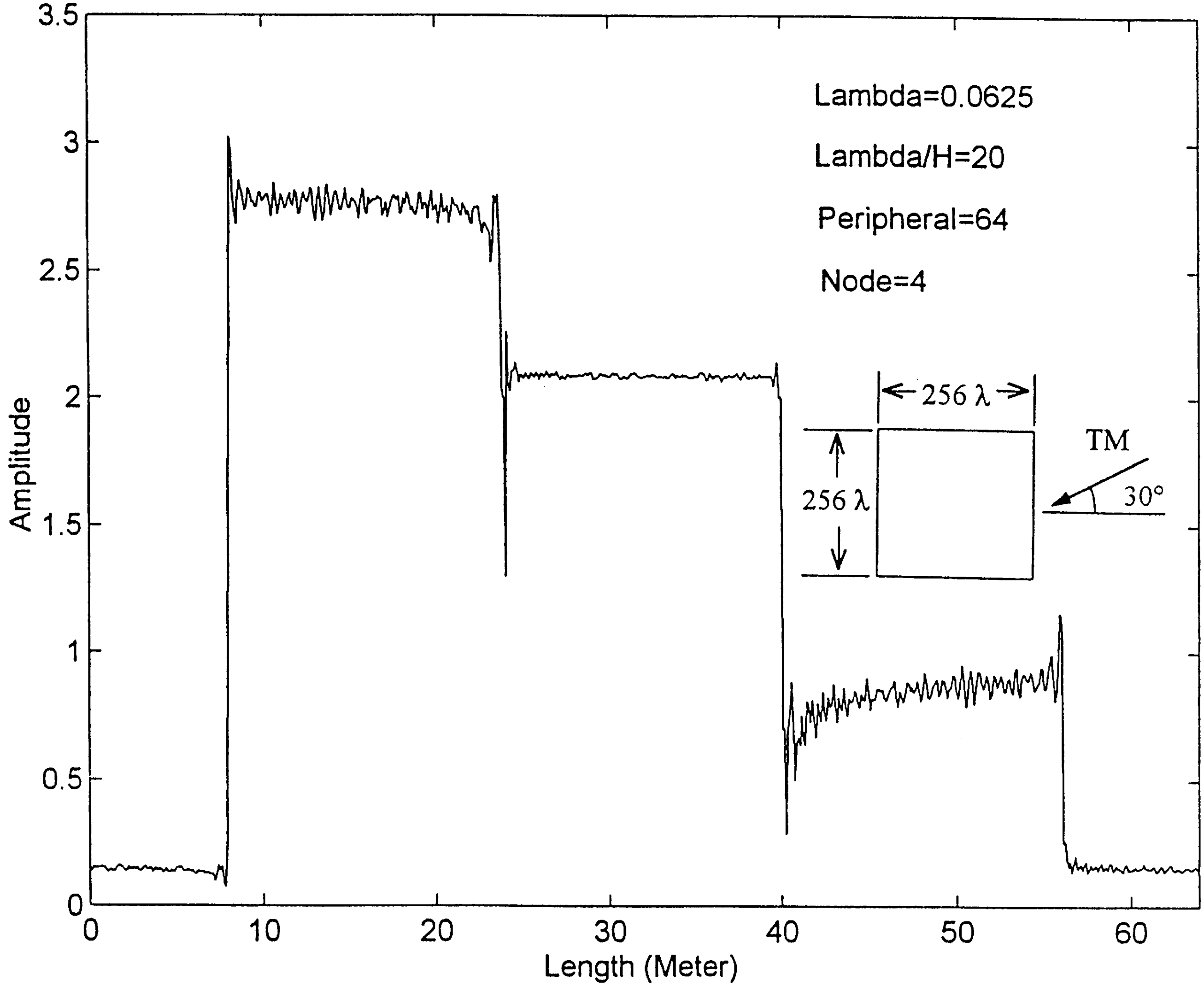

Fig. 4 Surface current distribution on a 256λ x 256λ square cylinder illuminated by a 30° TE wave incidence.

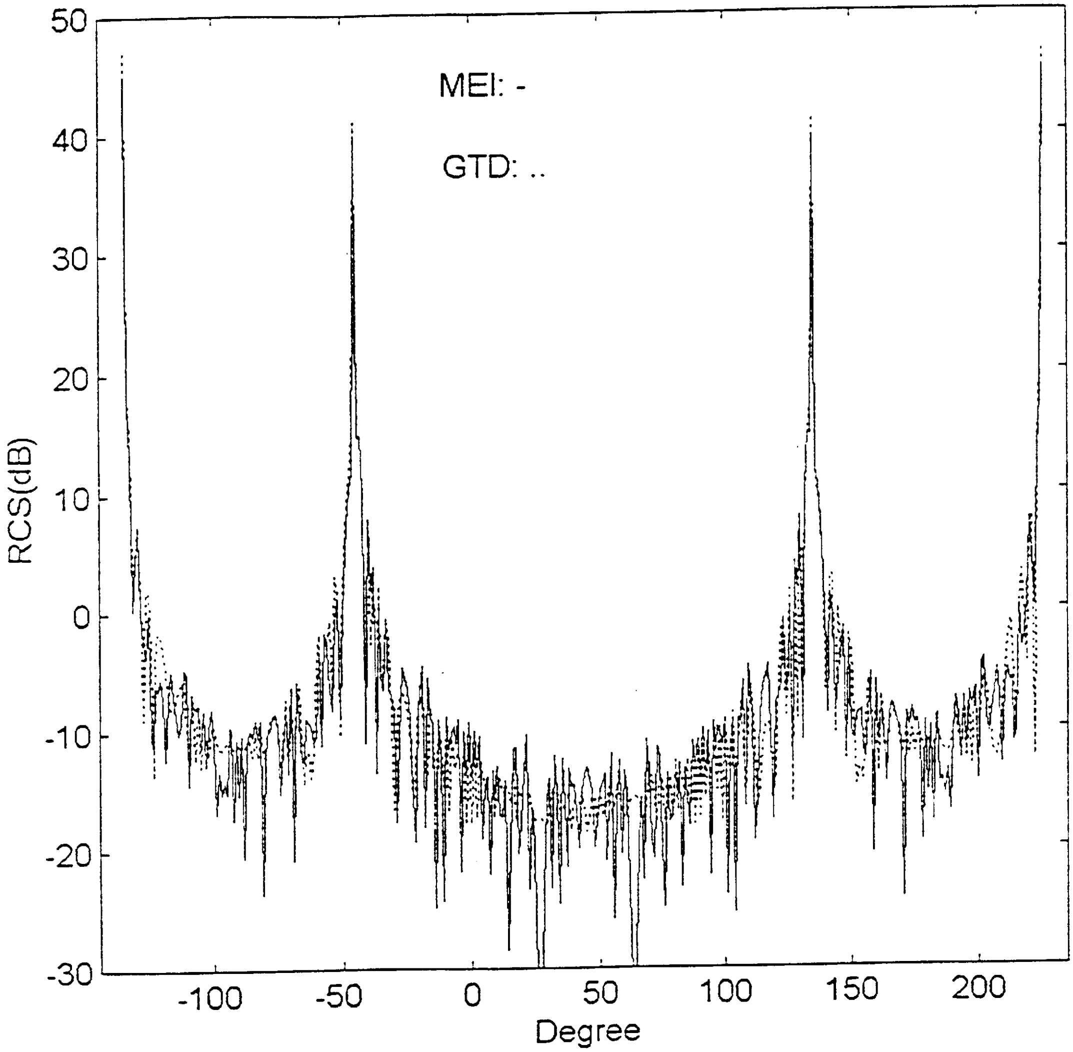

Fig. 5 Comparison of bistatic RCS of the scattering configuration in Fig. 4 illuminated by 45° TE wave incidence.