Two-Step Preconditioner of Multilevel Simple Sparse Method for Electromagnetic Scattering Problems

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Abstract **─** In order to efficiently solve the dense complex linear systems arising from electric field integral equations (EFIE) formulation of electromagnetic scattering problems, the multilevel simple sparse method (MLSSM) is used to accelerate the matrix-vector product operations. Because of the nature of EFIE, the resulting linear systems from EFIE formulation are challenging to solve by iterative methods. In this paper, the two-step preconditioner is used to alleviate the low convergence of Krylov subspace solvers, which combine the modified complex shifted preconditioner and sparse approximate inversion (SAI) preconditioner. Numerical examples demonstrate that the two-step preconditioner can greatly improve the convergence of the generalized minimal residual method (GMRES) for the dense complex linear systems and reduce the computationnal time significantly.

Index Terms **─** Electromagnetic (EM), generalized minimal residual method (GMRES), multilevel simple sparse method (MLSSM), preconditioner.

I. INTRODUCTION

The method of moments (MoM) [1-4] has found widespread application in a variety of electromagnetic problems. The resulting linear systems associated with the discretization of the electric field integral equation (EFIE) are large and dense for electrically large objects in electromagnetic scattering problems. It is basically impractical to solve the EFIE matrix equation using the direct method due to the memory usages of O(N^2) and computational complexity of O(N^3), where N is the number of unknowns. Making such solutions prohibitively expensive for large-scale problems, this difficulty can be circumvented by use of iterative solvers. One of the most popular techniques is the multilevel fast multipole algorithm (MLFMA) [5-6], which has complexity for a given accuracy. The MLFMA has been widely used in recent years to deal with electrically large problems due to its excellent computational efficiency. The multilevel fast far-field algorithm [7] was proposed by L. Rossi in 2000. The adaptive cross approximation method (ACA) is another fast iterative solution algorithm, which was proposed in 2000 by Bebendorf [8]. Similarly, the multilevel UV method has been successfully used to analyze the scattering from rough surfaces [9], propagation over terrain and urban environments, volume scattering from discrete scatterers. As opposed to MLFMA, the ACA and multilevel UV algorithm are purely algebraic and, therefore, do not depend on the Green's function.

The multilevel simple sparse method (MLSSM) was proposed by Canning and Adams [10-11]. Initially, the MLSSM is used to represent the impedance matrix sparsely in fast direct solution [12-14]. Based on the MLSSM, the iterative solution is introduced in [15] for EFIE formulation. It is well-known that EFIE provides a first-kind integral equation, which is ill-conditioned and gives rise to linear systems that are challenging to solve by iterative methods. Although using combined field integral equation (CFIE) can alleviate this difficulty [16], it is not suitable for an object with opened structure. Therefore, it is natural to use preconditioning techniques to improve the condition number of the system and accelerate the convergence rate of iterative solvers before iteration.

There are some simple preconditioners such as the diagonal or diagonal blocks of the coefficient

Submitted On: Nov 10, 2011
Accepted On: Dec 18, 2011

matrix, which can be effective only when the matrix has some degrees of diagonal dominance. Preconditioners based on incomplete LU factorizations have been successfully used on hybrid integral formulations [17], but they are sensitive to indefiniteness in the EFIE matrix, which leads to unstable triangular solvers and very poor preconditioners. Based on the hierarchical matrix, the H-LU preconditioner has been proposed in [18]. The SAI preconditioners are generally less prone to instabilities on indefinite systems [19-21], and outperform more classical approaches such as incomplete LU factorizations. The spectral multigrid and two-step preconditioner are proposed in [22-25], which seem to be efficient to electromagnetic problems.

In order to accelerate the convergence rate of fast iterative solvers of MLSSM, the two-step preconditioner is used in this paper. Firstly, the SAI preconditioner is adopted in the two-step preconditioner. Secondly, the modified complex shifted preconditioner is used to shift the smallest eigenvalues close to one. Numerical experiments demonstrate that the two-step preconditioner of MLSSM is more efficient than SAI preconditioner. Especially, the two-step preconditioner is suitable for multiple right hand vectors problem.

This paper is structured as follows: In Section II, the theory of the MLSSM is outlined. Then, the two-step preconditioner is presented in detail for the efficient solution of the dense linear system in Section III. In Section IV, some numerical results are presented to demonstrate the performance of the two-step preconditioner. Finally, conclusions are presented in Section V.

II. MULTILEVEL SIMPLE SPARSE METHOD OF EFIE

Consider a 3-D arbitrarily shaped perfectly electrically conducting (PEC) object immersed in a medium characterized by permittivity ε and permeability μ. The object is illuminated by an incident wave $\mathbf{E}^i$ that induces current $\mathbf{J}_s$ on the surface *S*. The current $\mathbf{J}_s$ satisfies the electric field integral equation:

$$jk\eta\hat{t}\cdot\int_S \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_s(\mathbf{r}')dS' = \hat{t}\cdot\mathbf{E}^i(\mathbf{r}) \, , \qquad (1)$$

where

$$\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \left[\bar{\mathbf{I}} + \frac{1}{k^2}\nabla\nabla\right]\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \, . \qquad (2)$$

The discretization of EFIE with MoM using planar Rao-Wilton-Glisson (RWG) basis functions for surface modeling is presented in [1]. The surface of the object is usually meshed with one-tenth of the wavelength for accuracy. The resulting linear systems from EFIE formulation after Galerkin's testing are briefly outlined as follows:

$$\sum_{n=1}^{N} Z_{mn} J_n = V_m, \qquad m = 1,2,...,N \, , \qquad (3)$$

where

$$Z_{mn} = \frac{jk}{4\pi}\Big(\iint_{S_m}\Lambda_m(\mathbf{r})\cdot\iint_{S_n}G(\mathbf{r},\mathbf{r}')\Lambda_n(\mathbf{r}')dS'dS - \frac{1}{k^2}\iint_{S_m}\nabla\bullet\Lambda_m(\mathbf{r})\cdot\iint_{S_n}G(\mathbf{r},\mathbf{r}')\nabla\bullet\Lambda_n(\mathbf{r}')dS'dS\Big) \, , \qquad (4)$$

and

$$V_m = \frac{1}{\eta}\iint_s \Lambda_m(\mathbf{r})\cdot\mathbf{E}^i(\mathbf{r})dS \, . \qquad (5)$$

For simplicity, let $\mathbf{Z}$ denote the coefficient matrix in (3), $\mathbf{J} = \{J_n\}$ and $\mathbf{V} = \{V_m\}$ in following. Then, the EFIE matrix equation (3) can be symbolically rewritten as:

$$\mathbf{Z}\bullet\mathbf{J} = \mathbf{V} \, , \qquad (6)$$

where $\mathbf{J}$ is the column vector containing the unknown coefficients of the surface current expansion with RWG basis functions. The impedance matrix $\mathbf{Z}$ is dense in the sense that all entries are nonzero.

The impedance matrix $\mathbf{Z}$ resulting from the discretization of EFIE formulation can be represented by the multilevel simply sparse method [14-15] based on a multilevel oc-tree as shown in Fig. 1. In [15], the structure of the MLSSM representation of the impedance matrix $\mathbf{Z}$ is given in a multilevel recursive manner

$$\mathbf{Z}_l = \hat{\mathbf{Z}}_l + \mathbf{U}_l\mathbf{Z}_{l-1}\mathbf{U}_l^{\mathrm{T}} \, , \qquad (7)$$

where $\mathbf{Z}_l$ is the reduced order impedance matrix and consists of only far-field interactions at level $l+1$, which will be compressed in the coarser levels recursively up to level-3. There is no level $L+1$ near-field interaction at the finest level L. Thus, $\mathbf{Z}_L$ is the impedance matrix $\mathbf{Z}$. In (7), $\hat{\mathbf{Z}}_l$ is the sparse matrix containing all near-neighbour interactions at level l of the oct-tree which were not represented at the finer level of the tree, $\mathbf{U}_l$ are block diagonal unitary matrices that compress interaction between sources in non-touching groups at level l. The pictorial representation of impedance matrix of EFIE in MLSSM is shown in Fig. 2.

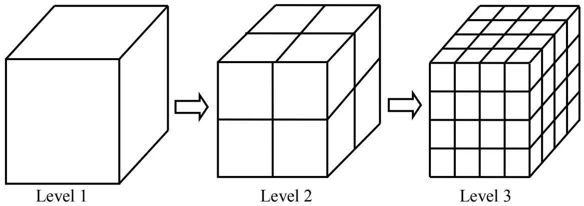

Fig. 1. Construction of an oc-tree.

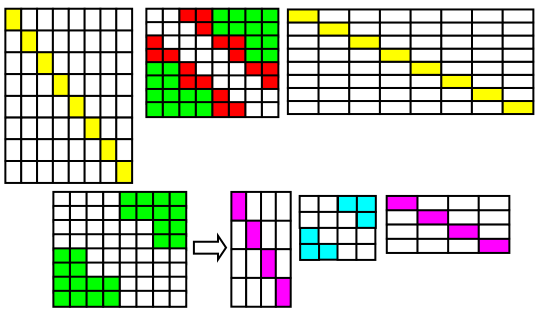

Fig. 2. The pictorial representations of impedance matrix of EFIE in MLSSM.

Based on the MLSSM representation of the impedance matrix **Z**, the efficient matrix vector product (MVP) is implemented as follows:

Subroutine MVP ($\mathbf{x}$, $\mathbf{y}$, l)
Begin $l = L$: 3: -1
    $\mathbf{y}_1 = \hat{\mathbf{Z}}_l \cdot \mathbf{x}$;
    $\mathbf{y}_2 = (\mathbf{U}_l)^{\mathrm{T}} \cdot \mathbf{x}$;
    Call MVP ($\mathbf{y}_2$, $\mathbf{y}_3$, l -1);
    $\mathbf{y} = \mathbf{y}_1 + \mathbf{U}_l \cdot \mathbf{y}_3$
End

III. TWO-STEP PRECONDITIONER

The linear system $\mathbf{Z} \bullet \mathbf{J} = \mathbf{V}$ resulting from EFIE with electromagnetic scattering MLSSM is often an ill-conditioned matrix and results in the low convergence of the Krylov subspace solvers, such as generalized minimal residual method (GMRES) [26]. In linear algebra, the convergence of the Krylov subspace solver is closely related to the condition number of impedance matrix **Z**. Denote the spectrum of **Z** in magnitude by

$$\sigma(Z)=\{ | \lambda_1 | \le | \lambda_2 | \le \cdots \le | \lambda_N | \} \ . \qquad (8)$$

Generally speaking, the condition number can be evaluated as follows:

$$\mathrm{cond}(\mathbf{Z}) = | \lambda_N | / | \lambda_1 | \ . \qquad (9)$$

In order to improve the convergence, the preconditioners are usually incorporated. According to (9), one can conclude that the smallest eigenvalues are responsible for slow convergence. The convergence of Krylov subspace solvers can be accelerated if by any means components in the residuals which correspond to the small eigenvalues can be removed during the iterations.

The SAI preconditioner can improve matrix condition number by clustering most of the large eigenvalues close to one, but leaving a few close to the origin. In [24], the modified complex shifted preconditioner was proposed, which can shift the smallest eigenvalues of impedance matrix **Z** close to a priori fixed constant. Therefore, we adopt the preconditioner in a two-step manner in order to accelerate the convergence of Krylov subspace solvers of MLSSM. Firstly, the SAI preconditioner is used to cluster most large eigenvalues close to one. The EFIE matrix equation (6) is transformed into an equivalent form as follows

$$\mathbf{P}_1 \mathbf{Z} \bullet \mathbf{J} = \mathbf{P}_1 \mathbf{V} \ , \qquad (10)$$

where $\mathbf{P}_1$ is the corresponding SAI preconditioner matrix. Secondly, the modified complex shifted preconditioner is utilized to shift some eigenvalues to one. Then, we obtain the equivalent form as follows

$$\mathbf{P}_2 \mathbf{P}_1 \mathbf{Z} \bullet \mathbf{J} = \mathbf{P}_2 \mathbf{P}_1 \mathbf{V} \ , \qquad (11)$$

where $\mathbf{P}_2$ is the corresponding modified complex shifted preconditioner matrix. For simplicity, the above combining preconditioner is referred to as two-step preconditioner.

In this paper, the construction of SAI preconditioner matrix $\mathbf{P}_1$ is referred to [21-22]. For the modified complex shifted preconditioner matrix $\mathbf{P}_2$, we have constructed in efficient manners as in [25]

$$\mathbf{P}_2 = \mathbf{I} + \lambda \mathbf{Y} \mathbf{E}^{-1} \mathbf{W}^{\mathrm{H}} \ , \qquad (12)$$

where $\mathbf{E} = \mathbf{W}^{\mathrm{H}} \mathbf{Z} \mathbf{Y}$, λ=(1.0,0.0) , **Y** and **W** are rectangular matrices with rank r, which contain r smallest eigenvalues of impedance matrix **Z**.

IV. NUMERICAL RESULTS

In this section, some numerical examples are simulated to demonstrate the efficiency of the two-step preconditioner. All computations are performed on Intel(R) Core(TM) 4 Quad CPU at 2.83 GHz and 8 GB of RAM in single precision. The restarted version of GMRES(m) is applied to solve linear systems, where m is the dimension size of Krylov subspace for GMRES and is set to be 30 in this paper. The iteration process is terminated when the normalized backward error is reduced by 10^{-3} for all examples, and the limit of the maximum number of iterations is set as 10000.

First, we consider the scattering of a perfectly electrically conducting (PEC) sphere with radius 1m at 300 MHz. The incident angles of plane wave are $\theta_i = 0^\circ$, $\phi_i = 0^\circ$. The sets of angles of interest for the bistatic RCS vary from 0 to 180 degree. As shown in Fig. 3, comparison with analytical solution from Mie series is made for the bistatic RCS of the sphere. It can be found that there is an excellent agreement between them and this demonstrates the validation of our MLSSM and the two-step preconditioner.

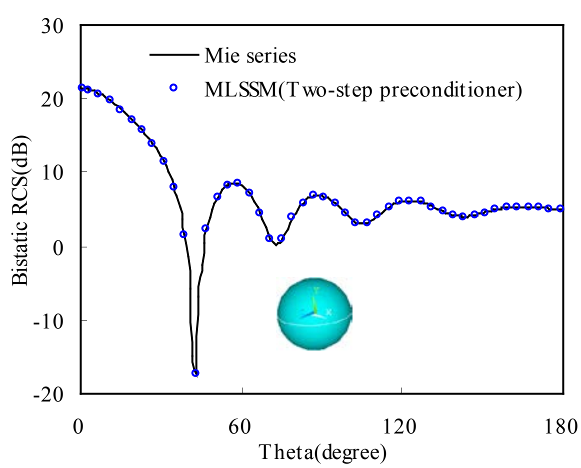

Fig. 3. Calculation results for bistatic RCS of the sphere with radius 1 m at 300 MHz.

Second, we consider a PEC plane as shown in Fig. 4. The length of the plane is 2.04 m in the x-axis direction, and width is 2.02 m in the y-axis direction. The frequency of the incident wave is 1GHz and the incident direction is $\theta_i = 60^\circ$, $\phi_i = 270^\circ$. The surface of the plane is meshed with one-tenth of the wavelength. The unknown of the PEC plane is 18264. The third example considers a VIAS geometry [27] as shown in Fig. 5. The frequency of the incident wave is 100 MHz and the incident direction is $\theta_i = 0^\circ$, $\phi_i = 0^\circ$. The unknown of the VIAS geometry is 15377.

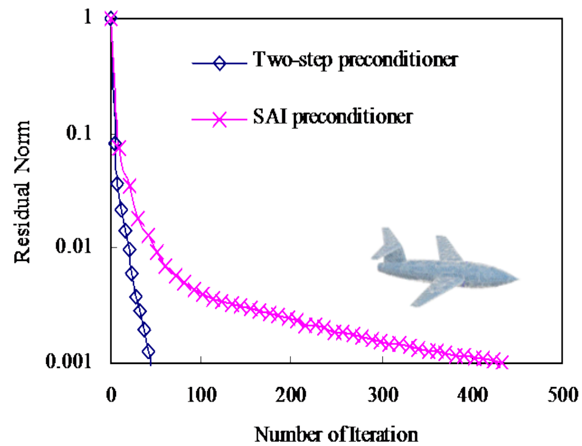

Fig. 4. Convergence history of the GMRES(m) for solving system on the plane.

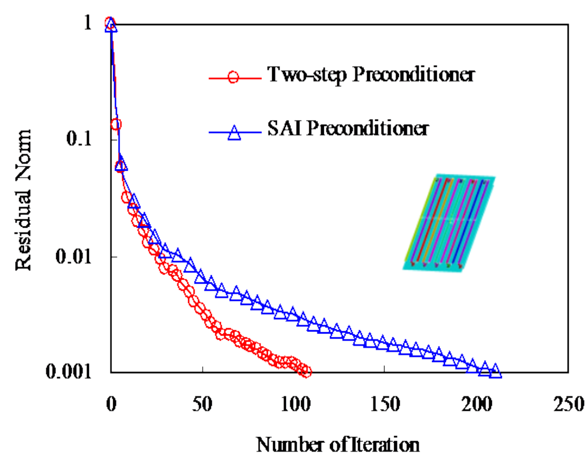

Fig. 5. Convergence history of the GMRES(m) for solving system on the VIAS geometry.

Table 1: Total number of iterations and solution time (in second) for the plane and VIAS geometry

Object	Unknown	Iteration Number		Solution Time	
		SAI	Two-step	SAI	Two-step
Plane	18264	442	44	195 s	22 s
VIAS	15377	215	107	63 s	31 s

The convergence histories of the GMRES algorithms with a two-step preconditioner and SAI preconditioner are displayed in Figs. 4-5 for the above examples. It can be observed that when compared with the SAI preconditioned GMRES (m), the two-step preconditioner can decrease the number of iterations by a factor of 10.0 on the plane case, 2.0 on the VIAS case. The solution times are given in Table 1 for the above examples. It can be seen from Table 1 that a two-step preconditioner can reduce the solution time and iteration number greatly.

In order to illustrate the performance of the two-step preconditioner further, the monostatic RCS of the above PEC sphere is calculated firstly. The interest angles vary from 0° to 180° in ϕ direction when θ_i is fixed at 0°. The error of monostatic RCS of the two-step preconditioner and Mie series is displayed in Fig. 6 for the metallic sphere. It can be seen that the maximum error of the monostatic RCS is less than 0.06 db and this demonstrates the validation of our MLSSM and the two-step preconditioner.

Then, the above plane and VIAS examples are calculated. The interest angles vary from 0° to 180° in ϕ direction when θ is fixed at 0°. The monostatic RCS of the two-step preconditioner and the SAI preconditioner are displayed in Figs. 7-8 for the above examples. The number of iterations is reported for the monostatic RCS calculation in Figs. 9-10.

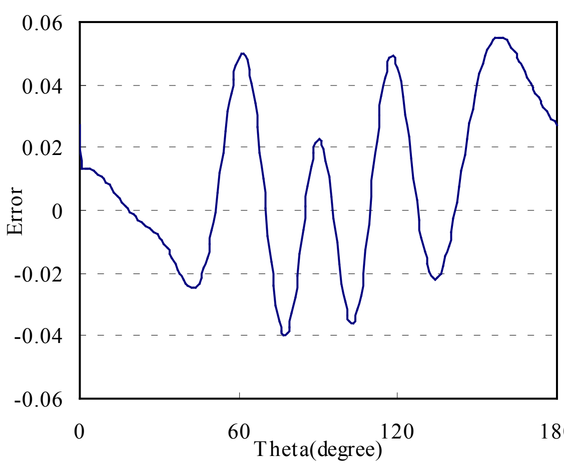

Fig. 6. The error of monostatic RCS of two-step preconditioner and Mie series for the PEC sphere.

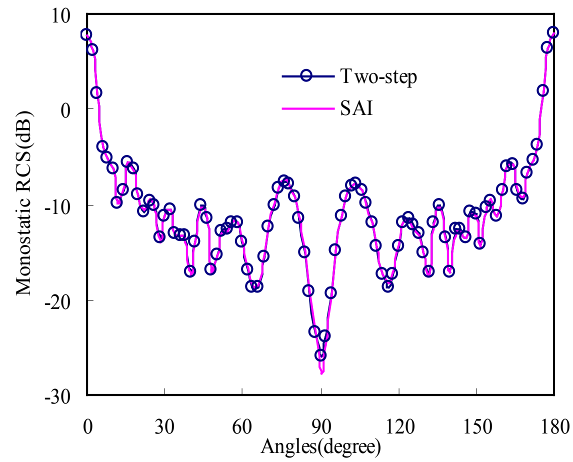

Fig. 7. The monostatic RCS of the PEC plane for $\theta = 0^{\circ}, 0^{\circ} \le \phi \le 180^{\circ}$.

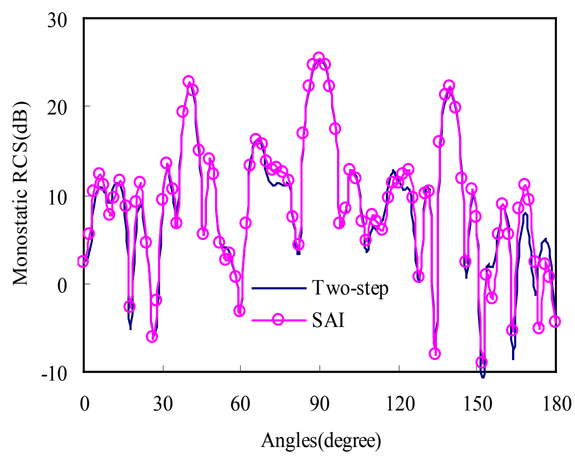

Fig. 8. The monostatic RCS of the PEC VIAS geometry for $\theta = 0^{\circ}, 0^{\circ} \le \phi \le 180^{\circ}$.

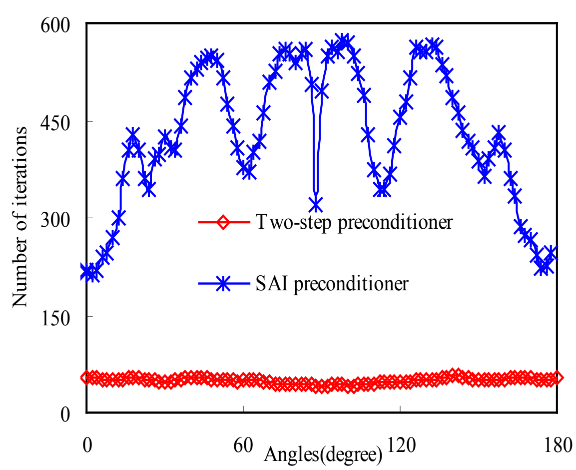

Fig. 9. Number of GMRES iterations for solving monostatic RCS on the plane.

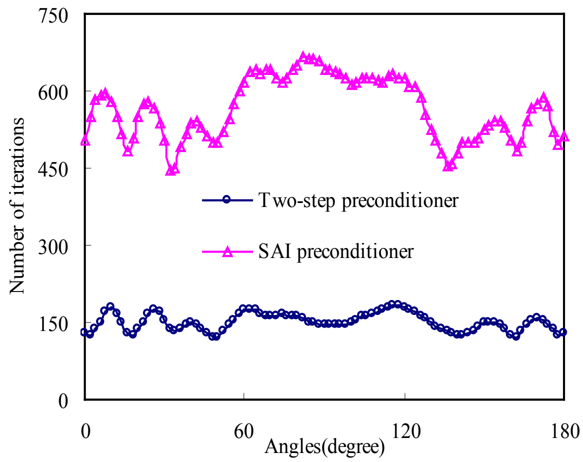

Fig. 10. Number of GMRES iterations for solving monostatic RCS on the VIAS geometry.

From Figs. 9-10, it can be seen that the number of iterations of the SAI preconditioner varies largely with respect to incident angles (i.e. RHS-vectors), while that of the two-step preconditioner is more similar and almost constant for each example. This is the advantage of the two-step preconditioner. Therefore, the two-step preconditioner is suitable for the multiple right hand vectors problem, such as the calculation of the monostatic RCS.

V. CONCLUSION

Because of the nature of the EFIE formulation, the linear system resulting from the electromagnetic scattering MLSSM is often an ill-conditioned matrix and results in the low convergence of the Krylov subspace solvers. In this paper, the two-step preconditioner is used to accelerate the convergence of the Krylov subspace solvers in MLSSM. The numerical examples demonstrate that the two-step preconditioner is more effective than SAI preconditioner in terms of the CPU time in iterative procedure. Especially, the two-step preconditioner is suitable for the multiple right hand vectors problem.

ACKNOWLEDGMENT

We would like to thank the support of Major State Basic Research Development Program of China (973 Program: 2009CB320201), Natural Science Foundation of 60871013, Jiangsu Natural Science Foundation of BK2009387 and NUST 2010ZYTS027.

REFERENCES

[1] S. Rao, D. Wilton, and A. Glisson, “Electromagnetic Scattering by Surfaces of Arbitrary Shape,” *IEEE Trans. Antennas and Propag.*, vol. 30, no. 3, pp. 409-418, May 1982.

[2] K. A. Michalski and D. L. Zheng, “Electromagnetic Scattering and Radiation by Surfaces of Arbitrary Shape in Layered Media, Part I: Theory,” *IEEE Trans. Antennas and Propag.*, vol. 38, no. 3, pp. 335-344, March 1990.

[3] K. A. Michalski and D. L. Zheng, “Electromagnetic Scattering and Radiation by Surfaces of Arbitrary Shape in Layered Media, Part II: Implementation and Results for Contiguous Half-Spaces,” *IEEE Trans. Antennas and Propag.*, vol. 38, no. 3, pp. 345-352, 1990.

[4] D. Ding, J. Ge, and R. Chen, “Well-Conditioned CFIE for Scattering from Dielectric Coated Conducting Bodies above a Half-Space,” *Applied Computational Electromagnetic Society (ACES) Journal*, vol. 25, no. 11, pp. 936 - 946, November 2010.

[5] W. C. Chew, J. M. Jin, E. Michielssen, and J. Song, *Fast Efficient Algorithms in Computa-tional Electromagnetics*, Boston, MA, Artech House, 2001.

[6] J. M. Song, C. C. Lu, and W. C. Chew, “Multilevel Fast Multipole Algorithm for Electromagnetic Scattering by Large Complex Objects,” *IEEE Trans. Antennas Propagat.*, vol. 45, no. 10, pp. 1488-1493, 1997.

[7] L. Rossi, P. J. Cullen, and C. Brennan, “Implementation of a Multilevel Fast Far-Field Algorithm for Solving Electric-Field Integral Equations,” *IEE Proc., Microw. Antennas Propag.*, vol. 147, iss. 1, pp. 19-24, 2000.

[8] M. Bebendorf, “Approximation of Boundary Element Matrices,” *Numer. Math.*, vol. 86, no. 4, pp. 565–589, 2000.

[9] K. L. Lai, L. Tsang, and C. -J. Ong, “Application of the Multilevel UV Method to Analyze Large Microstrip Patch Structures,” *IEEE Antennas Wireless Propag. Lett.*, vol. 4, no. 1, pp. 471-474, 2005.

[10] F. X. Canning and K. Rogovin, “A Universal Matrix Solver for Integral-Equation Based Problems,” *IEEE Antennas and Propagation Magazine*, vol. 45, iss. 1, pp. 16-19, Feb. 2003.

[11] A. Zhu, R. J. Adams, and F. X. Canning: “Modified Simply Sparse Method for Electromagnetic Scattering by PEC,” *IEEE Antennas and Propagation Society Internatio-nal Symposium*, Washington, DC, pp. 427-430, 2005.

[12] R. J. Adams, A. Zhu, and F. X. Canning, “Sparse Factorization of the TMz Impedance Matrix in an Overlapped Localizing Basis, *Progress in*

Electromagnet Research, vol. 61, pp. 291-322, 2006.

[13] Y. Xu, X. Xin, R. J. Adams, S. D. Gedney, and F.X. Canning, "Sparse Direct Solution of the Electric Field Integral Equation using Nonoverlapped Localizing LOGOS Modes," *Micro. Opt. Tech. Lett*., vol. 2, pp. 303-307, Feb. 2008.

[14] Y. Xu, X. Xin, and R. J. Adams, "A Sparse Factorization for Fast Computation of Localizing Modes," *IEEE Trans. Antennas Propagat.*, vol. 58, no. 9, pp. 3044-3049, 20120.

[15] X. Q. Hu, Y. Xu, and R. S. Chen, "Fast Iterative Solution of Integral Equation with Matrix Decomposition Algorithm and Multilevel Simple Sparse Method," *IET Microwaves, Antennas, and Propagation,* vol. 5, no. 13, pp. 1583-1588, Oct. 2011.

[16] J. M. Song, C. C. Lu, and W. C. Chew, "Multilevel Fast Multipole Algorithm for Electromagnetic Scattering by Large Complex Objects," *IEEE Trans. Antennas Propagat.*, vol. 45, no. 10, pp. 1488-1493, 1997.

[17] K. Sertel and J. L. Volakis, "Incomplete LU Preconditioner for FMM Implementation," *Microwave and Optical Technology Letters*, 26(7):265–267, 2000.

[18] T. Wan, R. S. Chen, and X. Q. Hu, "Data-Sparse LU-Decomposition Preconditioning Combined with MLFMM for Electromagnetic Scattering Problems," *IET Microwaves, Antennas & Propagation,* vol. 5, no. 11, pp. 1288-1296, Aug. 2011.

[19] B. Carpentieri, I. S. Duff, L. Griud, and G. Alleon, "Combing Fast Multipole Techniques and an Approximate Inverse Preconditioner for Large Electromagnetism Calculations," *SIAM J. Sci. Comput.*, 27(3):774–792, 2005.

[20] Y. Xie, J. He, A. Sulllvan, and L. Carin, "A Simple Preconditioner for Electric-Field Integral Equations," *Micro. Opt. Tech. Lett*., vol. 30(1), pp. 51-54, 2001.

[21] J. Lee, J. Zhang, and C. –C. Lu, "Sparse Inverse Preconditioning of Multilevel Fast Multipole Algorithm for Hybrid Integral Equations in Electromagnetics," *IEEE Transactions on Antennas and Propagat.*, vol. 52(9), pp. 2277-2287, 2004.

[22] P. L. Rui, R. S. Chen, D. X. Wang, and E. K. N. Yung, "Spectral Two-step Preconditioning of Multilevel Fast Multipole Algorithm for the Fast Monostatic RCS Calculation," vol. 55, no. 8, *IEEE Transactions on Antennas and Propagation*, August 2007.

[23] P. L. Rui and R. S. Chen, "A Spectral Multigrid Method Combined with MLFMA for Solving Electromagnetic Wave Scattering Problems," *IEEE Transactions on Antennas and Propagation*, September 2007.

[24] D. Z. Ding, R. S. Chen, Z. H. Fan, and P. L. Rui, "AA Novel Hierarchical Two-Level Spectral Preconditioning Technique for Electromag-netic Wave Scattering," *IEEE Transactions on Antennas and Propagation*, vol. 56, no. 4, April 2008.

[25] X. Q. Hu, M. Chen, D. Z. Ding, and R. S. Chen, "A Modified Complex Shifted Preconditiner Combined with Sparse Approximate Inversion Preconditiner for Electromagnetic Scattering," *Micro. Opt. Tech. Lett*., vol. 53, no. 1, pp. 55-58, January 2011.

[26] Y. Saad and M. Schultz, "GMRES: A Generalized Minimal Residual Algorithm for Solving Nonsymmetric Linear Systems," *SIAM J. Sci. Stat. Comput.*, vol. 7, no. 3, pp. 856-869, 1986.

[27] J. Cheng, S. A. Maloney, R. J. Adams, and F. X. Canning, "Efficient Fill of a Nested Re-presentation of the EFIE at Low Frequencies," *IEEE AP-S International Symposium* San Diego, CA, 2008.

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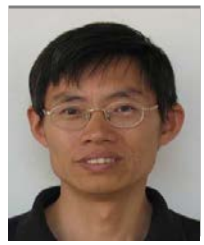

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