SCATTERING FROM A PERFECT ELECTRIC CONDUCTOR CYLINDER WITH AN INHOMOGENEOUS COATING THICKNESS OF GYROELECTRIC CHIRAL MEDIUM: EXTENDED MODE-MATCHING METHOD

Dajun Cheng and Yahia M. M. Antar
Department of Electrical and Computer Engineering
Royal Military College of Canada
Kingston, Ontario K7K 5L0, Canada

ABSTRACT. Based on the eigenfunction expansion of electromagnetic waves in the gyroelectric chiral medium, an extended mode-matching method is developed to study the electromagnetic scattering of a perfect electric conductor (PEC) circular cylinder with an inhomogeneous coating thickness of gyroelectric chiral medium. Excellent convergence property of the bistatic echo width is numerically verified, which establishes the reliability and applicability of the present extended mode-matching method for two-dimensional problem of gyroelectric chiral medium.

1 INTRODUCTION

The vector wave functions, which were first proposed by Hansen to study the electromagnetic radiation problems [1], are important concepts in electromagnetics. This concept, which has been extensively developed by Stratton [2], Morse and Feshbach [3], and Tai [4] in studying electromagnetic boundary-value problems, seems to gain increasing interest and importance. The vector wave functions have found versatile applications and present great advantages when compared with other methods (e.g., three-dimensional moment method [5], coupled-dipole method [6], and integral-equation technique [7]). However, the vector wave functions in any given complex media need to be developed, in order to provide methodological convenience in studying the electromagnetic properties of these materials. Moreover, to establish the reliability and applicability of the vector wave functions in studying the physical properties of complex media, convergence properties of the series involved must be extensively examined.

Basically, there are five analytical and numerical methods, which are based on the eigenfunction solution of the wave equation, to investigate the electromagnetic phenomena. These include the mode-matching method [8], the perturbation approach [9], the T-matrix method [10], the point-matching method [11], and the multipole technique [12]. Despite of the fact that the mode-matching method can provide rigorous criteria for other numerical methods, it is only applicable to simple boundary-value problems which allow the Helmholtz equation to have a separation of variable-based solution. The perturbation method, which involves a Taylor expansion of the fields on the boundary, requires the smallness of the boundary perturbation so that the higher order terms can be neglected. Although the T-matrix method have been widely employed to study the electromagnetic boundary-value problems of isotropic media, this formulation, derived from the Huygens's principle and extinction theorem, requires that the Green dyadic must be expressible in terms of the eigenfunctions. Furthermore, due to the limited knowledge about the Huygens's principle and extinction theorem in complex material, it is often very difficult to obtain the T-matrix formulation. These constraints on the T-matrix method make it unsuitable to investigate the boundary-value problems of complex materials where we can not obtain the eigenfunction expansion of Green dyadic. (For complex materials, the solution of the source-incorporated problems, which involves the Green dyadic, is obviously much more difficult than the source-free one.) Although the point-matching method is not limited by the same constraint condition as T-matrix method, it is well-known that it is more time-consuming for non-near-circular or -spherical geometry problems. The multipole technique, which requires the knowledge of the eigenfunction expansion of the field distribution of unit source, is difficult to explore for investigating the physical phenomena of complex materials. Considering the applicability scope of the above methods, other computational approaches based on the vector wave functions, which could provide advantages over these already existing methods, still need to be studied so as to provide the methodological convenience in investigating and exploring the physical phenomena involved in complex materials.

With recent advances in polymer synthesis techniques, increasing attention is being attracted to the analysis of interaction of electromagnetic waves with chiral medium in order to determine how to use this class of material to provide better solutions to current engineering problems [13, 14]. Therefore, chirality management, i.e., the management of the effects of chirality , seems to have potential applications in the control of the physical behaviors of devices that are made of chiral material, because the lectromagnetic properties of chiral devices are directly connected with the chiral parameter. However, only limited methods exist for chirality management once the chiral material is created. One exception is through the introduction of certain forms of controllable anisotropy that can be realized either by employing the electro-optic and piezoelectric effects, or by introducing certain forms of externally biased controllable magnetic fields. With chirality management as their motivation for their investigation, Engheta, Jaggard, and Kowarz proposed the concept of a Faraday chiral medium [15] and examined the propagation characteristics of electromagnetic plane waves in an unbounded Faraday chiral medium that blends gyrotropic effects with those of optical activity. Recently, the field representations [16] and the dyadic Green function [17] in a gyroelectric chiral medium have been presented in terms of the cylindrical vector wave functions based on a spectral angular expansion method and Ohm-Rayleigh method, respectively. Nevertheless, much effort is still necessary in order to achieve a thorough understanding of chirality management.

In the present investigation, an extended mode-matching method is developed to study the electromagnetic scattering of a PEC circular cylinder with an inhomogeneous coating thickness of gyroelectric chiral medium, based on the field representations in gyroelectric chiral medium in terms of the cylindrical vector wave functions [16]. The extended mode-matching method, by taking the integration of the azimuthal variable from 0 to 2π, makes the continuity of the tangential fields satisfied at the surface of the scatterer. Excellent convergence of the cylindrical vector wave functions in conjunction with the extended mode-matching method is numerically verified, and this establishes the reliability and applicability of the present method and computation.

In what follows, the harmonic $\exp(-i\omega t)$ time dependence is assumed and suppressed throughout.

2 PRELIMINARY: EIGENFUNCTION EXPANSION OF ELECTROMAGNETIC WAVES

For the sake of completeness and self-consistency, field representations in gyroelectric chiral medium will be briefly presented. From a phenomenological point of view, a homogeneous gyroelectric chiral medium can be characterized by the set of constitutive relations [15-17]:

$$\boldsymbol{D}=[\epsilon]\cdot\boldsymbol{E}+i\xi_c\boldsymbol{B} \tag{1a}$$

$$\boldsymbol{H}=i\xi_c\boldsymbol{E}+\boldsymbol{B}/\mu \tag{1b}$$

where $[\epsilon]=\epsilon_t[\boldsymbol{I}]_t+\epsilon_z\boldsymbol{e}_z\boldsymbol{e}_z+ig[\boldsymbol{I}]_t\times\boldsymbol{e}_z$ is the modified permittivity tensor, taking into account the contributions due to the chirality $[\boldsymbol{I}]_t=\boldsymbol{e}_x\boldsymbol{e}_x+\boldsymbol{e}_y\boldsymbol{e}_y$ denotes the transverse unit dyadic, and $\boldsymbol{e}_j$ denotes the unit vector in the j direction. Based on the concept of characteristic waves and the method of angular spectral expansion [18], it has been shown [16] that the electromagnetic waves in gyroelectric chiral medium can be represented in terms of the cylindrical vector wave functions as

$$\boldsymbol{E}(\boldsymbol{r})=\pi\sum_{q=1}^{2}\int_{-\infty}^{\infty}dk_z\sum_{n=-\infty}^{\infty}i^n e_{qn}(k_z)\cdot[A_q^e(k_z)\boldsymbol{M}_n^{(1)}(k_z,k_{\rho q})+B_q^e(k_z)\boldsymbol{N}_n^{(1)}(k_z,k_{\rho q})+C_q^e(k_z)\boldsymbol{L}_n^{(1)}(k_z,k_{\rho q})] \tag{2a}$$

$$\boldsymbol{H}(\boldsymbol{r})=\pi\sum_{q=1}^{2}\int_{-\infty}^{\infty}dk_z\sum_{n=-\infty}^{\infty}i^n e_{qn}(k_z)\cdot[A_q^h(k_z)\boldsymbol{M}_n^{(1)}(k_z,k_{\rho q})+B_q^h(k_z)\boldsymbol{N}_n^{(1)}(k_z,k_{\rho q})+C_q^h(k_z)\boldsymbol{L}_n^{(1)}(k_z,k_{\rho q})] \tag{2b}$$

where the expansion coefficients $A_q^e(k_z)$, $B_q^e(k_z)$, $C_q^e(k_z)$ have been explicitly given in [16]. Here, $k_{\rho q}(q=1,2)$ obeys the characteristic equation presented in [16]. The explicit expressions $A_q^h(k_z)$, $B_q^h(k_z)$, $C_q^h(k_z)$ can be easily obtained from Maxwell's equations and (2a).

Since Bessel, Neumann, and Hankel functions of the same order satisfy identical differential equation, the first kind of vector wave functions in equations (2a) and (2b) can be generalized to the three other sets, corresponding to Neumann and Hankel functions.

3 EXTENDED MODE-MATCHING METHOD AND NUMERICAL RESULTS

In this section, we will develop the extended mode-matching method to study the electromagnetic scattering of a PEC circular cylinder with an arbitrary coating thickness of gyroelectric chiral medium. For simplicity, we first fix the coordinate system so that the incident wave is along the +x axis, the PEC rod is bounded by the surface of $\rho=a$, and the outer surface of the coating is bounded by $\rho=f(\phi)$, where $f'(\phi)$ is a single value and continuous function of ϕ. Here, the coated PEC cylinder lies along the z axis.

Since an arbitrary polarized electromagnetic wave in free space can be decomposed into TM_z and TE_z polarized waves which are independent and dual with each other, we will only consider the TM_z incident case without losing any generality. The incident TM_z wave with unit amplitude of electric field is expanded in terms of the circular cylindrical vector wave functions as

$$\boldsymbol{E}^{inc}(\boldsymbol{r})=\boldsymbol{e}_z e^{ik_0 x} = \int_{-\infty}^{\infty} dk_z \sum_{n=-\infty}^{\infty} i^n \delta(k_z) \boldsymbol{N}_n^{(1)}(k_z,k_\rho)/k_0 \tag{3a}$$

$$\boldsymbol{H}^{inc}(\boldsymbol{r})=i\boldsymbol{e}_y e^{ik_0 x}/\eta_0 = \int_{-\infty}^{\infty} dk_z \sum_{n=-\infty}^{\infty} i^{n-1} \delta(k_z) \boldsymbol{M}_n^{(1)}(k_z,k_\rho)/(k_0\eta_0) \tag{3b}$$

where $k_\rho=(k_0^2-k_z^2)^{1/2}$, and $\delta(\cdot)$ is the Dirac delta function. Here, $k_0=\omega(\varepsilon_0\mu_0)^{1/2}$ and $\eta_0=(\mu_0/\varepsilon_0)^{1/2}$ represent the wave number and wave impedance of free space, respectively. The scattered electromagnetic waves may have TM_z and TE_z components and should be expanded as

$$\boldsymbol{E}^{sca}(\boldsymbol{r})=\int_{-\infty}^{\infty} dk_z \sum_{n=-\infty}^{\infty} i^n [a_n \boldsymbol{M}_n^{(3)}(k_z,k_\rho) + b_n \boldsymbol{N}_n^{(3)}(k_z,k_\rho)] \tag{4a}$$

$$\boldsymbol{H}^{sca}(\boldsymbol{r})=\int_{-\infty}^{\infty} dk_z \sum_{n=-\infty}^{\infty} i^{n-1}/\eta_0 \cdot [a_n \boldsymbol{N}_n^{(3)}(k_z,k_\rho) + b_n \boldsymbol{M}_n^{(3)}(k_z,k_\rho)] \tag{4b}$$

According to the cylindrical vector wave functions in the gyroelectric chiral medium as introduced in the previous section, the fields in the coating region can be represented as

$$\boldsymbol{E}^{int}=\sum_{q=1}^{2}\sum_{n=-\infty}^{\infty} i^n \{e_{qn}^{(1)}[A_q^e \boldsymbol{M}_n^{(1)}(k_{\rho q}) + B_q^e \boldsymbol{N}_n^{(1)}(k_{\rho q}) + C_q^e \boldsymbol{L}_n^{(1)}(k_{\rho q})] + e_{qn}^{(3)}[A_q^e \boldsymbol{M}_n^{(3)}(k_{\rho q}) + B_q^e \boldsymbol{N}_n^{(3)}(k_{\rho q}) + C_q^e \boldsymbol{L}_n^{(3)}(k_{\rho q})]\} \tag{5a}$$

$$\boldsymbol{H}^{int}=\sum_{q=1}^{2}\sum_{n=-\infty}^{\infty} i^n \{e_{qn}^{(1)}[A_q^h \boldsymbol{M}_n^{(1)}(k_{\rho q}) + B_q^h \boldsymbol{N}_n^{(1)}(k_{\rho q}) + C_q^h \boldsymbol{L}_n^{(1)}(k_{\rho q})] + e_{qn}^{(3)}[A_q^h \boldsymbol{M}_n^{(3)}(k_{\rho q}) + B_q^h \boldsymbol{N}_n^{(3)}(k_{\rho q}) + C_q^h \boldsymbol{L}_n^{(3)}(k_{\rho q})]\} \tag{5b}$$

where $e_{qn}^{(1)}$ and $e_{qn}^{(3)}$ (q= 1, 2) are expansion coefficients. Here, $k_z=0$ for the cylindrical vector wave functions and their weighted coefficients have been suppressed for the sake of writing simplicity.

The boundary conditions at the boundary $\rho=a$ are

$$E_z^{int}|_{\rho=a}=0 \tag{6a}$$

$$E_\phi^{int}|_{\rho=a}=0 \tag{6b}$$

And the boundary conditions at outer surface of the scatterer $\rho=f(\phi)$ are

$$H_\rho^{int}\sin\theta + H_\phi^{int}\cos\theta = (H_\rho^{inc}+H_\rho^{sca})\sin\theta + (H_\phi^{inc}+H_\phi^{sca})\cos\theta \tag{7a}$$

$$H_z^{int}=H_z^{inc}+H_z^{sca} \tag{7b}$$

$$E_z^{int}=E_z^{inc}+E_z^{sca} \tag{7c}$$

$$E_\rho^{int}\sin\theta + E_\phi^{int}\cos\theta = (E_\rho^{inc}+E_\rho^{sca})\sin\theta + (E_\phi^{inc}+E_\phi^{sca})\cos\theta \tag{7d}$$

where all the field components are evaluated at $\rho=f(\phi)$, and

$$\theta=\tan^{-1}\left(\frac{f'(\phi)}{f(\phi)}\right)$$

To numerically solve Eqs. (6) and (7), the infinite series involved must be truncated. In what follows, the infinite summation is truncated such that the series is taken to be summed up from -N to N. These truncated equations can be easily solved analytically for the expansion coefficients of the scattered fields. For the sake of consistency, detail for the formulations of the solution procedure are organized in the Appendix. After carefully examining the final numerical results, excellent convergent properties of these truncated series are established, which make the truncating process reasonable.

The bistatic echo width, which represents the density of the power scattered by the cylindrical object, is defined as

$$A_\sigma(\phi)=\lim_{\rho\to\infty}2\pi\rho\frac{Re\{[\boldsymbol{E}^{sca}(\boldsymbol{r})]\times[\boldsymbol{H}^{sca}(\boldsymbol{r})]^*\}\cdot\boldsymbol{e}_\rho}{Re\{[\boldsymbol{E}^{inc}(\boldsymbol{r})]\times[\boldsymbol{H}^{inc}(\boldsymbol{r})]^*\}\cdot\boldsymbol{e}_x} \qquad (8)$$

where the asterisk indicates complex conjugate, and $Re[.]$ denotes real part of the complex function. Using the asymptotic expression of the Hankel function in the far region, the expression (8) can be rewritten in a more explicit form

$$A_\sigma(\phi)=4k_0[|\sum_{n=-\infty}^{\infty}a_n e^{in\phi}|^2+|\sum_{n=-\infty}^{\infty}b_n e^{in\phi}|^2] \qquad (9)$$

To validate this extended mode-matching process, numerical results of the present method for the solution of a PEC circular cylinder with a coaxial coating of gyroelectric chiral medium and those calculated by the conventional mode-matching method are illustrated in Figure 1. Excellent agreement between the results is obtained.

Before actual computation for the scattering of a PEC circular cylinder with an inhomogeneous coating thickness of gyroelectric chiral medium, convergence test of the results involved should be carefully examined. It was found from many computed numerical results that for a properly truncated number N, reliable results can be obtained for various incident and scattering angle. Table I typically presents the numerical results for the convergence test for a PEC circular cylinder with a gyroelectric chiral coating of elliptical cross section. The convergence check indicates that the present extended mode-matching method can be reliably utilized to investigate the two-dimensional boundary-value problem of multilayered gyroelectric chiral objects. To provide criteria for other numerical method, Figure 2 illustrates the bistatic echo width of a PEC circular cylinder with a gyroelectric chiral coating of elliptical cross section.

It is observed that for the case where the scattering structure is symmetric with respect to the x axis (i.e., the major axis of the coating is along the x axis or y axis), the bistatic echo width of this structure is not symmetric with respect to the x axis, similar to the case of a gyromagnetic circular cylinder [19-21]. Straightforward mathematical analysis of the present formulation reveals that this asymmetric phenomenon of the bistatic echo width is attributed to the existence of the irrotational vector wave functions in the eigenfunction expansion of the electromagnetic waves inside the coating of the gyroelectric chiral medium, i.e., $C_q^p(k_z=0)\neq0$, for =1 or 2, and p=e or h.

For the case of a TE_z incident plane wave, numerical solution to the scattering by the coated PEC cylindrical structure can be similarly implemented following the line described previously.

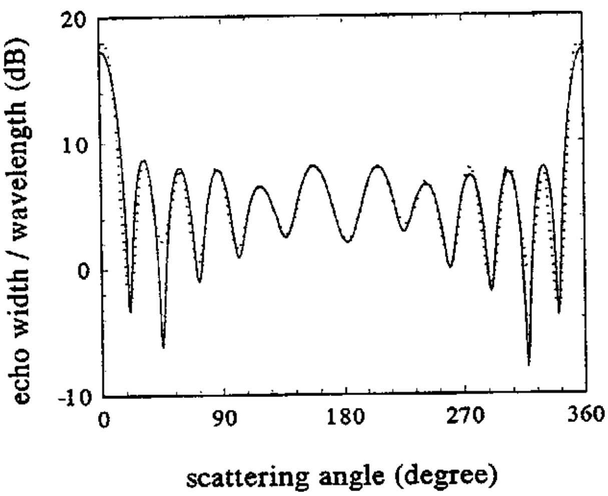

Figure 1 Validation of the extended mode-matching method for scattering by a PEC circular cylinder with a coaxial coating of gyroelectric chiral medium. The radius of the PEC circular cylinder is $0.8\lambda_0$, and the outer surface of the structure is $1.1\lambda_0$. The coating material has the parameter values of $\varepsilon/\varepsilon_0=2.5$, $\varepsilon_z/\varepsilon_0=2.1$, $g/\varepsilon_0=0.12$, $\mu/\mu_0=1.1$, and $\xi_c=0.002\Omega^{-1}$. The solid and dashed lines correspond to the numerical results computed by the extended mode-matching method and the conventional mode-matching method [16], respectively.

Table I Convergence test for the electromagnetic scattering by a PEC circular cylinder with a coating of elliptical cross section of gyroelectric chiral medium, due to a normally incident TM_z polarized plane wave. The constitutive parameters of the coating are taken to be the same as those in Figure 1. The elliptical-shaped coating has a semi major axis of $1.4\lambda_0$, and semi minor axis $1.1\lambda_0$. The radius of the conducting cone is $0.8\lambda_0$. The major axis of the elliptically-shaped cross section of the coating takes an angle of $2\pi/9$ with respect to the x axis.

$A_\sigma/\lambda_0\,(dB)$	ϕ=0°	ϕ=45°	ϕ=90°	ϕ=135°	ϕ=180°
N=9	.15875D+02	.16907D+01	.26482D+01	.44871D+01	.27669D+01
N=12	.15782D+02	.20219D+01	.23953D+01	.39844D+01	.30772D+01
N=15	.15776D+02	.20310D+01	.24316D+01	.40921D+01	.31177D+01
N=18	.15779D+02	.20360D+01	.24111D+01	.41045D+01	.31128D+01
N=20	.15779D+02	.20360D+01	.24111D+01	.41045D+01	.31128D+01

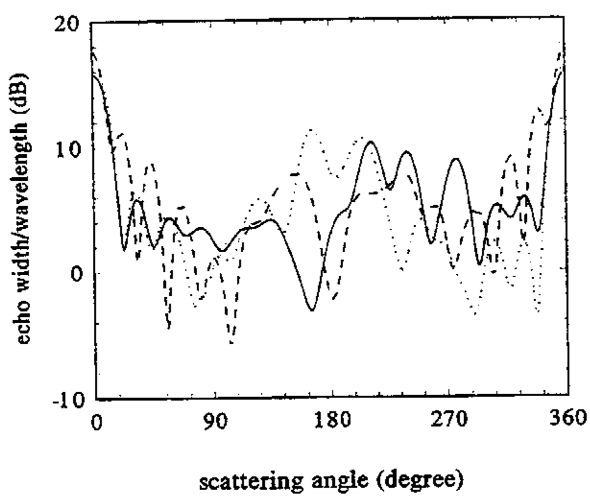

Figure 2 Scattering pattern of a PEC circular cylinder with a coating of elliptical cross section, due to a normally incident TM_z polarized plane wave. The geometrical and constitutive parameters of the gyroelectric chiral coating are taken to be as the same as those of Table I. The dotted line corresponds to the case where the major axis of the coating is along the x axis, the dashed line to the case where the major axis is along the y axis, and the solid line to the case where the major axis takes an angle of $2\pi/9$ with respect to the x axis.

4 CONCLUDING REMARKS

So far there is a gap in associating the microscropic structure parameters with the macroscopic medium parameters and the present work does not address this point but simply assumes the medium parameters known. However, a gyroelectric chiral medium could be manufactured in the ways as described in Ref. 15-17 in detail, and the parameter values for the calculation example are typically chosen without sacrificing both generality and the purpose of the present study. In fact, such a medium having the assumed parameter values can be produced by properly embedding helix elements in a gyroelectric medium on the condition that $\mu_0|\xi_c|^2/\varepsilon_0<|\mu/\mu_0|^2+|\min(\varepsilon/\varepsilon_0,\varepsilon_z/\varepsilon_0)|^2$.

In the present investigation, based on the cylindrical vector wave functions, an extended mode-matching method is proposed to study the two-dimensional electromagnetic scattering of an infinitely-long PEC circular cylinder with an inhomogeneous coating thickness of gyroelectric chiral medium. Excellent convergence property of the echo width of this scattering structure is numerically verified, which establishes the reliability and applicability of the present formulation and method.

It should be pointed out that the extended mode-matching method presented here, although does not require the eigenfunction expansion of Green dyadic, is only applicable to small-aspect-ratio scatterers as is the conventional T-matrix method. For a large-aspect-ratio scatterer, the time it takes to compute convergence is prohibitive. However, this method is obviously superior to the conventional mode-matching method which can only be applicable to circular cylindrical structure [8], the perturbation method [9] which is only suitable to near-circular cylindrical structure, and the T-matrix method [10] and multipole technique [12], both of which require the knowledge of a source-incorporated solution, and may be regarded as the modified form of point-matching method [11], since they can be utilized under the same conditions and have the same challenge for complex structure. Using the addition theorem of the cylindrical vector wave functions and the formulations for a single scatterer, a homogenization theory for the gyroelectric chiral composite media would be established, where the pair distributed function [20] can result from experiment, theoretical investigation, or numerical results.

It is of interest to note that the cylindrical vector wave functions can be expanded as discrete sums of the spherical vector wave functions [21], therefore the strategy of the present extended mode-matching method may be extended to solve the problems of three-dimensional finite-domain structures of the gyroelectric chiral medium. Although excellent convergence and efficiency of the present cylindrical vector wave functions in tackling the two-dimensional physical phenomena have been demonstrated by numerical computation, one should carefully examine the convergence and efficiency of the theory in actual computation when using the spherical vector wave functions to study three-dimensional electromagnetic phenomena involving the gyroelectric chiral medium. Nevertheless, it is believed that the cylindrical vector wave functions and the extended mode-matching method would be helpful in analyzing, interpreting and exploiting the physical phenomena associated with the boundary-value problems of layered structures as well as multi-scatterers consisting of gyroelectric chiral media.

5 APPENDIX

For simplicity, we introduce

$$Y_{qn}^{p(j)}(\rho)=\frac{in}{\rho}Z_n^{(j)}(k_{\rho q}\rho)A_q^p(k_z=0) + k_{\rho q}Z_n^{(j)'}(k_{\rho q}\rho)C_q^p(k_z=0) \qquad (A.1)$$

$$X_{qn}^{p(j)}(\rho)=k_{\rho q}Z_n^{(j)}(k_{\rho q}\rho)B_q^p(k_z=0) \qquad (A.2)$$

$$V_{qn}^{p(j)}(\rho)=\frac{in}{\rho}Z_n^{(j)}(k_{\rho q}\rho)C_q^p(k_z=0) - k_{\rho q}Z_n^{(j)'}(k_{\rho q}\rho)A_q^p(k_z=0) \qquad (A.3)$$

where $\rho=f(\phi)$ or $\rho=a$, q=1 or 2, p=e or h, and $Z_n^{(j)}(\cdot)=J_n(\cdot)$ or $H_n^{(2)}(\cdot)$ for j=1, 4, respectively.

From the boundary conditions Eqs. (6a) and (6b) at $\rho=a$, we have

$$[e_1^{(3)}]=[A^{11}][e_1^{(1)}]+[A^{12}][e_2^{(1)}] \qquad (A.4)$$

$$[e_2^{(3)}]=[A^{21}][e_1^{(1)}]+[A^{22}][e_2^{(1)}] \qquad (A.5)$$

where $[e_q^{(j)}]$ (for j=1 or 3, q= 1 or 2) are column vectors of the expansion coefficients for the internal fields of the coating, and

$$[A^{11}]=[[X^{2(3)}]^{-1}[X^{1(3)}]-[V^{2(3)}]^{-1}[V^{1(3)}]]^{-1} \cdot[[V^{2(3)}]^{-1}[V^{1(1)}]-[X^{2(3)}]^{-1}[X^{1(1)}]]$$

$$[A^{12}]=[[X^{2(3)}]^{-1}[X^{1(3)}]-[V^{2(3)}]^{-1}[V^{1(3)}]]^{-1} \cdot[[V^{2(3)}]^{-1}[V^{2(1)}]-[X^{2(3)}]^{-1}[X^{2(1)}]] \qquad (A.7)$$

$$[A^{21}]=[[X^{1(3)}]^{-1}[X^{2(3)}]-[V^{1(3)}]^{-1}[V^{2(3)}]]^{-1} \cdot[[V^{1(3)}]^{-1}[V^{1(1)}]-[X^{1(3)}]^{-1}[X^{1(1)}]] \qquad (A.8)$$

$$[A^{22}]=[[X^{1(3)}]^{-1}[X^{2(3)}]-[V^{1(3)}]^{-1}[V^{2(3)}]]^{-1} \cdot[[V^{1(3)}]^{-1}[V^{2(1)}]-[X^{1(3)}]^{-1}[X^{2(1)}]] \qquad (A.9)$$

and the matrices involved in Eqs. (A.6)-(A.9) are all diagonal with the diagonal elements given as

$$(V_p^{q(j)})_{mn}=V_{qn'}^{p(j)}(\rho=a)\delta(m'-n') \qquad (A.10)$$

$$(X_p^{q(j)})_{mn}=X_{qn'}^{p(j)}(\rho=a)\delta(m'-n') \qquad (A.11)$$

for $m,n\in[1, 2N+1]$ and $m'=m-(N+1)$, $n'=n-(N+1)$.

After substituting Eqs. (A.4) and (A.5) into Eqs. (7a-7d), multiplying both sides of the resulting equations with $e^{-im\phi}(m=-N,-N+1,...,N-1,N)$ and integrating with respect to ϕ from 0 to 2π, we end up with

$$\sum_{q=1}^{2}[I^{qe}][e_q^{(1)}]=[I^{(2)}][a] \qquad (A.12)$$

$$\sum_{q=1}^{2}[A^{qh}][e_q^{(1)}]=\frac{k_0}{i\omega\mu_0}[I^{(5)}][a] \qquad (A.13)$$

$$\sum_{q=1}^{2}[A^{qe}][e_q^{(1)}]-[I^{(5)}][b]=[Q][I] \tag{A.14}$$

$$\sum_{q=1}^{2}[I^{qh}][e_q^{(1)}]-\frac{k_0}{i\omega\mu_0}[I^{(2)}][b]=[P][I] \tag{A.15}$$

where $[a]$ and $[b]$ are column vectors of the expansion coefficients of the scattered waves, respectively, $[I]$ is the unit vector, and

$$[A^{qp}]=[C_p^{q(1)}]+[C_p^{1(3)}][A^{1q}]+[C_p^{2(3)}][A^{2q}] \tag{A.16}$$

$$[I^{qp}]=[B_p^{q(1)}]+[B_p^{1(3)}][A^{1q}]+[B_p^{2(3)}][A^{2q}] \tag{A.17}$$

Here, the elements of the matrices involved are given as

$$(I^{(5)})_{mn}=\int_{\phi=0}^{2\pi} i^{n'}e^{i(n'-m')\phi}k_0H_{n'}^{(1)}(k_0\rho)d\phi \tag{A.18}$$

$$(P)_{nm}=\frac{1}{i\omega\mu_0}\int_{\phi=0}^{2\pi} i^{n'}e^{i(n'-m')\phi}\cdot[\frac{in'}{\rho}J_{n'}(k_0\rho)\sin\theta-k_0J_{n'}'(k_0\rho)\cos\theta]d\phi \tag{A.19}$$

$$(Q)_{nm}=\int_{\phi=0}^{2\pi} i^{n'}e^{i(n'-m')\phi}J_{n'}(k_0\rho)d\phi \tag{A.20}$$

$$(B_p^{q(j)})_{mn}=\int_{\phi=0}^{2\pi} i^{n'}e^{i(n'-m')\phi}\cdot[Y_{qn'}^{p(j)}(\rho)\sin\theta+V_{qn'}^{p(j)}(\rho)\cos\theta]d\phi \tag{A.21}$$

$$(I^{(2)})_{mn}=\int_{\phi=0}^{2\pi} i^{n'}e^{i(n'-m')\phi}\cdot[\frac{in'}{\rho}H_{n'}^{(1)}(k_0\rho)\sin\theta-k_0H_{n'}^{(1)'}(k_0\rho)\cos\theta]d\phi \tag{A.22}$$

$$(C_p^{q(j)})_{mn}=\int_{\phi=0}^{2\pi} i^{n'}e^{i(n'-m')\phi}X_{qn'}^{p(j)}(\rho)d\phi \tag{A.23}$$

for q=1 or 2, and p=e or h.

Straightforward algebraic manipulation for (A.12)-(A.15) yields the column vector of the expansion coefficients of the scattered waves

$$[a]=[\frac{i\omega\mu_0}{k_0}[I^{(2)}]^{-1}[D^{(1)}]-[I^{(5)}]^{-1}[D^{(2)}]]^{-1}\cdot[\frac{i\omega\mu_0}{k_0}[I^{(2)}]^{-1}[P]-[I^{(5)}]^{-1}[Q]] \tag{A.24}$$

$$[b]=[\frac{ik_0}{\omega\mu_0}[D^{(1)}]^{-1}[I^{(2)}]+[D^{(2)}]^{-1}[I^{(5)}]]^{-1}\cdot[[D^{(1)}]^{-1}[P]-[D^{(2)}]^{-1}[Q]] \tag{A.25}$$

where

$$[D^{(2)}]=[A^{1e}][C^{(2)}]+[A^{2h}][C^{(1)}] \tag{A.26}$$

$$[D^{(1)}]=[I^{1h}][C^{(2)}]+[I^{2h}][C^{(1)}] \tag{A.27}$$

and

$$[C^{(2)}]=[[I^{2e}]^{-1}[I^{1e}]-[A^{2h}]^{-1}[A^{1h}]]^{-1}\cdot[[I^{2e}]^{-1}[I^{(2)}]-\frac{k_0}{i\omega\mu_0}[A^{2h}]^{-1}[I^{(5)}]] \tag{A.28}$$

$$[C^{(1)}]=[[I^{1e}]^{-1}[I^{2e}]-[A^{1h}]^{-1}[A^{2h}]]^{-1}\cdot[[I^{1e}]^{-1}[I^{(2)}]-\frac{k_0}{i\omega\mu_0}[A^{1h}]^{-1}[I^{(5)}]] \tag{A.29}$$

Equations (A.24) and (A.25) give the complete solutions of the expansion coefficients of the scattered waves (4a) and (4b), which make the bistatic echo width expression (9) easily evaluated.

6 REFERENCES

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