Interval-Based Robust Design of a Microwave Power Transistor

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Abstract **–** An interval-based approach aimed at the robust design of a specific performance of a Double Hetero-junction Bipolar Transistor (DHBT) for microwaves applications is presented. The robust design is obtained by looking at the range of the performance function by means of an overestimation, given in analytical form, of its amplitude. The proposed approach is described by referring to two theoretical performance functions to show the reliability for both the univariate and multivariate cases. The worst case approach is considered in order to study the minimum variation of the max oscillation frequency of the DHBT, obtained by a regression model from numerical results, in presence of given parameters variations. The physical and geometrical parameters affecting the performance are regarded as implicitly uncorrelated and uniformly distributed in an assigned range and therefore all their combinations are kept into account. The implemented approach permits to achieve a greater robustness of the solution without assuming approach-specific settings and additional computations dependent on designer's ability and can be used to maximize the production yield.

Keywords: Robust design, uncertain parameters, and optimization.

I. INTRODUCTION

The real behavior of a component is inevitably different from that considered in the design process owing to the uncertainties in the values of physical and geometrical parameters, to the effective operating conditions and to the drift and aging effects. Such an inconvenience may be faced up during the prototyping process of the component by a costly and time consuming dynamic adjustment of the parameters values whose convergence is based on the designer ability. However, it is possible to obtain a component realization, satisfying the imposed constraints even in presence of parameters changes, if in the early design phase such variations are properly taken into account. It is therefore possible to achieve a robust design that is the chosen combination of the design parameters ensures that component performance presents the minimal variations with respect to the parameters changes. The possibility to accomplish a robust design is particularly relevant in those fields, as in the dimensioning of an electronic device, in which the realization of prototypes is expensive and lengthy [1].

As shown in [2], an innovative approach, based on the use of Interval Analysis, leads to a robust design of a component able to satisfy the desired constraints even when the geometric dimensions, the physical properties or the operating conditions assume any possible value in an assigned range. For a given Performance Function (PF) described by a polynomial form, it furnishes the Most Robust Stationary Solution (MRSS), i.e. the set of nominal parameters such that its first derivative is zero, and an over-bounding of the PF. This systematic approach leads to the quick identification of the most suitable combination of the parameters values thus allowing to increase the production yield, reduce the optimization time and consequently the overall time-to-market process.

In this paper the main properties of the interval-based robust design approach are discussed by considering two theoretical performance functions in order to show the reliability for both the univariate and the multivariate cases. The method is then applied to the design of a PF represented by the max oscillation frequency of a Double Hetero-junction Bipolar Transistor (DHBT) for microwave applications. In particular, the dependency of the PF with respect to the influencing factors, given by a polynomial form obtained by interpolating the numerical results of a physical simulator [1], is analyzed. It is shown that the application of the interval-based method allows achieving more general and approach-independent information on the robustness of a particular solution with a slight investment in terms of computations.

The proposed approach can also be extended to other regression models describing further relevant performances controlling the electrical and thermal behaviour of the DHBT, such as common emitter breakdown voltage, max collector current density, etc. However, we explicitly remark that the main purpose of the present work is to show the effectiveness of the interval-based approach, rather than to perform a systematic and exhaustive design of the electronic device and hence only the variability of the max oscillation frequency with respect to physical and geometrical characteristics is discussed.

The paper is organised as follows. After a brief presentation of the Interval-based design approach in sect. II, two theoretical applications will be illustrated in sect. III. In sect. IV the model based design of the max oscillation frequency of a DHBT is discussed and in Sect V the main conclusions are drawn.

II. INTERVAL-BASED ROBUST DESIGN

The Performance Function (PF) describes the device performance as a function of ν design parameters, $\underline{x} = (x_1, x_2, \cdots, x_\nu)$. Let us suppose that the objective is to find a solution, i.e. a set of nominal parameters values, which satisfies assigned design constraints. A *robust solution* is one which guarantees that the constraints are fulfilled also in presence of assigned parameters variations $\underline{\Delta} = (\Delta_1, \Delta_2, \cdots, \Delta_\nu)$ [3]. Around such solution the range of the PF is generally narrow, tending to a point if the PF is locally flat. Not all the robust solutions have the same characteristics. A robust solution which implies that the PF variations are localised at the boundary of the Region of Acceptability (ROA) may become a non robust one if one of the parameters exhibits a variation greater that the expected one. It is possible to discriminate the level of solution robustness by looking at the range of the PF. In fact, a robustness index can be simply obtained by considering the amplitude of the range function with respect to a given parameter variation. The lower is the amplitude of the range, the greater is the robustness. For example, if the PF is a function of one parameter *x*, as shown in Fig. 1, the robustness index with respect to a variation of ±Δ around the nominal solution x_0 is given by the value $w(f_X)$ correspondent to,

$$w\left(f_{\underline{X}}\right) = \max_{\underline{x}\in\underline{X}_0} f\left(\underline{x}\right) - \min_{\underline{x}\in\underline{X}_0} f\left(\underline{x}\right) \qquad (1)$$

$w(f_X)$ is the range width of $f(x)$ when $x \in [x_0 - \Delta, x_0 + \Delta] = X$ and f_X represents the range of the PF for such variation.

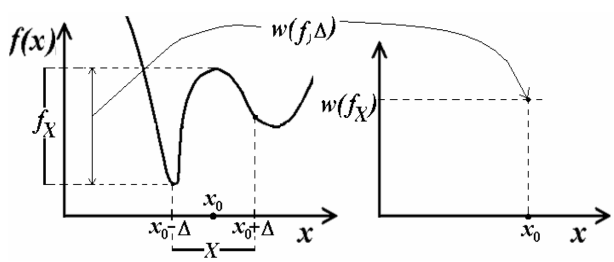

Fig. 1. A monodimensional PF *f(x)* and the space of the range amplitude for a given parameter variation Δ.

In the design process it would be useful to have an algorithm that furnishes $w(f_X)$ to obtain a biunique correspondence as shown in Fig. 1. Indeed, it is not easy to obtain $w(f_X)$ and an approximation of it is generally adopted, typically in discrete way, by computing the equation (1) for each point. Moreover, the algorithms available in the literature lead to an overestimation of the actual robustness of the PF, due to an intrinsic characteristic of equation (1). In fact, the research of the range of *f(x)* is conditioned by the presence of local minima/maxima and the quality of the result is somehow discretionary, since it depends on the choice of the parameters of the searching algorithm [4]. As a consequence, it may happen to select a robust one as a nominal solution that actually is not robust. Therefore, an underestimation of the robustness index must be adopted in order to guarantee its reliability and it can be obtained by means of an overestimation of equation (1). In fact, since the lower is the amplitude of the range, the greater is the robustness, an overestimation of equation (1) leads to an underestimation of the robustness of the nominal solution.

The Interval Analysis (IA) is an arithmetic that furnishes a reliable inclusion of the true range of a function for a given interval of values of the variables. Therefore the overestimation of the range amplitude can be achieved by exploiting the peculiarities of the IA and, in particular, the "over-bounding" of the function [5-6]. The function bounding and a generic over-bounding in presence of a given uncertainty of the variable are depicted in Fig. 2.

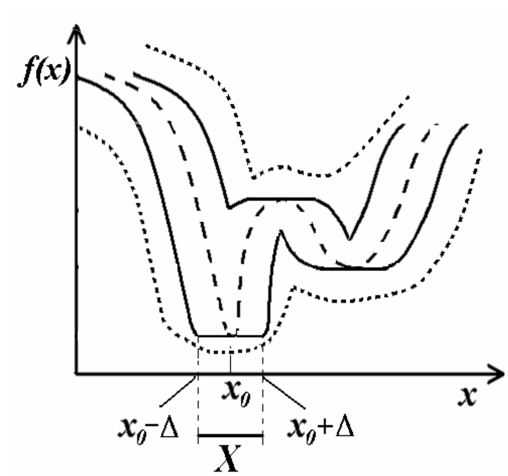

Fig. 2. Bounding (continuous lines) and over-bounding (dotted lines) of *f(x)* for a parameter variation ±Δ.

The bounding is given by the two solid curves: the upper-bound, that is the locus of the maxima of the function *f(x)* when the parameter *x* spans the "moving" interval $[x_0 - \Delta, x_0 + \Delta] \equiv X$, and the lower-bound that is the locus of the minima for the same moving interval. Instead, an over-bounding is given by the two dotted curves: they include the function bounding, i.e. represent an overestimation of the upper-bound and an underestimation of the lower-bound.

A possible over-bounding can be easily obtained by applying the Interval Arithmetic to *f(x)* when the variable *x* is substituted by the interval *X* [5]. In fact IA is arithmetic defined on sets of intervals rather then sets of real numbers. An interval *X* is an ordered pair of real numbers $X=[a,b]$ such that, $X=[a,b]=\{x|a<x<b,\ with\ a,x,b\in\Re\}$, and all the values in *X* are equally probable. The sets of intervals on $\Re$ is denoted as $I\Re$. The interval width is defined as $w(X)=b-a$. In the following we will refer to a symmetric interval $X=[x-\Delta,x+\Delta]$, centred around the nominal point *x*, whose width is 2Δ. In presence of multivariate function, the IA treats the variables as uncorrelated. In presence of parameters variations, IA permits a straight determination of an interval that certainly includes the true range of a function; thanks to the "inclusion property" [5] and it can be suitably adopted in a worst-case design [7].

If the IA is applied to the Taylor series expansion of the PF around a nominal solution we obtain an interval, and if the nominal solution varies we obtain an interval function named Interval Taylor Extension (ITE) [2]. As an example, for a PF of a single parameter and for a generic point x_0 representing a particular nominal solution, we have the following ITE,

$$\begin{aligned}F_{ITE}(X)&=\sum_{k=0}^{\infty}\frac{f(x_0)^{(k)}}{k!}(X-x_0)^k=\\&=\sum_{k=0}^{n}\alpha_k Y^k+\frac{f(X)^{(n+1)}}{(n+1)!}\Delta^{n+1}Y^{n+1}\in I\Re\end{aligned}\tag{2}$$

where $Y=[-1,1]\in I\Re$ is a constant interval and $\alpha_k=\Delta^k\frac{f^{(k)}(x_0)}{k!}$.

It results that $\forall x\in X=[x_0-\Delta,x_0+\Delta]\subset\Re$ $f(x)\in F_{ITE}(X)$, where *X* is the compact given by a tolerance Δ on the nominal parameter. Therefore, ITE is an inclusion of the range of *f(x).* The Width of ITE (WITE) for polynomial PF is characterised by the following properties [2]:

a) It is a continuous, non differentiable function which can be expressed in symbolic form;
b) It presents local minima positioned in the stationary points of the corresponding PF;
c) A maximum variation of the parameters can be found such that WITE reaches its absolute minimum in correspondence of the most robust stationary point;
d) Representing a valuable robustness index, it furnishes an accurate means for classifying the relative robustness of the stationary points.

Therefore, thanks to the property (a), the robustness of the nominal solution can be evaluated by considering a continuous, non differentiable function, $w(F_{ITE}(X))$ or WITE. In particular, if $f(\underline{x})=f(x_1,x_2,\cdots,x_\nu)$ is a ν-variate polynomial function of *n-th* order, then $w(F_{ITE}(\underline{X}))$ is,

$$w(F_{ITE}(\underline{X}))=\sum_{1\le i_1+\ldots+i_\nu\le n}\beta_{i_1\ldots i_\nu}\left|\alpha_{i_1\ldots i_\nu}\right|\ge 0\tag{3}$$

with $\beta_{i_1\ldots i_\nu}=\begin{cases}2 & if\ i_k\ odd,\ k=1,2,\ldots,\nu\\1 & otherwise\end{cases}$ and

$$\alpha_{i_1\ldots i_\nu}=\frac{\Delta_1^{\,i_1}\cdots\Delta_\nu^{\,i_\nu}}{i_1!\cdots i_\nu!}\frac{\partial^{i_1}}{\partial x_1^{\,i_1}}\left(\cdots\left(\frac{\partial^{i_\nu}f(\underline{x}_0)}{\partial x_\nu^{\,i_\nu}}\right)\right).$$

Besides, thanks to the properties (b) and (c), it is possible to obtain robust solution by solving the following minimization problem [2],

$$\min_{\underline{x}_0\in\Re^n}\left(w\left(F_{ITE}(\underline{X})\right)\right)\tag{4}$$

rather than equation (1). In this way the problem of local maxima/minima point presents in equation (1) is avoided and the discretionary choice of the parameter of the searching algorithm is limited to the external minimum. It is useful to remark that equation (3) is an overestimation of $w(f_{\underline{X}})$ and the robustness index represented by WITE gives an underestimation of the effective robustness, as a result of "monotonic inclusion" [5]. Therefore equation (4) is not equivalent to equation (1), but the same overestimation guarantees that the true variation of the PF is certainly lower than that indicated by the WITE index, i.e. the particular solution is more robust than that pointed out. As a result, the reliability of the solution increases. Moreover, the robustness index represented by WITE has an analytic expression that can be treated in a symbolic way for any PF. Finally, this methodology can be extended also to generic functions which not necessarily are expressed in a polynomial form.

III. THEORETICAL EXAMPLES

A. A 9-th order univariate polynomial function

In order to show the properties of the proposed approach for the monodimensional-case, let us consider the following 9-th order PF,

$$\begin{aligned}f(x)=&7.9\times10^{-8}x^9-9.8\times10^{-6}x^8+5.1\times10^{-4}x^7-0.014x^6+\\&+0.24x^5-2.4x^4+13x^3-33.8x^2+25.7x+21.3\end{aligned}\tag{5}$$

In the interval $x\in[1, 24]$ it shows 3 minima and 4 maxima, two of which are located at the extremes of the compact as evidenced in Fig. 3.

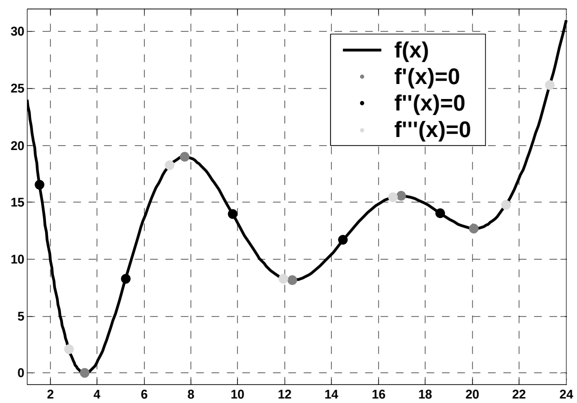

Fig. 3. 9-th order univariate polynomial function.

The Taylor series expansion of such a function around the nominal solution x_0 can be expressed as,

$$f(x)=\sum_{k=0}^{9}\frac{f^{(k)}(x_0)}{k!}(x-x_0)^k \tag{6}$$

Let us choose the interval $X\in I\Re$ characterised by a radius Δ coincident with the variation of the design parameter and centred in its nominal value. By substituting *x* with *X* in equation (6) and proceeding with the IA we obtain the $F_{ITE}(X)$ corresponding to equation (5). It results $\forall x\in X=[x_0-\Delta,x_0+\Delta]\in I\Re$

$$f(x)\in F_{ITE}(X)=\sum_{k=0}^{9}\frac{f^{(k)}(x_0)}{k!}(X-x_0)^k\ , \tag{7}$$

or $f_X\subseteq F_{ITE}(X)$. Moreover, we get $w(f_X)\leq w(F_{ITE}(X))$ (overbounding IA property). By simple algebra the $F_{ITE}(X)$ can be rewritten as,

$$F_{ITE}(X)=\sum_{k=0}^{9}\Delta^k\frac{f^{(k)}(x_0)}{k!}([-1,1])^k=\sum_{k=0}^{n}\alpha_k Y^k \tag{8}$$

with the same significance of α_k and *Y* as in the previous section. In this case the WITE is given by,

$$\begin{aligned} w(F_{ITE}(X)) &= \sum_{j=1}^{9}\beta_j|\alpha_j| = \\ &= (|\alpha_1|+|\alpha_3|+|\alpha_5|+|\alpha_7|+|\alpha_9|)+ \\ &+2(|\alpha_2|+|\alpha_4|+|\alpha_6|+|\alpha_8|)\geq 0\ . \end{aligned} \tag{9}$$

In the examined range $w(F_{ITE}(X))$ has 31 non derivable points, 5 of which corresponding to the roots of *f'(x)* in the range [1,24], i.e. $x_k\in\{20.08,16.97,12.32,7.74,3.48\}\subset[1,24]$. These are points of minimum of $w(F_{ITE}(X))$ for each considered Δ (Fig. 4).

The absolute minimum is one of these points and $w(F_{ITE}(X))$ gives a precise information concerning the relative robustness of the stationary points, as evidenced by the light grey curves in Fig. 4. In particular, the relative magnitude of the robustness coincides with the values of the curves (light grey squares) in such points.

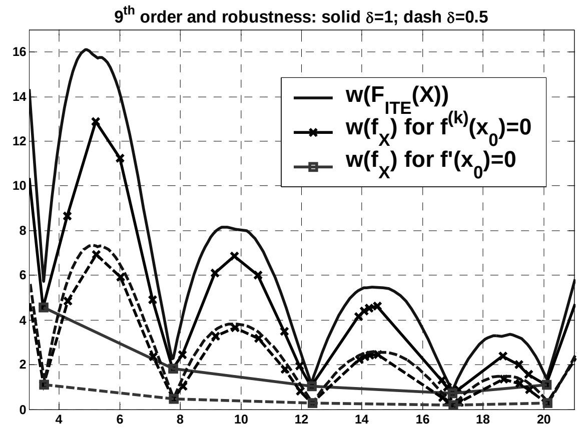

Fig. 4. WITE and $w(f_X)$ for the 9-th polynomial function.

The MRSS is found in x_0=16.97 for $\Delta\in\{0.5,1\}$. It is also evident that the amplitude of the PF range, i.e. the solution of (1), can be obtained by means of a discrete analysis for each nominal point: it corresponds to finding the difference between the max and min in the considered interval. The resulting curve, obtained by linear interpolation between two contiguous points, is discontinuous. Its level of accuracy can be improved by considering a greater number of points. The $w(F_{ITE}(X))$ instead, is a continuous function (curves without marker in Fig. 4) described by a symbolic expression which is valid for each nominal solution. Besides, in addition to the sorting in terms of robustness of the stationary points, an over-bounding is achievable without additional computational efforts (Fig. 5).

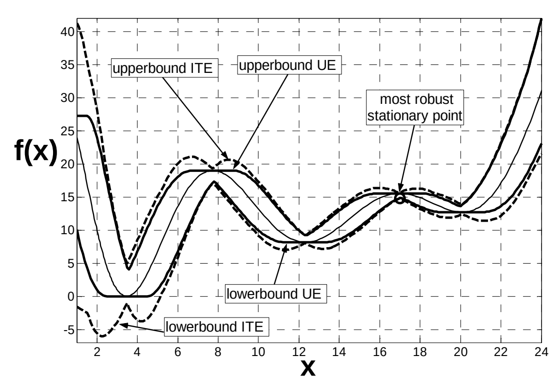

Fig. 5. ITE function bounding for δ=1.

B. A 3-rd order bivariate polynomial function

The properties and the reliability of the ITE are kept also in presence of multidimensional PFs. In order to show the simplicity of the proposed approach also for multivariate problems, we take into consideration the case of 2-variate function $f(\underline{x})$ that is the case of dependency on 2 design parameters. Now we suppose that $f(\underline{x})$ is the following 3^{rd} order polynomial PF (Fig. 6),

$$f(x_1,x_2)=\sum_{\substack{i_1,i_2=0\\ i_1+i_2\le n}}^{n} a_{i_1 i_2\ldots i_\nu}\, x_1^{\,i_1} x_2^{\,i_2} = 3+2x_1^2x_2 + \\ -5x_1^2 + x_1x_2 - 4x_2^2 + 2x_1x_2^2 - 2x_1^3 \tag{10}$$

The PF (equation 10) has a local maximum in [0, 0] for $(x_1,x_2)\in[-5{,}5]\times[-5{,}5]\subset\Re^2$ where it shows also a very smooth region.

It can be expressed in the $\Re$ domain by means of its complete Taylor series around a nominal solution $\underline{x_0}=(x_{10},x_{20})\in\Re^2$,

$$f(\underline{x})=\sum_{k=0}^{3}\frac{\left[\left((\underline{x}-\underline{x}_0)\cdot\nabla\right)^k f\right](\underline{x}_0)}{k!}= \\ =\sum_{k=0}^{3}\frac{\left[\left((x_1-x_{10})\frac{\partial}{\partial x_1}+(x_2-x_{20})\frac{\partial}{\partial x_2}\right)^k f\right](\underline{x}_0)}{k!} \tag{11}$$

then, by substituting the vector $\underline{x}\in\Re^2$ with the interval vector $\underline{X}=\left([x_{10}-\Delta_1,x_{10}+\Delta_1],[x_{20}-\Delta_2,x_{20}+\Delta_2]\right)\in I\Re^2$ we obtain the ITE of equation (10) [8],

$$F_{ITE}(\underline{X})=\sum_{k=0}^{3}\frac{\left[\left((\underline{X}-\underline{x}_0)\cdot\nabla\right)^k f\right](\underline{x}_0)}{k!}= \\ =\sum_{k=0}^{3}\frac{\left[\left([-\Delta_1,\Delta_1]\frac{\partial}{\partial x_1}+[-\Delta_2,\Delta_2]\frac{\partial}{\partial x_2}\right)^k f\right](x_{10},x_{20})}{k!}= \\ =\sum_{k=0}^{3}\frac{\left[(\underline{\Delta Y}\cdot\nabla)^k f\right](\underline{x}_0)}{k!} \tag{12}$$

with $\underline{\Delta Y}=(\Delta_1Y_1,\Delta_2Y_2)$ and Y_i=[-1,1]$\in I\Re$ $\forall i$=1,2.

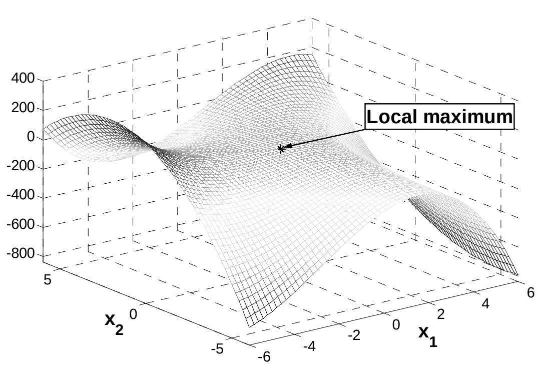

Fig. 6. A bi-variate PF.

Due to the IA properties, the previous computed $F_{ITE}(\underline{X})$ contains the PF in equation (10) $\forall\underline{x}\in\underline{X}=\left([x_{10}-\Delta_1,x_{10}+\Delta_1],[x_{20}-\Delta_2,x_{20}+\Delta_2]\right)$, $f(\underline{x})\in F_{ITE}(\underline{X})$, i.e. it is an overbounding of equation (10) in presence of a Δ_i variation around the nominal parameter solution x_{0i}, $\forall i$=1,2.

By using the binomial coefficients to express the power of a sum, the equation (12) leads to the following expression,

$$F_{ITE}(\underline{X})= \\ =\sum_{k=0}^{3}\frac{\sum_{h=0}^{k}\binom{k}{h}\Delta_1^{\,k-h}\Delta_2^{\,h}\frac{\partial^{k-h}}{\partial x_1^{\,k-h}}\left(\frac{\partial^h f(x_{10},x_{20})}{\partial x_2^{\,h}}\right)Y_1^{\,k-h}Y_2^{\,h}}{k!}= \\ =\sum_{k=0}^{3}\sum_{h=0}^{k}\alpha_{k,h}Y_1^{\,k-h}Y_2^{\,h} \tag{13}$$

where $\alpha_{k,h}=\frac{\Delta_1^{\,k-h}\Delta_2^{\,h}}{h!(k-h)!}\frac{\partial^{k-h}}{\partial x_1^{\,k-h}}\left(\frac{\partial^h f(x_{10},x_{20})}{\partial x_2^{\,h}}\right)$ is a real number and $Y_1=Y_2$=[-1,1] are constant intervals.

The equation (13) can be useful to understand the resulting analytic expression of its width, given by equation (3). In fact, for n=3 the width of $F_{ITE}(X)$ can be expressed as follows,

$$w(F_T(X_1,X_2))=2|\alpha_{10}|+2|\alpha_{11}|+|\alpha_{20}|+2|\alpha_{21}|+ \\ +|\alpha_{22}|+2|\alpha_{30}|+2|\alpha_{31}|+2|\alpha_{32}|+2|\alpha_{33}| \tag{14}$$

The equation (14) is a positive non-differentiable function with potential minima in $\alpha_{k,h}$=0. In particular, by considering k=1 and h=0,1, the following system must be solved,

$$\begin{cases}\alpha_{10}=0\\ \alpha_{11}=0\end{cases}. \tag{15}$$

The system in equation (15) corresponds to cancelling the gradient of f($\underline{x}$),

$$\begin{cases}\delta_1\dfrac{\partial f(x_1,x_2)}{\partial x_1}=0\\ \delta_2\dfrac{\partial f(x_1,x_2)}{\partial x_2}=0\end{cases}\Rightarrow\begin{cases}\dfrac{\partial f(x_1,x_2)}{\partial x_1}=0\\ \dfrac{\partial f(x_1,x_2)}{\partial x_2}=0\end{cases} \tag{16}$$

and to find a stationary point of $f(\underline{x})$, if the Hessian matrix eigenvalues have equal sign [9], such occur in the local maximum in Fig. 6. In fact, if we look at the Fig. 7. where the WITE obtained for $\Delta_1=\Delta_2=1$ is depicted, it is possible to verify that the width of $F_{ITE}(X)$ is minimum just in [0,0] and it corresponds to the MRSS.

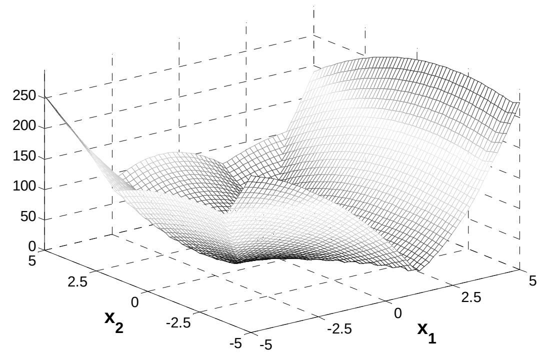

Fig. 7. WITE for the PF of the application II.

Furthermore, if the contour plot of WITE is kept in to account (Fig. 8), we can obtain additional information about the behaviour of the PF without computational efforts.

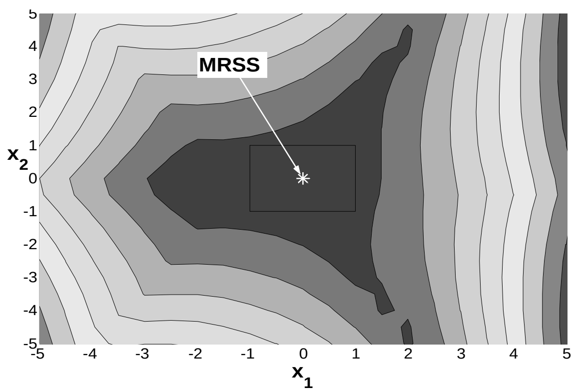

Fig. 8. WITE contour plot for the PF of the application II and the MRSS (*) in the variability box (rectangle).

In fact it is possible to highlight the flat region around the stationary point of $f(\underline{x})$ by considering the wide equipotent area around the MRSS. Such an information can be give to the designer a degree of freedom for his choice, that can adopt, for example, a wider tolerances on the nominal parameter to perform a lower cost design or that can pick up a nominal solution between the commercial value that are in the equipotent area of robustness.

IV. ROBUST DESIGN OF THE MAX OSCILLATION FREQUENCY OF A DHBT

In order to apply the proposed method to a real design problem we consider the max oscillation frequency of a Double-Heterojunction Bipolar Transistors (DHBT). Such devices have been proposed for microwave power applications (up to 20GHz), e.g., in airborne radars or mobile phones, because of their high output power, and superior power efficiency with respect to Single Heterojunction Bipolar Transistors (SHBT) [1] and [10].

The second Heterojunction between base and collector, which is added in DHBT in order to increase the common-emitter breakdown voltage, however, perturbs the electron flow across this junction, a problem that can be tackled by means of a GaAs spacer between the base and collector (Fig. 9). In order to examine the critical dependence of the DHBT performances on the physical and geometrical parameters of the spacer and collector, without recurring to lengthy and costly experimental realizations, simple behavioural mathematical models are considered [1]. In particular, polynomial forms interpolating the numerical values obtained from suitable simulation experiments (for specified operating points of the device) are employed to study the variability of relevant performance characteristics as a function of some major factors describing the structure and the processing of the component through a Design of Experiment (DoE) approach [11].

In our study we follow the same approach and focus our attention to the PF represented by the max oscillation frequency f_{max} of the DHBT. Indeed, the procedure can also be extended to other relevant performances, such as common emitter breakdown voltage, max collector current density and static current gain, controlling the electrical and thermal behaviour of the DHBT. However, since the main goal of the present work is to highlight the effectiveness of the interval-based approach, rather than to perform an in-depth design of the DHBT, only the variability of the f_{max} with respect to physical and geometrical characteristics is discussed.

In particular, we use the same interpolating polynomial adopted in [1] and compare our results with those reported there. With reference to Fig. 9, the following expression describes the influence on f_{max} of the impurity concentration (x_1) and the thickness of the base–collector spacer (x_2), the impurity concentration (x_3) and thickness of the collector (x_4),

$$f_{\max} = 66.42 + 0.3823x_2 + 6.334x_3 - 10.95x_4 + \\ -1.181x_1^{\,2} - 1.515x_2^{\,2} + 6.487x_3x_4 \qquad (17)$$

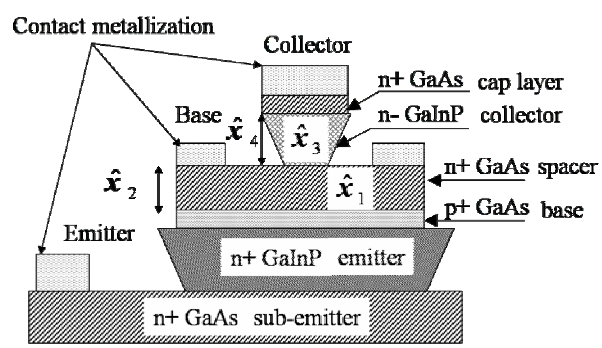

Fig. 9. Schematic setup of a collector-up DHBT [1].

where the variables have been normalized. In Table the adopted intervals of variation of the four actual parameters values $\hat{x}_i$ are reported. As usual in experiment design in presence of non-isodimensional and inhomogeneous factors [12], they are normalized in [-1, 1] by using the coded values x_i, $\forall i=1,\ldots,4$.

Table 1. Interval of variation of the four considered factors.

	values	
	min	**max**
$\hat{x}_1$ [log[cm^{-3}]]	log(5×10^{15})	log(1×10^{18})
$\hat{x}_2$ [nm]	15	60
$\hat{x}_3$ [log[cm^{-3}]]	log(5×10^{15})	log(8×10^{16})
$\hat{x}_4$ [μm]	0.2	1.2
x_i , ∀ i=1,2,3,4	-1	1

By imposing the minimization of the f_{max} variations in presence of an assigned uncertainty on the four design parameters $\underline{\delta} = (\log 2, 30\%, \log 1.5, 15\%)$, the objective function to be analysed is given by,

$$\min\left[\max_{\underline{x}\in D} f_{\max}(\underline{x}) - \min_{\underline{x}\in D} f_{\max}(\underline{x})\right] = \min\left(\Delta f\left(\underline{x}_0, \underline{\delta}\right)\right). \quad (18)$$

It is an optimization problem on a discrete parameter space defined by the set of pairs of the min/max values achieved by the performance function in the hyper-cube whose side length is given by the variations vector $\underline{\delta}$ moving in the hyper-space $D \subseteq \Re^4$ around a nominal solution $\underline{x}_0$. By adopting the ITE approach, the problem can be more easily formulated as an unconstrained optimization problem corresponding to the search of the minimal amplitude of the ITE, $w(F_{ITE}(\underline{X}))$. In particular, the ITE can be expressed as,

$$\begin{aligned} F_{ITE}(\underline{X}) &= \sum_{k=0}^{2} \frac{\left[\left(\left(\underline{X}-\underline{x}_0\right)\cdot\nabla\right)^k f\right]\left(\underline{x}_0\right)}{k!} = f\left(\underline{x}_0\right) + \\ &+ \left[\left(\left(\underline{X}-\underline{x}_0\right)\cdot\nabla\right) f\right]\left(\underline{x}_0\right) + \frac{1}{2}\left[\left(\left(\underline{X}-\underline{x}_0\right)\cdot\nabla\right)^2 f\right]\left(\underline{x}_0\right) \end{aligned} \quad (19)$$

where $\underline{X}-\underline{x}_0 = [-\delta_1,\delta_1],[-x_{20}\delta_2, x_{20}\delta_2],[-\delta_3,\delta_3],[-x_{40}\delta_4, x_{40}\delta_4]$ due to the presence of absolute, x_1 and x_3, and relative, x_2 and x_4, tolerances.

By employing the dependences of the four variables the ITE becomes,

$$\begin{aligned} F_{ITE}(\underline{X}) &= f\left(\underline{x}_0\right) + \delta_1[-1,1] f_{x_1}\left(\underline{x}_0\right) + \\ &+ \delta_2 |x_{20}| [-1,1] f_{x_2}\left(\underline{x}_0\right) + \delta_3[-1,1] f_{x_3}\left(\underline{x}_0\right) + \\ &+ \delta_4 |x_{40}| [-1,1] f_{x_4}\left(\underline{x}_0\right) + 2\delta_1^2 [0,1] f_{x_1 x_1}\left(\underline{x}_0\right) + \\ &+ 2\delta_2^2 x_{20}^2 [0,1] f_{x_2 x_2}\left(\underline{x}_0\right) + 2\delta_3\delta_4 |x_{40}| [-1,1] f_{x_3 x_4}\left(\underline{x}_0\right) \end{aligned} \quad (20)$$

whereas the remaining terms of the expression are null.

The equation (20) gives an over-bounding of the performance function: given the particular nominal solution, $\underline{x}_0$ and by assigning, through the vector $\underline{\delta}$, the variation that each parameter can assume, the performance function will be certainly included in the range of values defined by the particular interval $F_{ITE}(\underline{X}_0)$. By using just the over-bounding of the function we have an overestimation of the maximum value of the equation (17) and an over-estimation of its minimum value: their difference can be used to evaluate the robustness of each solution. In such a way we can use the problem of equation (4) to obtain a robust solution. In particular, if we adopt an easy uniform grid of 11 points for each coded parameter in the range [-1, 1] and we evaluate the minimum of the 11^4 ITE amplitudes, we obtain that it is in the nominal solution reported in Table at the ITE column. In such nominal point the max oscillation frequency is f_{max}=74.455 GHz and the equation (20) gives the inclusion of the range of possible values that the performance of equation (17) can assume in presence of the considered $\underline{\delta}$ variation. It is reported in the last row of Table 1. Moreover, by looking at Table 2, we can observe that in this particular solution point the amplitude of the range of the performance function is almost 3.251 GHz. Instead, the Optimal Robust Solution (ORS), reported in the same Table at the column ORS, is achieved by the authors in [1] through an adaptive random search. Indeed, this approach is not easy in the same way and depends on the choice of suitable setting parameters. A nominal value of f_{max}=74.786 GHz is obtained for the PF and an inclusion of the range of its possible values as the interval $F_{ITE}(\underline{X})$=[72.777,76.223] GHz is also achieved. This last interval is easily obtained by evaluating once the equation (20). Therefore, the range of the PF is 3.446 GHz, bigger than that obtained by the ITE solution. Hence, if we adopt the ORS approach the robustness decreases of about 6% with respect to that achieved by the ITE.

Table 2. Actual nominal values and interval of inclusion of the PF for ITE and ORS.

	ITE	**ORS**
$\hat{x}_{10}$ [cm^{-3}]	7.07×10^{16}	5.7×10^{16}
$\hat{x}_{20}$ [nm]	37.5	40
$\hat{x}_{30}$ [cm^{-3}]	4.59×10^{16}	2.5×10^{16}
$\hat{x}_{40}$ [μm]	0.4	0.33
$F_{ITE}(\underline{X}_0)$ [GHz]	[72.593,74.844]	[72.777,76.223]

The ORS solution is more favourable than that based on the ITE approach if the maximum value of the oscillation frequency is the first designer's objective. In fact, in such solution the PF range is larger but it is also shifted toward higher frequency values. The designer must decide which aspect is prevalent for his scope. Actually, the ITE approach furnishes, without particular settings and additional computations dependent on designer's ability, a look-up table indicating the range of the PF and its amplitude in presence of a given parameter variation $\underline{\delta}$ for each considered nominal solution. By using such table the designer can choose a solution rather than another by exploiting at the same time the information concerning the maximization of the PF and the minimization of its variation.

V. CONCLUSIONS AND REMARKS

An interval-based approach to the robust design with applications to a specific performance of a Double Hetero-junction Bipolar Transistor (DHBT) for microwaves applications has been proposed. The considered performance function is the max oscillation frequency of the DHBT, obtained by a regression model from numerical results. The use of the Interval Analysis allows to efficiently implement the worst case approach for determining the minimum variation of the performance in presence of uncertain parameters. The physical and geometrical parameters affecting the performance are considered implicitly uncorrelated and uniformly distributed in an assigned range and therefore all their combinations are kept into account. The robust design is obtained by means of an overestimation of the amplitude of the performance function range. The procedure allows to achieve a greater robustness of the solution without assuming approach-specific settings and performing additional computations dependent on designer's ability. The implemented approach can also be extended to other single relevant performances controlling the electrical and thermal behaviour of the device or employed in a multi-objective optimization problem. This last aspect is now under study and will be dealt with in forthcoming communications.

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