Dynamic Operation of Coils with Ferromagnetic Cores Taking Magnetic Hysteresis into Account

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***Abstract*-The computation of electromagnetic field diffusion in ferromagnetic and conductive media is coupled to circuit analysis problems, involving coils with iron cores. The classical Preisach model provides the relationship between magnetic field intensity and flux density at the local level, inside the core. Combined with a 2D eddy current simulation, carried out by a time stepping method, the dynamic operation of the core is modelled. The circuit equations are solved taking the effects of hysteresis and skin effects caused by eddy currents induced in core sheets into account.**

I. Introduction

The aim of this paper is to elaborate a dynamic hysteresis model for ferromagnetic sheets within the cores of coils in power circuits and to implement it in the equation system of the respective circuits to simulate their dynamic operation.

It is based on the static hysteresis characteristic of the ferrous material of the core described by the classical Preisach model [1] on one hand and numerical computation of the electromagnetic field diffusion in the given geometry performed by finite difference [2,3] on the other. The classical Preisach model provides the relationship between magnetic field intensity and magnetisation or flux density at the local level. Included in the governing equation system of the time-varying electromagnetic field, it enables eddy current simulation [4,6-8] taking hysteresis into account.

The result, a model that provides the relationship between magnetic field intensity at the surface of the sheets and total magnetic flux through the cross section of the core, yields the relationship between the current in the coil and the induced voltage. It is implemented in the circuit analysis program together with circuit equations to simulate circuit dynamic operation.

Two problems are solved, one involving a coil with ferromagnetic core made of sheets supplied from a sinusoidal voltage through a rectifying diode and one simulating the dynamic operation of a transformer supplying one-way rectifier. The variation in time of the current through the coils, the flux through the cross section of the core and the voltage applied to the coil, respectively to the diode are plotted as well as the trace of the operation point in the magnetisation characteristic of the core. The resistance of the wire and leakage inductances are also taken into account.

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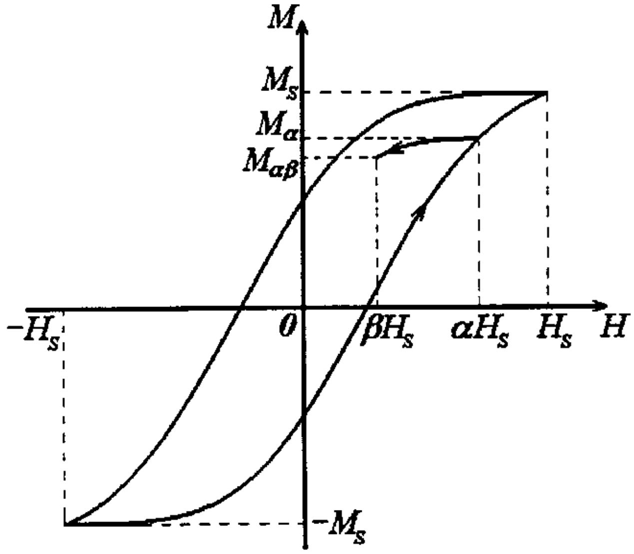

Fig. 1. Measurements used for the Preisach-model.

II. Experimental Construction of the Classical Preisach Model

At very low frequencies (10 Hz), the eddy currents induced in the core of a transformer made of thin, insulated sheets can be ignored and the magnetic flux considered to be uniformly distributed over the cross-section S of the core (quasi-static regime). Measuring and integrating the voltage e_2 induced in the secondary coil (w_2), the magnetic flux of the core can be computed as:

$$\varphi(t) = \varphi_0 - \frac{1}{w_2}\int_0^t e_2(t)\,dt \ , \qquad (1)$$

the magnetization being:

$$M(t) = \frac{\varphi(t)}{\mu_0 S} - H_0(t) \ , \qquad (2)$$

the value of the field intensity at each point of the core cross-section being, in quasi-static operation, equal to that at the surface of the sheets (H_0).

The classical Preisach-model requires the values of magnetisation along the first order reversal curves (branches of minor loops starting in points of the major hysteresis loop). The major loop is obtained with the input pattern:

$$H_0(t) = H_S \sin\left(2\pi f t - \pi/2\right) \ , \qquad (3)$$

where H_S is the value of input (field intensity) at which the output (magnetization) reaches saturation (M_S). If the Preisach-array has $2n$ lines, the distance between two successive reversal points has to be:

$$\Delta H = \frac{1}{n} H_S \quad . \tag{4}$$

Values of output have to be determined for inputs situated at (see Fig.1):

$$\begin{cases} \alpha H_S = k\Delta H \\ \beta H_S = l\Delta H \end{cases} ; \quad \begin{cases} \alpha = k/n \; ; \; k = \overline{-n, n} \\ \beta = l/n \; ; \; l = \overline{-n, k} \end{cases} . \tag{5}$$

Magnetization values in the reversal points situated on the major loop are determined as:

$$\begin{aligned} \varphi_\alpha &= \varphi_{-1} - \frac{1}{w_2} \int_0^{t_\alpha} e_2(t)dt \\ t_\alpha &= \frac{1}{2\pi f}\left(\arcsin \alpha + \frac{\pi}{2}\right) \\ M_\alpha &= \frac{\varphi_\alpha}{\mu_0 S} - \alpha H_S \end{aligned} \tag{6}$$

The required first order reversal curves are obtained with the input function:

$$H_0(t) = H_S\left[\frac{\alpha - 1}{2} + \frac{\alpha + 1}{2}\sin\left(2\pi f t + \pi/2\right)\right] . \tag{7}$$

Along these curves, the values of magnetization are determined as:

$$\begin{aligned} \varphi_{\alpha\beta} &= \varphi_\alpha - \frac{1}{w_2} \int_0^{t_{\alpha\beta}} e_2(t)dt \\ t_{\alpha\beta} &= \frac{1}{2\pi f}\left(\arcsin \frac{2\beta - \alpha + 1}{\alpha + 1}\alpha - \frac{\pi}{2}\right) \\ M_{\alpha\beta} &= \frac{\varphi_{\alpha\beta}}{\mu_0 S} - \beta H_S \end{aligned} \tag{8}$$

The output values determined in the way described above fill the Preisach-array, necessary for the numerical implementation of the Preisach model. For this purpose at least $n = 100$ is required to ensure sufficient accuracy, but measurements can be performed for a lower value of n, followed by the determination of the intermediary output values by numerical (e.g. spline) interpolation.

III. Simulation of Coil Dynamic Operation

For the layout shown in Fig. 2, the following values of parameters are considered:

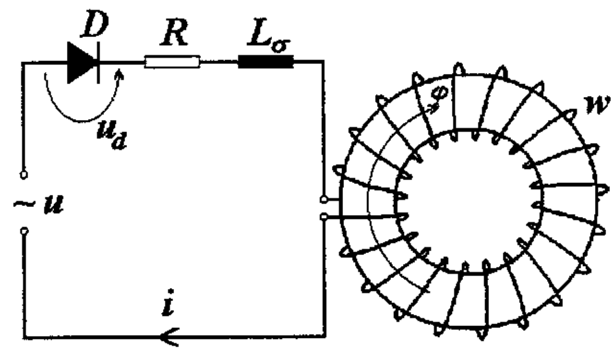

Fig. 2. Circuit layout

- winding:
  - number: $w = 200$;
  - leakage inductance: $L\sigma = 1$ mH.
- core:
  - cross-section: $S = 200$ mm^2 (10 sheets 20 mm wide, 1 mm thick each);
  - mean radius: $(r_i + r_e)/2 = 20$ mm;
  - medium length: $l = 125.7$ mm;
  - conductivity: $\sigma = 5 \cdot 10^6$ S/m.
- total resistance of the circuit: $R = 10\ \Omega$.
- supply voltage:
  - magnitude: $U_0 = 100\ \Omega$;
  - frequency: $f = 200$ Hz.

The circuit equations are:

- conducting diode

$$\begin{cases} \dfrac{di}{dt} = \dfrac{u(t) - Ri}{L_\sigma + \dfrac{w^2}{l}\dfrac{d\varphi}{dH_0}} \; ; \quad i > 0 \\ u_d = 0 \end{cases} \tag{9}$$

- blocked diode:

$$\begin{cases} u_d = u(t) - w\dfrac{d\varphi}{dt} \; ; \quad u_d < 0 \\ i = 0 \end{cases} \tag{10}$$

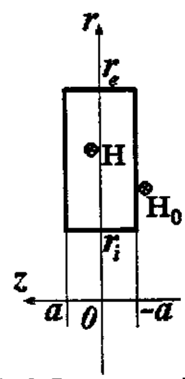

Fig. 3. Core cross-section

The source-field at the surface of the core sheets:

$$H_0(r,t) = \frac{wi(t)}{2\pi r} ; r \in [r_i, r_e] , \tag{11}$$

the geometry of the cross section of the core being plotted on Fig. 3.

The orientations of the magnetic field intensity and eddy current density vectors are:

$$\begin{aligned} \mathbf{H} &= \mathbf{u}_\varphi H(r, z, t) \\ \mathbf{J} &= -\mathbf{u}_r J_r(r, z, t) + \mathbf{k} J_z(r, z, t) \end{aligned} \tag{12}$$

The differential equation in cylindrical co-ordinates written for the magnetic field intensity inside the core [5]:

$$\frac{\partial H}{\partial t} = \frac{\dfrac{\partial^2 H}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial H}{\partial r} - \dfrac{H}{r^2} + \dfrac{\partial^2 H}{\partial z^2}}{\sigma\mu_0\left(1 + \dfrac{\partial M}{\partial H}\right)} \quad , \qquad (13)$$

$$r \in [r_i, r_e] \quad , \quad z \in [0, a] \quad , \quad t \geq 0$$

with the boundary conditions:

$$\left.\begin{aligned} H(r,a,t) &= H_0(r,t) \\ \frac{\partial H}{\partial z}(r,0,t) &= 0 \end{aligned}\right\} \quad r \in [r_i, r_e] \qquad\qquad (14)$$
$$\left.\begin{aligned} H(r_i,z,t) &= H_0(r_i,t) \\ H(r_e,z,t) &= H_0(r_e,t) \end{aligned}\right\} \quad z \in [0, a] \quad ; \quad t \geq 0$$

is solved in time-domain by means of the finite difference method, followed at every time-step by the computation of the total flux of the core and its derivative with respect to the variation of source-field intensity if the diode is conducting or with respect to time, if the diode is blocked.

Having these derivatives, the circuit equations (9) and (10) can be solved (e.g. by Runge-Kutta method) to obtain the dynamic behaviour of the coil taking the ferromagnetic characteristics of its core into account.

Regarding the implementation of the method, it is important that the Preisach model has to be provided with an interpolation routine, so that its output is continuous, because discontinuities would affect the stability of the algorhytm for solving the circuit equations.

Continuity of output can be ensured by running four Preisach models at every spatial grid-point, sandwiching the output for the actual variation of input between two neighbouring curves, obtained for input histories passing through exact input steps. Both neighbouring curves need two models, one trailing and one going ahead of the actual input variation. Interpolating the value of the output between the outputs of the four neighbouring models, it becomes continuous even in higher order reversal points situated between input steps.

The scalar Preisach model [1] has been implemented using a Gaussian distribution of the elementary hysteresis switches:

$$\mu(\alpha\ \beta) = e^{-\frac{(\alpha-\beta)^2}{a} - \frac{(\alpha+\beta)^2}{b}} \quad , \qquad (15)$$
$$\alpha \in [-1,1]; \qquad \beta \in [-1,1]$$

with $a = 0.2$ and $b = 0.4$, $n = 100$ field intensity steps of $\Delta H = 100$ *A/m* and magnetization scale $sm = 705$ *A/m*.

The results are shown on Figs. 4 - the waveforms of the current through the circuit, the voltage on the diode and the flux through the core and 5 - the magnetizing characteristic of the core (it can be observed that flux variation takes place even with blocked diode - zero source field - until the eddy currents within the core sheets disappear). In Fig. 4. u^* stands for the restriction of the supply voltage to the time while the diode is blocked. The distributions of the effective values of the electromagnetic field quantities inside the core sheets are plotted in Figs. 5, 6, and 7.

IV. Simulation of Transformer Dynamic Operation

For the layout shown in Fig. 8, the parameter values are:

- windings: numbers: $w_1 = 360$, $w_2 = 90$; resistance: $R_1 = 1.8\ \Omega$, $R_2 = 0.2\ \Omega$; leakage inductance: $L\sigma_1 = 10$ mH, $L\sigma_2 = 0.63$ mH.
- core: cross-section: $S = 1280$ mm^2 (10 sheets, 32 mm wide, 4 mm thick each); medium length: $l = 400$ mm (curvature ignored), conductivity: $\sigma = 5 \cdot 10^6$ S/m.
- load resistor: $R_S = 5\ \Omega$.
- supply voltage: magnitude: $U_0 = 300$ V; frequency: $f = 50$ Hz.

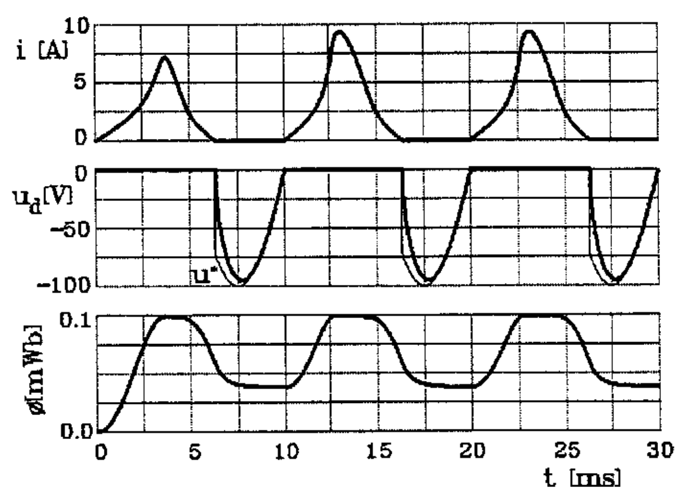

Fig. 4. Resulting waveforms of the current, voltage on the diode and flux through the core of the coil

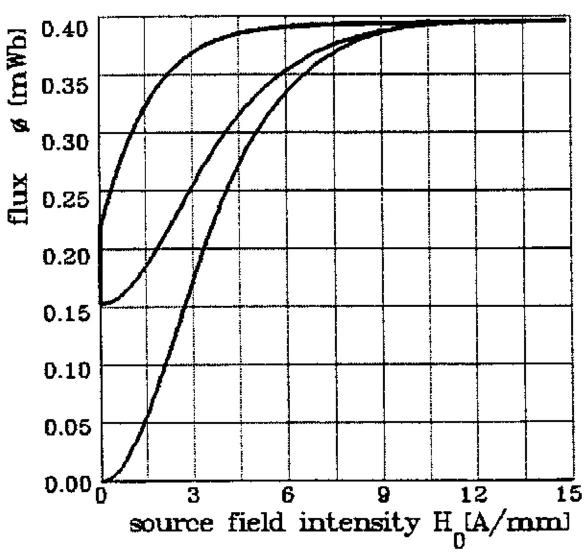

Fig. 5. Core magnetizing characteristic

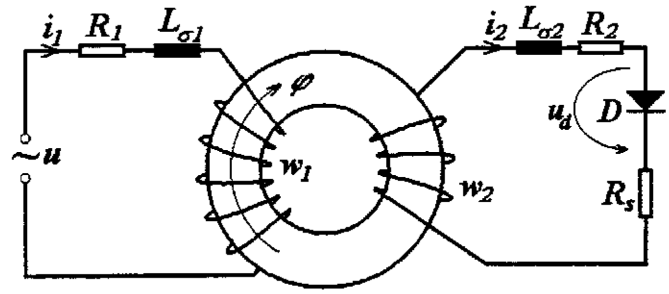

Fig. 8. Circuit layout for transformer simulation

The circuit equations are:

- conducting diode:

$$\begin{cases} \dfrac{di_1}{dt} = \dfrac{1}{L_{\sigma 1}}\left(u(t) - R_1 i_1 - w_1 \dfrac{d\varphi}{dt}\right) \\ \dfrac{di_2}{dt} = \dfrac{1}{L_{\sigma 1}}\left(-(R_2 + R_S) i_2 - w_2 \dfrac{d\varphi}{dt}\right); \; i_2 > 0 \\ u_d = 0 \end{cases} \quad (16)$$

- blocked diode:

$$\begin{cases} \dfrac{di_1}{dt} = \dfrac{1}{L_{\sigma 1}}\left(u(t) - R_1 i_1 - w_1 \dfrac{d\varphi}{dt}\right) \\ i_2 = 0 \qquad\qquad\qquad ; \; u_d < 0 \\ u_d = -w_2 \dfrac{d\varphi}{dt} \end{cases} \quad (17)$$

The source-field intensity at the surface of the core is needed as the boundary condition for computation of the electromagnetic field intensity inside the core:

$$H_0(t) = \frac{w_1 i_1(t) + w_2 i_2(t)}{l} \; . \quad (18)$$

According to the simplifying assumptions (Fig. 9):

$$\mathbf{H} = \mathbf{k} H(x, y, t) \; . \quad (19)$$

and the differential equation for the magnetic field inside the core:

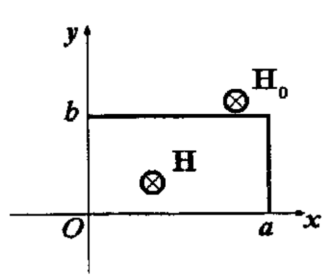

Fig. 3. Core sheet cross-section

$$\frac{\partial H}{\partial t} = \frac{\dfrac{\partial^2 H}{\partial x^2} + \dfrac{\partial^2 H}{\partial y^2}}{\sigma \mu_0 \left(1 + \dfrac{\partial M}{\partial H}\right)} \; , \quad (20)$$

$$(x, y) \in [0, a] \times [0, b] \; , \; t \ge 0$$

with boundary conditions:

$$\left. \begin{array}{l} H(b, x, t) = H(a, y, t) = H_0(t) \\ \dfrac{\partial H}{\partial x}(0, y, t) = 0 \; ; \; y \in [0, b] \\ \dfrac{\partial H}{\partial y}(x, 0, t) = 0 \; ; \; x \in [0, a] \end{array} \right\} ; \quad t \ge 0^{\cdot} \quad (21)$$

The non-linear term $\partial M / \partial H$ from the denominator of equation (20) is evaluated by means of the classical Preisach model taking actual and past local values of field intensity into account, the solution is formulated in the time domain using finite differences with an alternating direction and fractional step method [2]. A quarter of the sheet cross-section has been discretized with a grid consisting of 17 points in the x direction (sheet width) and 5 in the y direction (sheet thickness).

The time step for the combined (circuit analysis - field diffusion) algorithm had to be set as small as $2 \cdot 10^{-6}$ s to ensure stability, because the current derivatives appear implicitly on the right hand side of the equations (16) and (17), toughening the stability conditions to be fulfilled by the solving algorithms.

The simulation has been performed starting with the moment of connection of the transformer to the supply.

The resulting waveforms of the currents in the coils and of the magnetic flux in the core are plotted in Fig. 10, while

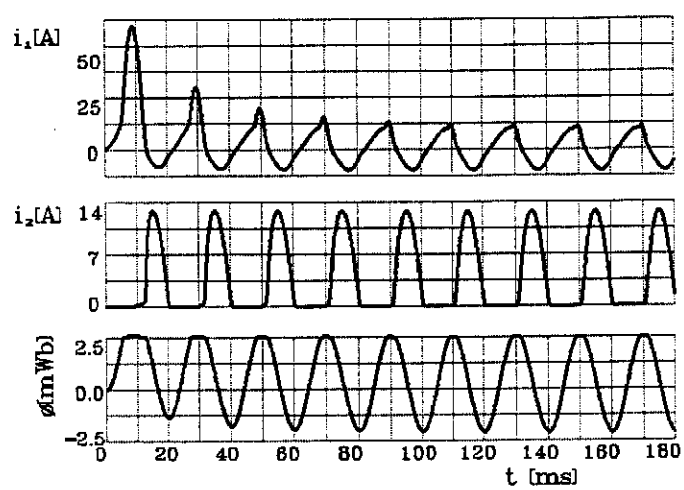

Fig. 10. Current and flux waveforms

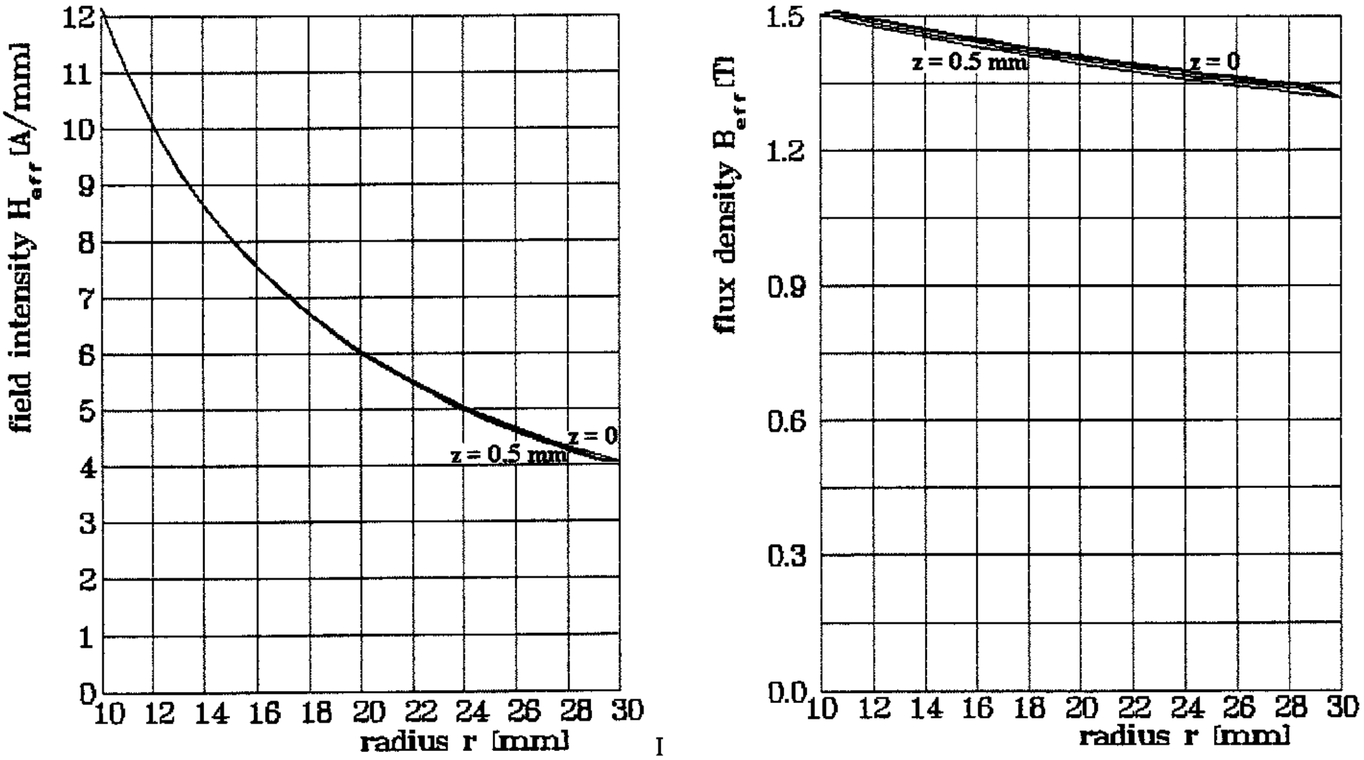

Fig. 6. Distribution of magnetic field intensity and flux density inside the core

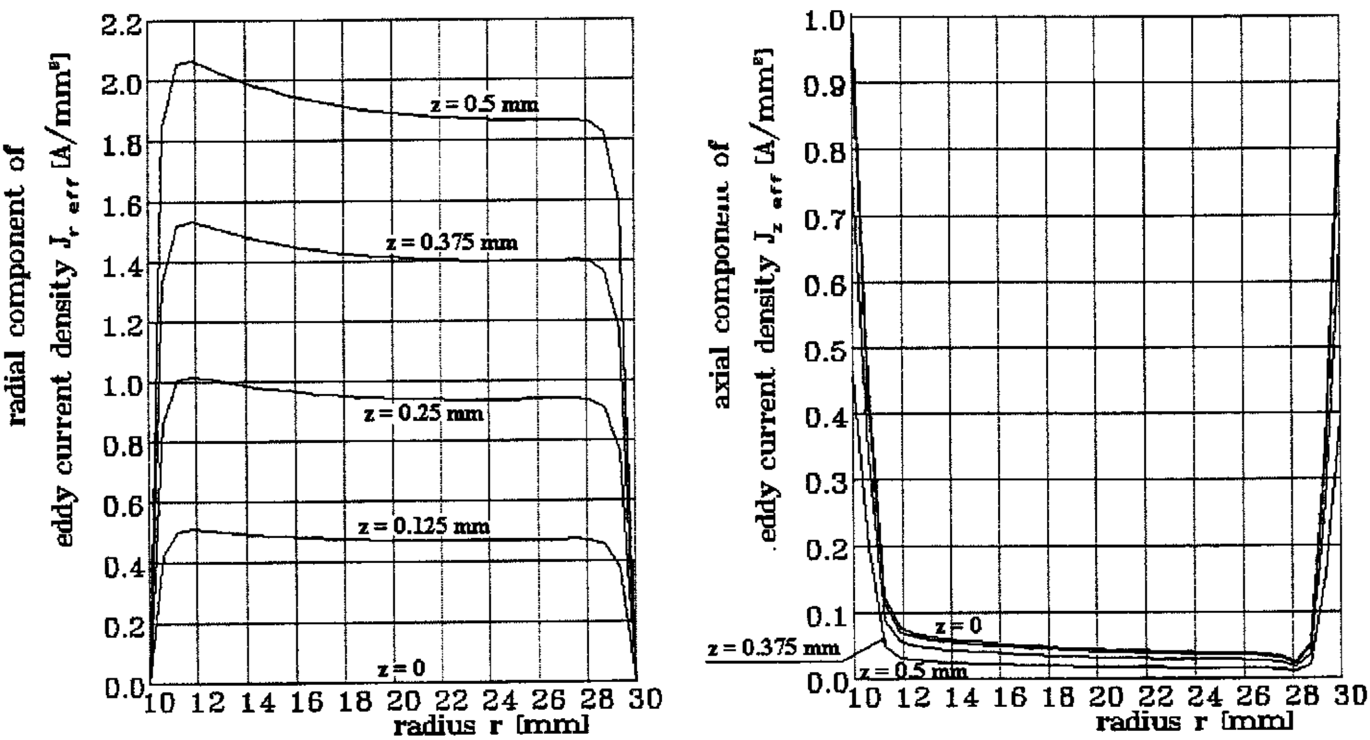

Fig. 7. Distribution of effective values of the radial and axial components of the eddy currentdensity inside the core

Regarding the geometry of the core, the following simplifying assumptions are made:

- the curvature of the core is ignored. It is considered to be straight with length equal to the torus mean perimeter.
- the magnetic field intensity vector as well as the magnetization and flux density vectors are oriented along the longitudinal axis of the sheets (as if they were infinite).

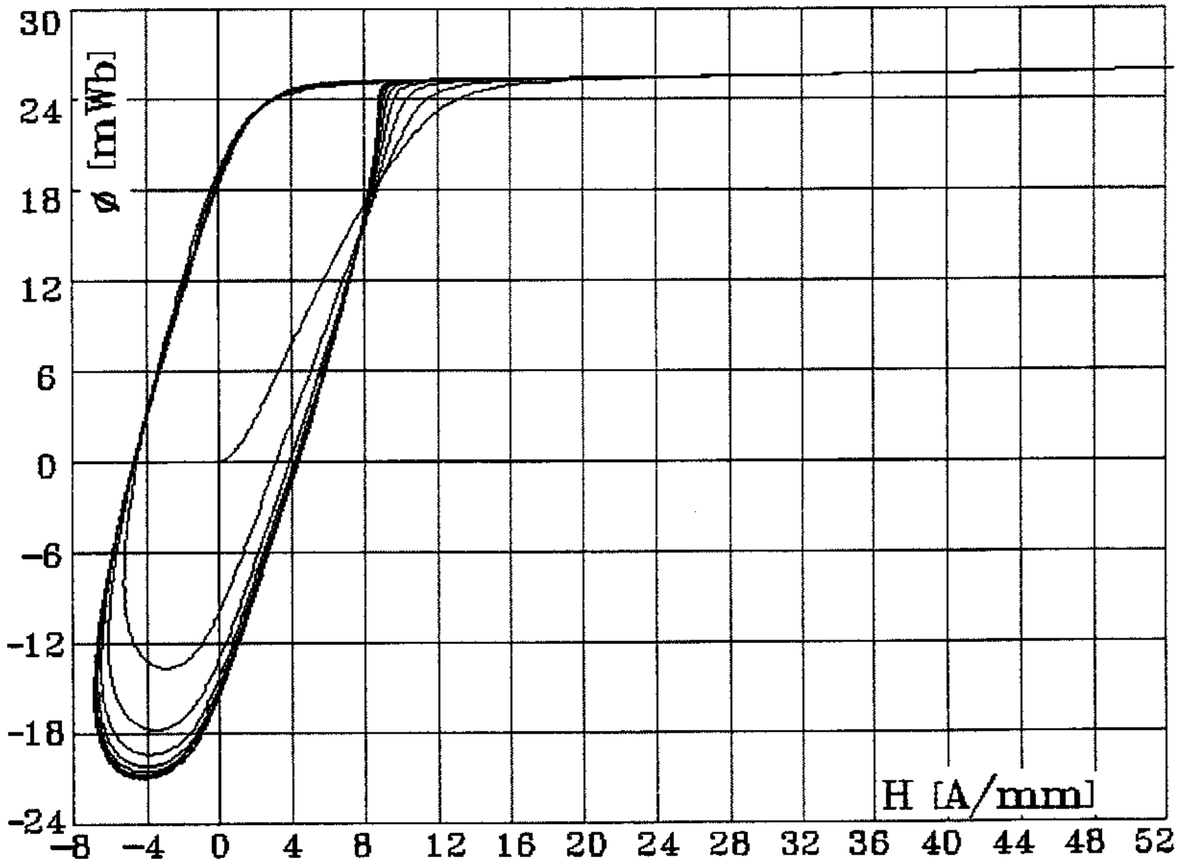

Fig. 11. Core magnetizing loops

the magnetizing loops of the core can be seen in Fig. 11.

The simulated transformer operates with over-saturated core.

V. Conclusions

The presented model is suitable for accurate simulation of the operation of coils with ferromagnetic cores as well as that of transformers in non-harmonic regimes [9].

It takes the effects of core-saturation, hysteresis and skin-effect due to eddy currents into account. The price of the enhanced accuracy is the relatively great amount of memory and computing time required.

Fairly accurate results can be expected in the cases of non-oriented core materials, in which the effects of domain wall motions can be ignored at relatively low frequencies and can be treated as homogeneous materials [4,10].

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