Speedup Using a Modal Frequency Method for Finite Element Analysis of a Dual–Mode Microwave Filter

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Abstract—**Computer time required for finite element analysis of microwave filters is reduced by more than an order of magnitude by using modal frequency rather than direct frequency methods. In the conventional direct frequency method, the number of unknowns is equal to the number of edge degrees of freedom. Instead, the new modal frequency method first computes the 3D modes and then uses them as basis functions, thereby greatly reducing the number of degrees of freedom. The two methods are applied to the European benchmark problem of a dual–mode microwave filter. The modal frequency method obtains essentially the same results as the direct frequency method, but when analyzing 201 frequencies it yields a speedup factor of 15.**

INTRODUCTION

Microwave resonators are often used to make filters. If the coupling coefficients are small and the quality factor Q is high, then narrow passbands or stopbands may require analysis at a hundreds of frequencies. In conventional direct frequency finite element analysis (FEA), total solution time is directly proportional to the number of frequencies analyzed, and may therefore be hundreds of times longer than the solution at one frequency.

To reduce computer time, several methods of reduced order modeling have been used. Asymptotic waveform analysis (AWE) has been used for several years [1], and it has been successfully combined with finite element analysis [2]. The most recent method of AWE is called PVL for Padé via Lanczos process [3]–[6]. While PVL can extend the frequency range compared to standard AWE methods, as of now no AWE method is guaranteed to find all resonances over a specified frequency range.

In a new technique called modal frequency FEA [7], a 3D real Lanczos eigenvalue analysis with Sturm sequencing is first performed to reliably find *all* resonances of low loss devices. The resulting eigenvectors are used as basis functions for solutions over a range of frequencies, thereby possibly saving computer time if S–parameters are needed for a large number of frequencies.

This paper begins with a review of the modal frequency method of FEA and some of its recent applications. Then the new modal frequency method is applied to a benchmark problem from a European magazine. The problem is a two–port dual–mode cylindrical six–cavity filter with iris coupling to rectangular waveguides. The modal frequency results will be compared with those obtained by the conventional direct frequency method and by measurements.

MODAL FREQUENCY FINITE ELEMENT ANALYSIS

Conventional direct frequency FEA consists of solving the complex matrix equation [7], [8] with angular frequency ω:

$$[\, -\omega^2[M] + j\omega[C] + [K]\,]\{u\} = \{P\} \tag{1}$$

where [M] is the permittance matrix (proportional to permittivity), [C] is the conductance matrix (proportional to conductivity), and [K] is the reluctance matrix (inversely proportional to permeability). For the 3D edge finite elements used here, the unknown vector {u} consists of edge magnetic vector potentials $\overline{A}$. The electric field is then $-j\omega\overline{A}$. {P} is the excitation vector, which for S–parameter computations is located at the ports. The {u} vector has as many degrees of freedom as there are finite element edge unknowns, which usually number in the tens of thousands. Note that the left hand matrix changes with frequency and thus solution time is proportional to the number of frequencies analyzed.

Instead of solving (1) directly, we can first compute the real eigenvalues and eigenvectors, denoted by $\{\phi_i\}$, of the 3D finite element model. Then a *modal frequency* solution is assumed to be a linear combination of the eigenvectors, expressed as [7]:

$$\{u\} = [\phi]\{q\} \tag{2}$$

where the matrix $[\phi]$ is made up of m columns of individual orthogonal eigenvectors $\{\phi_i\}$, and the vector {q} contains all of the coefficients. If there are n direct degrees of freedom in a problem (the length of the column vector {u}), then $[\phi]$ is an (n x m) matrix. This transformation can be highly accurate when all n eigenvectors of the system are used. In many cases only a small approximation is introduced if a limited number of eigenvectors in a specified frequency range is used.

The frequency range of the eigensolution should include all modes that are expected to be excited. All of the

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real modes over any finite frequency range are rigorously computed in our software [7] using a Sturm sequenced Lanczos algorithm. Then the final solution is obtained by substituting (2) into (1):

$$-\omega^2[M][\phi]\{q\} + j\omega[C][\phi]\{q\} + [K][\phi]\{q\} = \{P\} \quad (3)$$

Premultiplying both sides by $[\phi]^T$ results in:

$$-\omega^2[\phi]^T[M][\phi]\{q\} + j\omega[\phi]^T[C][\phi]\{q\} + [\phi]^T[K][\phi]\{q\} = [\phi]^T\{P\} \quad (4)$$

where the three new *modal* matrices are:

$$[m] = [\phi]^T[M][\phi] \quad (5)$$
$$[c] = [\phi]^T[C][\phi] \quad (6)$$
$$[k] = [\phi]^T[K][\phi] \quad (7)$$

Thus (4) can be rewritten as the *modal frequency equation*:

$$(-\omega^2[m] + j\omega[c] + [k])\{q\} = \{p\} \quad (8)$$

Recall that the unknown $\{q\}$ vector is of length equal to the selected number of real 3D modes. This reduction in the number of unknowns from those of $\{u\}$ of (1) can often lead to a substantial computational speedup compared with direct frequency FEA.

Because real modes are assumed, the modal frequency method is applicable only to low loss problems. While dielectric and ferrite material losses can be analyzed directly, wall losses can only be analyzed indirectly using equivalent lossy air or other filler material. Also, real modes cannot represent radiation boundaries, and thus antennas cannot be analyzed by modal frequency FEA.

A recent paper [7] describes the theory of modal frequency FEA in detail. It includes a discussion of S–parameter computations using approximate port boundary conditions.

The same paper also applies modal frequency FEA to computing the S–parameters of two filters. One filter is a cutoff–coupled rectangular dielectric resonator filter, and the other is a coax–fed rectangular box containing cylindrical dielectric resonators. Modal frequency FEA results obtained are similar to those obtained by direct frequency and by measurements. Compared with direct frequency FEA, modal frequency FEA obtained speedup factors ranging from 1.4 to 4 for analyses at approximately 100 frequencies.

Another recent paper [9] applies modal frequency FEA to two other filters. One is the ACES/TEAM 19 single cavity filter, for which accurate S–parameters were obtained with a speedup of 10 over direct frequency FEA. The other example analyzed was an improper variant of a dual–mode filter; here the proper geometry is used in the analysis via both direct frequency and modal frequency FEA.

DESCRIPTION OF THE DUAL–MODE MICROWAVE FILTER

Dual–mode microwave filters use two similar propagating waveguide modes to produce a bandpass filter. The transmission coefficient often shows excellent rejection outside the desired passband [10].

The earliest dual–mode filters produced their dual modes by means of adjustable screws inserted in waveguide walls [11]. Recently, an improved design uses rotated apertures to produce dual couter–rotating modes in a circular waveguide [12]. Usually multiple apertures are placed along the length of the filter, where each aperture is rotated sequentially. The rotations are either all in the clockwise direction or all counterclockwise.

The dual–mode filter analyzed in this paper has elliptical apertures that are rotated sequentially in the counterclockwise direction [13]. Figs. 1 and 2 show the outside of the filter, which is made up of six cylindrical cavities fed by rectangular waveguide at both ends. The rectangular waveguide cross section is 1.905 by 0.9525 cm. The cylindrical cavities are all of diameter 2.40 cm. The geometry was entered via solid modeling commands in the preprocessing software. The filter is entirely filled with air and all of its walls are made of aluminum. Thus the filter is here assumed to be lossless.

Fig. 3 is an isometric view at the same angles of Fig.1, but showing the inside of the filter. There are seven elliptical apertures at positions and major axis angles [13] listed in Table 1. Fig. 4 is a front view of the inside of the filter, showing the ellipses. The two large tilted ellipses of Table 1 have a major axis of 2.4 cm and a minor axis of 2.1 cm. The two large verical ellipses are 2.4 by 2.0414 cm. The two end ellipses are 1.278 by 0.4 cm, and the single center ellipse is 0.87 by 0.4 cm.

TABLE 1. Elliptical apertures in filter

z min (cm)	z max (cm)	major axis angle (deg.)
0.750	0.923	180 (end ellipse)
1.4729	1.523	135 (large tilted ellipse)
2.0728	2.132	90 (large vertical ellipse)
2.6827	2.8342	90 (center ellipse)
3.3841	3.4441	90 (large vertical ellipse)
3.994	4.044	45 (large tilted ellipse)
4.5939	4.7669	0 (end ellipse)

Fig. 5 shows the measured S–parameters. They were obtained using an experimental filter that is specified to

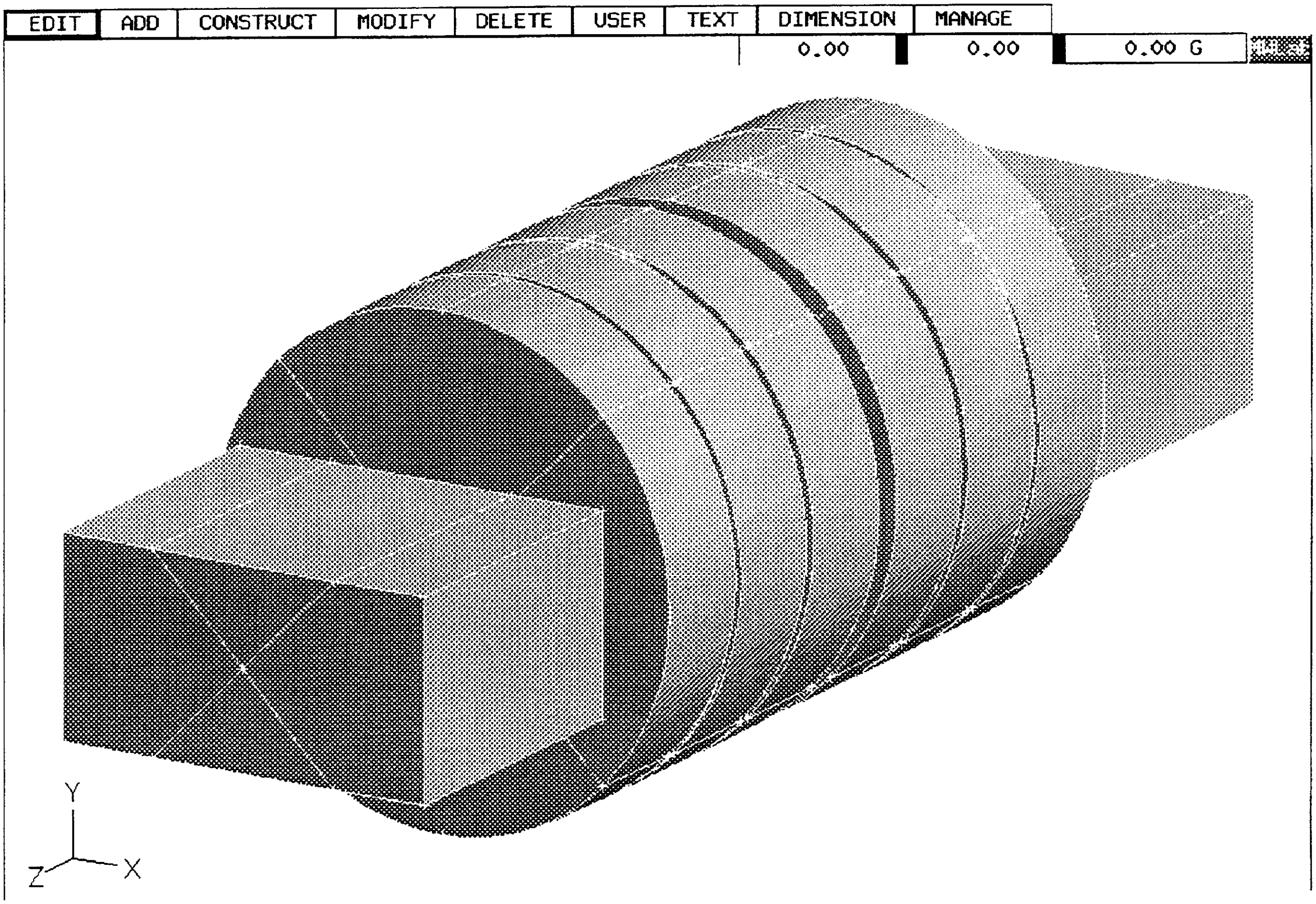

Fig. 1. Isometric view of outside of solid geometry model of dual−mode filter.

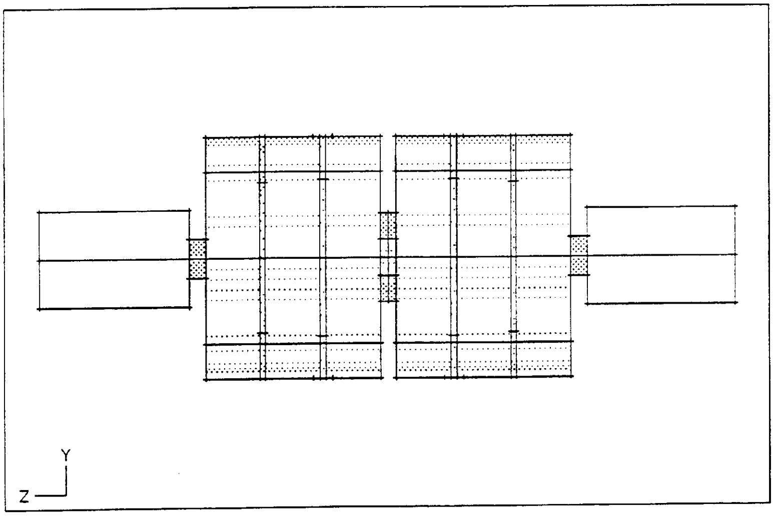

Fig. 2. Side view of dual−mode filter of Fig. 1.

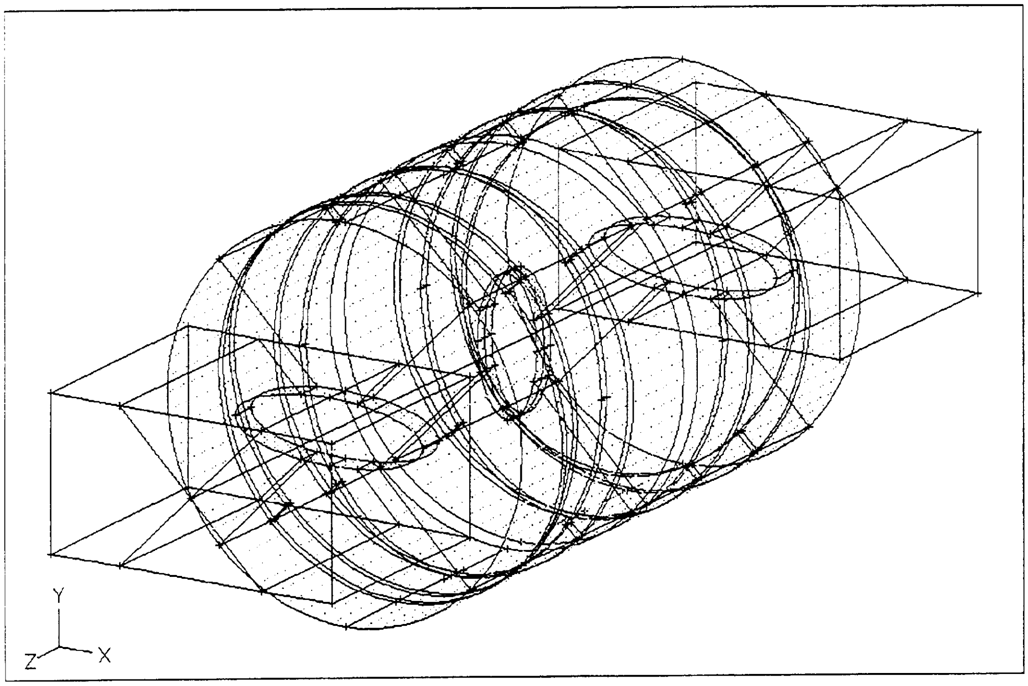

Fig. 3. Isometric view of inside of solid geometry model of dual–mode filter.

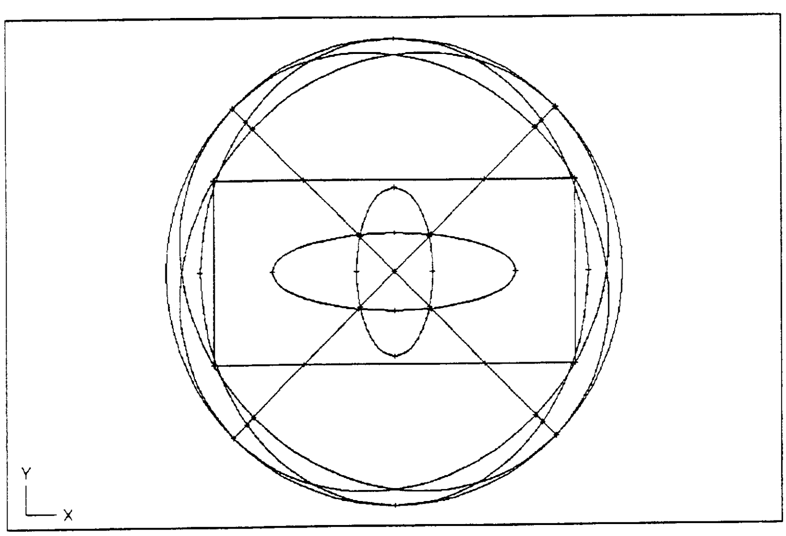

Fig. 4. Front view of inside of solid geometry model of dual–mode filter.

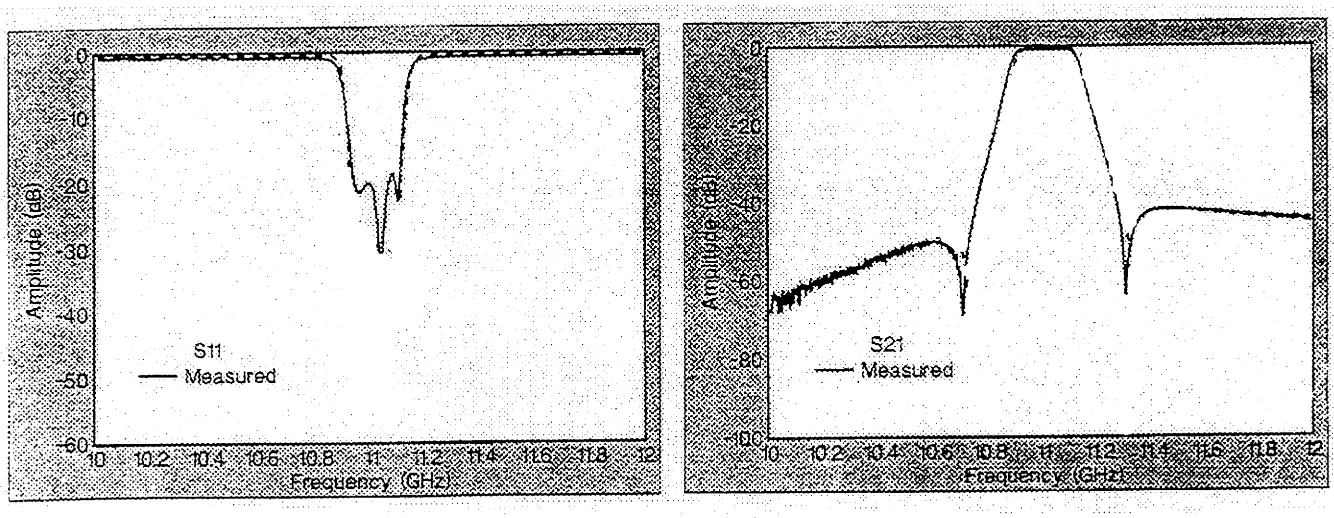

Fig. 5. Measured S−parameters of dual−mode filter [13].

obey the above dimensions within a tolerance better than plus or minus ten microns.

3D FINITE ELEMENT MODEL AND COMPARATIVE RESULTS

Even though the six cylindrical cavities of Figs. 1 through 4 are all symmetric about several mirror planes, the rotation of the elliptical aperture axes means that the filter possesses no plane of symmetry. Thus the entire filter must be modeled, here using finite elements.

Fig. 6 shows the finite element model, which consists of 10,169 second order edge (H1−curl) tetrahedrons. They have a total of 64,450 edge degrees of freedom.

The model of Fig. 6 was submitted to our software, which computed the fields and S−parameters using equations given previously [7]. The S−parameters were computed using the software's direct frequency capability of (1) and its modal frequency capability of (8) at 201 frequencies from 10 to 12 GHz, spaced by 10 MHz.

The computed S−parameters are shown in Fig. 7 for direct frequency and in Fig. 8 for modal frequency FEA.

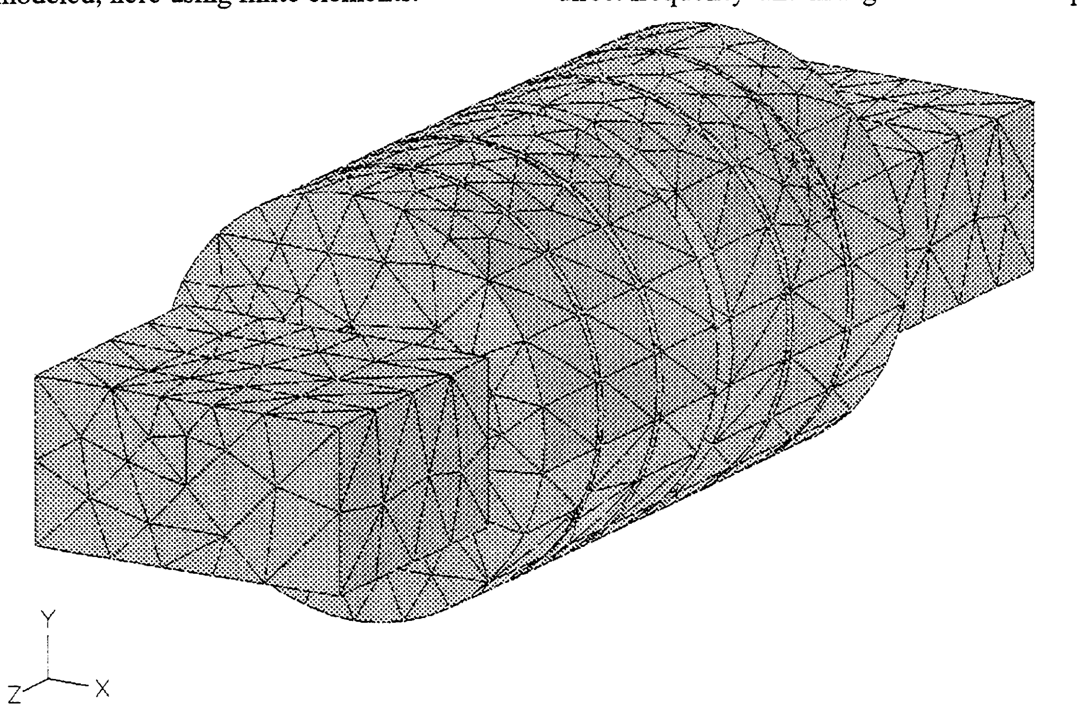

Fig. 6. Finite element model of filter, made up of 10,169 H1−curl tetrahedrons.

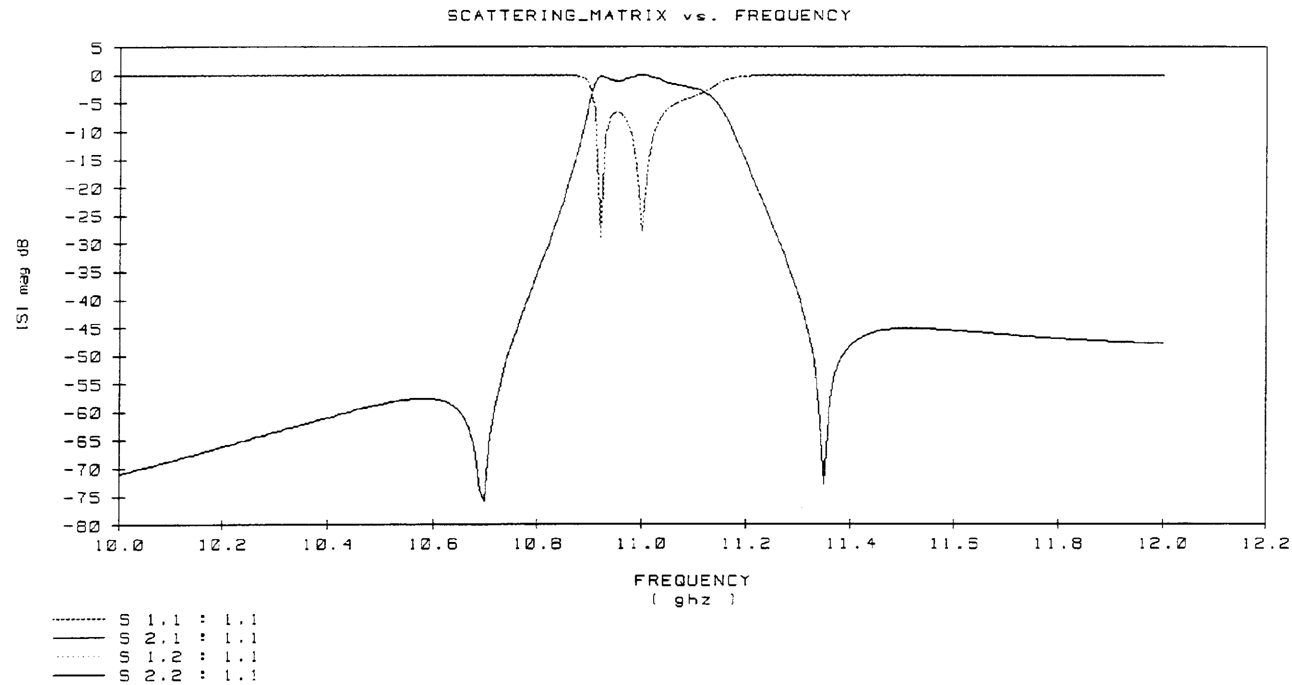

Fig. 7. S−parameters computed using direct frequency method at 201 frequencies.

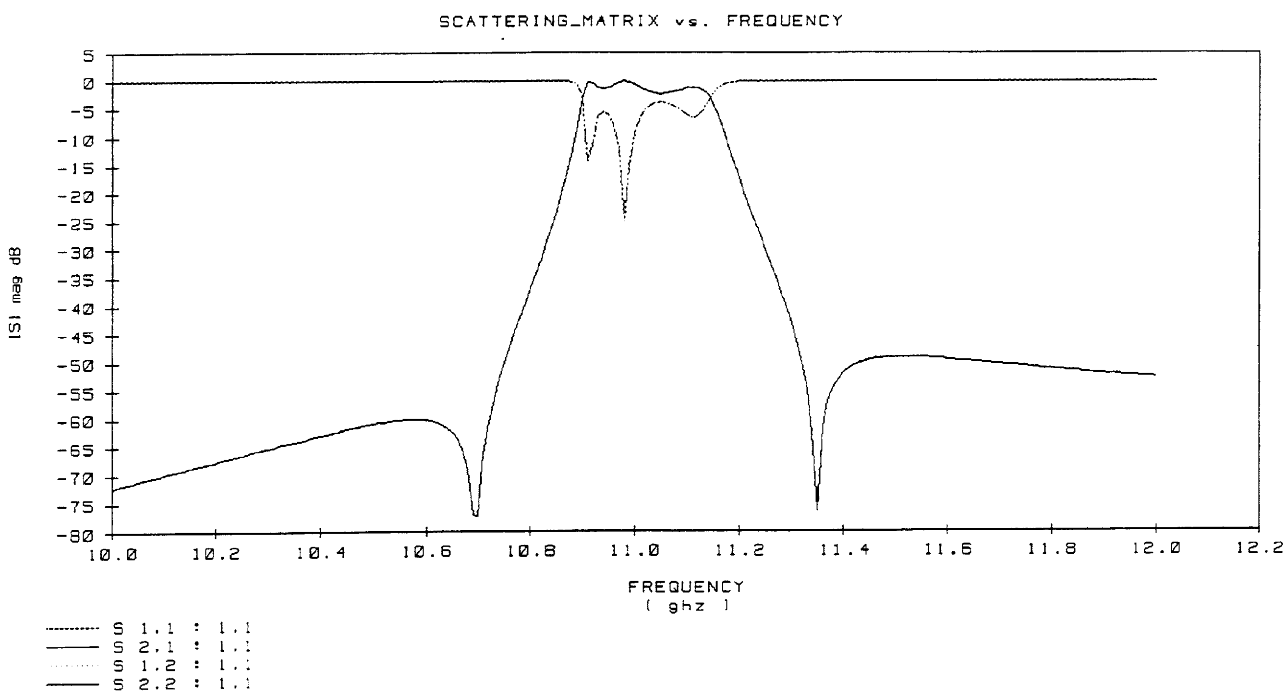

Fig. 8. S−parameters computed using modal frequency method at 201 frequencies.

Comparing Figs. 5, 7, and 8 shows that the direct and modal results are very similar, both for S11 and S21. The measured S21 of Fig. 5 appears similar to the computed S21 of Figs. 7 and 8. The measured S11 of Fig. 5 differs significantly from the computations of Figs. 7 and 8 in the passband region.

There are several possible explanations for the difference between the measured and computed S11. One is that the measurements may have been made on a filter with a greater diameter, evidently denoted as x.xxx in [13], than the 2.40 cm assumed in Fig. 1. Another possibility is that more finite elements may be needed, whether direct frequency FEA or modal frequency FEA is used. The expected accuracy of the computed and measured S parameters is on the order of plus or minus 5 db.

COMPUTER PERFORMANCE

Both the direct frequency and modal frequency computations of Figs. 7 and 8 were made on a Hewlett–Packard model 735/125 workstation. The CPU time was 40,718 seconds for the direct frequency analysis and 2,767 seconds for the modal frequency analysis. The modal frequency analysis first searched for all 3D modes from 5 to 12.5 GHz, and found ten modes at frequencies ranging from 8.27 to 11.36 GHz.

The software developed here is also available on other high–performance computers including several types of Cray parallel processors. On the Cray, a vectorized and parallelized sparse solver has been developed [14]. The software includes not only the solid geometry preprocessing of Figs. 1 through 4, but also the automatic mesh generation of Fig. 6, as well as postprocessing capabilities such as the graphs of Figs. 7 and 8.

Eigenvalue extraction and other computationally intensive tasks in the software are carried out using special vector kernels that have been optimized for various high–performance computer platforms. The most important vector kernels are usually BLAS (basic linear algebraic subroutines). Level I–type kernels include SAXPY and DOT [14], [15]. Block kernels are also specific to the particular hardware and include double kernels such as multiplications of scalars and vector blocks. BLAS level II and especially level III–type kernels are not favored in commercial finite element systems, because they are usually incompatible with simple fortran matrix storage schemes.

Recently, sparse matrix methods have been implemented that greatly enhance performance for very large finite element matrices. Sparse BLAS kernels such as SAXPI have become heavily used in sparse matrix routines. Also enhancing performance of sparse matrix processing are new resequencing methods based on minimum degree ordering [14], [16].

CONCLUSION

A dual–mode microwave filter has been analyzed by both conventional direct frequency FEA and by a new modal frequency FEA technique. Both methods have obtained S21 at 201 frequencies that agrees well with measurements, but their S11 disagrees somewhat with measurements. While the modal frequency method obtained essentially the same S–parameters as the direct frequency method, it has achieved a speedup factor of 15. Thus modal frequency FEA is attractive for analysis of filters and other low–loss resonant microwave devices.

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