The Applied Computational Electromagnetics Society

NEWSLETTER

Vol.2 No.1 May 1987

EDITORS

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Volume 2 Number 1 May 1987

PRESIDENT'S CORNER

E.K. Miller

For the edification of those of you who weren't able to attend, the second ACES Meeting held recently in Monterey was by all accounts a great success. Attendance was in the 125+ range with over 40 presentations plus other demonstrations and discussion concerning software exchange, establishing validation and benchmarks for computer models, and setting future directions for ACES itself.

In order to survey attendee/member interest, a brief questionnaire was handed out at the Meeting with the results of the 70+ returned summarized below. If you would like to provide your own input, please complete the version included in this Newsletter and return it to me [E.K. Miller, Rockwell Science Center, P.O. Box 1085, 1049 Camino dos Rios, Thousand Oaks, CA 91360, Telephone (805) 373-4297] so that we may develop a representative measure of member interests. The raw numbers are included here for ease of preparation and interpretation, and since multiple replies were possible the totals may not always sum to the same number.

We divided the questionnaire into three main parts as follows:

(1) Applications Areas of Interest
(2) Comments concerning the annual Meeting
(3) Evaluation of the importance of possible ACES activities and areas in which the respondent would be willing to help.

MODELING INTERESTS

Applications:

Antennas	66
Scattering	48
Propagation	31
Other	16

Other included near-field applications, EMP, EMC, Interaction/Coupling, etc.

Model Types Used:

Integral Equation	57
Differential Eq.	17
GTD	37
Other	17

Other included PO/PTD, Hybrid modeling, modal expansions, etc.

Modeling Domain:

Frequency	43
Time	16

COMMENTS/SUGGESTIONS FOR ANNUAL MEETINGS (REVIEWS)

Length

Too Short	3
Too Long	1
About Right	74

Presentation Time

Too Short	26
Too Long	1
About Right	49
Other	2+

Other included comments that we need more breads, and perhaps 20 minutes per presentation rather than the 15 allowed this year. We might conclude that 15-20 minutes is a preferred presentation time.

Possible Use of Multiple Sessions

Avoid at all costs	36
Two OK	41
More than two OK	2

Future Locations for Meetings

West Coast	65
Other	16

Other suggestions included East Coast (1), Hawaii (3), "Someplace Warm" (4), Mid-West (3), etc.

Time of Year Preferred

Jan/Feb/Mar	75
Summer	6
Fall	3

RECOMMENDED ACES FUNCTIONS AND VOLUNTEER ACTIVITIES

ACES Functions

Respondents were asked to rank order the relative importance they attach to several functions that have been discussed for ACES. We included four possibilities for ACES to undertake on the questionnaire:

(1) Promote Software Exchange
(2) Develop Procedures for Code Validation/Calibration
(3) Establish a Solved-Problems Library
(4) Collect Modeling Guidelines/Difficulties

as well as:

(5) Other (to be specified).

The results are as follows (number of responses in order of most important to least important with four categories):

Software Exchange	25,14,17,19 (195)
Validation/Calibration	20,23,21,11 (202)
Solved-Problems Library	8,16,20,31 (135)
Modeling Guidelines	27,24,14,10 (218)
Other (Creative Codes)	1

with the numbers in parentheses being the weighted totals

We observe that three of the suggested functions exhibit similar support, with a solved-problems library being of significantly less interest. I must admit that this outcome is somewhat disappointing to me personally since one of the things that I have been trying to get started for a long time is a growing library of solved problems organized topically into a sort of handbook. I remain convinced of that value of doing this, and conclude that perhaps the lower value placed on this activity reflects the ineffectiveness of a presentation that I made at this year's ACES Review on just this topic.

In any case, we do have to target our attention and resources because there's only so much that a new organization can expect to accomplish. These initial results of our questionnaire indicate members' concerns to focus essentially on software use and exchange, i.e., how do we use codes and how do we know their results are right? As we collect further opinions from other ACES members, we should be able to select those areas for attention that will most contribute to increasing the value of ACES to present and future members.

Volunteer Activities

Respondents to the questionnaire were also asked to indicate at least one area (committee) to which they would be willing to donate some time and effort. Our most critical need was, and remains, that of obtaining help in producing the Newsletter. But there are a number of other areas where help is sorely needed as well. Included here is a list of the activities included in the questionnaire and the number of volunteers who expressed a willingness to work. Please feel free to add your own name! Those indicating such interest can expect to hear from a committee chairman in each of these areas in the near future.

Publications	12
Membership	8
Software Exchange	25
Technical Activities	21
Nominating	1
Constitution/Bylaws	3
Finance	1
Publicity	7
Conferences/Meetings	13
Other (European Liaison)	1

The names of Newsletter volunteers have been passed on to the new Newsletter Editor, David Stein, to whom we should give a Hip-Hip, Hooray!, and from whom you who checked publications as a volunteer activity should already have heard. We should also thank Bob Bevensee for his contributions to the Newsletter and past Review as well as Dick Adler who has been a lynch-pin in holding our Monterey meetings together.

There are a few other odds and ends left over from the '87 Meeting. One is that our Administrative Committee (or ADCOM) is short handed because the term of one of the original members (Janet MacDonald) was for one year. We discovered that the process of getting prospective ADCOM members and officers is somewhat inconvenient, requiring the exchange of paperwork, etc. prior to the annual meeting (it helps to read the constitution and bylaws!). In any case, it would be more convenient if we were to establish a procedure for mail nomination and elections, something which we are working to establish. We are also still pushing paper to implement the final steps required for having a bona-fide legal organization. By the time you read this we can hope that this last step has finally been accomplished.

Well, that's all for now. Incidentally, you might have noticed that I have a new address (again). I joined the Rockwell Science Center April 15 after having spent less than two years at the University of Kansas. We (my wife and I) found that we missed California more than we had anticipated. Since the Science Center has a very nice auditorium, we are considering the possibility of holding a future Review there.

FROM THE EDITOR

I am most privileged to present Vol. 2, No. 1 of the ACES Newsletter, which may be the final issue published under that title if we implement our proposed name change to ACES Journal, ACES Review, or something similar. The papers featured in this issue discuss viewpoints and electromagnetic modeling tools which have been under-represented among papers previously submitted. These papers are not only of interest in themselves but also indicate ways in which we can broaden our scope. My apologies for the delay in publication, but if certain initiatives pay off this summer and early fall, the delay will be our last one.

One of our major goals is to increase the diversity of papers with regard to applications and to techniques. Many of my recent initiatives have been toward this end. Applications of interest include antennas, networks, radar cross section, shielding, radiation hazards, and other electromagnetic propagation (including effects of terrain and atmosphere). The techniques include methods-of-moments, diffraction theories, physical optics, modal expansion methods, perturbation theory, integral- and differential-equation methods, and hybrid methods -- as well as other techniques in the time or frequency domain. It is emphasized that these lists are not exhaustive and that increased emphasis in certain areas does not imply a corresponding shift of emphasis away from other areas. ("Emphasis," however, is determined primarily by the types of papers submitted, not by editorial policy.) The ultimate objective is broad appeal of the ACES Newsletter among users and developers of computational tools for electromagnetics. A possible spinoff is the promotion of interdisciplinary efforts in computational electromagnetics.

On a related note, we seek to publish different types of material. Our "mainstay" will continue to be full-length papers as well as short notes -- which may address general computational methods, particular computer codes, or particular applications. (Even unsuccessful applications are legitimate for publication, provided that a reasonable expectation of success -- and a reasonable effort -- are reflected.) However, we are also interested in featuring tutorials and in making the "Pandora's Box" column viable.

In addition, other types of material have been suggested by various ACES members:

Listings of short codes, and sample runs (Donn Campbell)

Reprints, of ACES-relevant papers first printed in other journals, and of classical analyses widely referenced but in books now out of print (Bob Dehoney)

Corrections of errors in widely-used reference material (Bob Dehoney)

Measured data, for code validation purposes (Bob Dehoney)

Comparison of the results of two or more computer codes, as exercised on standard benchmark geometries (Art Ludwig)

All of these suggestions represent opportunities to expand our service to the applied computational electromagnetic community. Even government agencies, which to my knowledge have no central "clearinghouse" for parallel efforts in electromagnetics software development, might benefit from code validation against measured data or against other codes. For these reasons, material in any of these categories will be considered for publication in future issues, subject to certain caveats. First, the reprinting of previously-published material requires written authorization by the rightful owners. Finally, we shall seek alternate ways to accommodate lengthy codes and voluminous data, should the need arise.

This diversity in content is best achieved by a corresponding diversity in sources. Therefore, when the new ACES brochure is completed, I plan to send copies to several universities (electrical engineering and physics departments), to prominent individuals in computational electromagnetics, and to the developers of electromagnetics computer software (for antennas, radar cross section, and related areas). Many of the recipients of these brochures may want to publish in the ACES Newsletter. (Recently, Bob Dehoney pointed out that graduate theses might provide an excellent source for papers.) Furthermore, I plan to send brochures to certain other electromagnetics-related publications. Inasmuch as we do not duplicate their efforts, they might make potential authors aware of our existence. I anticipate that the ACES brochure will be available long before press time for the November 1987 issue.

(If you live outside the United States, I would be most grateful if you help identify the university electrical engineering and physics departments in your respective nations.)

A word of caution is in order. Although we have identified several new potential sources of papers, this does not mean that we may become complacent. ACES still depends on contributions from the current membership. All of us know what happens when everyone thinks, "Let George do it." Furthermore, the new brochure will not necessarily be available tomorrow or next week.

At the same time, and on behalf of the ACES membership, I thank those who contributed material, suggestions, and/or effort to the current ACES Newsletter issue. I am most indebted to Dick Adler, Trish Adler, Bob Bevensee, Ted Roach, and Ed Miller for their administrative support.

During the coming months, I encourage all of you to share your suggestions and comments. For your convenience, I again provide my address and telephone number (Post Office Box 541, Holloman Air Force Base, New Mexico 88330; 505-434-3338). However, in late July, I will be inaccessible by telephone inasmuch as I shall be moving (and transitioning to civilian life). You may continue using my mailing address; mail will be forwarded. When I am re-settled in early August, I shall send my new telephone number and address to all ACES members. Meanwhile, let's begin thinking about the November 1987 issue!

David E. Stein
Editor-in-Chief

1. At press time, the 4th Annual Review of Progress in Applied Computational Electromagnetics is scheduled for 22-24 March 1988 at the Naval Postgraduate School, Monterey, CA.

2. The ACES SOFTWARE LIBRARY is taking form. It includes, at present, three types of available software:

a. A list of EM programs (on ACES SOFTWARE FORMS) which is generally available for use by ACES members, and possibly others. Some restrictions may apply on availability, such as DoD agencies and their contractors.

b. Software which is available from and being distributed by specified ACES members to ACES members ONLY.

c. Software and Documentation which is available from ACES for ACES members ONLY. MININEC3 and GRAPS are the first offerings in this category.

3. Over 80% of all non-U.S. checks we receive cost ACES bank charges of at least $25. Effective immediately, we are only accepting International Money Orders, payable in U.S. currency, for the appropriate U.S. amount for all foreign transactions.

PANDORA'S BOX

by Dawson Coblin

According to Greek mythology, Pandora opened her box only long enough for evil to escape and roam the world. By closing it, she trapped hope in the box. So man is doomed to live in an evil world without hope. In their less sanguine moments, code users feel the same way; lost, abandoned and despairing. It is intended that in this column Pandora's box can be opened again and hope allowed to escape.

The purpose of this column is to concentrate on unsuccessful applications of commonly used codes. The goal will be to determine areas where the application may have forced the code to break down and make suggestions for improving the results. The success of this approach depends on the responsiveness of the ACES members to share their less successful attempts and quandaries.

The membership is therefore solicited to send their problem cases to me for review. Please include the name of the code used (and version, if applicable), the specifics of the test case, examples of the output, a list of the problems and contradictions observed, and your name, address, and telephone number. Please respond to the following address:

R.D. Coblin
O/6242;B/130
Lockheed Missiles & Space Co.
P.O. Box 3504
Sunnyvale, CA 94088-3504.

MODELING NOTES

The primary purpose of ACES and the Newsletter is to foster information exchange among workers involved in developing and applying computer codes to model electromagnetic problems.

This section features short articles about particular aspects of the more popular codes and short notes which summarize user experience with specific codes. To facilitate the submission of short notes in a standard form which can be easily referenced later, we include the ACES MODELING SHORT-NOTES form for 1-3 page submittals.

Readers are encouraged to report their code experiences in these ACES MODELING SHORT-NOTE forms and send them to the ACES Secretary, whose address is listed in the FRONTISPIECE. Camera-ready SHORT-NOTE forms are preferred.

AVAILABLE SOFTWARE

We would like to recognize the hard work that the Chairman of this committee, *TED ROACH*, has done. He has basically single-handedly made this committee a success. We appreciate his ideas, efforts, and achievements.
KEEP UP THE GOOD WORK!!!

ACES LIBRARY - NEW ITEMS

CURRENT INDEX OF ITEMS IN LIBRARY:

ITEM #	DESCRIPTION	COMPUTER
001	MININEC2	IBM PC
002	MININEC2F frequency sweep	IBM PC
003	ENHANCED MININEC2 double ARRAY size to 20 wires, etc.	IBM PC
004	ENHANCED MININEC2	IBM PC
005	THIN WIRE MININEC2	IBM PC
006	NEC2	DEC VAX
007	NEC3	DEC VAX
008	MININEC3	IBM PC/XT or AT
009	GRAPS Graphical plotting system	IBM PC/XT or AT

The following items have been added since the November 1986 NL:

010	IGUANA 4.1	IBM PC
011	NAC3	IBM PC
	Dr. Anders MOM code for thin wire antennas, compiled for fully expanded PC up to 800 segments	
012	SIGDEMO	IBM PC
	Demo disk for Network Analysis, Nodal Analyses/Filter design. A fast, easy to use, moderately priced commercial program	
013	Misc BASIC programs	IBM PC
	RF designers toolbox SWR from Ric Thowless QD-CPLR from Ted Roach	
014	AT-ESP	IBM PC/XT or AT
	Ray Luebbers' full PC implementation of the mainframe ESP Code including graphics using VDI drivers	

ACES SOFTWARE FORM

Software Number: #010 - IGUANA 4.1

Machine: IBM PC/XT or AT (or compatible)

Directory Listing:

140 FILES - Approx 1.5M Bytes
.EXE with auxiliary programs and sample data sets

Description: Interactive Graphics Utility for Automated NEC Analysis is a system to partially automate data entry and display processes for NEC/MININEC3.

a) Features:

1) Creation of 3-D models
2) Generation of NEC wire cards for (1)
3) User entry & maintenance of other required NEC input information
4) Translate & transmit data to a NEC host computer
5) Capture & Display of NEC output
6) MININEC3
7) GRAPS

b) Configuration:

IBM PC-XT/256K RAM
Color Graphic Adapter (CGA)
Two Serial ports
Microsoft Mouse (Bus Version) *
Parallel dot matrix printer
HP 7470 plotter *
GRAFBAR GP-7 Digitizer *
A-B Switch Box (DB25) *
* optional

c) Software Language Required: none

d) Formatted:
5 1/4" floppy
DOS 2.1

e) Available from:

James C. Logan
NOSC, Code 822 (T)
271 Catalina Blvd.
San Diego, CA 92152

f) Access:
DoD agencies & their contractors only. Foreign requests must go through diplomatic channels.

g) Documentation:

Provided with software.

h) Cost:
Send 4 preformatted unlabled (DOS2.1) floppies in self-addressed, stamped return mailer suitable for 8x11 manuals (2.7 lbs) and include a request in writing.

NOTE: At press-time, both IGUANA 4.0 and 4.1 require additional modifications to correct coding deficiencies. Until this is able to be completed, distribution of IGUANA is on hold.

ACES SOFTWARE FORM

Software Number: #011 - NAC-3 Vers. 1.10

Machine: IBM PC-AT

Directory Listing:

NAC3-MAIN.FOR	NAC3-CODE.OBJ	NAC3-INFO.FOR
NAC3-INDF.FOR	NAC3-CODE.EXE	

Description: NAC-3 performs a frequency domain analysis of thin wire antennas and scatterers of arbitrary geometry in isotropic but arbitrary lossy propagation media. The MoM code utilizes the point matching technique and is based on 3-term hyperbolic basis functions of complex arguments. NAC-3 is highly modular and consists of pre-processor, core and post-processor permitting the core to be moved to mainframes for extensive number crunching. NAC-3 has been optimized for efficiency, numerical stability and accuracy as well as fast execution on mainframes, minis and PCs. VLF limit for dipoles is L/W1=1.E-8 single precision and 1.E-16 double precision, limits for loops are C/W1=1.E-4 and 1.E-8 respectively.

a) Features:

1) 800 segments std. (adjustable)
2) Max size of matrix (adjustable)
   a) 200 Unknowns (in-core)
   b) 800 Unknowns (off-core)
   c) Automatic off-core solution if insufficient in-core memory
3) 3-D Geometry creation modules
4) Multiple loads (3 modes) and excitations (4 modes, mixable)
5) Perfect and reflective coefficient finite ground, cliff modeling for second degree horizontal curves.
6) Frequency sweeping. Start, Stop, increments not array limited.
7) Disk file store and recall of
   a) Geometry
   b) Currents and impedances
   c) Far fields
8) Interactive mode and graphics
   a) Menu driven screens
   b) Full input data testing and error analysis
   c) 3-D Geometry view
   d) Display of currents/charge
   e) Linear and log, polar and rectangular far field plots.
   f) Smith chart, VSWR plot

b) Configuration:

512K RAM (matrix size 150x150 in-core)
640K RAM (matrix size 200x200 in-core)
Add on RAM as VDISK suggested to speed up off-core solutions
8087 co-processor required
HERCULES card for graphics
Dot matrix IDS - or Epson compatible printer

c) Software Language Required:

STAND ALONE versions NAC3-CODE.EXE for either 512K or 640K RAM options, IBM PROFESSIONAL FORTRAN 77 compiler Version 1.00, if array dimensions need to be adjusted to fit the available core memory.

d) Formatted: 5 1/4" floppy 1.2M 2HD DOS 3.0

e) Available from:

Dr. Roger Anders
Applied Electromagnetics Engineering
Vorder Halden 11
D-7777 Salem 1, West Germany

f) Access:
public domain software
no restrictions

g) Documentation:
Provided with purchase.

h) Cost:
Available on request.

ACES SOFTWARE FORM

Software Number: #012 - SIGDEMO

Machine: IBM PC

Directory Listing:

SIGDEMO.EXE
DSIGNODE.EXE
FILTDEMO.EXE
SIGNET.DOC
STRIP.COM

Description: Demo disk of an excellent and reasonably priced S-Parameter ladder network analyzer, Nodal analyses and filter design. Circuits on demo disk may contain only resistors, capacitors & S-Parameter devices. Full system includes inductors, stripline, transformers, etc. Excellent for antenna tuners, couplers, matching networks, etc.

a) Features:

1) Fast, easy to edit & use
2) Network analyses, s-parameter output
3) Calculate & plot gain/loss, phase, group delay, Smith Chart, stability factor, etc.
4) Enter device S-Parameter into disk file
5) Auto-optimization of S21 max, S11 min, etc.
6) 5 yrs free updates
7) Filter Designs HP, LP, BP, BS Filters, stores file on disk for direct recall into SIGNET program. Demo doesn't give component values.
8) Not copy protected
9) Fast inline compiled for 8087 on purchased complete program

b) Configuration:

EXE files only
8087 required on NODAL Analyses
256K bytes min.

IBM PC, XT, AT
Med. Res, IBM graphics or CGA, Mouse opt.

c) Software Language Required:
2.0 or later

d) Formatted:
DOS 2.0 or later
360K, 5 1/4" disk

f) Access:
None

e) Available from:

MicroCube Corp.
P.O. Box 488
Leesburg, VA 22075
(703) 777-7157

h) Cost:
$10.00 to ACES members for DEMO disk. List price less than $1890. I can offer 10% discount to ACES members and more off if we can get some quantity.

g) Documentation:

On disk

ACES SOFTWARE FORM

Software Number: #013 - MISCELLANEOUS

Machine: IBM PC

Directory Listing:

SWR.BAS
QD-CPLR.BAS

Description: Miscellaneous Short BASIC Programs for antenna & RF design. We will add programs from time to time as they become available until the disk if full.

a) Features:

1) SWR, Power, loss, Magnitude etc., conversions by Ric Thowless
2) QD-CPLR; lumped element Quadrature coupler design from July/Aug - Sept/Oct 1979 RF Design Magazine; article by C.Y. Ho & Bob Furlow, program by T. Roach

b) Configuration:

IBM PC or compatible; 128K

c) Software Language Required:

BASICA; GWBASIC

d) Formatted:
DOS 2.0
360K

e) Available from:

T.H. Roach
MicroCube Corp.
P.O. Box 488
Leesburg, VA 22075
(703) 777-7157

f) Access:
None

g) Documentation:
Programs to be self explanatory on disk.

h) Cost:
$5.00; covers disk mailer & shipping

ACES SOFTWARE FORM

Software Number: #014 - AT-ESP

Machine: IBM PC/XT or AT

Directory Listing:

ESP Source	128K	ESPPLOT.EXE	92K
ESPPLOT SOURCE	15K	Data Files for example	
ESP.EXE	341K	Inputs and Outputs	

Description: Full PC implementation of the mainframe Electromagnetic Surface Patch (ESP) Code developed at Ohio State including graphics using VDI drivers.

a) Features:

1) 60 wire modes
2) 65 plate modes, and
3) 2 attachment modes.
4) may be increased to ~200 modes total.
5) 1/4 λ per wire mode
6) 24 plate modes / sq. λ
7) CRT graphics
8) printer/plotter graphics

b) Configuration:

512 (640K suggested)

Math coprocessor required

Drivers available for most graphics devices.
Hard drive suggested

c) Software Language Required:

IBM Professional or
Ryan/McFarland FORTRAN

none required for .EXE files

d) Formatted:
5 1/4" floppy 1.2 Meg
(360K DSDD on request)

e) Available from:

Raymond Luebbers
Penn State University
CSSL, EE Dept.
University Park, PA 16802

f) Access:
Public Domain

g) Documentation:

E.H. Newman, "A User's Manual for ESP," Ohio State University ElectroScience Lab Report 713402-1, July 1981.

h) Cost:
$500

A proposed supplemental information form for RCS prediction software is featured in the next three pages. Reader comments, suggestions, and submissions (wherever possible) are encouraged.

*NOTE: The term "RCS prediction software" includes general-purpose electromagnetic modeling codes with RCS prediction capability.

INFORMATION ON OTHER SOFTWARE SOURCES

Ted Roach

Other software useful to the antenna and RF design engineer is available from the following sources. Some of it is not RF related. Some of it is commercial software. Some is very inexpensive, others very expensive. Most of this is for use on an IBM PC.

An important parameter of our antenna analyses is the antenna impedance characteristics. We need to match impedance of these over some required bandwidth, tune these, couple to them, diplex them, etc. To accomplish this, and for other work in RF, we ought to have a good ladder network, nodal analyses and filter design software tools. I have listed below some programs that are practically free. Some of the others are commercial programs that are moderate to very expensive. Usually, the less expensive the fewer features and more unhandy to use. These are usually frequency domain analyses programs. I personally use the SIGNET series of analyses tools and would be totally lost without them.

I currently need to do some work in pulsed RF audio signals and suspect the SPICE programs will be most useful for transient analyses. I understand that these were derived mainly from the U.C. Berkeley version 2G.6 of SPICE as public domain. If someone has this version or a modified public domain version for the PC, it would be nice to include it in the exchange library. Two commercial versions are included in my list and I suspect many more programs are available. Please let me know what else is available along these lines.

There is a propagation path loss program that doesn't take terrain into consideration. I remember using a program years ago on an HP-9825 that accepted terrain details and predicted path loss. One of these would be nice in our library.

The remaining list has miscellaneous programs dealing with SWR, Smith Charts, mixer intermod, couplers, etc. of general use. I note that some of our RF and Microwave Magazines are again providing useful software programs and even making them available on disk for a very small fee, much less than it would cost to enter the code yourself. Finally, I list again the EE Public Domain library which is making many of these programs available for $10.00 per disk.

USEFUL INFORMATION NOT RF RELATED:

1. PC-SIG - up to 700 public domain disks for IBM PC's, currently is putting out an occasional brochure for software purchasers that describes the newest listings. Costs are typically $6.00 per disk. They have versions of PC-Write and PC-Type which may be useful for members to generate articles for the newsletter. Their index disks include a neat program for searching for key words in the index and printing our the number of the diskette.

PC-SIG
1030D E. Duane Ave.
Sunnyvale, CA 94086
(408) 730-9291

2. Macintosh Users - one public domain software exchange is:

Public Domain Exchange
2074 S. Walsh
Santa Clara, CA 95050
(408) 496-0624

COMMERCIAL CIRCUIT SIMULATION:

1. SPICE - U.C. Berkeley v. 2G.6 - This public domain program seems to be the grandaddy of all the SPICE programs. But I don't have a copy nor and address to obtain it from. Maybe one of the members can help.

2. PRE-SPICE:	LIBR, Editors, Monte Carlo drive	$125
IS-SPICE:	AC,DC Transient analyses SPICE	$ 95
INTU-SCOPE:	Graphics, Fourier Anal. etc.	$175
for PC:	from IntuSoft San Pedro, CA	$395 combined
3. P-SPICE		$950
Probe graphics post-processor		$450
for PC		$1400
(VAX prices are double)		

from: Microsim Corp.
23175 La Codema Dr.
Laguna Hills, CA 92653
(714) 770-3022

PROPAGATION PATH ANALYSES:

1. 100 mHz - 3.5 gHz to 1000 miles
for typical earth, does not include path topological data entry.

for PC or C-64, TI-99/4
RF Design Aug 86
Available from EE Public Domain Library
Disk #8

LADDER/NODAL ANALYSES:

1. The first version that I used was from the Hewlett Packard Users group catalog, used on an HP-9845. In HP BASIC, very reasonable.

2. Another early version listed in the 1975 Microwave Journal Engineers Handbook and Buyers Guide, by R.G. Tipping of RCA - listing in FORTRAN IV. This same issue has 16 other short programs in FORTRAN and BASIC concerned with S-parameter, impedance and math functions.

3. RF Design - Nov. 1986 - Ladder program by Kenneth Wyatt of TRW in BASIC for IBM PC (with code corrections in the FEB 87 issue) $10.00 from:

Ken Wyatt
56 Aspen Dr.
Woodland Park, CO 80863

4. RF Design - DEC. 1986. Nodal analyses by Bert Erickon of GE Corp. in BASIC. The article provides a quick review of history of program and references along with listing of code easily arranged for use on any computer. EE Public Domain Disk #8.

5. ALMOND - by Chris Trask, Microwave Analyses using S-parameters - from EE Public Domain Library. $10.00 + $5.00 for book; Disk 1 or Disk 9 (compiled). See address below.

COMMERCIAL PROGRAMS FOR PC's - compiled with many features:

1. Touchstone - from EESof on HP - $11,800

2. Compact - from CCC Compact; $9,700 for PC, HP computers, plus $2,500 for Filter design.

3. SIGNET, SIGNODE & Filter at less than $1890.00. Very complete, easy to learn and use, See demo disk in ACES library #012.

MISCELLANEOUS PROGRAMS:

1. Microwaves - Dec. 1977. 9 FORTRAN Subroutines, Smith Chart calculations. p. 172 Prof. W.J. Remillard at North Eastern Univ. Boston

2. The same issue - article by D.L. Cheadle of W/J, includes a program to study mixer 3rd order intermod distortion, listing in BASIC (this has been severely modified by MicroCube for use on IBM PC)

3. CAD program design of Stripline coupled lines in Microwaves and RF. Dec. 1986 p. 91. by U. Bochtler, F. Endress and W. Stuttgart. Source code available for $2.00 from Microwaves and RF Magazine. see article.

4. Lumped element Quadrature Couplers - program by MicroCube Corporation from article in RF Design, Ho and Furlow; on ACES library disk #013.

5. EE Public Domain Library
   36 Irene Lane East
   Plainview, NY 11803 (516) 822-1697
   Now up to 10 disks - many to do with RF design matching, intermod frequencies, filters, Smith Chart. Disks are $10.00 each.

6. MICRO - A series of programs for microstrip design and analyses. Including line width, impedance, spacing, etc., low pass, bandpass, bandstop filter design, stepped impedance transformer and hybrid ring design, coupled lines plus three new programs. For IBM PC. I have not used it but this one looks very useful. Price: $195.00 from Volunteer Regional Software Distribution Centers.

T.H. Stanford
454 West Blueridge Place
Escondido, CA 92026 (619) 741-7783
(Some discount for quantities of 4 or more.)

VOLUNTEER REGIONAL SOFTWARE DISTRIBUTION CENTERS

Bob Noel

One of the problems facing the Software Exchange Committee is the way to distribute the software that is in existence in the ACES Software Library. A possible solution is to ask some of the members of ACES volunteer to be "Distribution Centers". This would mean that these people would volunteer to electronically send, via modems, software to members of ACES who request it.

This would require volunteers in each geographical area of the country (So. California, No. California, Texas, Virginia, Florida, Massachusetts, etc.) who have access to an IBM PC or PC-compatible equipped with a modem. It would be very helpful to also have a hard disk. The contents of the ACES library would be read on to each of these computers.

Members would be notified as to the contents of the library, the names of the people around the country to call, and the phone numbers to reach them. Once the requester reached the volunteer by telephone, the arrangement as to baud rate, parity, number of bits, etc. could be agreed upon. The volunteer could then give the requester the number to call to reach the modem, so that the requester pays for the phone line while files are being transferred.

If a requester does not have access to a modem, then written requests with formatted disks could still be made to the volunteers. This also should be a quick process.

Because of the limited number of people who are members and who don't have the codes that are in the library, this shouldn't be a very time-consuming activity for the volunteers. Also, the number of codes in the library would be a limiting factor for a short time.

Membership in ACES will grow as the availability of these codes becomes more widespread. This will generate further income for the society and will enable the consideration of purchasing time on a computer bulletin board service in the future. Also with more society income, we may want to consider buying "site-licenses" for software that is not free, to distribute to ACES members. This plan may serve to get the software exchange activities of ACES off the ground.

BASIC HELP FOR NEC USERS

Chuck Vandament

Chuck Vandament presented four IBM-PC programs at the conference in LLNL in the spring of 1985. Written in BASIC(A), they form an elemental set of pre- and post-processors to use with NEC. They are a big help if you don't have IGUANA or some other program. The source code is delivered on the disk so you can modify it to suit your needs, but it should play on most machines. Chuck has shipped about fifty copies to individuals. He is on the ACES Software committee and will ship you a copy if you send him a blank disk and mailer.

WIREMODL is an easy way to construct wire models; you specify the locations of the wire ends, then connect the dots. The output from this program is a set of GW "cards".

SEGMENTR will look at any NEC input set and adjust the segments to fit the maximum length guidelines as a function of frequency. If you need smaller segments near a feedpoint or junction, you must adjust them manually. This program analyzes the GW cards and tells you the number of the wire in which the longest segment occurs, the length of the longest segment, the number of the wire with the thickest segment along with the L/a ratio, and the total number of segments in the whole model.

PERSPLOT will draw a picture of the model on your screen if you have some sort of graphics card in your PC. The model can be viewed from user specified viewing points to help detect modeling errors. The program also writes and output file which can be used by some plotters and printer-plot programs such as Golden Software's Plotcall (TM) program.

PTRNPLOT will plot a pattern on a plotter or printer (also using a printer-plot program). It does a pretty fair job of interpolating between NEC calculated pattern increments of up to 5 degrees to make a smooth plot on the paper or screen.

QUICK NOTE ON QUICK-BASIC 3.0 FOR IBM-PC USERS

Ted Roach

We have finally obtained a copy of Microsoft Quick Basic 3.0 with its improvements over QB 2. Version 3.0 uses direct IEEE format for the 8087 Math co-processor without converting from and to Microsoft format. Recall Bill Seabreeze's paper at the 1987 ACES Conference reporting on his modifications of MININEC3 which he expanded using QB 2. to use 640K of RAM. His test antenna, a 31 element YAGI with 217 segments required about 4 hours and 38 minutes run time on his Leading Edge P.C. with 7.16 mHz clock and 8087 math co-processor. Note also he couldn't get it to run on an IBM PC AT at that time.

With the program re-compiled using QB version 3.0, the program now requires 1 hour and 30 minutes on his Leading Edge. -- A speed improvement of 3:1!!! I think we may have a winner here.

Also, Bill retried his old enhanced MININEC3 compiled with QB version 2 for no co-processor on a PC AT. This time, however, he used the AT without the CONFIG.SYS installed. The AT running at 8mHz, zero wait state with no co-processor also required about 1 hour and 30 minutes run time for the 31 element test antenna. Apparently the CONFIG.SYS took up some of the same memory that the MININEC program arrays needed. Bill also just completed a run of the 31 element test antenna on the PC AT with the 80287 co-processor installed and with QB 3.0 compiled for co-processor. Computer time now was reduced to 49 minutes, or about half of the time required for the run without the co-processor.

Other Notes:

1. Quick-Basic is a little hard to get used to. I have some difficulty pointing at ICONS. I think you have to be a little retarded to use these. Someone should put a contract out on whoever invented WINDOWS.

2. Turbo-Basic by Borland is reported to be much more user friendly than Quick-Basic but may have a problem. I do not have first hand experience with this and am reporting hearsay. The problem is that it inexplicably will begin dumping an infinite series of numbers to screen or printer. Also, it apparently only has 15 digit output format for numbers. I expect Borland will fix these problems and it might be worth waiting to see. Speed should be about the same as QB 3.0 for MININEC type programs when used with the co-processor since it also uses IEEE format for handling the co-processor.

3. QB 3.0 includes a re-dimensioning instruction. Perhaps now we can compile a program that requests how much memory your machine has available, then automatically dimensions the arrays to fit and tells you haw many wires and segments you can have. Neat if it works! Note also - QB 3.0 comes in two versions, one with the 8087 libraries and one without. The executable run time compiler is included. The list price is about $99.50 for the whole works. I got mine at 20% off. What a bargain, and some mail order houses are advertising for under $60.

4. A comparison of Quick Basic and Turbo Basic is given in the June 1987 issue of BYTE Magazine, "Reviewer's Notebook" page 227. The Savage benchmark was used and the programs were compared in speed and accuracy, with and without co-processor.

5. During the next few months, we will try to get the time-hungry executable files for MININEC in the ACES library updated with the QB3.0 compiler.

CODE ANALYSIS

Ted Roach

Request for Large Machine Analysis of 31 Element YAGI design presented in Bill Seabreeze's paper at the 1987 ACES Conference

Results of the antenna analysis on Bill's expanded MININEC3 using corrected element lengths came out very close to the published measurements for the antenna. However, the uncorrected elements lengths gave and additional 1dB of gain that all experimenters indicated was not possible. Results for a shorter 18 element YAGI using the same program came out exactly. Questions to be answered are:

1. Should double precision math be used when you have such a large number of pulses (217)?

2. Or should double precision math be used if antenna gain exceeds a certain number of dB?

3. Or would the 31 element YAGI actually give the 20.7 dB of gain if elements were longer or the feed point exactly matched, or if the whole thing were elevated above ground some exact distance?

4. Or should we just say the results are good enough?

In any case the expanded MININEC3 program appears to provide excellent results for a large number of elements just as is. However, in the interest of getting the above questions resolved it would be interesting to run the antenna design on a larger computer. If one of our members wishes to explore this further, note that the design details of the antenna are provided in the December 1986 issue of HAM Radio Magazine. Or I will provide a copy of the article upon request.

NEC–3 ERROR ALERT!!! *

G. J. Burke
Lawrence Livermore National Laboratory
Livermore, CA 94550

An error has been found in NEC–3 in the calculation of the radiation pattern for an antenna when using the radial wire ground screen approximation. The reflection coefficients for this case were the reciprocal of the correct values. The present incorrect statements at lines 165 and 166 of subroutine FFLD are

```
RRV=(ROZ+ZSCRN*ZRSIN)/(-ROZ+ZSCRN*ZRSIN)                FF 165
RRH=(ZSCRN*ROZ+ZRSIN)/(ZSCRN*ROZ-ZRSIN)                 FF 166
```

The correct expressions are

```
RRV=-(ROZ-ZSCRN*ZRSIN)/(ROZ+ZSCRN*ZRSIN)                FF 165
RRH=(ZSCRN*ROZ-ZRSIN)/(ZSCRN*ROZ+ZRSIN)                 FF 166
```

The same expressions are used in NEC–2 but are correct there since the sign of ROZ is changed before the image field is computed. The ground-screen reflection coefficients for the near field in NEC–3 are correct. It should be remembered, however, that a vertical monopole at the center of the radial wire ground screen will see an infinite wire density at the reflection point and, hence, the current will be the same as over a perfectly conducting ground.

The effect of this error on the antenna gain will be greater for sparse ground screens and poor ground. For dense ground screens the reflection coefficient is near unity so using the reciprocal makes little difference. In any case it should be remembered that the radial wire ground screen approximation is only valid for relatively dense screens and neglects diffraction from the screen edge.

The following NEC data set can be used to test the ground screen approximation:

```
CETEST OF RADIAL WIRE GROUND SCREEN APPROXIMATION
GW1,5,0.,0.,0.,0.,0.,2.5,.001,
GE1,
FR0,0,0,0,30.,
GN0,100,0,0,10.,.01,3.,.0001,
EX0,0,1,0,1.,
RP4,10,1,1000,0.,0.,10.,0.,
EN
```

* Work performed under the auspices of the U. S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

The input impedance for this monopole, using single precision NEC–3 on a VAX computer, was $40.1092 + j23.5529$ ohms. The antenna gains before and after correcting the error in reflection coefficients are show below.

Antenna gain computed by NEC–3 single precision with error in reflection coefficients for radial wire ground screen:

– – ANGLES – –		– POWER GAINS –		
THETA	PHI	VERT.	HOR.	TOTAL
DEGREES	DEGREES	DB	DB	DB
0.00	0.00	-999.99	-999.99	-999.99
10.00	0.00	-12.14	-999.99	-12.14
20.00	0.00	-6.02	-999.99	-6.02
30.00	0.00	-2.37	-999.99	-2.37
40.00	0.00	0.24	-999.99	0.24
50.00	0.00	2.26	-999.99	2.26
60.00	0.00	2.98	-999.99	2.98
70.00	0.00	2.68	-999.99	2.68
80.00	0.00	-0.05	-999.99	-0.05
90.00	0.00	-999.99	-999.99	-999.99

Antenna gain computed by NEC–3 single precision after correction of error in reflection coefficients:

– – ANGLES – –		– POWER GAINS –		
THETA	PHI	VERT.	HOR.	TOTAL
DEGREES	DEGREES	DB	DB	DB
0.00	0.00	-999.99	-999.99	-999.99
10.00	0.00	-12.20	-999.99	-12.20
20.00	0.00	-6.15	-999.99	-6.15
30.00	0.00	-2.61	-999.99	-2.61
40.00	0.00	-0.14	-999.99	-0.14
50.00	0.00	1.63	-999.99	1.63
60.00	0.00	2.45	-999.99	2.45
70.00	0.00	2.19	-999.99	2.19
80.00	0.00	-0.42	-999.99	-0.42
90.00	0.00	-135.40	-999.99	-135.40

MININEC3 UPDATED

This continues the update mechanism for MININEC3 (see ACES NEWSLETTER Vol. 1 No. 2, p.27). The following update will bring the distributed code up to version 10. To add lines of code, it is necessary to use the renumber option in BASIC to step the lines by mc than one, add the appropriate lines, and then renumber by ones. Remember that the code as received will not run under the regular BASIC interpreter. You must compile it or reduce the la arrays which cause it to exceed BASIC's 64K memory limit.

```
1 REM ****** MININEC(3) **********  NOSC CODE 822 (JCL CHANGE 10) 5-1-87
.
.
.
342 REM ********** ADDITION OF LOADS **********
343 IF NL=0 THEN 377
344 F5=2*P*F*1000000!          <----- ADD *1000000 FACTOR
345 FOR I=1 TO NL
346 IF LS(I)<1 THEN 366        <----- CHANGE LINE
347 REM ----- S-PARAMETER LOADS
348 U1=0
349 U2=0
350 D1=0
351 D2=0
352 S=-1                       <----- CHANGE LINE
353 FOR J=0 TO LS(I) STEP 2
354 S=-S                       <----- MOVE OLD LINE 359 TO HERE
355 U1=U1+LA(1,I,J)*S*F5^J
356 D1=D1+LA(2,I,J)*S*F5^J
357 L=J+1
358 U2=U2+LA(1,I,L)*S*F5^L
359 D2=D2+LA(2,I,L)*S*F5^L
360 NEXT J
361 J=LP(I)
362 D=D1*D1+D2*D2
363 LI=(U2*D1-D2*U1)/D
364 LR=(U1*D1+U2*D2)/D
365 GOTO 369
366 LR=LA(1,I,1)
367 LI=LA(2,I,1)
368 J=LP(I)
369 F2=1/M
370 IF C%(J,1)<>-C%(J,2) THEN 372
371 IF K<0 THEN F2=2/M
372 ZR(J,J)=ZR(J,J)+F2*LI
373 ZI(J,J)=ZI(J,J)-F2*LR
374 NEXT I
.
.
.
```

LIGHTNING CURRENT REDISTRIBUTION

BY
W. P. GEREN
B. G. MELANDER
D. L. HALL

SCIENCE AND ENGINEERING ASSOCIATES, INC
701 DEXTER AVENUE NORTH
SEATTLE, WA 98109
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ABSTRACT

This paper presents a multilevel transfer function approach to determining lightning coupling to aircraft circuitry. The first level transfer function relates the airframe current distribution to the lightning channel current. The second level, which is the principal topic of this paper, relates the current in individual wires and shields in various locations within the aircraft to the total current flowing through the surrounding airframe. The third and innermost level relates voltages and currents at the pin level to wire and shield currents using a transmission line model. The source for each successive level is obtained from the currents computed in the preceding level.

The major part of this paper describes a technique for calculating the current distribution on a structure made up of strips having arbitrary conductivities and thicknesses. It can be used to determine currents on wires or shields located within an aircraft, for example in a cockpit or landing gear bay. The fundamental assumptions limiting the approach are that the structure can be locally approximated as two dimensional, the structure is electrically small in the transverse dimension, and, for fields within a closed surface, the structural thickness is small compared to a skin depth.

The REDIST code developed for the two-dimensional current calculations is described along with verification established by comparison with analytical and measured results.

INTRODUCTION

Maximum current and voltage levels on aircraft components or LRU's (line replaceable units) must be predicted for an aircraft struck by lightning to adequately protect the LRU's and thus the aircraft. A multilevel transfer function approach to lightning coupling analysis has been proposed in the past (e.g. Auckland and Wallenberg). This approach is shown schematically in Figure 1. The approach consists of three levels of analysis. The first is to determine the exterior airframe current using method-of-moments codes, either wire grid (e.g. WIRANT) or surface patch (e.g. NEC or SRC). The next level is to calculate interior fields and induced currents on wiring and shields within regions of the aircraft such as the cockpit or fuselage interior. Transfer functions modelling this coupling have been determined in the past by using simple models or direct measurements of aperture, joint, or diffusion coupling. Use of more detailed models specific to the geometry under consideration is a more accurate way to determine the shield transfer functions. The third level in the lightning coupling analysis is to determine the transfer function onto LRU pins by using transmission line models of the wiring with specified LRU load impedances.

This multilevel analysis approach assumes that the coupling among the various levels is negligible, i.e. that the shields and wire bundles inside the aircraft do not have an appreciable effect on the exterior airframe currents and that the wiring inside the shields does not perturb the shield currents. In cases for which the levels are coupled, the model can be modified to include the interlevel coupling. For example, internal metal structure or wire bundle shields in a mostly graphite aircraft will carry considerable current at low frequency. The internal fields will be dependent on the current carried by this structure at low frequencies rather than diffusion through the graphite skin panels. By including some detail of the internal structure in the aircraft WIRANT model, for example, cables in a graphite wing, the inter-level coupling is simulated in the model. Thus, the multilevel approach has the flexibility to accurately model lightning current flow through a mixed material aircraft.

The majority of this paper concerns itself with calculations of shield current transfer functions for shields located in regions of mixed materials. A similar problem was analyzed by Fisher (2) and Burrows (3) for metallic structures of constant cross-section. Their approach was to segment the structure into longitudinal strips, compute the corresponding self and mutual inductances, and obtain the current division among the strips by solving the resulting impedance matrix equation relating the currents to the voltage drop along the structure. This is a two-dimensional quasistatic approximation which yields good results for magnetic fields external to a conducting surface.

With the advent of graphite-epoxy materials in aircraft structures, it has become necessary to predict the distribution of currents among electrically connected conductors having dissimilar conductivities. In addition, since the conductivity of graphite-epoxy is much lower than aluminum, it is necessary to compute internal magnetic fields. This was done in the Burrows-Fisher approach by adding resistive terms to the impedance matrix, corresponding to the sheet resistance of the material. The method of dividing the structure into strips and computing the impedance matrix may be described as the circuit analysis approach to current redistribution. An alternative approach is to pose the problem as an electromagnetic boundary-value problem and use the method-of-moments to compute current distributions. This technique is the fields analysis approach.

An existing method-of-moments approach which could be used for the two-dimensional analysis is the code, WIRANT, since it is able to incorporate arbitrary materials within a model. WIRANT represents an object under analysis such as an aircraft by a wire grid model. However, results obtained with this code for circuit wiring near other structure are not reliable due to field leakage through the grid. This is because the fields exhibit an artificial spatial variation near flat surfaces represented by wire grids due to field concentration at the grid elements. An example of this irregularity is seen in Figure 2a. The example shows an aluminum wire over an aluminum ground plane. This effect can be minimized by including more wires in a model. Wires need to be spaced about as close as the spacing of the shield and the plane. This method soon becomes prohibitive due to the large number of elements needed. Because of this, we have opted to develop and use a two-dimensional magnetostatic code, which represents surfaces as flat strips. Such a code has demonstrably superior performance for accurately computing interior fields and near-surface fields, in cases where the two-dimensional magnetostatic approximation is justified. Figure 2b shows the same case as in 2a using REDIST to calculate the fields. Note that the fields are very uniform around the plane, not localized as in WIRANT.

For metallic and graphite-epoxy materials, there are three frequency regimes of interest within the lightning spectrum. They are, in order of increasing frequency, resistive current division, inductive current division, and diffusion. In the first two, the current density is constant across the sheet thickness, and the current distribution around the structure perimeter is determined by either resistive division, at low frequency, or by inductive division, at frequencies for which the structure inductance dominates its impedance. The diffusion regime is defined as those frequencies for which the sheet thickness is comparable to, or greater than, the skin depth of the material. In this case, the current density across the sheet is concentrated on the outer surfaces. For closed structure, the fields are excluded from the interior regions at high frequency.

The fields internal to a closed metallic structure are insignificant throughout the lightning frequency range because of the long diffusion time through the surface. However, in realistic aircraft structures there are always joints and other discontinuities to produce internal fields through aperture coupling. On the other hand, structure consisting in part or whole of graphite epoxy will have appreciable internal fields which are produced by diffusion through the graphite or direct internal current conduction. Provided that the sheet thickness is less than the skin depth of graphite-epoxy, the internal fields can be computed by current division among the structural members using REDIST.

APPROACH

The basic approach to calculating the currents and fields surrounding a complex multiconductor two dimensional structure is to describe the structure as a series of electrically-connected two dimensional thin strips. The problem is then set up to solve using method of moments techniques. The assumptions fundamental to this approach include:

(1) The structure can be locally approximated as two-dimensional;

(2) Current flow is along the axis of the structure;

(3) The cross-section of the structure is electrically small in the frequency range of interest;

(4) The structure can be approximated by thin strips of appropriate conductivity. The thickness is assumed much less than the skin depth of the material at the frequencies analyzed.

Using these basic assumptions, the electromagnetic equations to be solved are as follows:

$$\nabla^2 A_z = -\mu_o J_z \qquad (1)$$

with the boundary conditions on the conductor surface given by

$$E_z + j \cdot w \cdot A_z = \text{constant} \qquad (2)$$

$$E_z = Z_s * J_z \qquad (3)$$

where Az = axial component of vector potential

Jz = axial component of current density

Ez = axial component of electric field

Zs = surface impedance of conductor

For thin sheets, sheets thinner than the skin depth at the frequency of interest, the surface impedance is given by

$$Z_s = N/(2*\text{TANH}(\Upsilon * T/2)) \qquad (4)$$

where

$\Upsilon = \sqrt{j \omega \mu \sigma}$

T = sheet thickness in meters

μ_o = permeability of free space (4π x E-7)

$N = \sqrt{j \omega \mu / \sigma}$

Equation (1) is the usual magnetostatic approximation for fields in the presence of a current source for electrically small sources. Equation (2) is a consequence of assuming equipotential surfaces. This is the same assumption Burrows made in his analysis (3) and implies this analysis is only valid for good conductors. Equation (4) gives the expression for the surface impedance. This equation was derived for thin sheets with thicknesses smaller than the skin depth and assumes constant Ez across the sheet.

The solution to equation (1) can be written in terms of a 2-D Green's function as shown in equation (5).

$$A_z = \mu_o \int \text{LN}(1/(|\vec{X} - \vec{X}'|)) \; J_z(\vec{X}) \; D\vec{X}' \qquad (5)$$

The integral is performed over the width of the current sheet assuming no variation over the thickness, T.

Using the method of moments approach, the conducting surface is subdivided into small strips assumed to carry constant current density. The integration in equation (5) then becomes a sum of integrations over constant current carrying strips. This integration can be done exactly using the above assumptions. Therefore, Az can be expressed as a sum over the unknown currents times an analytic function dependent on relative location of the current strips, their orientation, and their width.

The method of moments approach for calculating Az coupled with equations (2)-(4) give a matrix equation to be solved for the current density in each strip as shown in equation (6).

$$J_x * N/(2*\text{TANH}(\Upsilon * T/2)) + j\omega \sum_j J_j F_{ij} = \text{CONSTANT} \qquad (6)$$

where Fij is a function of the strip width, orientation, and location. It is derived from the exact integration of equation (5) over a strip of constant current.

The REDIST code was developed to implement the solution of this equation. The boundary conditions of equation (3) and (4) are imposed at the centers of the strips. To use the code, the geometry is input with the various conductors broken into thin strips. The currents on each strip are calculated for each frequency chosen. To check that the strip division is an adequate representation of the geometry, strips are further subdivided until the current distribution no longer changes. For many geometries this process only involves a couple of trials. The code can also calculate the electric and magnetic fields throughout the geometry. Examples are given in the succeeding sections.

VERIFICATION OF REDIST

In order to verify the accuracy of REDIST, it was compared against test cases which were relevant to the geometries of interest, as well as calculable by simple analytic expressions. Two classes of tests were defined, i.e., a wire over a ground plane and the fields in a rectangular trough. The REDIST and analytic calculation comparison was excellent for all cases considered.

Wire over ground plane: Three cases were considered for this geometry, namely, inductive division between a wire and ground plane, resistive division between wire and a ground plane, and a resistive wire over a perfect conductor. The geometry and parameters chosen for each case are shown in Figure 3. The first two cases yield almost frequency-independent transfer functions for the wire. The transfer functions for these cases are shown in Figures 4a and 4b. The transfer function for the third case is shown in Figure 4c. It varies proportional to frequency at low frequency where the wire is resistive and levels off when the wire inductance becomes important. The comparison of the REDIST and analytic results are listed below:

Table 1

Comparison of REDIST and Analytic Calculations
Wire Over Ground Plane

Inductive division:

analytic result = -35.0 dB
REDIST result = -35.3 dB

Resistive division:

analytic result = -54.9 dB
REDIST result = -54.9 dB

Resistive wire over perfect conductor:

Frequency(Hz)	Amplitude(dB)		Phase(degrees)	
	analytic	REDIST	analytic	REDIST
1E1	-130.63	-130.48	90.00	89.90
1E2	-110.63	-110.49	89.99	89.96
1E3	-90.63	-90.50	89.91	89.89
1E4	-70.63	-70.50	89.04	89.01
1E5	-50.75	-50.63	80.44	80.19
1E6	-36.47	-36.50	30.71	30.00
1E7	-35.17	-35.29	3.40	3.24

Fields in rectangular trough: The magnetic field at the bottom center of a rectangular trough was calculated for a range of depth-to-width ratios and compared with an analytic result obtained by a conformal transformation. All REDIST calculations were performed with the same segmentation of the geometry.

Table 2
Comparison of REDIST and Analytic Field Calculations
Rectangular Trough

Depth/Width	analytic	REDIST	Error
1.	6.36E-2	6.45E-2	1.4%
1.67	7.83E-3	8.08E-3	3.2%
2.5	5.71E-4	5.50E-4	-3.7%
3.33	4.17E-5	3.93E-5	-5.8%

SCALE MODEL TEST

A scale model test of REDIST was made with a rectangular box. The box was coated with copper paint to make the box slightly resistive (the total box resistance was measured to be 0.25 ohms). The box geometry and test setup is shown in Figure 5. The box measured 6 inches square in cross-section and 15 inches long. A wire was laid along one face, 1.5 inches from the face and terminated in a variety of resistances to the ground plane. The box was enclosed in an outer chicken wire box mounted on an aluminum sheet ground plane. The box was taped to the ground plane with copper tape and further painted with silver paint to maintain good joints to the ground plane.

This setup is similar to one which might be used for lightning simulation testing of an object. More uniform fields and current distribution are provided by the coaxial transmission line setup of the test object and the return path. The electric and magnetic fields distribution approximate that for an object such as an aircraft in free space.

The transfer function of the current on the wire relative to the input current was measured for three terminating resistances on the wire, no load, 0.1 ohms, and 0.2 ohms. The measured waveforms are shown in Figures 6a-c. The waveforms were measured from 1 KHz to 100 MHz. These were limits imposed at the lower frequency by noise and at the high frequency by limitations of the current probes (they were rated to 140 MHz). The resonances seen in the data result from transmission line resonances due to the length of the box and the coax cables used for measurement.

Two REDIST models of the test setup were run. The first included the box, the wire, and an added lumped inductance due to the coax used to measure the wire current. This geometry is shown in Figure 7a. The second model included the return cage as shown in Figure 7b. The return cage made no difference on the transfer functions calculated for the wire. This is due to the symmetry of the geometry. The resistance of the box was measured to be 0.25 ohms. The resistance of the wire and short length of coax was measured to the ground plane and was 0.062 ohms. These resistances were used to determine the input parameters for thickness for the REDIST models. The wire was modelled as a hexagon with a radius, thickness, and conductivity corresponding to a 22 AWG wire. Extra lumped resistance was added to the model to include the short length of coax and the added resistors used for the three tests. An added lumped inductance was added to the REDIST model equal to the measured inductance of the short coax length. Thus all input parameters to REDIST were measured resistances, inductances, and lengths. No other parameters were included.

Comparisons of the three termination resistances are shown in Figures 6a-c with the triangles in the figures representing calculated values. A more detailed comparison is shown in Table 3. The results are very good at all frequencies. The largest error is at 100 KHz where the agreement is off by at most 1.8 dB. This amounts to an error of 23%. Other frequency results show agreement ranging from 0.1 dB to 0.8 dB. This is excellent agreement showing that REDIST can predict currents in simple geometries. More extensive testing of REDIST with more complex test setups and including more use of different materials is needed to benchmark the code for reliability in use as an aircraft coupling code.

Table 3

Comparison REDIST and Measured Data

R load = 0, no extra load

Frequency (Hz)	Calculated (dB)	Measured (dB)
1E3	-1.9	-1.8
1E4	-2.0	-2.3
1E5	-5.7	-7.5
1E6	-23.2	-22.7
1E7	-44.0	-43.2

R load = 0.1 ohms

Frequency (Hz)	Calculated (dB)	Measured (dB)
1E3	-4.3	-4.3
1E4	-4.4	-4.6
1E5	-6.9	-8.7
1E6	-23.2	-23.0
1E7	-44.0	-43.2

R load = 0.2 ohms

Frequency (Hz)	Calculated (dB)	Measured (dB)
1E3	-6.2	-6.3
1E4	-6.3	-6.6
1E5	-8.0	-9.7
1E6	-23.3	-23.2
1E7	-44.0	-43.4

MODELLING EXAMPLE

An example of the use of REDIST is given in this section. This example is meant to be simple and instructive in the use of this code. The model is shown in Figure 8. The model is a box with three metal sides and one graphite epoxy side. The model geometry is read from a data file. The data input includes the box geometry, the number of segments the box is to be divided

into (variable depending on frequency and geometry), the thickness of the box walls, and the conductivity.

Inside the box is a test wire in various locations as shown by the numbers in the figure. REDIST calculates the current on the test wire relative to the input current for the total structure and writes the result to a data file. This transfer function is shown in Figure 9.

These transfer functions can be multiplied by the transfer function from a WIRANT model, for example, and then multiplied by the lightning threat spectrum, for a complete analysis of shield currents. The WIRANT model current transfer functions can be made more accurate by including wire models of the metal walls of this geometry as part of an aircraft model. The total current flowing through these walls and the graphite is then added in WIRANT and used as input to the REDIST model. This gives a two level transfer function approach to calculating shield currents using WIRANT and REDIST.

A simpler and more severe estimate of shield currents is to simply multiply the shield transfer function obtained from REDIST by the source current, e.g. a double exponential lightning spectrum. The resulting current pulse waveforms calculated on the test wire in the various locations are shown in Figure 10. The peak current levels vary over an order of magnitude. The induced current is small for locations well away from the graphite lid and near the metal bottom plate and increases steadily as the wire is moved toward the graphite away from the metal. These results show that the REDIST code could be useful in evaluating potential wiring locations.

The currents on the other sections of the box can be output as well. Both electric and magnetic fields components can be calculated at specified points. The magnetic field inside the box is shown in Figure 11a and 11b. The contours shown are the same levels for the two cases shown. The fields plots shows the field leakage through the graphite top at low frequency (11a) and the exclusion of these fields at higher frequency (11b).

LIMITATIONS OF REDIST

There are several limitations to the use of the REDIST code. The first is that the geometry under study be approximately two dimensional. Often the current flow is in one predominant direction so this condition is satisified. The second limitation is high frequency validity. The upper frequency limit is determined when the thickness of the conductors is about equal to the skin depth of the conductor. For many instances this upper limit is above the lightning frequency spectrum for thin metallic skins and webbing and most graphite structure. Only thicker busbars and larger structural members may not fit this requirement. Another frequency limitation is that the entire structure is to be considered electrically small. The last limitation applies to the geometrical complexity to be modelled. The number of segments used in the model is limited by the capacity of the computer and time restrictions. Models have been run to date on IBM-PC compatible computers. The largest model run has been 82 segments. This model took about 3 1/2 hours to run 51 frequencies. Fairly large models (100 or so segments) can be easily run using REDIST on microcomputers making this code inexpensive to use for lightning coupling analysis.

SUMMARY

This paper has described an approach to calculating currents on a multiconductor structure in the event of a lightning strike using multiple transfer functions. The approach is based on a code developed to calculate transfer functions on shields or structure within an aircraft structure such as in a cockpit or landing gear area. The approach can calculate the current on any piece of the structure, wire or shield as a function of frequency. These currents can then be multiplied by transfer function results from an aircraft model using for example, WIRANT, and then multiplied by the source current. The resulting frequency domain waveform can then be Fourier transformed to give the current pulse expected on that shield or wire. This code is expected to improve the estimates of lightning currents induced on internal or external wires or shields for aircraft lightning protection with special application to mixed graphite/metal aircraft.

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3. Wallace, B.J.; Burrows, B.J.; Zeitler, R.T.; Ledwig, A.J.; Wiles, K.G.; "Composite Forward Fuselage Systems Integrations"; AFFDL-TR-78-110, Volume 2, May 1978.

MULTILEVEL LIGHTNING COUPLING ANALYSIS

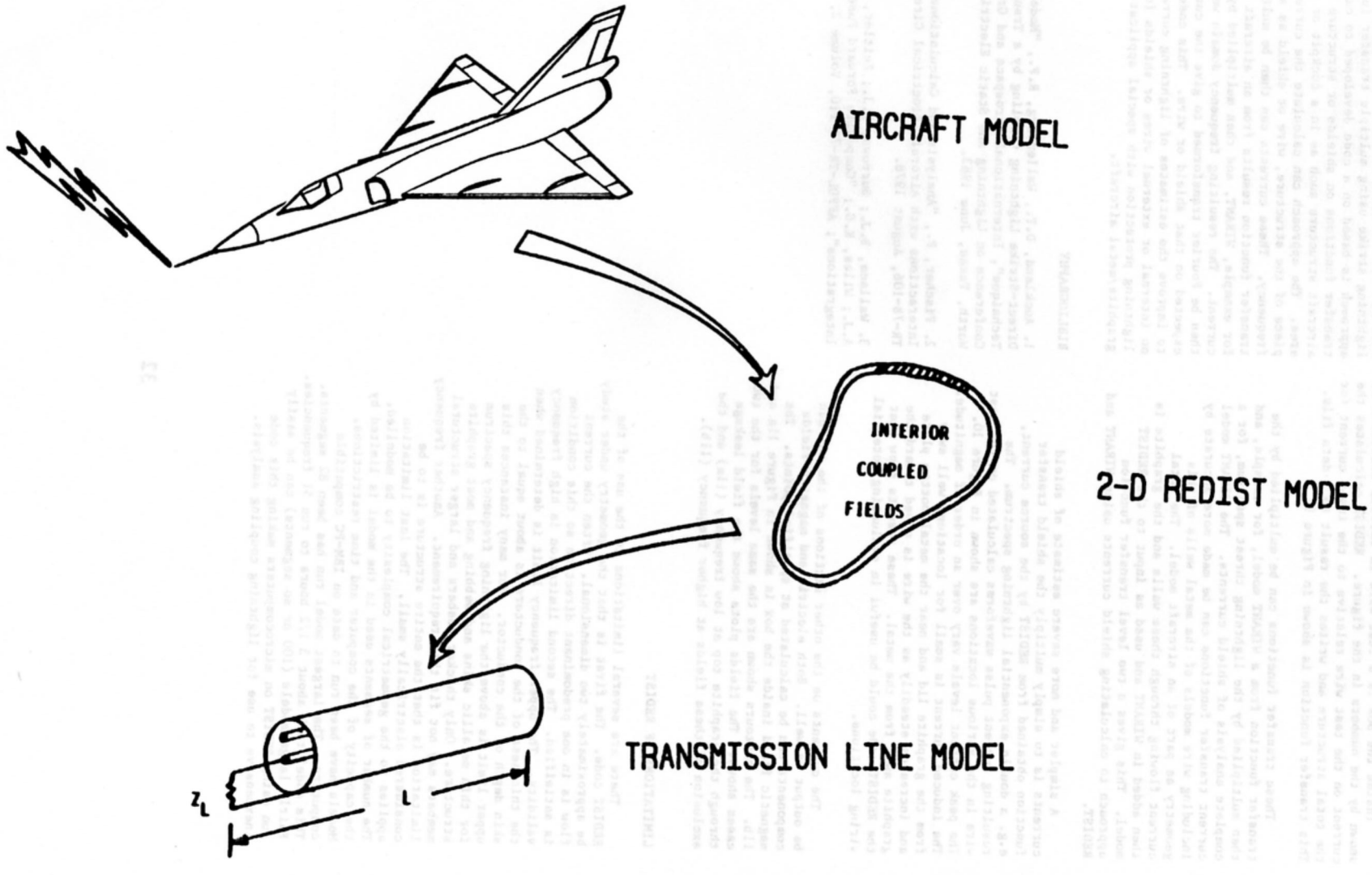


Figure 1 - Multilevel Lightning Coupling Analysis Approach

ALUMINUM WIRE OVER ALUMINUM PLANE

CONTOURS OF CONSTANT VECTOR POTENTIAL

CALCULATIONS DONE WITH WIRANT CODE

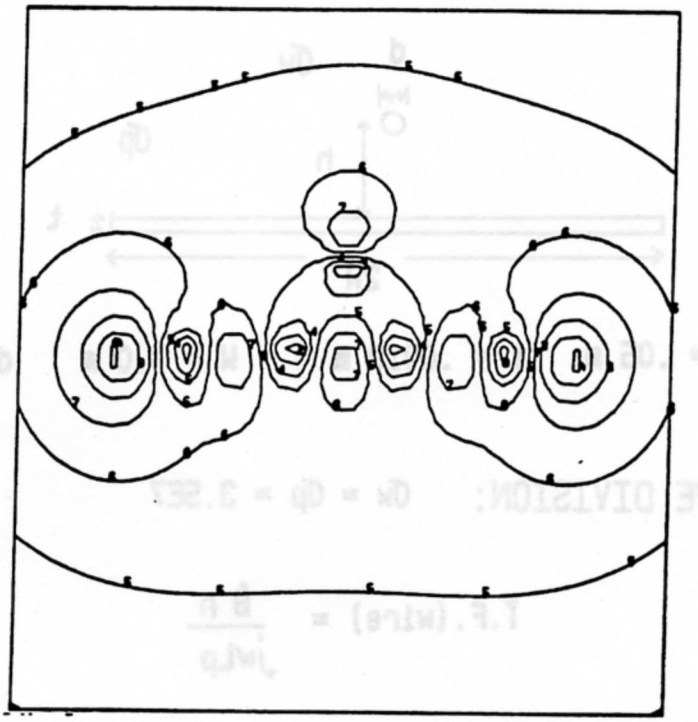


Figure 2a - Magnetic Field Contour Plot for Wire over Groundplane Using WIRANT Code

CALCULATIONS DONE WITH REDIST CODE

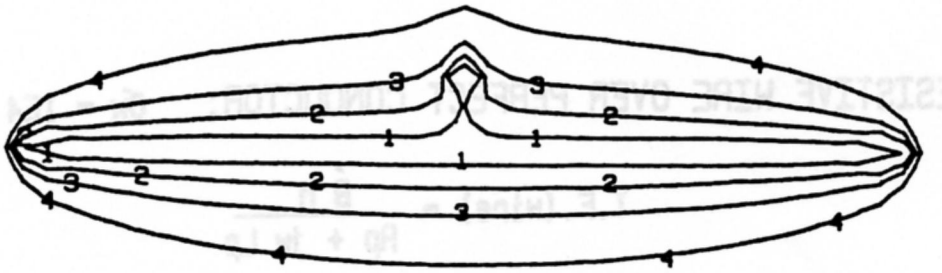
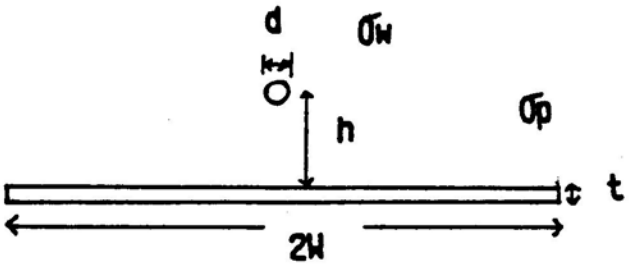


Figure 2b - Magnetic Field Contour Plot for Wire over Groundplane Using REDIST Code

WIRE OVER GROUNDPLANE

REDIST CODE VERIFICATION



h = .05 m t = .0013 m W = 1.0 m d = .012 m

INDUCTIVE DIVISION: $\sigma_w = \sigma_p = 3.5E7$

$$T.F.(wire) = \frac{\dot{B}\, h}{jwLp}$$

RESISTIVE DIVISION: $\sigma_w = 1E2 \quad \sigma_p = 1E3$

$$T.F.(wire) = \frac{Rp}{Rw + Rp}$$

RESISTIVE WIRE OVER PERFECT CONDUCTOR: $\sigma_w = 1E4 \quad \sigma_p = 1E12$

$$T.F.(wire) = \frac{\dot{B}\, h}{Rp + jw\, Lp}$$

where $\dot{B} = \frac{j w \mu_o}{2\pi W}$ $\qquad Lp = \frac{\mu_o}{2\pi} \ln \frac{4h}{d}$

Figure 3 - Wire Over Groundplane Geometry and Parameters for REDIST Code Verification

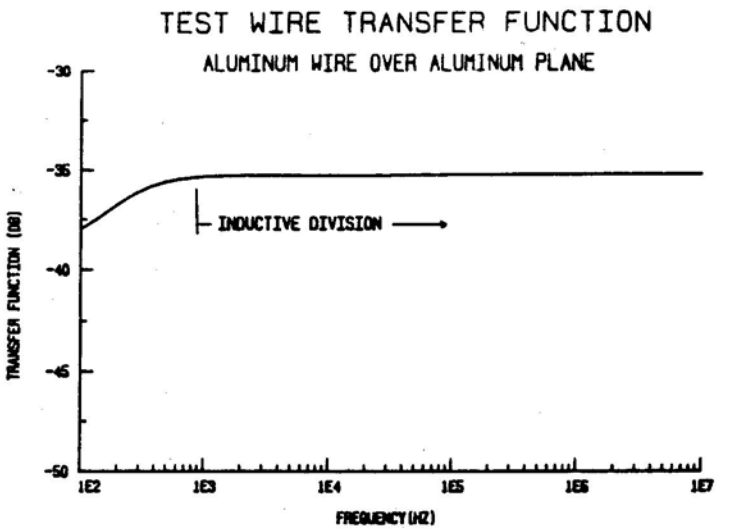


Figure 4a - Wire Over Groundplane -- Inductive Division

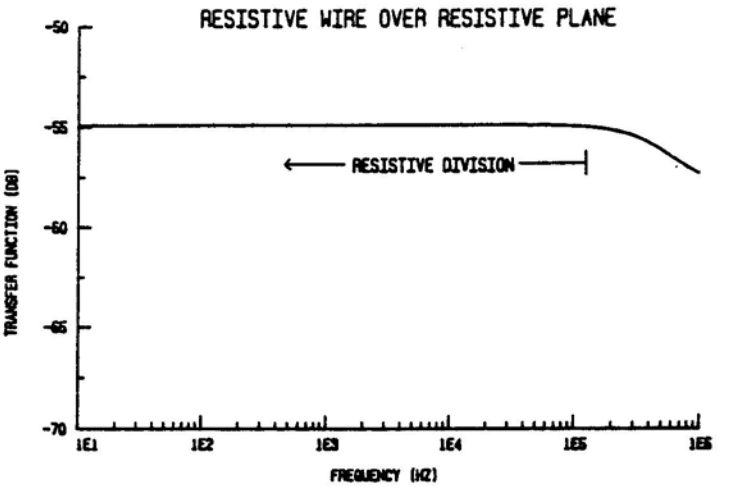


Figure 4b - Wire Over Groundplane -- Resistive Division

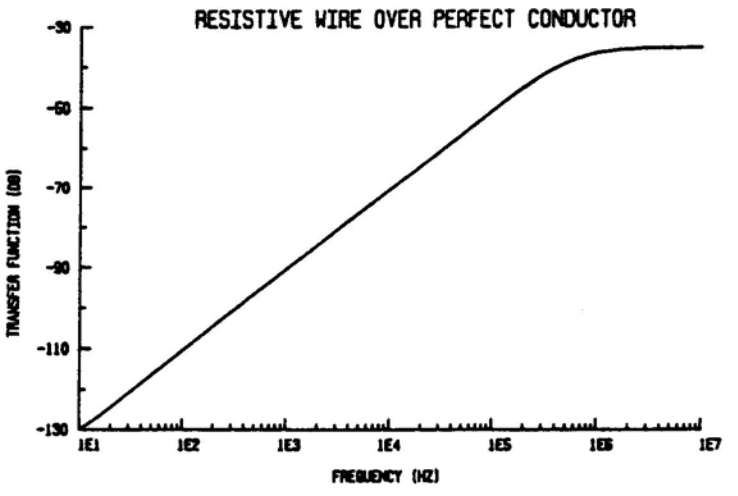


Figure 4c - Resistive Wire Over Perfect Conductor

SCALE MODEL TEST GEOMETRY

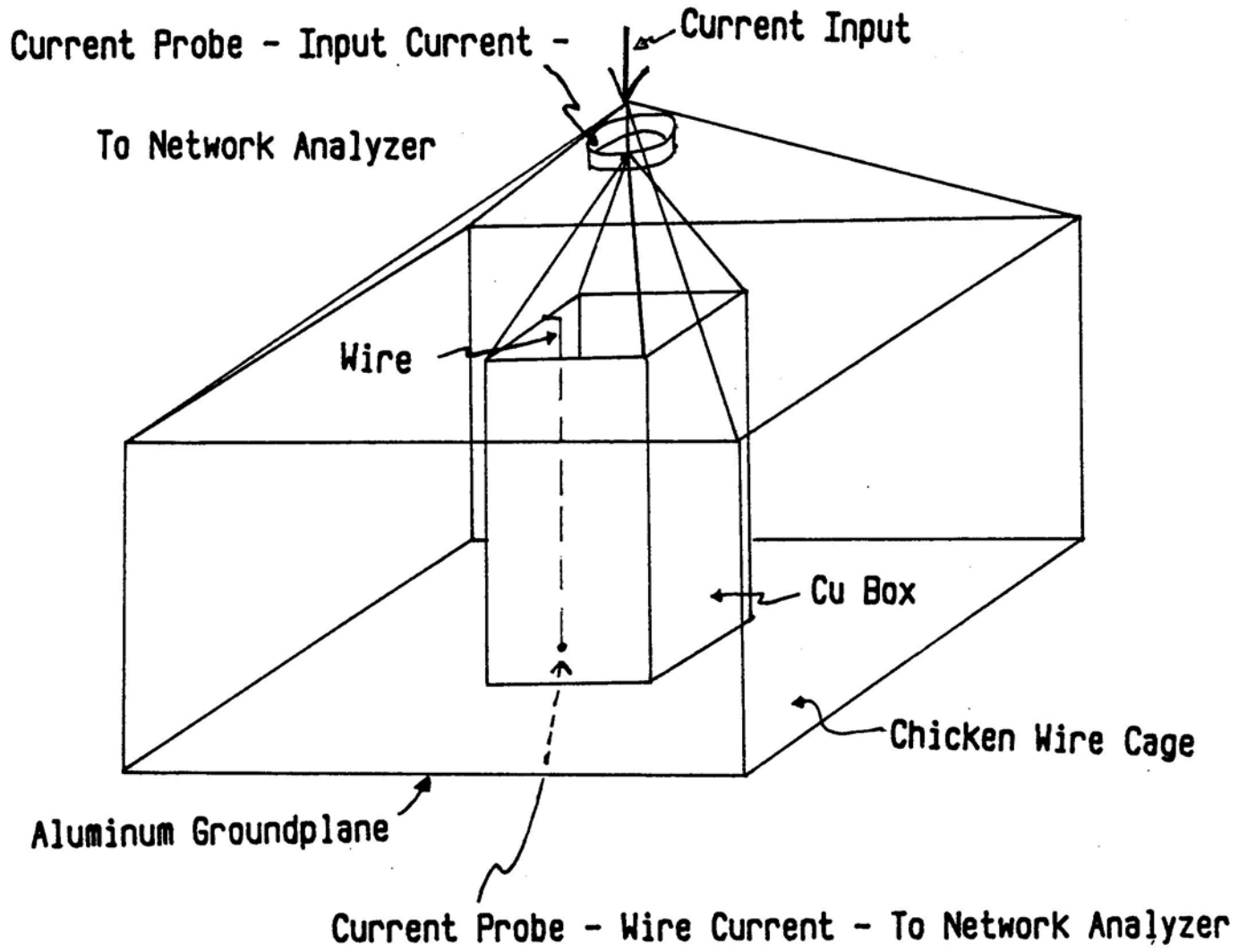


Figure 5 - Scale Model Test Geometry

WIRE TRANSFER FUNCTION

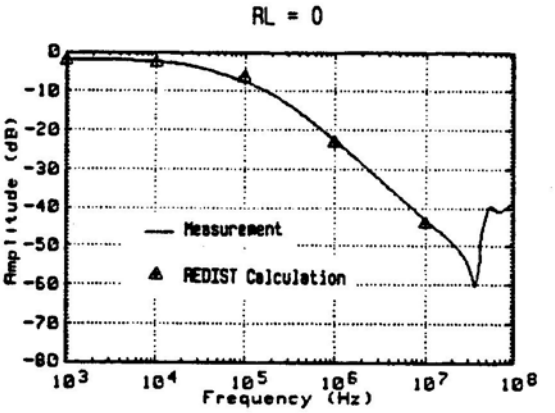


Figure 6a - Wire Transfer Function Measurement Compared to REDIST Calculations -- No Load Case

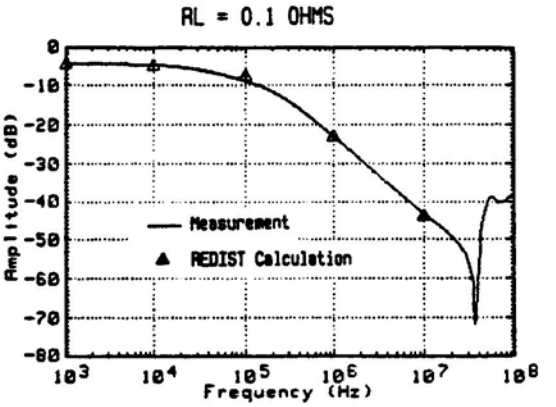


Figure 6b - Wire Transfer Function Measurement Compared to REDIST Calculations -- Load Resistance = 0.1 ohms

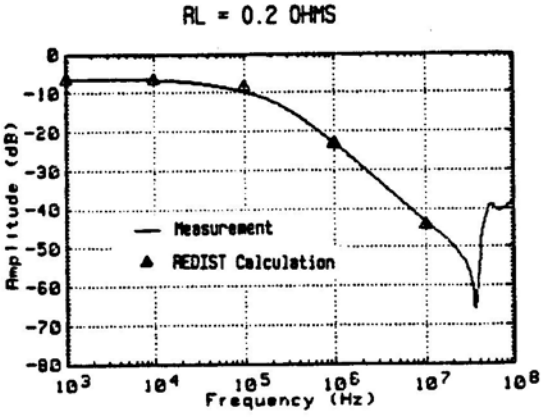


Figure 6c - Wire Transfer Function Measurement Compared to REDIST Calculations -- Load Resistance = 0.2 ohms

SCALE MODEL TEST

REDIST MODEL

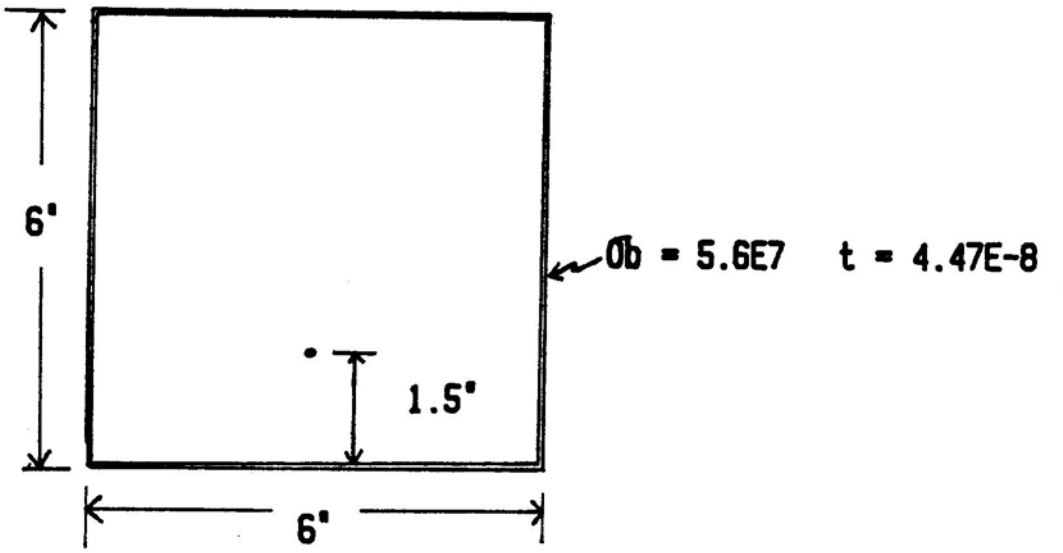


Figure 7a - REDIST Geometry Used for Scale Model Test Calculations

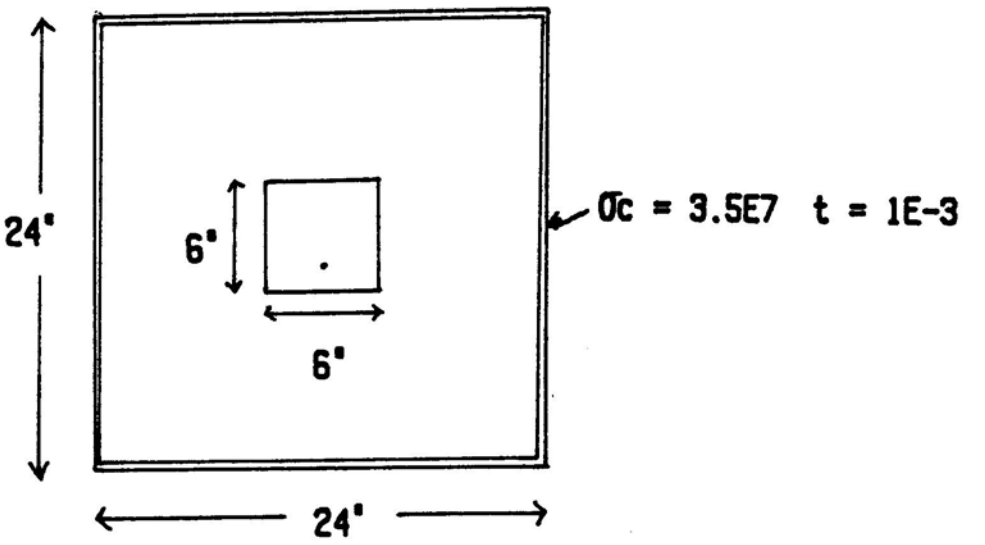
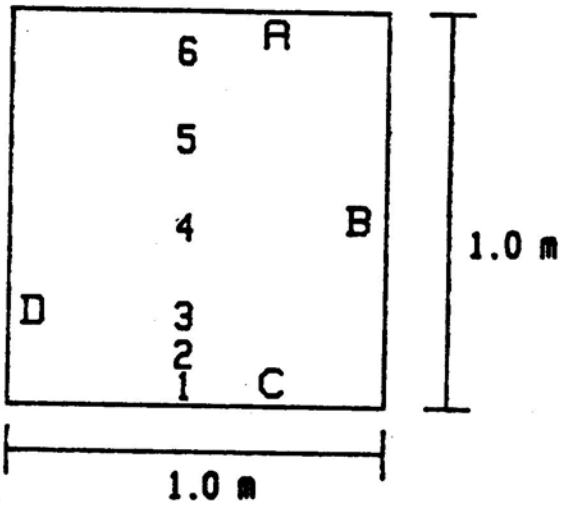


Figure 7b - REDIST Geometry Used with Return Cage Used for Scale Model Test Calculations

METAL BOX WITH GRAPHITE LID

REDIST MODEL



Wire Locations Denoted by #s 1-6

WALL	STRUCTURAL PARAMETERS	
	THICKNESS (m)	CONDUCTIVITY (mho/m)
A	.0023	1.5 E4
B	.0013	3.5 E7
C	.0013	3.5 E7
D	.0013	3.5 E7

Figure 8 - Geometry of Metal Box with Graphite Lid

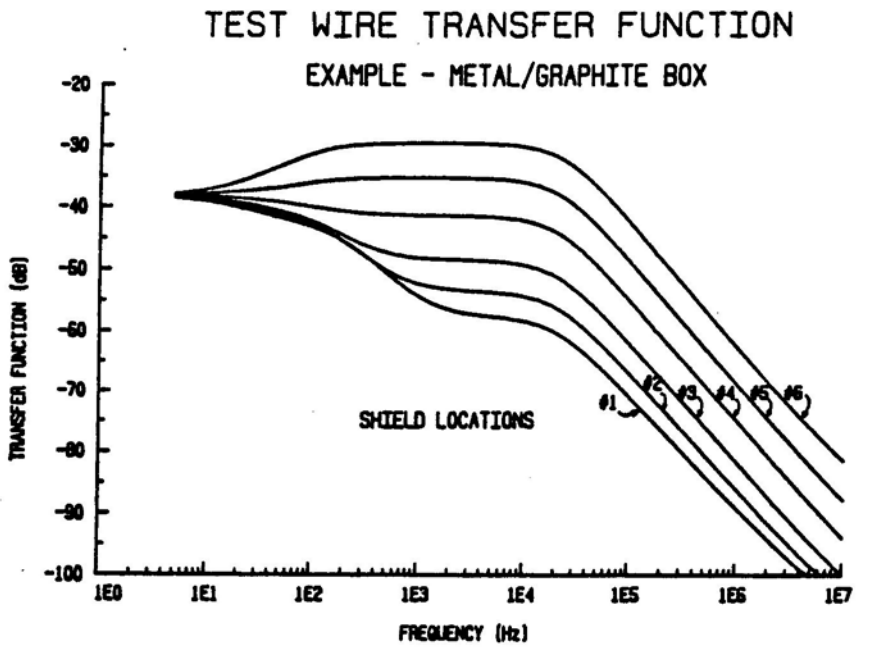


Figure 9 - Wire Transfer Function for Metal/GE Box for Several Wire Locations

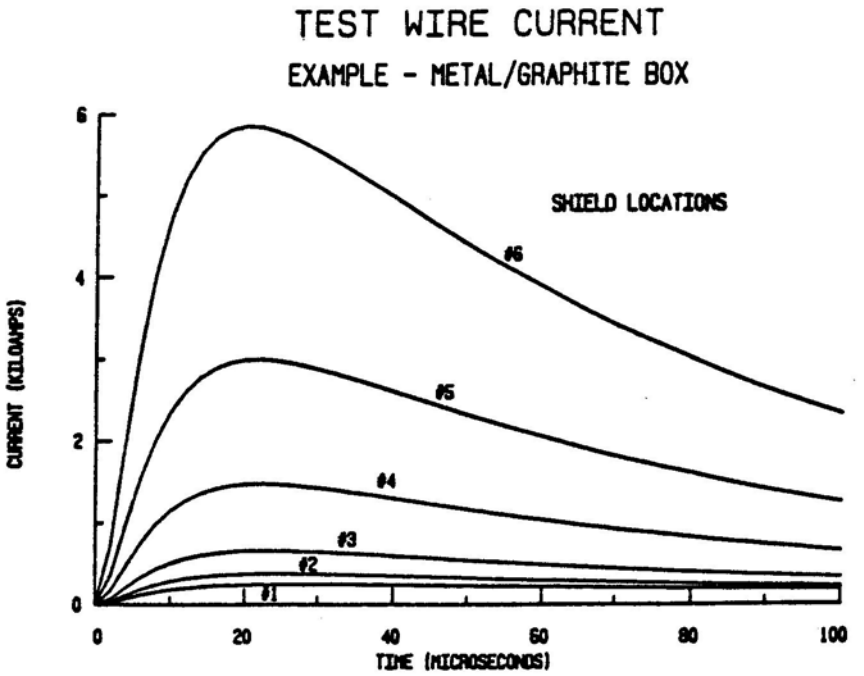


Figure 10 - Wire Current for Metal/GE Box for Several Wire Locations

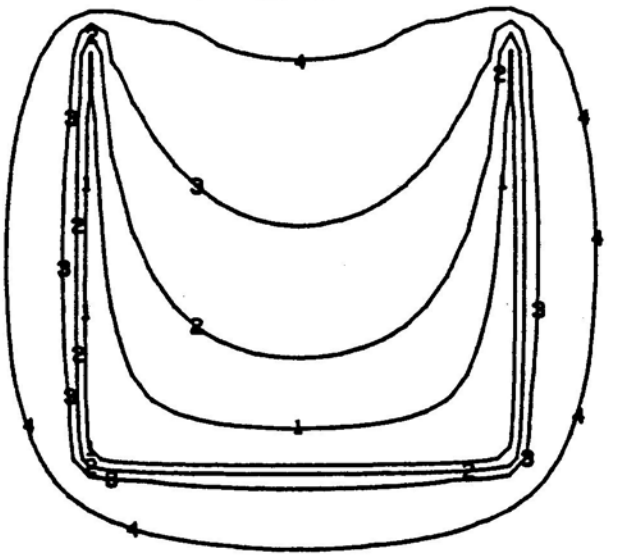


Figure 11a - Magnetic Field Contours for Metal/GE Box at 1 KHz

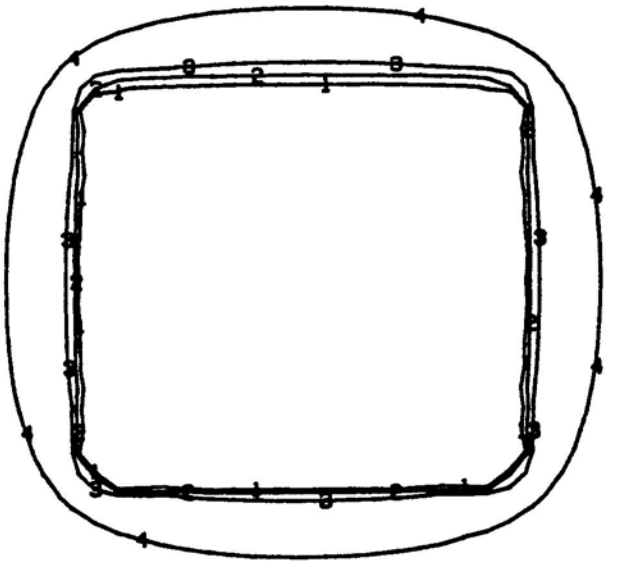


Figure 11b - Magnetic Field Contours for Metal/GE Box at 1 MHz

ITERATIVE METHODS: WHEN TO USE THEM FOR COMPUTATIONAL ELECTROMAGNETICS

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ABSTRACT: The question of when to use an iterative method is addressed from the perspective of the electromagnetics (EM) computer code user. Recent research suggests that, at least in some situations, iterative methods offer significant advantages over direct methods of solution. The circumstances surrounding these situations are investigated, and the features favorable to iterative methods are identified. The paper also includes suggestions for incorporating iterative algorithms into existing computer codes.

1. INTRODUCTION

There has been considerable research activity directed toward the development of iterative techniques for electromagnetic (EM) radiation and scattering problems [1]-[24]. However, from the perspective of the EM computer code user, the fundamental question of when to use an iterative solution method is seldom addressed. There are generally three situations for which iteration may be recommended over direct methods of solution. This paper will investigate these situations and attempt to clarify the degree to which iterative methods may actually be useful. In circumstances where iteration is believed to offer advantages over direct methods, suggestions are given for implementing an iterative algorithm within an existing code. Many iterative algorithms are in widespread use, and no comprehensive survey is made here. However, the observed performance of one iterative algorithm, the conjugate gradient (CG) method, is reported for electromagnetics applications.

Recent research suggests that iterative methods offer advantages over direct methods in three situations. The first involves ill-conditioned equations. Since direct methods of solution fail for badly conditioned systems, it is often argued that iteration may be the only stable way to solve these equations. The second situation arises if the convergence of an iterative method is very rapid, either as a result of a good initial estimate of the solution, minimal accuracy requirements, or other unique properties of that particular problem. If the convergence rate of an iterative algorithm is fast, the computational cost (i.e., CPU time) may favor the use of iteration over a direct method. Finally, some equations discretize to produce sparse matrices or matrices with special

structure. Iterative algorithms can easily exploit any redundancy in the matrix elements to reduce computer memory requirements, yielding benefits for large-order systems (reduced CPU time due to fewer out-of-core storage transfers).

Although these situations may favor the use of iteration, the tradeoff between iterative and direct methods of solution is also dependent on the machine in use (the word length, the amount of available fast memory, and the presence or absence of specialized architectures such as pipeline processors). Thus, it is impossible to make definitive recommendations on the general use of iterative solution methods. Instead, the information presented here is intended to aid the user in making this decision for himself.

To understand the role of iteration in computational EM, it is helpful to note that the numerical solution process essentially consists of two different parts. The first is the discretization process, where the original equation to be solved (differential equation, integral equation, etc.) is converted to a discrete system of finite order (a matrix equation). The second is the solution process, where the resulting matrix equation may be treated using either direct or iterative methods. The position adopted here is that, in general, any combination of discretization procedure and solution algorithm may be used. For clarity, we make a distinction between the terms iterative algorithm (a "black box" useful for solving any discrete system) and iterative technique (i.e., procedures such as the Spectral-iterative technique (SIT) [3], [4], [23] that combine specific discretizations with specific iterative algorithms). Note also that the term direct method usually refers to general Gaussian elimination, although specialized direct methods are also available for the treatment of banded, Toeplitz, and other special systems.

2. ITERATION AND ILL-CONDITIONED SYSTEMS

Gaussian elimination fails because of an accumulation of round-off error when applied to a system whose condition number exceeds a certain limit depending on the machine word length and the matrix order [2], [4], [26]. If implemented in a robust manner, iterative methods do not generally suffer from a buildup of round-off error, although they do suffer from round-off error within each iteration step. Because of the inherent limitation of Gaussian elimination, iterative methods are sometimes recommended for the treatment of ill-conditioned systems [2], [17]. However, it is only in special cases that iteration will likely be successful for the treatment of badly conditioned equations.

Generally, the convergence rate of iterative algorithms depends on the condition number of the system matrix, and in practice is slow for systems that are ill-conditioned [27]. If

the equation is very badly conditioned, convergence may slow to the point of stagnation [12], [21]. However, if the excitation (i.e., right-hand side of the system) is orthogonal to the eigenvectors that correspond to the near-zero eigenvalues, the convergence rate may be acceptable. For such an excitation, the near-zero eigenvalues will be invisible to the iterative process, so that the system appears to be better conditioned than it actually is. (We will refer to such eigenvectors and eigenvalues as extraneous). Contrived examples illustrating the successful iterative solution to badly conditioned systems usually involve extraneous eigenvalues [2].

Extraneous eigenvalues may arise in electromagnetics applications, especially in circumstances where a high-order discretization is used within the numerical modeling procedure. For instance, consider the numerical treatment of the electric-field integral equation (EFIE), whose spectrum consists of an infinite set of discrete eigenvalues. The discretization process requires the systematic projection of the eigenvalues onto the matrix operator, so that a relatively low-order matrix model of the original equation captures the dominant eigenvalues [24]. As the order of the matrix equation is successively increased, more and more eigenvalues are projected from the continuous operator to the matrix operator. Depending on the problem excitation, eventually the additional eigenvalues will be extraneous. The EFIE has the property that the eigenvalues appearing in the matrix model tend to spread in the complex plane as more and more are projected from the integral operator [24]. If the equation is over-discretized, so that a very large matrix is used in a problem with relatively few important eigenvalues, the condition of the system may degrade because of the extraneous eigenvalues. But since the extraneous eigenvalues are invisible, iterative algorithms can provide a stable solution process, even if the condition number is such that Gaussian elimination fails.

Thus, it is possible that iteration can be used to solve certain systems that are too ill-conditioned for treatment by direct methods. However, in the absence of theoretical support indicating that near-zero eigenvalues (or very large eigenvalues) are extraneous, there is little reason to expect iteration to suceed in the solution of a very ill-conditioned system. In addition, round-off errors may prevent the successful solution of a system even though the near-zero eigenvalues are extraneous.

3. SPEED OF ITERATION VS. GAUSSIAN ELIMINATION

There are a variety of factors that affect the efficiency of iteration compared to Gaussian elimination. Iteration can be terminated after a few digits of accuracy are obtained in the solution, which may be all that is desired or needed for the application at hand. If a good initial estimate of the solution is available, an iterative algorithm may be able to refine the result to necessary accuracy in far less time than required for

the Gaussian elimination solution of the same system (which does not make use of the initial estimate). Sometimes, the iterative algorithm may converge at a very fast rate, reducing the need for a good initial estimate (for instance, fast convergence may result if only a few eigenvectors are excited by the right-hand side). In other words, there are circumstances where iterative algorithms require less computation than Gaussian elimination. It is difficult, however, to identify these situations except by trial and error. In practice, it is much more likely that the iterative algorithm requires more computation than elimination, which helps explain why iteration is not often recommended for the treatment of general matrix equations (as opposed to systems with sparsity or special structure, as discussed in Section 4) [26]. A second difficulty lies with the fact that many of the iterative algorithms in widespread use will diverge for the indefinite complex-valued equations usually arising from EM problems [1]-[4], [6], [9]-[11], [19].

The conjugate gradient (CG) method [25]-[28] is one iterative algorithm that has generated widespread interest for EM applications. For general complex-valued systems, the CG method can be guaranteed to converge if applied to the normal equations (the system obtained by premultiplying the original matrix equation by the transpose-conjugate of the system matrix). In a recent article discussing the convergence of the CG method when used with EM integral equations, it was observed that the algorithm typically required N/3 iterations to produce a reasonably accurate solution, where N is the order of the matrix equation [21]. (The details of this study are summarized in Section 6.) These findings indicate that, for EM problems likely to be encountered in practice, the CG method will typically require twice as much computation per solution as Gaussian elimination (unless a very accurate initial estimate of the solution is available). Thus, it appears that the use of the CG method will not be cost-effective unless some storage reduction feature is present in the problem (see Section 4).

There is a significant drawback when using iterative methods to treat a system for multiple excitations. EM scattering problems frequently require the treatment of numerous independent excitations (i.e., waves incident on a scatterer from many different directions). If Gaussian elimination is used to factorize the system matrix into a lower triangular and upper triangular part, an implicit inverse matrix is available that can be used to treat any number of right-hand sides efficiently. Iterative methods do not generate an implicit inverse, and thus suffer in comparison when used for problems requiring the treatment of multiple right-hand sides. As a result, when comparing the speed of an iterative algorithm to that of Gaussian elimination, the number of right-hand sides likely to need treatment must be taken into account.

An example of an iterative technique whose utility seems to lie primarily with relative speed is the hybrid-iterative method (HIM) recently proposed for the solution of certain EM scattering

problems [15], [20]. Specifically, the approach involves the iterative solution of the magnetic-field integral equation (suitably discretized) for perfectly conducting scatterers. An asymptotic approximation is used to construct an accurate initial estimate of the solution, after which a simple iterative algorithm is used to refine the result. In a variety of examples the procedure appears to be more efficient than a comparable approach using Gaussian elimination, at least for single excitations [20]. The efficiency of the iterative procedure is apparently due to the good initial estimate of the solution. A potential drawback to the procedure is that the simple iterative algorithm involved will sometimes diverge.

4. ITERATION TO EXPLOIT STORAGE REDUCTION FEATURES

In contrast to the HIM discussed above, most of the iterative methods proposed for computational EM problems exploit some type of structure or sparsity in the discrete system [1], [3]-[7], [9]-[13], [22]. General purpose Gaussian elimination algorithms require the full NxN matrix to be stored in computer memory, which places a bottleneck on the solution process. Even if the system matrix posesses significant structure, general direct methods require the storage of the redundant elements. Although specialized direct methods are sometimes appropriate for the problem of interest (such as those for treating Toeplitz systems [29]), iterative methods offer the possibility of treating more general systems. For example, slightly perturbed Toeplitz systems (frequently arising in certain EM scattering problems [16]) can not usually be treated with conventional Toeplitz routines.

Differential equations discretize to produce sparse matrix equations, and this sparsity can be exploited with specialized direct solvers or with iteration. (Direct solvers are used in conjunction with schemes for optimum row and column ordering to compress the bandwidth and minimize the fill-in during elimination [30].) Although direct methods have grown in popularity for the treatment of sparse systems, iterative algorithms preserve more of the original sparsity and remain the method of choice for many problems [27]. Differential equation methods are directly applicable to closed-region EM problems [31], and progress has been made toward applying them to open-region EM problems [32].

Often, open-region EM scattering problems are posed in terms of integral equations. Unfortunately, seldom in practice do the associated matrices posess significant structure. The primary exception involves integral equations with convolutional kernels, which when applied to certain geometries discretize to yield matrices with discrete-convolutional symmetries. An example is the Toeplitz structure seen in linear antenna problems [29]. Procedures such as the K-space method [1], the Spectral-iterative technique [2], [4], [6], [9], [23] and the Discrete-convolution

method [5], [18] all exploit discrete-convolutional structure via an iterative solution algorithm. The types of scattering geometries treated in this manner include flat plates and surfaces of constant curvature [9], [13], dielectric bodies [4], [6], [7], [16], and frequency selective surfaces [3], [19]. Details on this type of discretization procedure are available in the literature [1], [3]-[7], [9]-[13], [22], [23]. Unfortunately, arbitrarily shaped structures that are convenient to analyze with surface integral equations (i.e., bent wires, airplanes, etc.) do not fall into the class that naturally produce discrete-convolutional symmetries in the associated system matrix. This may limit the use of iteration for treating integral equations.

5. REMARKS ON ITERATIVE ALGORITHMS AND ELECTROMAGNETICS

There are a variety of iterative algorithms in use in all areas of science and engineering, and many of these are described in detail in texts on numerical analysis [25]-[27]. The simplest iterative algorithm is the Jacobi procedure [26]. The K-space method [1], the Spectral-iterative technique [3], [4], the Discrete-convolution method [5], [18], and the Hybrid-iterative method [15], [20] all initially used a form of the Jacobi algorithm for the solution of the discrete system. The Jacobi algorithm has low storage overhead and can be implemented in a few lines of code. Unfortunately, the Jacobi algorithm is not theoretically guaranteed to converge for general complex-valued equations. The EM research community has directed considerable effort toward improving the convergence of this type of algorithm, without any effective remedy for most of the problems of interest [6], [9], [11], [19].

In recent years, the convergence problems associated with the simple Jacobi procedure have been circumvented by the use of the conjugate gradient (CG) algorithm [25]-[28]. The CG method is based on the minimization of a quadratic functional, and if implemented in a robust manner will never diverge. It requires more storage and computational overhead than the Jacobi procedure, and if used with the normal equations is subject to more round-off error per iteration step. For ill- conditioned systems and finite precision machine arithmetic, the CG method sometimes stagnates. However, in most cases the improvement in convergence reliability (posessing an algorithm that will not diverge) appears to outweigh these potential drawbacks.

As mentioned in Section 3, the convergence behavior of the CG method has been observed when used to treat integral equations representing a variety of EM scattering problems [21]. Most of the time, the algorithm requires between N/4 and N/2 iterations to converge to necessary accuracy, where N is the order of the matrix equation under consideration. This behavior is based upon the use of a zero initial estimate of the solution and a discretization involving subsectional expansion and testing functions. The convergence rate is actually determined by the

number of important eigenvalues (as opposed to extraneous eigenvalues) projected onto the system matrix, and thus is not really a function of matrix order. However, for a discretization density of approximately 10 cells per linear wavelength, and a plane wave excitation, the number of important eigenvalues usually falls in the range from N/4 to N/2. For higher cell densities, the convergence rate is faster relative to matrix order.

6. INCORPORATION OF ITERATION INTO EXISTING CODES

The incorporation of iteration into an existing computer code need not be a difficult task. In general, an iterative algorithm can be a "black box" much like the Gaussian elimination routines used, for example, in the LINPACK library [33]. Iterative algorithms only require the presence of an implicit matrix operator, which is a subroutine that when given a column vector returns the product of the NxN system matrix with the column vector. (If the CG algorithm is used with the normal equations, a second subroutine providing the transpose-conjugate of the system matrix is also required.) Thus, any structure or sparsity may be taken into account in the operator subroutine, and need not affect the organization of the iterative algorithm itself. For instance, the discrete-convolutional structure arising from the SIT (see Section 4) can be exploited using the fast-Fourier transform (FFT) as an efficient alternative to conventional matrix multiplication [23]. The FFT can be implemented within the operator subroutine, and kept entirely invisible to the part of the code that performs the iteration.

Because simple iteration schemes sometimes diverge, it might be desirable to construct a package of several different iterative algorithms that could be used in conjunction with each other in an attempt to optimize the speed of solution. If a simple algorithm was found to be diverging, another could be invoked until one with favorable convergence properties is identified. This approach would eliminate one of the drawbacks of the iterative techniques used in the past, i.e., the algorithm might only converge for certain ranges of parameters, leaving the user unable to treat the other cases [1] [6], [9], [19]. At the same time, it would permit the use of simple algorithms for problems where they do converge at a reasonable rate, reducing the overhead as much as possible.

7. SUMMARY

The above discussion has attempted to clarify the use of iterative methods in computational electromagnetics. It is apparent that there are situations where iterative solution methods offer an improvement in efficiency over Gaussian elimination. In practice, this improvement is based upon the

relative speed and storage constraints associated with a given procedure in a specific machine environment. Because these constraints are not universal, the EM computer code user must draw his own conclusion based upon the specific problem of interest. In the absence of one of the three favorable situations outlined above, it is likely that iterative algorithms may prove inferior to direct methods of solution, especially if multiple excitations are to be treated.

The author welcomes feedback from the reader concerning the use of iterative methods for electromagnetics applications.

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The Radial Functions of Spheroidal Wave Functions For High Aspect Ratio

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ABSTRACT

Since the series of the spheroidal Neumann functions and the associated Legendre functions of the second kind converge very poorly at small radial argument ξ for both prolate and oblate, the series expansions of the prolate radial functions in powers of (ξ^2-1), and of the oblate radial functions in powers of (ξ^2+1) are particularly useful in calculating the spheroidal radial functions of the second kind, $R_{mn}^{(2)}$. This paper presents new techniques for obtaining the functions $R_{mn}^{(2)}$ and their coefficients a_r^{mn} contained in the first part and b_r^{mn} in the second part of the functions $R_{mn}^{(2)}$ for prolate as well as the coefficients α_r^{mn} and β_r^{mn} for oblate. In order to compute the scattering by spheroids with high aspect ratio especially for large size parameter, the explicit forms of a_r^{mn} and α_r^{mn} generated therefrom are listed up to r=12, the analytical forms of the coefficients b_m^{mn} (r=m case for b_r^{mn}) at any number m are strictly derived. Having the spheroidal eigenvalues λ^{mn}, the spheroidal angular functions S_{mn}, and the spheroidal radial functions R_{mn}, together with the boundary conditions matching, we can completely solved the problems of electromagnetic scattering from dielectric or metallic spheroids.

I. INTRODUCTION

The spheroidal differential equation for radial functions R_{mn} is expressed as

$$\frac{d}{d\xi}(\xi^2 \mp 1)\frac{dR_{mn}}{d\xi} - (\lambda_{mn} - c^2\xi^2 \pm \frac{m^2}{\xi^2 \mp 1})R_{mn} = 0 \qquad (1)$$

where size parameter $c = \kappa(a^2 - b^2)^{\frac{1}{2}}$: κ is wave number, a is semi-major axis, and b is semi-minor axis. Radial coordinate $\xi = a/(a^2 - b^2)^{\frac{1}{2}}$ for prolate and $\xi = b/(a^2 - b^2)^{\frac{1}{2}}$ for oblate. λ_{mn} are eigenvalues of the spheroidal differential equation, with $n \geq m$. In Eq. (1) the upper sign pertains to the prolate coordinate and the lower one to the oblate one. If replacing c by -ic and ξ by $i\xi$ in the equation, we can convert the prolate differential equation into the oblate one. The spheroidal radial functions R_{mn} for large values of ξ, in general, can be obtained by means of some integral relations,[1] as follows:

$$R_{mn}^{(j)}(c,\xi) = \frac{1}{\sum_{r=0,1}^{\infty}{}' d_r^{mn}(c)\frac{(2m+r)!}{r!}}\left(\frac{\xi^2-1}{\xi^2}\right)^{m/2}\sum_{r=0,1}^{\infty}{}' i^{r+m-n} d_r^{mn}(c)\frac{(2m+r)!}{r!}\, z_{m+r}^{(j)}(c\xi) \qquad (2$$

where $d_r^{mn}(c)$ are the expansion coefficients of the spheroidal angle functions S_{mn} with respect to the associated Legendre functions P_{m+r}^{m}, r is the summation index, and the prime over the summation symbol indicates that the summation is over only odd number of r when (n-m) is odd, and over only even when (n-m) is even. Superscript j for the radial functions $R_{mn}(c,\xi)$ and for the (m+r)th order spherical Bessel functions $z_{m+r}(c\xi)$ implies which of the four kinds of spherical Bessel functions is referred to, that is, spherical Bessel $j_{m+r}(c\xi)$, Neumann $n_{m+r}(c\xi)$, and Hankel functions of the first and of the second kind, $h_{m+r}^{(1)}(c\xi)$ and $h_{m+r}^{(2)}(c\xi)$, in order of j=1,2,3,and 4, respectively.

Because the spherical Bessel functions of the first kind, $j_{m+r}(c\xi)$, converge absolutely for whatever value of ξ and whichever kind of spheroid, Eq. (2) is always applicable to the spheroidal radial functions of the first kind, $R_{mn}^{(1)}$, and the oblate radial functions $R_{mn}^{(1)}(-ic,i\xi)$ can be generated just by replacing c by -ic and ξ by $i\xi$ in the equation. But for the spheroidal radial functions of the second kind, $R_{mn}^{(2)}$, as aspect ratio increases, the resulting series of spherical Neumann functions converge poorly while value of ξ approches unity for prolate and zero for oblate, respectively. In fact, the expansion of $R_{mn}^{(2)}$ is an asymptotic series being not absolutely convergent for any finite value of ξ, as shown by Morse and Feshbach.[2] The functions $R_{mn}^{(3)}(c,\xi)$ and $R_{mn}^{(4)}(c,\xi)$ in Eq. (2) are, therefore, also not appropriate for small values of ξ. I had developed the series expansion in powers of (ξ^2-1) for the prolate radial functions and that in powers of (ξ^2+1) for the oblate ones instead of Eq. (2), to evaluate $R_{mn}^{(2)}$. Since the radial functions $R_{mn}^{(3)}$ and $R_{mn}^{(4)}$ are defined from the relations

$$R_{mn}^{(3)(4)}(c,\xi) = R_{mn}^{(1)}(c,\xi) \pm iR_{mn}^{(2)}(c,\xi) \qquad (3-$$

$$R_{mn}^{(3)(4)}(-ic,i\xi) = R_{mn}^{(1)}(-ic,i\xi) \pm iR_{mn}^{(2)}(-ic,i\xi) \qquad (3-$$

the discussion is only confined to the functions $R_{mn}^{(2)}$.

II. POWER-SERIES EXPANSIONS OF $R_{mn}^{(2)}$

The procedures for derivation of the functions $R_{mn}^{(2)}$ are cited from Flammer's book,[3] with giving more details in some processes and the much higher order terms than in that book.

The radial functions $R_{mn}^{(1)}(c,z)$ and the angle functions $S_{mn}(c,z)$, both satisfy the same differential equation, namely —

$$\frac{d}{dz}(1-z^2)\frac{dS_{mn}(c,z)}{dz} + (\lambda_{mn} - c^2z^2 - \frac{m^2}{1-z^2})S_{mn}(c,z) = 0$$

$$\frac{d}{dz}(z^2-1)\frac{dR_{mn}(c,z)}{dz} - (\lambda_{mn} - c^2z^2 + \frac{m^2}{z^2-1})R_{mn}(c,z) = 0 \qquad (4)$$

so they should be proportional to each other:

$$R_{mn}^{(1)}(c,z) = \left[\kappa_{mn}^{(1)}(c)\right]^{-1} S_{mn}(c,z) \qquad (5)$$

where joining factors $\kappa_{mn}^{(1)}(c)$ are in the forms:

$$\kappa_{mn}^{(1)}(c) = \frac{(2m+3)(n+m+1)!\sum_{r=1}^{\infty}{}' d_r^{mn}(c)\frac{(2m+r)!}{r!}}{2^{n+m}d_1^{mn}(c)c^{m+1}m!(\frac{n-m-1}{2})!(\frac{n+m+1}{2})!} \quad \text{for } (n-m) \text{ odd} \qquad (5\text{-}1)$$

$$\kappa_{mn}^{(1)}(c) = \frac{(2m+1)(n+m)!\sum_{r=0}^{\infty}{}' d_r^{mn}(c)\frac{(2m+r)!}{r!}}{2^{n+m}d_0^{mn}(c)c^{m}m!(\frac{n-m}{2})!(\frac{n+m}{2})!} \quad \text{for } (n-m) \text{ even} \qquad (5\text{-}2)$$

The series expansions of $S_{mn}(c,z)$ in powers of $(1-z^2)$ can be derived with the use of the associated Legendre functions represented by hypergeometric functions, as found to be:

$$S_{mn}(c,z) = z(1-z^2)^{\frac{1}{2}m}\sum_{k=0}^{\infty} c_{2k}^{mn}(c)(1-z^2)^k \quad \text{for } (n-m) \text{ odd} \qquad (6\text{-}1)$$

$$S_{mn}(c,z) = (1-z^2)^{\frac{1}{2}m}\sum_{k=0}^{\infty} c_{2k}^{mn}(c)(1-z^2)^k \quad \text{for } (n-m) \text{ even} \qquad (6\text{-}2)$$

in which

$$c_{2k}^{mn}(c) = \frac{1}{2^m k!(m+k)!} \sum_{r=k}^{\infty} \frac{(2m+2r+1)!}{(2r+1)!}(-r)_k (m+r+\tfrac{3}{2})_k d_{2r+1}^{mn}(c) \quad \text{for } (n-m) \text{ odd} \tag{6-3}$$

$$c_{2k}^{mn}(c) = \frac{1}{2^m k!(m+k)!} \sum_{r=k}^{\infty} \frac{(2m+2r)!}{(2r)!}(-r)_k (m+r+\tfrac{1}{2})_k d_{2r}^{mn}(c) \quad \text{for } (n-m) \text{ even} \tag{6-4}$$

where the symbol with subscript k stands for

$$(x)_k = x(x+1)(x+2)\cdots(x+k-1), \text{ and } (x)_0 = 1 \tag{6-5}$$

Since $|\xi| > 1$, the functions $S_{mn}(c,\xi)$ might be written as the series expansions in powers of (ξ^2-1) instead of $(1-\xi^2)$ with the proper change of the "phase" $(1-\xi^2)^{\frac{1}{2}m}$. Utilizing the relation in Eq. (5) and the adjusted functions $S_{mn}(c,\xi)$, we find that

$$R_{mn}^{(1)}(c,\xi) = \left[\kappa_{mn}^{(1)}(c)\right]^{-1} \xi(\xi^2-1)^{\frac{1}{2}m} \sum_{k=0}^{\infty} (-1)^k c_{2k}^{mn}(c)(\xi^2-1)^k \quad \text{for } (n-m) \text{ odd} \tag{7-1}$$

$$R_{mn}^{(1)}(c,\xi) = \left[\kappa_{mn}^{(1)}(c)\right]^{-1} (\xi^2-1)^{\frac{1}{2}m} \sum_{k=0}^{\infty} (-1)^k c_{2k}^{mn}(c)(\xi^2-1)^k \quad \text{for } (n-m) \text{ even} \tag{7-2}$$

Using the Wronskian identity

$$\Delta\left[R_{mn}^{(1)}(c,\xi), R_{mn}^{(2)}(c,\xi)\right] = R_{mn}^{(1)}(c,\xi)\frac{dR_{mn}^{(2)}(c,\xi)}{d\xi} - R_{mn}^{(2)}(c,\xi)\frac{dR_{mn}^{(1)}(c,\xi)}{d\xi} = \frac{1}{c(\xi^2-1)} \tag{8-1}$$

and the definite integral of ξ from infinity to the certain value of ξ, one obtains

$$R_{mn}^{(2)}(c,\xi) = R_{mn}^{(1)}(c,\xi)\int_{\infty}^{\xi} \frac{d\xi}{c(\xi^2-1)\left[R_{mn}^{(1)}(c,\xi)\right]^2} \tag{8-2}$$

where disposal of the order for the integral limits is in accordance with the asymptotic property of $R_{mn}^{(2)}(c,\xi)$ containing Neumann functions $n_{m+r}(c\xi)$, that is —

$$n_{m+r}(c\xi) \xrightarrow[c\xi \to \infty]{} \frac{1}{c\xi} \sin\left[c\xi - \tfrac{1}{2}(m+r+1)\pi\right] \longrightarrow 0 \tag{8-3}$$

With the substitution of Eqs. (7-1) and (7-2) in Eq. (8-2), the integrand can be developed in Taylor series expansions:

$$\frac{1}{c(\xi^2-1)\left[R_{mn}^{(1)}(c,\xi)\right]^2} = \frac{\left[\kappa_{mn}^{(1)}(c)\right]^2}{c\xi^2(\xi^2-1)^{m+1}} \sum_{r=0}^{\infty} \frac{a_r^{mn}(c)}{r!}(\xi^2-1)^r \qquad \text{for (n-m) odd} \qquad (9\cdot$$

$$\frac{1}{c(\xi^2-1)\left[R_{mn}^{(1)}(c,\xi)\right]^2} = \frac{\left[\kappa_{mn}^{(1)}(c)\right]^2}{c(\xi^2-1)^{m+1}} \sum_{r=0}^{\infty} \frac{a_r^{mn}(c)}{r!}(\xi^2-1)^r \qquad \text{for (n-m) even} \qquad (9\cdot$$

where

$$a_r^{mn}(c) = \frac{d^r}{dx^r}\left\{ \frac{1}{\left[\sum_{k=0}^{\infty}(-1)^k c_{2k}^{mn}(c)x^k\right]^2} \right\}_{x=0}, \qquad (9\cdot$$

which we will discuss in Section III.

These integral recursion relations are useful to the integral in Eq. (8-2):

$$\int \frac{d\xi}{\xi^2(\xi^2-1)^{m+1-r}} = \frac{1}{\xi(\xi^2-1)^{m-r}} + (2m-2r+1)\int \frac{d\xi}{(\xi^2-1)^{m+1-r}} \qquad \text{for } \begin{matrix}(n-m)\text{ odd}\\ r < m\end{matrix} \qquad (1($$

$$\int \frac{d\xi}{(\xi^2-1)^{m+1-r}} = -\frac{\xi}{(2m-2r)(\xi^2-1)^{m-r}} - \frac{(2m-2r-1)}{(2m-2r)}\int \frac{d\xi}{(\xi^2-1)^{m-r}} \qquad \text{for } \begin{matrix}(n-m)\text{ even}\\ r < m\end{matrix} \qquad (1($$

$$\int \frac{d\xi}{\xi^2(\xi^2-1)^{m+1-r}} = \frac{(\xi^2-1)^{r-m}}{\xi} - (2r-2m-1)\int (\xi^2-1)^{r-(m+1)}d\xi \qquad \text{for } \begin{matrix}(n-m)\text{ odd}\\ r > m\end{matrix} \qquad (1($$

$$\int \frac{d\xi}{(\xi^2-1)^{m+1-r}} = \frac{\xi(\xi^2-1)^{r-m}}{(2r-2m)} - \frac{(2r-2m+1)}{(2r-2m)}\int (\xi^2-1)^{r-m}d\xi \qquad \text{for } \begin{matrix}(n-m)\text{ even}\\ r > m\end{matrix} \qquad (1($$

Inserting Eqs. (9-1) and (9-2) in Eq. (8-2), using the relations in Eqs. (10-1)-(10-

and then integrating, we get

$$R_{mn}^{(2)}(c,\xi) = Q_{mn}(c)R_{mn}^{(1)}(c,\xi)\left\{\frac{1}{2}\log\frac{\xi+1}{\xi-1}\right\} + g_{mn}(c,\xi) \qquad (1:$$

where

$$Q_{mn}(c) = \frac{\left[\kappa_{mn}^{(1)}(c)\right]^2}{c}\sum_{r=0}^{m} a_r^{mn}(c)\frac{(-1)^{m-r+1}(2m-2r+1)!}{r!\left[2^{m-r}(m-r)!\right]^2}. \qquad \text{for (n-m) odd} \qquad (1:$$

$$Q_{mn}(c) = \frac{\left[\kappa_{mn}^{(1)}(c)\right]^2}{c}\sum_{r=0}^{m} a_r^{mn}(c)\frac{(-1)^{m-r+1}(2m-2r)!}{r!\left[2^{m-r}(m-r)!\right]^2} \qquad \text{for (n-m) even} \qquad (1:$$

One should note that the summation is over from r=0 to r=m. The functions $g_{mn}(c,\xi)$ can be written in the forms:

$$g_{mn}(c,\xi) = (\xi^2 - 1)^{-\frac{1}{2}m} \sum_{r=0}^{\infty} b_r^{mn}(\xi^2 - 1)^r \qquad \text{for (n-m) odd} \qquad (11\text{-}3)$$

$$g_{mn}(c,\xi) = \xi(\xi^2 - 1)^{-\frac{1}{2}m} \sum_{r=0}^{\infty} b_r^{mn}(\xi^2 - 1)^r \qquad \text{for (n-m) even} \qquad (11\text{-}4)$$

they satisfy the inhomogeneous radial equation

$$\left\{ \frac{d}{d\xi}(\xi^2 - 1)\frac{d}{d\xi} - (\lambda_{mn} - c^2\xi^2 + \frac{m^2}{\xi^2 - 1}) \right\} g_{mn}(c,\xi) = 2Q_{mn}(c)\frac{d}{d\xi}R_{mn}^{(1)}(c,\xi) \qquad (11\text{-}5)$$

Substituting Eqs. (11-3) and (11-4) in the differential equation (11-5), we can get the recursion relations for the prolate coefficients b_r^{mn}:

$$4r(r-m)b_r^{mn} + \left[(2r-m-1)(2r-m-2) - \lambda_{mn} + c^2\right]b_{r-1}^{mn} + c^2 b_{r-2}^{mn} = 2\left[\kappa_{mn}^{(1)}\right]^{-1} Q_{mn}(-1)^{r-m}(2r-m)c_{2r-2m}^{mn}$$

$$+ 2\left[\kappa_{mn}^{(1)}\right]^{-1} Q_{mn}(-1)^{r-m-1}(2r-m-1)c_{2r-2m-2}^{mn} \qquad \text{for (n-m) odd} \qquad (12\text{-}1)$$

$$4r(r-m)b_r^{mn} + \left[(2r-m)(2r-m-1) - \lambda_{mn} + c^2\right]b_{r-1}^{mn} + c^2 b_{r-2}^{mn} = 2\left[\kappa_{mn}^{(1)}\right]^{-1} Q_{mn}(-1)^{r-m}(2r-m)c_{2r-2m}^{mn}$$

$$\text{for (n-m) even} \qquad (12\text{-}2)$$

where we have employed the series expansions of $R_{mn}^{(1)}(c,\xi)$ of Eqs. (7-1) and (7-2) to the right-hand side of Eq. (11-5).

While m is not equal to zero, by inserting in the Wronskian identity the series expansions of $R_{mn}^{(1)}(c,\xi)$ and $R_{mn}^{(2)}(c,\xi)$ of Eq. (11) as well as subsequent Eqs. (11-1)-(11-4), and by using Taylor expansion deduced from Eq. (9-3):

$$\frac{1}{\left[\sum_{k=0}^{\infty} (-1)^k c_{2k}^{mn}(c)(\xi^2 - 1)^k\right]^2} = \sum_{r=0}^{\infty} \frac{a_r^{mn}(c)}{r!}(\xi^2 - 1)^r \qquad (13\text{-}1)$$

the initial prolate coefficients b_0^{mn} are found by equating the lowest power of (ξ^2-1) on both sides of the Wronskian identity, that is

$$b_0^{mn} = -\frac{\kappa_{mn}^{(1)}(c)}{2mcc_0^{mn}} \qquad (m > 0) \qquad \text{for both (n-m) odd and even} \qquad (13\text{-}2)$$

The coefficients b_m^{mn} are not determinable from the recursion formulae in Eqs. (12-1) and (12-2) because of the factor (r-m) of b_r^{mn} in these equations. We will use other method to obtain the coefficients b_m^{mn} in section IV.

Power-series expansions of the oblate radial functions can be generated with the substitution of the corresponding terms:

$$c \longrightarrow -ic \qquad (14\text{-}1)$$

$$\xi \longrightarrow i\xi \qquad (14\text{-}2)$$

$$d_r^{mn}(c) \longrightarrow d_r^{mn}(-ic) \qquad (14\text{-}3)$$

$$c_{2k}^{mn}(c) \longrightarrow c_{2k}^{mn}(-ic) \qquad (14\text{-}4)$$

$$\kappa_{mn}^{(1)}(c) \longrightarrow \kappa_{mn}^{(1)}(-ic) \qquad (14\text{-}5)$$

Thus for the oblate radial functions $R_{mn}^{(1)}$, we have

$$R_{mn}^{(1)}(-ic,i\xi) = \left[i^{-m-1}\kappa_{mn}^{(1)}(-ic)\right]^{-1}\xi(\xi^2+1)^{\frac{1}{2}m}\sum_{k=0}^{\infty}c_{2k}^{mn}(-ic)(\xi^2+1)^k \quad \text{for (n-m) odd} \qquad (15\text{-}1)$$

$$R_{mn}^{(1)}(-ic,i\xi) = \left[i^{-m}\kappa_{mn}^{(1)}(-ic)\right]^{-1}(\xi^2+1)^{\frac{1}{2}m}\sum_{k=0}^{\infty}c_{2k}^{mn}(-ic)(\xi^2+1)^k \quad \text{for (n-m) even} \qquad (15\text{-}2)$$

The oblate functions $R_{mn}^{(2)}$ can be obtained from the relation

$$R_{mn}^{(2)}(-ic,i\xi) = Q_{mn}^{*}(-ic)R_{mn}^{(1)}(-ic,i\xi)\left[\tan^{-1}\xi - \tfrac{1}{2}\pi\right] + g_{mn}^{*}(-ic,i\xi) \qquad (16)$$

with

$$Q_{mn}^{*}(-ic) = -\frac{\left[i^{-m-1}\kappa_{mn}^{(1)}(-ic)\right]^2}{c}\sum_{r=0}^{m}\alpha_r^{mn}(-ic)\frac{(2m-2r+1)!}{r!\left[2^{m-r}(m-r)!\right]^2} \quad \text{for (n-m) odd} \qquad (16\text{-}1)$$

$$Q_{mn}(-ic) = \frac{\left[i^{-m}\kappa_{mn}^{(1)}(-ic)\right]^2}{c}\sum_{r=0}^{m}\alpha_r^{mn}(-ic)\frac{(2m-2r)!}{r!\left[2^{m-r}(m-r)!\right]^2} \quad \text{for (n-m) even} \qquad (16\text{-}2)$$

where

$$\alpha_r^{mn}(-ic) = \frac{d^r}{dx^r}\left\{\frac{1}{\left[\sum_{k=0}^{\infty}c_{2k}^{mn}(-ic)x^k\right]^2}\right\}_{x=0} \qquad (16\text{-}3)$$

The functions $g^{*}_{mn}(-ic,i\xi)$ can be written in the forms:

$$g^{*}_{mn}(-ic,i\xi) = (\xi^2+1)^{-\frac{1}{2}m}\sum_{r=0}^{\infty}\beta^{mn}_{2r}\xi^{2r} \qquad \text{for (n-m) odd} \quad (16\text{-}4)$$

$$g^{*}_{mn}(-ic,i\xi) = \xi(\xi^2+1)^{-\frac{1}{2}m}\sum_{r=0}^{\infty}\beta^{mn}_{2r}\xi^{2r} \qquad \text{for (n-m) even} \quad (16\text{-}5)$$

they satisfy another inhomogeneous radial equation

$$\left\{\frac{d}{d\xi}(\xi^2+1)\frac{d}{d\xi} - \left[\lambda_{mn}(-ic) - c^2\xi^2 - \frac{m^2}{\xi^2+1}\right]\right\}g^{*}_{mn}(-ic,i\xi) = -2Q^{*}_{mn}(-ic)\frac{d}{d\xi}R^{(1)}_{mn}(-ic,i\xi) \qquad (16\text{-}6)$$

Similar to prolate, we can get the recursion relations for the oblate coefficients β^{mn}_{2r}:

$$(2r+1)(2r+2)\beta^{mn}_{2r+2} + \left[2r(2r-2m+1)+m(m-1)-\lambda_{mn}(-ic)\right]\beta^{mn}_{2r} + c^2\beta^{mn}_{2r-2} = -\frac{2Q^{*}_{mn}(-ic)}{i^{-m-1}\kappa^{(1)}_{mn}(-ic)}\times$$

$$\left[\sum_{k=r-m}^{\infty}c^{mn}_{2k}(-ic)(2k+m+1)\frac{(k+m)!}{r!(k+m-r)!} - \sum_{k=r-m+1}^{\infty}c^{mn}_{2k}(-ic)(2k+m)\frac{(k+m-1)!}{r!(k+m-r-1)!}\right]$$

for (n-m) odd (17-1)

$$(2r+2)(2r+3)\beta^{mn}_{2r+2} + \left[(2r+1)(2r-2m+2)+m(m-1)-\lambda_{mn}(-ic)\right]\beta^{mn}_{2r} + c^2\beta^{mn}_{2r-2} =$$

$$-\frac{2Q^{*}_{mn}(-ic)}{i^{-m}\kappa^{(1)}_{mn}(-ic)}\times\left[\sum_{k=r-m+1}^{\infty}c^{mn}_{2k}(-ic)(2k+m)\frac{(k+m-1)!}{r!(k+m-r-1)!}\right] \qquad \text{for (n-m) even} \quad (17\text{-}2)$$

The initial coefficients β^{mn}_{0} are found by inserting $R^{(1)}_{mn}(-ic,i\xi)$ and $R^{(2)}_{mn}(-ic,i\xi)$ in the corresponding Wronskian identity for oblate and then setting ξ equal to zero. Noticing that

$$R^{(1)}_{mn}(-ic,i0) = 0 \qquad \text{for (n-m) odd} \quad (18\text{-}1)$$

we thus obtain

$$\beta^{mn}_{0} = -\left[cR^{(1)'}_{mn}(-ic,i0)\right]^{-1} \qquad \text{for (n-m) odd} \quad (18\text{-}2)$$

$$\beta^{mn}_{0} = -Q^{*}_{mn}(-ic)R^{(1)}_{mn}(-ic,i0) + \left[cR^{(1)}_{mn}(-ic,i0)\right]^{-1} \qquad \text{for (n-m) even} \quad (18\text{-}3)$$

The remaining coefficients β_{2r}^{mn} are completely determinable from the recursion formulae in Eqs. (17-1) and (17-2).

III. THE COEFFICIENTS a_r^{mn} AND α_r^{mn}

The explicit forms of the prolate coefficients a_r^{mn} in Flammer's book are listed up to r=4. According to the empirical formula given by Asano *et al.*,[4] the terminated number M (m=0,1,2,...,M) for the spheroid with high aspect ratio is proportional to the size parameter c in the form:

$$M \simeq c \tag{19}$$

In order to compute the light scattering by such a spheroid with large size parameter, I had derived the expressions of a_r^{mn} up to very large number r.

Defining

$$f(x) = \frac{1}{\left(\sum_{k=0}^{\infty} (-1)^k c_{2k}^{mn}(c) x^k\right)^2} \tag{20-1}$$

applying Taylor series expansion to it:

$$\begin{aligned} f(x) &= f(0) + \frac{f'(x)}{1!}x + \frac{f''(x)}{2!}x^2 + \ldots + \frac{f^{(r)}(x)}{r!}x^r + \ldots \\ &= a_0^{mn} + \frac{a_1^{mn}}{1!} + \frac{a_2^{mn}}{2!} + \ldots + \frac{a_r^{mn}}{r!} + \ldots \end{aligned} \tag{20-2}$$

and assuming

$$G = \sum_{k=1}^{\infty} (-1)^k \times \left(\frac{c_{2k}^{mn}}{c_0^{mn}}\right) x^k \tag{20-3}$$

then developing f(x) by binomial expansion:

$$f(x) = (c_0^{mn})^{-2}(1 + G)^{-2} = (c_0^{mn})^{-2} \sum_{\ell=0}^{\infty} (-1)^{\ell}(\ell+1)G^{\ell} \tag{20-4}$$

where we have used the series identity for $|G| < 1$, we can obtain a_r^{mn} by equating the power of x^r on both sides in Eq. (20-2).

The coefficients a_r^{mn} are listed below up to r=12; for convenience, we have omitted the superscripts mn of the coefficients c_{2k}^{mn} (k=0,1,2,$\cdots$) in the following expressions.

$$a_0^{mn} = \frac{1}{c_0^2} \tag{21-1}$$

$$a_1^{mn} = \frac{1!}{c_0^2}\left(\frac{2c_2}{c_0}\right) \tag{21-2}$$

$$a_2^{mn} = \frac{2!}{c_0^2}\left(\frac{3c_2^2}{c_0^2} - \frac{2c_4}{c_0}\right) \tag{21-3}$$

$$a_3^{mn} = \frac{3!}{c_0^2}\left(\frac{4c_2^3}{c_0^3} - \frac{6c_2c_4}{c_0^2} + \frac{2c_6}{c_0}\right) \tag{21-4}$$

$$a_4^{mn} = \frac{4!}{c_0^2}\left[\frac{5c_2^4}{c_0^4} - \frac{12c_2^2c_4}{c_0^3} + \frac{3(2c_2c_6+c_4^2)}{c_0^2} - \frac{2c_8}{c_0}\right] \tag{21-5}$$

$$a_5^{mn} = \frac{5!}{c_0^2}\left[\frac{6c_2^5}{c_0^5} - \frac{20c_2^3c_4}{c_0^4} + \frac{12(c_2c_4^2+c_2^2c_6)}{c_0^3} - \frac{6(c_2c_8+c_4c_6)}{c_0^2} - \frac{2c_{10}}{c_0}\right] \tag{21-6}$$

$$a_6^{mn} = \frac{6!}{c_0^2}\left[\frac{7c_2^6}{c_0^6} - \frac{30c_2^4c_4}{c_0^5} + \frac{10(3c_2^2c_4^2+2c_2^3c_6)}{c_0^4} - \frac{4(6c_2c_4c_6+3c_2^2c_8+c_4^3)}{c_0^3} + \frac{3(2c_2c_{10}+2c_4c_8+c_6^2)}{c_0^2} - \frac{2c_{12}}{c_0}\right] \tag{21-7}$$

$$a_7^{mn} = \frac{7!}{c_0^2}\left[\frac{8c_2^7}{c_0^7} - \frac{42c_2^5c_4}{c_0^6} + \frac{30(2c_2^3c_4^2+c_2^4c_6)}{c_0^5} - \frac{20(c_2c_4^3+3c_2^2c_4c_6+c_2^3c_8)}{c_0^4} + \frac{12(2c_2c_4c_8+c_2c_6^2+c_2^2c_{10}+c_4^2c_6)}{c_0^3} - \frac{6(c_2c_{12}+c_4c_{10}+c_6c_8)}{c_0^2} + \frac{2c_{14}}{c_0}\right] \tag{21-8}$$

$$a_8^{mn} = \frac{8!}{c_0^2}\left[\frac{9c_2^8}{c_0^8} - \frac{56c_2^6c_4}{c_0^7} + \frac{21(5c_2^4c_4^2+2c_2^5c_6)}{c_0^6} - \frac{30(2c_2^2c_4^3+4c_2^3c_4c_6+c_2^4c_8)}{c_0^5} + \frac{5(12c_2c_4^2c_6+12c_2^2c_4c_8+6c_2^2c_6^2+4c_2^3c_{10}+c_4^4)}{c_0^4} - \frac{12(2c_2c_4c_{10}+2c_2c_6c_8+c_2^2c_{12}+c_4c_6^2+c_4^2c_8)}{c_0^3} + \frac{3(2c_2c_{14}+2c_4c_{12}+2c_6c_{10}+c_8^2)}{c_0^2} - \frac{2c_{16}}{c_0}\right] \tag{21-9}$$

$$a_9^{mn} = \frac{9!}{c_0^2}\left\{\frac{10c_2^9}{c_0^9} - \frac{72c_2^7c_4}{c_0^8} + \frac{56(3c_2^5c_4^2+c_2^6c_6)}{c_0^7} - \frac{14(10c_2^3c_4^3+15c_2^4c_4c_6+3c_2^5c_8)}{c_0^6} + \frac{30(c_2c_4^4+6c_2^2c_4^2c_6+4c_2^3c_4c_8+2c_2^3c_6^2+c_2^4c_{10})}{c_0^5} - \frac{20(3c_2c_4c_6^2+3c_2c_4^2c_8+3c_2^2c_4c_{10}+3c_2^2c_6c_8+c_2^3c_{12}+c_4^3c_6)}{c_0^4} + \frac{4(6c_2c_4c_{12}+6c_2c_6c_{10}+3c_2c_8^2+3c_2^2c_{14}+6c_4c_6c_8+3c_4^2c_{10}+c_6^3)}{c_0^3} - \frac{6(c_2c_{16}+c_4c_{14}+c_6c_{12}+c_8c_{10})}{c_0^2} + \frac{2c_{18}}{c_0}\right\} \qquad (21\text{-}10)$$

$$a_{10}^{mn} = \frac{10!}{c_0^2}\left\{\frac{11c_2^{10}}{c_0^{10}} - \frac{90c_2^8c_4}{c_0^9} + \frac{36(7c_2^6c_4^2+2c_2^7c_6)}{c_0^8} - \frac{56(5c_2^4c_4^3+6c_2^5c_4c_6+c_2^6c_8)}{c_0^7} + \frac{21(5c_2^2c_4^4+20c_2^3c_4^2c_6+5c_2^4c_6^2+10c_2^4c_4c_8+2c_2^5c_{10})}{c_0^6} - \frac{6(20c_2c_4^3c_6+30c_2^2c_4c_6^2+30c_2^2c_4^2c_8+20c_2^3c_4c_{10}+20c_2^3c_6c_8+5c_2^4c_{12}+c_4^5)}{c_0^5} + \frac{10(12c_2c_4c_6c_8+6c_2c_4^2c_{10}+2c_2c_6^3+6c_2^2c_4c_{12}+6c_2^2c_6c_{10}+3c_2^2c_8^2+2c_2^3c_{14}+3c_4^2c_6^2+2c_4^3c_8)}{c_0^4} - \frac{12(2c_2c_4c_{14}+2c_2c_6c_{12}+2c_2c_8c_{10}+c_2^2c_{16}+2c_4c_6c_{10}+c_4c_8^2+c_4^2c_{12}+c_6^2c_8)}{c_0^3} + \frac{3(2c_2c_{18}+2c_4c_{16}+2c_6c_{14}+2c_8c_{12}+c_{10}^2)}{c_0^2} - \frac{2c_{20}}{c_0}\right\} \qquad (21\text{-}11)$$

$$a_{11}^{mn} = \frac{11!}{c_0^2}\left\{\frac{12c_2^{11}}{c_0^{11}} - \frac{110c_2^9c_4}{c_0^{10}} + \frac{90(4c_2^7c_4^2+c_2^8c_6)}{c_0^9} - \frac{72(7c_2^5c_4^3+7c_2^6c_4c_6+c_2^7c_8)}{c_0^8} + \frac{56(5c_2^3c_4^4+15c_2^4c_4^2c_6+6c_2^5c_4c_8+3c_2^5c_6^2+c_2^6c_{10})}{c_0^7} - \frac{42(c_2c_4^5+10c_2^2c_4^3c_6+10c_2^3c_4c_6^2+10c_2^3c_4^2c_8+5c_2^4c_4c_{10}+5c_2^4c_6c_8+c_2^5c_{12})}{c_0^6} + \frac{30(6c_2c_4^2c_6^2+4c_2c_4^3c_8+12c_2^2c_4c_6c_8+6c_2^2c_4^2c_{10}+2c_2^2c_6^3+4c_2^3c_4c_{12}+4c_2^3c_6c_{10}+2c_2^3c_8^2+c_2^4c_{14}+c_4^4c_6)}{c_0^5} - \frac{20(6c_2c_4c_6c_{10}+3c_2c_4c_8^2+3c_2c_4^2c_{12}+3c_2c_6^2c_8+3c_2^2c_4c_{14}+3c_2^2c_6c_{12}+3c_2^2c_8c_{10}+c_2^3c_{16}+c_4c_6^3+3c_4^2c_6c_8+c_4^3c_{10})}{c_0^4} + \frac{12(2c_2c_4c_{16}+2c_2c_6c_{14}+2c_2c_8c_{12}+c_2c_{10}^2+}{c_0^3}$$

$$\frac{c_2^2c_{18}+2c_4c_6c_{12}+2c_4c_8c_{10}+c_4^2c_{14}+c_6c_8^2+c_6^2c_{10})}{c_0^2} - \frac{6(c_2c_{20}+c_4c_{18}+c_6c_{16}+c_8c_{14}+c_{10}c_{12})}{c_0}$$

$$+ \frac{2c_{22}}{c_0}\Bigg) \tag{21-12}$$

$$a_{12}^{mn} = \frac{12!}{c_0^2}\Bigg(\frac{13c_2^{12}}{c_0^{12}} - \frac{132c_2^{10}c_4}{c_0^{11}} + \frac{55(9c_2^8c_4^2+2c_2^9c_6)}{c_0^{10}} - \frac{30(28c_2^6c_4^3+24c_2^7c_4c_6+3c_2^8c_8)}{c_0^9} + \frac{18(35c_2^4c_4^4+$$

$$\frac{84c_2^5c_4^2c_6+28c_2^6c_4c_8+14c_2^6c_6^2+4c_2^7c_{10})}{c_0^8} - \frac{56(3c_2^2c_4^5+20c_2^3c_4^3c_6+15c_2^4c_4c_6^2+15c_2^4c_4^2c_8+$$

$$\frac{6c_2^5c_4c_{10}+6c_2^5c_6c_8+c_2^6c_{12})}{c_0^7} + \frac{7(30c_2c_4^4c_6+90c_2^2c_4^2c_6^2+60c_2^2c_4^3c_8+120c_2^3c_4c_6c_8+$$

$$\frac{60c_2^3c_4^2c_{10}+20c_2^3c_6^3+30c_2^4c_4c_{12}+30c_2^4c_6c_{10}+15c_2^4c_8^2+6c_2^5c_{14}+c_4^6)}{c_0^6} - \frac{30(4c_2c_4c_6^3+$$

$$\frac{12c_2c_4^2c_6c_8+4c_2c_4^3c_{10}+12c_2^2c_4c_6c_{10}+6c_2^2c_4c_8^2+6c_2^2c_4^2c_{12}+6c_2^2c_6^2c_8+4c_2^3c_4c_{14}+$$

$$\frac{4c_2^3c_6c_{12}+4c_2^3c_8c_{10}+c_2^4c_{16}+2c_4^3c_6^2+c_4^4c_8)}{c_0^5} + \frac{5(24c_2c_4c_6c_{12}+24c_2c_4c_8c_{10}+12c_2c_4^2c_{14}+$$

$$\frac{12c_2c_6c_8^2+12c_2c_6^2c_{10}+12c_2^2c_4c_{16}+12c_2^2c_6c_{14}+12c_2^2c_8c_{12}+6c_2^2c_{10}^2+4c_2^3c_{18}+12c_4c_6^2c_8+$$

$$\frac{12c_4^2c_6c_{10}+6c_4^2c_8^2+4c_4^3c_{12}+c_6^4)}{c_0^4} - \frac{4(6c_2c_4c_{18}+6c_2c_6c_{16}+6c_2c_8c_{14}+6c_2c_{10}c_{12}+3c_2^2c_{20}+$$

$$\frac{6c_4c_6c_{14}+6c_4c_8c_{12}+3c_4c_{10}^2+3c_4^2c_{16}+6c_6c_8c_{10}+3c_6^2c_{12}+c_8^3)}{c_0^3} + \frac{3(2c_2c_{22}+2c_4c_{20}+$$

$$\frac{2c_6c_{18}+2c_8c_{16}+2c_{10}c_{14}+c_{12}^2)}{c_0^2} - \frac{2c_{24}}{c_0}\Bigg) \tag{21-13}$$

The explicit forms of the oblate coefficients $\alpha_r^{mn}(-ic)$ can be easily found from those of $a_r^{mn}(c)$ by the relationship

$$\alpha_r^{mn}(-ic) = (-1)^r a_r^{mn}(-ic) \tag{22}$$

IV. THE COEFFICIENTS b_m^{mn}

The functions $R_{mn}^{(2)}(c,\xi)$ can be expressed in terms of the associated Legendre functions of the first kind, P_{m+r}^m, and of the second kind, Q_{m+r}^m:

$$R_{mn}^{(2)}(c,\xi) = \frac{1}{\kappa_{mn}^{(2)}(c)}\left\{ \sum_{r=-2m+1}^{\infty}{}' d_r^{mn}(c) Q_{m+r}^m(\xi) + \sum_{r=2m+1}^{\infty}{}' d_{r}^{mn}(c) P_{r-m-1}^m(\xi)\right\} \quad \text{for } (n-m) \text{ odd} \qquad (23\text{-}1)$$

$$R_{mn}^{(2)}(c,\xi) = \frac{1}{\kappa_{mn}^{(2)}(c)}\left\{ \sum_{r=-2m}^{\infty}{}' d_r^{mn}(c) Q_{m+r}^m(\xi) + \sum_{r=2m+2}^{\infty}{}' d_{r}^{mn}(c) P_{r-m-1}^m(\xi)\right\} \quad \text{for } (n-m) \text{ even} \qquad (23\text{-}2)$$

the implications of the above-cited symbols refer to Flammer's book, the joining factors $\kappa_{mn}^{(2)}(c)$ are given by

$$\kappa_{mn}^{(2)}(c) = -\frac{2^{n-m}(2m)!\left(\frac{n-m-1}{2}\right)!\left(\frac{n+m+1}{2}\right)! d_{-2m+1}^{mn}(c)}{(2m-3)(2m-1)m!(n+m+1)!c^{m-2}} \sum_{r=1}^{\infty}{}' d_r^{mn}(c)\frac{(2m+r)!}{r!} \quad \text{for } (n-m) \text{ odd} \qquad (23\text{-}3)$$

$$\kappa_{mn}^{(2)}(c) = \frac{2^{n-m}(2m)!\left(\frac{n-m}{2}\right)!\left(\frac{n+m}{2}\right)! d_{-2m}^{mn}(c)}{(2m-1)m!(n+m)!c^{m-1}} \sum_{r=0}^{\infty}{}' d_r^{mn}(c)\frac{(2m+r)!}{r!} \quad \text{for } (n-m) \text{ even} \qquad (23\text{-}4)$$

For $\xi > 1$, from the definition of the functions Q_{m+r}^m:

$$Q_{m+r}^m(\xi) = (\xi^2-1)^{\frac{1}{2}m}\frac{d^m Q_{m+r}(\xi)}{d^m\xi} \qquad (24\text{-}1)$$

and the expression of Q_{m+r} in terms of the Legendre functions of the first kind:

$$Q_{m+r}(\xi) = \frac{1}{2}P_{m+r}(\xi)\log\frac{\xi+1}{\xi-1} - \sum_{k=0}^{\left\lfloor (m+r-1)/2\right\rfloor}\frac{(2m+2r-4k-1)}{(m+r-k)(2k+1)}P_{m+r-2k-1}(\xi) \qquad (24\text{-}2)$$

where $\lfloor\nu\rfloor$ denotes the integer part in ν, we can get the formula for Q_{m+r}^m, as follows:

$$Q_{m+r}^m(\xi) = \frac{1}{2}P_{m+r}^m(\xi)\log\frac{\xi+1}{\xi-1} + \frac{1}{2}\sum_{\ell=1}^{m} c_m^{\ell}(\ell-1)!(-1)^{\ell-1}\left[\frac{(\xi-1)^{\ell}-(\xi+1)^{\ell}}{(\xi^2-1)^{\frac{1}{2}\ell}}\right]P_{m+r}^{m-\ell}(\xi)$$
$$- \sum_{k=0}^{\left\lfloor (r-1)/2\right\rfloor}\frac{(2m+2r-4k-1)}{(m+r-k)(2k+1)}P_{m+r-2k-1}^m(\xi) \qquad (24\text{-}3)$$

By virtue of the hypergeometric functions, the expression $(\xi - 1)^{\ell} - (\xi + 1)^{\ell}$ can be written in powers of (ξ^2-1):[5]

$$(\xi - 1)^{\ell} - (\xi + 1)^{\ell} = -2^{\ell}\, F\left(-\frac{\ell}{2}, \frac{-\ell+1}{2}, -\ell+1, 1 - \xi^2\right) \qquad \text{for } \ell \text{ odd} \qquad (25\text{-}1)$$

$$(\xi - 1)^{\ell} - (\xi + 1)^{\ell} = -2^{\ell}\xi\, F\left(\frac{-\ell+1}{2}, \frac{-\ell+2}{2}, -\ell+1, 1 - \xi^2\right) \qquad \text{for } \ell \text{ even} \qquad (25\text{-}2)$$

Since the series expressions of above hypergeometric functions are truncated at

$$k = \frac{\ell-1}{2} \qquad \text{for } \ell \text{ odd} \qquad (25\text{-}3)$$

$$k = \frac{\ell-2}{2} \qquad \text{for } \ell \text{ even} \qquad (25\text{-}4)$$

where k is summation index for the series expansion of the hypergeometric functions, further developments of the expressions (25-1) and (25-2) lead to

$$(\xi - 1)^{\ell} - (\xi + 1)^{\ell} = -2^{\ell}\ell \sum_{k=0}^{(\ell-1)/2} \frac{(\ell-k-1)!}{(\ell-2k)!k!2^{2k}} (\xi^2 - 1)^k \qquad \text{for } \ell \text{ odd} \qquad (25\text{-}5)$$

$$(\xi - 1)^{\ell} - (\xi + 1)^{\ell} = -2^{\ell}\xi \sum_{k=0}^{(\ell-2)/2} \frac{(\ell-k-1)!}{(\ell-2k-1)!k!2^{2k}} (\xi^2 - 1)^k \qquad \text{for } \ell \text{ even} \qquad (25\text{-}6)$$

The desired forms for $Q^m_{m+r}(\xi)$ in Eq. (24-3) can be obtained by using transformations from the associated Legendre functions of the first kind to the hypergeometric functions

$$P^{\nu}_{\mu}(\xi) = \frac{(\mu+\nu)!}{2^{\nu}\nu!(\mu-\nu)!}\xi(\xi^2 - 1)^{\frac{1}{2}\nu} F\left(\frac{\nu-\mu+1}{2}, \frac{\nu+\mu+2}{2}, \nu+1, 1 - \xi^2\right) \qquad \text{for } (\mu-\nu) \text{ odd} \qquad (26\text{-}1)$$

$$P^{\nu}_{\mu}(\xi) = \frac{(\mu+\nu)!}{2^{\nu}\nu!(\mu-\nu)!}(\xi^2 - 1)^{\frac{1}{2}\nu} F\left(\frac{\nu-\mu}{2}, \frac{\nu+\mu+1}{2}, \nu+1, 1 - \xi^2\right) \qquad \text{for } (\mu-\nu) \text{ even} \qquad (26\text{-}2)$$

Now we can determine the prolate coefficients b^{mn}_{m} by comparing Eq. (11) with Eqs. (23-1) and (23-2) after expressing the functions $Q^m_{m+r}(\xi)$ and $P^m_{r-m-1}(\xi)$ in powers of (ξ^2-1) with the use of Eqs. (24-3), (25-5), (25-6), (26-1) and (26-2).

It is obvious that b^{mn}_{m} in Eqs. (27-1) and (27-2) are composed of three parts, that is: (1). the part containing the coefficients d^{mn}_{2r+1} or d^{mn}_{2r}, which is derived from the functions $P^m_{m+r-2k-1}(\xi)$ contained in $Q^m_{m+r}(\xi)$. In Flammer's book, he missed the factor (2k+1) in the denominator of Eq. (27-1) and gave the wrong upper limits of summation: $\left[(m+2r)/2\right]$

and $\left((m+2r-1)/2\right]$, instead of correct limits r and $\left[(2r-1)/2\right]$ in Eqs. (27-1) and (27-2), respectively; (2). the part containing the coefficients d^{mn}_{2r+1} or d^{mn}_{2r}, which is derived from the functions $P^{m-\ell}_{m+r}(\xi)$ contained in Q^{m}_{m+r} with the substitutions of Eqs. (25-5) and (25-6) in Eq. (24-3). After simplications on mathematics, the terms in this part can be written in different forms depending upon whether the summation index ℓ appeared in the second part of Eq. (24-3) and (n-m) are odd or even; (3). the part containing the coefficients $d^{mn}_{\rho|2r-1}$ or $d^{mn}_{\rho|r}$. This part can be obtained by developing the functions $P^{m}_{r-m-1}($ in terms of (ξ^2-1) with the aid of the hypergeometric functions.

The complete expressions of b^{mn}_{m} are in the forms:

$$b^{mn}_{m} = -\frac{1}{\kappa^{(2)}_{mn}}\left\{\sum_{r=0}^{\infty} d^{mn}_{2r+1}\left[\sum_{k=0}^{r}\frac{(2m+4r-4k+1)(2m+2r-2k)!}{2^m m!(2k+1)(m+2r-k+1)(2r-2k)!}-\sum_{\ell=1}^{m} f_1(\ell)\right]\right.$$

$$\left.-\sum_{r=m+1}^{\infty} d^{mn}_{\rho|2r-1}\frac{(2r-2)!}{2^m m!(2r-2m-2)!}\right\} \qquad \text{for (n-m) odd} \qquad (27$$

$$b^{mn}_{m} = -\frac{1}{\kappa^{(2)}_{mn}}\left\{\sum_{r=0}^{\infty} d^{mn}_{2r}\left[\sum_{k=0}^{(2r-1)/2}\frac{(2m+4r-4k-1)(2m+2r-2k-1)!}{2^m m!(2k+1)(m+2r-k)(2r-2k-1)!}-\sum_{\ell=1}^{m} f_2(\ell)\right]\right.$$

$$\left.-\sum_{r=m+1}^{\infty} d^{mn}_{\rho|2r}\frac{(2r-1)!}{2^m m!(2r-2m-1)!}\right\} \qquad \text{for (n-m) even} \qquad (27$$

with

$$f_1(\ell) = -\frac{m!(2m+2r-\ell+1)!\left(r+\frac{\ell+1}{2}\right)!}{2^{m-2\ell+1}(m-\ell)!(2r+\ell+1)!\left(r+m-\frac{\ell}{2}\right)!}\sum_{k=0}^{(\ell-1)/2}\frac{\left(r+m-k+\frac{\ell}{2}\right)!}{(m-k)!\left(r+k-\frac{\ell-1}{2}\right)!(\ell-k)(\ell-2k)!k!2^{2k}}$$

$$\text{for } \ell \text{ odd} \qquad (2$$

$$f_1(\ell) = \frac{m!(2m+2r-\ell+1)!\left(r+\frac{\ell}{2}\right)!}{2^{m-2\ell+1}\ell(m-\ell)!(2r+\ell+1)!\left(r+m-\frac{\ell-1}{2}\right)!}\times$$

$$\left\{\frac{\left(r+m+\frac{\ell+1}{2}\right)!}{m!\left(r-\frac{\ell}{2}\right)!\ell!}+\sum_{k=0}^{(\ell-2)/2}\frac{\left(r+m-k+\frac{\ell-1}{2}\right)!\left[4(k+1)(\ell-k-1)+(\ell-2k-1)(\ell-2k-2)\right]}{(m-k-1)!\left(r+k-\frac{\ell-2}{2}\right)!(\ell-k-1)(\ell-2k-1)!(k+1)!2^{2k+2}}\right\}$$

$$\text{for } \ell \text{ even} \qquad (2$$

$$f_2(\ell) = -\frac{m!(2m+2r-\ell)!(r+\frac{\ell-1}{2})!}{2^{m-2\ell+1}(m-\ell)!(2r+\ell)!(r+m-\frac{\ell}{2})!}\sum_{k=0}^{(\ell-1)/2}\frac{(r+m-k+\frac{\ell}{2})!}{(m-k)!(r+k-\frac{\ell+1}{2})!(\ell-k)(\ell-2k)!k!2^{2k}}$$

for ℓ odd (27-5)

$$f_2(\ell) = \frac{m!(2m+2r-\ell)!(r+\frac{\ell}{2})!}{2^{m-2\ell+1}\ell(m-\ell)!(2r+\ell)!(r+m-\frac{\ell+1}{2})!}\sum_{k=0}^{(\ell-2)/2}\frac{(r+m-k+\frac{\ell-1}{2})!}{(m-k)!(r+k-\frac{\ell}{2})!(\ell-k)(\ell-2k-1)!k!2^{2k}}$$

for ℓ even (27-6)

It is worth pointing out that the coefficients b_m^{mn} derived by Flammer are only for $0 \leq m \leq 2$ without analytical expressions for the numbers m greater than 2.

Compared with the oblate coefficients β_m^{mn}, which can be directly obtained from the recursion relation, the expressions of the prolate coefficients b_m^{mn} are more complicated but still very useful.

V. CONCLUSION

Calculation mode for spheroidal radial functions of the second kind, $R_{mn}^{(2)}$, strongly depends upon the size parameter c and radial argument ξ. Only for large values of $c\xi$, the ordinary expansion in Eq. (2) is applicable. The expressions relating to the associated Legendre functions of the first and the second kind in Eqs. (23-1) and (23-2) converge well, so long as c is not too large for small values of ξ, that is, for values of ξ near, but not including, unity in the prolate case, and for values of ξ near, and including, zero in the oblate case. If values of ξ are close to unity for the prolate with high aspect ratio, the above-mentioned two modes break down. Then, the expansions derived in the preceding sections provide the powerful way for values of ξ slightly greater than unity but less than the first value of ξ,at which $R_{mn}^{(1)}(c,\xi)$ is zero,in the prolate case and for values of ξ greater than zero but less than the first value of ξ,at which $R_{mn}^{(1)}(-ic,i\xi)$ is zero,in the oblate case.

It is worth pointing that the new calculation mode for spheroidal radial functions in this paper has been employed to both prolate and oblate spheroids with aspect ratio as high as 20:1 and size parameter as large as 50∿60.

VI. ACKNOWLEDGMENT

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A Selective Survey of Computational Electromagnetics

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ABSTRACT

The continuing growth of computing resources is changing how we think about, formulate, solve, and interpret problems. In electromagnetics as elsewhere, computational techniques are complementing the more traditional approaches of measurement and analysis to vastly broaden the breadth and depth of problems that are now quantifiable. An attempt is made in this article to place some of the tools used in computational electromagnetics into perspective with respect to the different kinds of approaches that may be used and their computer-resource requirements. After a brief background discussion in Section 2, we review in Sections 3 and 4 respectively some of the analytical and the numerical issues involved in developing a computer model. In Section 5 we include some practical considerations from the viewpoint of computer-resource requirements, followed by a discussion of ways by which computer time might be reduced. Our presentation concludes with a brief examination of validation and error checking. Emphasis throughout is on review and summarization rather than detailed exposition.

I. INTRODUCTION

Computational ElectroMagnetics (CEM) may be broadly defined to be that branch of electromagnetics that intrinsically and routinely involves using a digital computer to obtain numerical results. With the evolutionary development of CEM during the past 20-plus years, two basic lines of improvement can be identified. One is due to advances taking place in computer hardware and software, providing tools of steadily growing power with little effort on the part of the electromagnetics community per se. The other line of improvement originates from within the electromagnetics discipline itself, where increasing awareness and utilization of numerical techniques has provided an expanding base of capability for solving problems in electromagnetics. The result has been to add the third tool of computational methods in EM specifically, and in science and engineering generally, to the two classical tools of experimental observation and mathematical analysis.

The goal of this article is to review concisely and conceptually some of the basic issues involved in CEM, to survey present capabilities and to contemplate future directions where appropriate. We attempt to accomplish this extremely broad task by introducing only the detail needed to illustrate the central ideas involved, and providing a selection of references from which the interested reader may obtain more information. Emphasis will be focussed on the underlying principles which unify the various modeling approaches used in electromagnetics while avoiding most of the detail that makes them different. In section 2 we begin with a brief discussion of the basic idea. In Sections 3 and 4 respectively, we examine some of the analytical and numerical issues involved in developing an electromagnetic computer model. Some of the practical implications of developing models in terms of the required computer resources are considered in Section 5, followed in Section 6 by a more detailed discussion of various means by

which computer-time requirements might be reduced. In Section 7 we examine errors encountered and model validation, followed by a concluding summary in Section 8. While we attempt to provide some perspective concerning the relationship between differential- (DE) and integral-equation (IE) modeling, more attention will be devoted to the latter.

II. BACKGROUND DISCUSSION

Electromagnetics is the scientific discipline that deals with electric and magnetic sources and the fields these sources produce in specified environments. Maxwell's Equations provide the starting point for the study of electromagnetic problems, together with certain principles and theorems such as superposition, reciprocity, equivalence, induction, duality, linearity, uniqueness, etc. derived therefrom [Harrington (1961), Van Nostrand Scientific Encyclopedia (1976)]. While a variety of specialized problems can be identified, a common ingredient of essentially all of them is that of establishing a quantitative relationship between a cause (forcing function or input) and its effect (the response or output).

A. Modeling as a Transfer-Function

This relationship may be viewed as a generalized transfer function (see Fig. 1) in

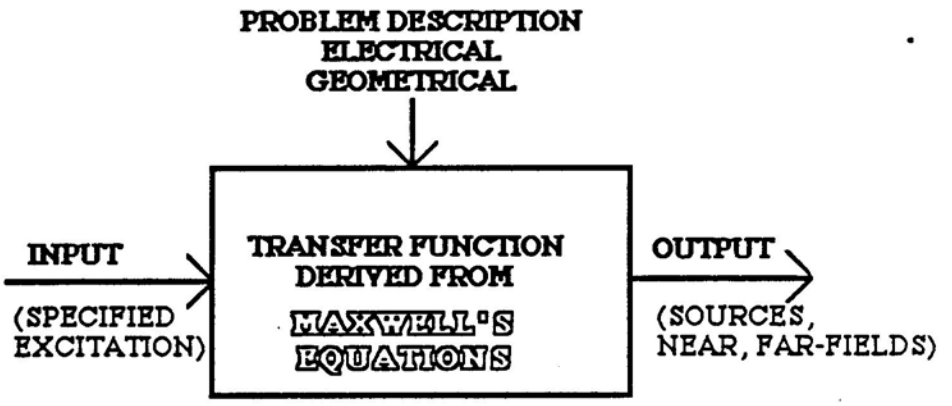

Figure 1. Transfer-function model for electromagnetics.

which two basic problem types become apparent. For the analysis or direct problem, the input is known and the transfer function is deriveable from the problem specification, with the output or response to be determined. For the case of the synthesis or inverse problem, two problem classes may be identified . The "easy" synthesis problem involves finding the input given the output and transfer function, an example of which is that of determining the source voltages which produce an observed pattern for a known antenna array. The "hard" synthesis problem itself separates into two problems. One is that of finding the transfer function, given the input and output, an example of which is that of finding a source distribution that produces a given far field. The other and still more difficult, is that of finding the object geometry which produces an observed scattered field from a known exciting field. The latter problem is the most difficult of the three synthesis problems to solve because it is intrinsically transcendental and non-linear. Furthermore, such problems are subject to uniqueness constraints which can impose difficulties and uncertainties in developing their solutions.

Electromagnetic transfer functions almost always include a field propagator, as the "cause" mentioned above normally involves a source whose fields produce the "effect". It is therefore valid to conclude that the essence of electromagnetics is the study and determination of field propagators, and it follows that CEM inevitably does also. This conclusion, while perhaps appearing transparent, is actually an extremely fundamental one as it provides a focus for what CEM is all about, and provides a basis for classification of model types as we now discuss.

B. Some Issues Involved in Developing a Computer Model

In order to establish an appropriate perspective for subsequent discussion, we briefly consider here a classification of model types, the steps involved in developing a computer model, the desirable attributes of a computer model, and finally the role of approximation throughout the modeling process.

1. Classification of Model Types

It is convenient to classify solution techniques for electromagnetic modeling in terms of the field propagator that might be used, the anticipated application, and the problem type for which the model is intended to be used as is outlined in Table 1.

2. Development of a Computer Model

Development of a computer model in electromagnetics or literally any other disciplinary activity can be decomposed into a small number of basic, generic steps. These steps might be described by different names, but would include at a minimum those outlined in Table 2. Note that by its nature, validation is an open ended process which cumulatively can absorb more effort than all the other steps together. The primary focus of the discussion which follows in this article is on areas (3) through (6).

3. Desirable Attributes of a Computer Model

A computer model must have some minimum set of basic properties to be useful. From the long list of attributes that might be desired, we consider: 1) accuracy, 2) efficiency, and 3) utility the three most important as summarized in Table 3. Accuracy is put foremost since results of insufficient or unknown accuracy have uncertain value and may even be harmful. On the other hand, a code that produces accurate results but at unacceptable cost will have hardly any more value. Finally, a code's applicability in terms of the depth and breadth of the problems for which it can be used determines its utility.

4. The Role of Approximation

As approximation is an intrinsic part of each of step involved in developing a computer model, we summarize some of the more commonly used approximations in Table 4. We note that the distinction between an approximation at the conceptualization step and during the the formulation is somewhat arbitrary, but choose to use the former category for those approximations that occur prior to the formulation itself.

III. ANALYTICAL ISSUES IN DEVELOPING A COMPUTER MODEL

As mentioned above, selection of a field propagator is a first step in developing an

electromagnetic computer model. Further attention here is limited to propagators which employ either the Maxwell curl equations or source integrals which employ a Green's function. We consider briefly first selection of the solution domain and then selection of the field propagator with discussion here limited to integral- and differential-equation models.

A. Selection of Solution Domain

Either the IE or DE propagator can be formulated in the time domain, where time is treated as an independent variable, or in the frequency domain where the harmonic time variation $\exp(i\omega t)$ is assumed). The numerical treatment of generic time- and frequency-domain operators denoted by $L(\omega)$ and $L(\tau)$ respectively is compared in Table 5.

Whatever propagator and domain are chosen, the analytically formal solution can be numerically quantified via use of the method of moments (MoM) [Harrington (1968)], leading ultimately to a linear system of equations as a result of developing a discretized and sampled approximation to the continuous (generally) physical reality being modeled. Developing the approach that may be best suited to a particular problem involves making tradeoffs among a variety of choices throughout the analytical formulation and numerical implementation. In the following discussion, we consider some aspects of these choices and their influence on the utility of the computer model which eventually results.

B. Selection of Field Propagator

We briefly discuss and compare below the characteristics of IE- and DE-based models in terms of their development and applicability.

1. Integral-equation (IE) model

The basic starting point for developing an IE model in electromagnetics is selection of a Green's function appropriate for the problem class of interest. While there are a variety of Green's functions from which to choose, a typical starting point for most IE MoM models is that for an infinite medium. Although the formulation might be accomplished in various ways, one of the more straightforward is based on the scalar Green's function and Green's theorem. This leads to the Kirchoff integrals [Stratton (1941), p. 464, et. seq.] from which the fields in a given contiguous volume of space can be written in terms of integrals over the surfaces which bound it and volume integrals over those sources located within it. Exceptions to this general rule are discussed below in Section 4.

Analytical manipulation of a source integral which incorporates the selected Green's function as part of its kernel function then follows, with the specific details depending on the particular formulation being employed. Perhaps the simplest is that of boundary-condition matching wherein the behavior required of the electric and/or magnetic fields at specified surfaces which define the problem geometry is explicitly imposed. Alternative formulations, for example, the Rayleigh-Ritz variational method and Rumsey's reaction concept might be used instead, but as pointed out by Harrington (1980), from the viewpoint of a numerical implementation any of these approaches lead to formally equivalent models.

This analytical formulation leads to an integral operator, whose kernel can include differential operators as well, which acts on the unknown source or field. Although it would be more accurate to refer to this as an integro-differential equation, it is usually called simply an integral equation. Two kinds of integral equations are obtained, one known as a Fredholm integral equation of the first kind in which the unknown appears only under the integral, and the other a second-kind equation in which the unknown also appears outside the integral. In Table 6 are included some of the integral equations used in CEM, including the magnetic-field integral equation (MFIE) and electric-field integral equation (EFIE) for perfect conductors for both the frequency domain and time domain.

2. Differential-Equation (DE) Model

A DE MoM model, being based on the defining Maxwell's Equations, requires intrinsically less analytical manipulation than does derivation of an IE model. Numerical implementation of a DE model however, can differ significantly from that used for an IE formulation in a number of ways for several reasons:

i. The differential operator is a local rather than global one in contrast to the Green's function upon which the integral operator is based. This means that the spatial variation of the fields must be developed from sampling in as many dimensions as possessed by the problem, rather than one less as the IE model permits if an appropriate Green's function is available.

ii. The integral operator includes an explicit radiation condition.

iii. The differential operator includes a capability to treat medium inhomogeneities, non-linearities, and time variations in a more straightforward manner than does the integral operator.

These and other differences between development of IE and DE models are summarized in Table 7, with their modeling applicability compared in Table 8.

IV. NUMERICAL ISSUES IN DEVELOPING A COMPUTER MODEL

A. Sampling functions

At the core of numerical analysis is the idea of polynomial approximation, an observation made by Arden and Astill (1969) in facetiously using the subtitle "Numerical Analysis or 1001 Applications of Taylor's Series". The basis idea is to approximate quantities of interest in terms of sampling functions, often polynomials, that are then substituted for these quantities in various analytical operations. Thus, integral operators are replaced by finite sums and differential operators are similarly replaced by finite differences. For example, use of a first-order difference to approximate a derivative of the function F(x) in terms of samples $F(x_+)$ and $F(x_-)$ leads to

$$dF(x)/dx \approx [F(x_+) - F(x_-)]/h;\ x_- \le x \le x_+ \qquad (1a)$$

and implies a linear variation for F(x) between x_+ and x_- as does use of the trapezoidal rule

$$\int_{x_-}^{x_+} F(x)dx \approx h[F(x_+) + F(x_-)]/2 \qquad (1b)$$

to approximate the integral of F(x), where $h = x_+ - x_-$. The central-difference approximation for the 2nd derivative,

$$d^2F(x)/dx^2 \approx [F(x_+) - 2F(x_0) + F(x_-)]/h^2; \quad (1c)$$

similarly implies a quadratic variation for F(x) around $x_0 = x_+ - h/2 = x_- + h/2$, as does use of Simpson's rule

$$\int_{x_-}^{x_+} F(x)dx \approx h[F(x_+) + 4F(x_0) + F(x_-)]/6 \quad (1d)$$

to approximate the integral. Other kinds of polynomials and function sampling can be employed, as discussed in a large volume of literature, some examples of which are Abramowitz and Stegun (1964), Acton (1970), and Press et. al. (1986). It is interesting to see that numerical differentiation and integration can be accomplished using the same set of function samples and spacings, differing only in the signs and values of some of the associated weights. Note also that when the function samples can be unevenly spaced, as in Gaussian quadrature, the result will always be more accurate (for well-behaved functions) for a given number of samples. This suggests the benefits that might be derived from using unequal sample sizes in MoM modeling should a systematic way of determining the best sampling scheme be developed.

B. The Method of Moments (MoM)

Numerical implementation of the moment method is a relatively straightforward, and an intuitively logical extension, of these basic elements of numerical analysis, as described in the well-known book by Harrington (1968) and discussed and used extensively in CEM [see for example Mittra (1973, 1975), Strait (1980), Strait and Adams (1980), Harrington et. al. (1981), Perini and Buchanan (1985), Ney (1985), Itoh (1986), Poggio and Miller (1987)]. Whether it is an integral equation, a differential equation, or another approach that is being used for the numerical model, there are three essential sampling operations that are involved in reducing the analytical formulation via the moment method to a computer algorithm as outlined in Table 9. We note that operator sampling can ultimately determine the sampling density needed to achieve a desired accuracy in the source-field relationships involving integral operators, especially at and near the "self term" where the observation and source points become coincident or nearly so. Whatever the method used for these sampling operations, they lead to a linear system of equations or matrix approximation of the original integral or differential operators. Since the operations and choices involved in developing this matrix description is common to all moment-method models, we discuss them in somewhat more detail below.

When using IE techniques, the coefficient matrix in the linear system of equations which results is most often referred to as an impedance matrix because in the case of the E-field form, its multiplication of the vector of unknown currents equals a vector of electric fields or voltages. The inverse matrix similarly is often called an admittance matrix because its multiplication of the electric-field or voltage vector yields the unknown-current vector. In this discussion we instead use the terms direct matrix and solution matrix since they are more generic descriptions whatever the forms of the originating integral or differential equations. As illustrated below, development of the direct matrix and solution matrix dominate both the computer time and storage requirements of numerical modeling.

In the particular case of an IE model, the coefficients of the direct or original matrix are

the mutual impedances of the multi-port representation which approximates the problem being modeled, and the coefficients of its solution matrix (or equivalent thereof) are the mutual admittances. Depending on whether a sub-domain or entire-domain basis has been used (see following section), these impedances and admittances represent either spatial or modal interactions among the N ports of the numerical model. In either case, these coefficients possess a physical relatability to the problem being modeled, and ultimately provide all the information available concerning any electromagnetic observables that are subsequently obtained.

Similar observations might also be made regarding the coefficients of the DE models, but whose multi-port representations describe local rather than global interactions. Because the DE model almost always leads to a larger, albeit less dense, direct matrix, its inverse (or equivalent) is rarely computed. It is worth noting that there are two widely used approaches for DE modeling, finite-difference (FD) and finite-element (FE) methods[Teng and Chang (1984), Mason and Anderson (1985)]. They differ primarily in how the differential operators are approximated and the differential equations are satisfied, although the FE method commonly starts from a variational viewpoint while the FD approach begins from the defining differential equations. The FE method is generally better suited for modeling problems with complicated boundaries to which it provides a piecewise linear approximation as opposed to the cruder stairstep approximation of FD.

1. Factors involved in choosing basis and weight functions

Basis- and weight-function selection plays a critical role in determining the accuracy and efficiency of the resulting computer model. One goal of the basis- and weight-function selection is to minimize computer time while maximizing accuracy for the problem set to which the model is to be applied. Another, possibly conflicting goal, might be that of maximizing the collection of problem sets to which the model is applicable. A third might be to replicate the problem's physical behavior with as few samples as possible. Some of the generic combinations of bases and weights that are used for MoM models are listed below in Table 10 [Poggio and Miller (1973)].

a. Basis-function Selection

We note that there are two classes of bases used in MoM modeling, sub-domain and entire-domain, functions. The former involves the use of bases which are applied in a repetitive fashion over sub-domains or sections (segments for wires, patches for surfaces, cells for volumes) of the object being modeled. The simplest example of a sub-domain basis is the single-term basis given by the "pulse" or stair-step function, which leads to a single, unknown constant for each sub-domain. Multi-term bases involving two or more functions on each sub domain and an equivalent number of unknowns, are more often used for sub-domain expansions.

The entire-domain basis on the other hand, uses multi-term expansions extending over the entire object, for example a circular harmonic expansion in azimuth for a body of revolution. As for sub-domain expansions, an unknown is associated with each term in the expansion. Examples of hybrid bases can also be found, where sub-domain and entire-domain bases are used on different parts of an object [Bornholdt and Medgyesi-Mitschang (1986)].

Although sub-domain bases are probably more flexible in terms of their applicability, they have a disadvantage generally not exhibited by the entire-domain form, which is the

discontinuity that occurs at the domain boundaries. This discontinuity arises because an n_s-term sub-domain function can provide at most n_s-1th continuity to an adjacent basis of the unknown it represents assuming one of the n_s constants is reserved for the unknown itself. For example, the three-term or sinusoidal sub-domain basis $a_i + b_i\sin(ks) + c_i\cos(ks)$ used for wire modeling can represent a current continuous at most up to its first derivative. This provides continuous charge density, but produces a discontinuous first derivative in charge equivalent to a tri-pole charge at each junction.

As additional terms are used to develop a sub-domain basis, higher-order continuity can be achieved in the unknown that the basis represents assuming still that one constant is reserved for the unknown. In the general case of the n_s-term sub-domain basis, up to n_s-1 constants can be determined from continuity conditions with the remainder reserved for the unknown. The kind of basis function employed ultimately determines the degree of fit that the numerical result can provide to the true behavior of the unknown for a given order of matrix. An important factor that should influence basis-function selection then is how closely a candidate function might resemble the physical behavior of the unknown it represents.

b. Weight-function Selection

The simplest weight that might be used is a delta function which leads to a point-sampled system of equations. But point sampling of the field operators can reveal any numerical anomalies that might arise as a result of basis-function discontinuities. Distributed, multi-term weight functions can also be employed on either a sub-domain or entire-domain basis, to provide a further smoothing of the final equations to be solved. One example of this is the special case where the same functions are used for both the bases and weights, a procedure known as Galerkin's method. The kind of testing function employed ultimately determines the degree to which the equations can be matched for a given basis function and number of unknowns. Some specific examples of basis- and weight-function combinations used in electromagnetics are summarized in Table 11.

2. Computing the Direct Matrix

We observe that obtaining the coefficients of the direct matrix in IE modeling is generally a two-step process. The first step is that of integrating the defining integral equation in which the unknown is replaced by the basis functions selected. The second step involves integration of this result multiplied by the weight function selected. When using delta-function weights this second step is numerically trivial. But when using non-delta weights, such as the case in a Galerkin approach where the same function is used for both basis and weights, this second step can be analytically and numerically challenging.

Among the factors affecting the choice of the basis and weight functions therefore, one of the most important is that of reducing the computational effort needed to obtain the coefficients of the direct matrix. This is one of the reasons, aside from their physical appeal, why sinusoidal bases are often used for wire problems. In this case, where piece-wise linear, filamentary current sources are most often used in connection with the thin-wire approximation, field expressions are available in easily evaluated, analytical expressions [Richmond (1965), Miller and Deadrick (1975)]. This is the case as well where Galerkin's method is employed [Richmond (1974)].

Aside from such special cases however, numerical evaluation of the direct matrix coefficients will involve the equivalent of point sampling of whatever order is needed to achieve the desired accuracy as illustrated below. Using a wire-like one dimensional problem to illustrate this point, we observe that at its most elementary level evaluation of the ij'th matrix coefficient then involves evaluating integrals of the form

$$Z_{i,j} = \int w_i(s) \int [b_j(s')K(s,s')ds']ds$$

$$\approx \sum p_m q_n w_i(s_n) b_j(s'_m) K(s_n, s'_m) \qquad (2)$$

$$= \sum\sum p_m q_n z(i,j,m,n);\ m=1,\ldots,M(i,j) \quad ;n=1,\ldots,N(i,j) \quad i,j=1,\ldots,N$$

where K(s,s') is the IE kernel function, and s_n and s'_m are the n'th and m'th locations of the observation and source integration samples. Thus, the final, direct-matrix coefficients can be seen to be "constructed" from sums of the more elementary coefficients z(i,j,m,n) weighted by the quadrature coefficients p_m and q_n used in the numerical integration, which will be the case whenever analytical expressions are not available for the $Z_{i,j}$. These elementary coefficients, given by $w_i(s_n)b_j(s'_m)K(s_n,s'_m)$ can in turn be seen to be simply products of samples of the integral-equation kernel or operator, and sampled basis and testing functions. It should be apparent from this expanded expression for the direct-matrix coefficients that interchanging the basis and weight functions leaves the final problem description unchanged, although the added observation that two different integral equations can yield identical matrices when using equivalent numerical treatments is less obvious [Wilton and Butler (1976), (1981)].

3. Computing the Solution Matrix

Once the direct matrix has been computed, the solution can be obtained numerically using various approaches. These range from inversion of the direct matrix to developing a solution via iteration as summarized in Table 12. A precautionary comment is in order with respect to the accuracy with which the solution matrix might be obtained. As computer speed and storage have increased, the number of uknowns employed in modeling have also increased, from a few 10s in earlier years to 1,000s now when using IE techniques. The increasing number of operations involved in solving these larger matrices increases sensitivity of the results to round-off errors. This can be especially the case when the direct matrix is not well conditioned. It is therefore advisable to perform some sensitivity analyses to determine the direct-matrix condition number and to ascertain the possible need for performing some of the computations in double precision.

4. Obtaining the Solution

When a solution matrix has been developed using inversion or factorization,subsequently obtaining the solution is computationally straightforward, involving multiplication of the right-hand-side (RHS) source vector by the solution matrix. When an iterative approach is used, a solution matrix is not computed but the solution is instead developed from RHS-dependent manipulation of the direct matrix. Motivation for the latter comes from the possibility of reducing the N_x^3 dependency of the direct procedure. As problem size increases, the computation cost will be increasingly

dominated by the solution time.

V. SOME PRACTICAL CONSIDERATIONS

Although the overall solution effort has various cost components, perhaps the one most considered is the computer time and storage required to obtain to obtain the numerical results desired. With the increasing computer memories becoming available, where even micro-computers and work stations can directly address giga-bytes, the memory costs of modeling are becoming generally less important than the time cost, with which we are primarily concerned with here.

A. Integral-equation modeling

1. Frequency domain

If we consider an IE model specifically, we can show that in general, the computer time associated with its application is dependent on the number of unknowns N_x in the frequency domain as

$$T_{IE,\omega} \approx A_{fill}N_x^2 + A_{solve}N_x^3 + A_{source}N_x^2N_{sources} + A_{field}N_xN_{sources}N_{fields} \propto (L/\Delta L)^{3d}, \text{ for } N_x > A_{fill}/A_{solve} \quad (3a)$$

where the A's are computer- and algorithm-dependent coefficients which account for computation of:

A_{fill}--the direct (impedance) matrix,

A_{solve}--the solution (admittance) matrix (assuming inversion or factorization),

A_{source}--the source distribution for one excitation,

A_{field}--one field value, where $A_{field} \leq A_{fill}$ depending on whether a near-field (=) or far-field (<) value is obtained.

2. Time domain

A similar relationship holds for a time-domain IE model which uses N_t time steps,

$$T_{IE,t} \approx A_{source}N_x^2N_tN_{sources} + A_{field}N_tN_{sources}N_{fields} \propto (L/\Delta L)^{2d+1+p}, \; 0 \leq p \leq 1 \text{ depending on the number of sources} \quad (3b)$$

where the A's account for computation of the time-domain terms equivalent to their frequency-domain counterparts above. Although a direct matrix may require solution initially prior to time-stepping the model, that is normally avoided by using $\delta t \leq \Delta x/c$, which yields an explicit solution.

As can be appreciated from these expressions, the number of unknowns that are required for these computations to be acceptably accurate have a strong influence on the computer time eventually needed.

B. Differential-equation modeling

1. Frequency domain

DE modeling is less commonly used in the frequency domain primarily

because the order of the matrix that results depends on $(L/\Delta L)^D$ rather than the usual $(L/\Delta L)^{D-1}$ dependency of an IE model. On the other hand, the matrix coefficients require less computation whether the DE model is based on a finite-difference or finite-element treatment. Furthermore, the matrix is very sparse because a differential operator is a local rather than global one as is the integral operator. Matrix fill time is therefore generally not of concern, and the overall computer time is dominated by the direct matrix solution time, given approximately by

$$T_{DE,\omega} \approx A_{solve} N_x W^2 = A_{solve} N_x^p;\ p = 1,\ 2,\ 7/3 \text{ for } d = 1,\ 2, \text{ and } 3 \qquad (3c)$$
$$\propto (L/\Delta L)^{3d-2}$$

Note that the banded nature of the DE direct matrix has been taken into account where the bandwidth varies as N_x^0, $N_x^{1/2}$, and $N_x^{2/3}$ respectively ($N_x^{[(d-1)/d]}$) in 1, 2, and 3 dimensions.

2. Time domain

Time-domain DE modeling can employ either implicit or explicit solution methods for developing the time variation of the solution. An explicit technique is one whereby the update at each time step is given in terms of solved-for past values of the unknowns and the present excitation, with no interaction permitted between unknowns within the same time step. An implicit technique on the other hand does allow for interaction of unknowns within the same time step, but can therefore require the solution of a matrix equation. In spite of this disadvantage, implicit techniques are important because they are not subject to Courant instability when $c\delta t > \Delta x$ as is an implicit approach.

The solution time for the explicit case is approximated by

$$T_{DE,t} \approx A_{step} N_x N_t \qquad (3d)$$
$$\propto (L/\Delta L)^{d+1}$$

while for the implicit case we have

$$T_{DE,t} \approx (A_{step} N_x^2 N_t + N_x^3)$$
$$\propto (L/\Delta L)^{2d+1}$$

assuming a banded matrix is employed to solve the implicit direct matrix.

C. Sampling requirements

We may estimate the number of samples needed to adequately model the spatial, temporal, and angular variation of the various quantities of interest in terms of an object characteristic length L, and sampling dimension d. This may be done from knowledge of the typical spatial and temporal densities determined from computer experiments, and/or from invocation of Nyquist-like sampling rates for field variations in angle as a function of aperture size. The resulting estimates are summarized in Table 13 and apply to both IE and DE models.

These may be regarded as wavelength-driven sampling rates, in contrast with the complexity-driven sampling rates that can arise due to problem variations that are small in scale compared with λ. Complexity-driven sampling would affect primarily N_x, resulting in larger values than those indicated above.

We note that the computer time is eventually dominated by computation of the solution

matrix, and can grow as f^3, f^6, and f^9 respectively for wire, surface and volume objects modeled using integral equations and matrix factorization or inversion. Thus, in spite of the fact that main-frame computer power has grown by a factor of about 10^6 from the UNIVAC-1 to the CRAY2, the growth in problem size is much less as illustrated by Fig. 2. The curves on this graph demonstrate emphatically the need for finding faster ways of performing the computations, some aspects of which we next discuss.

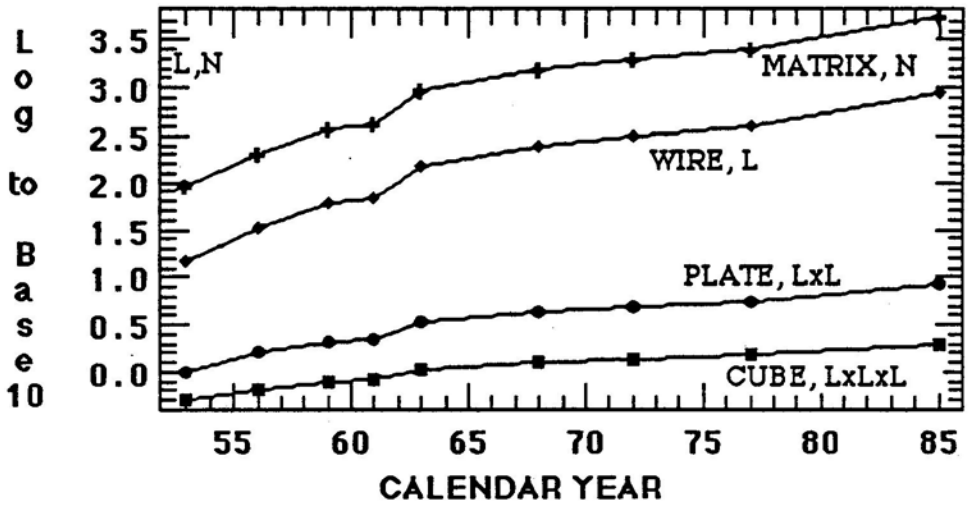

Figure 2. Time development of IE-based modeling capability for wire, plate, and penetrable cube of characteristic dimension L in wavelengths, and matrix or order N solvable in one hour of computer time using main-frame computers introduced in the years indicated.

VI. WAYS OF DECREASING COMPUTER TIME

The obvious drawback of direct moment-method models as N_x increases with increasing problem size and/or complexity suggests the need for less computationally intensive alternatives. There are various alternatives for decreasing the computer cost associated with solving electromagnetic problems using the method of moments. The basic intent in any case is either to reduce the direct cost of performing a given modeling computation, or to reduce the number of modeling computations needed to obtain a desired result. These might include analytical, computational, and experimental approaches or combinations thereof, some of which we briefly consider here.

A. Analytical

Analytical alternatives might include extensions of the basic approach such as use of specialized Green's functions, combination of two or more approaches in a hybrid formulation, or a variety of other possibilities, some of which are briefly discussed below and outlined in Table 15.

1. Specialized Green's functions

The basic integral equations presented above employ an infinite-medium Green's function (IMGF), use of which leads to a problem description in terms of only the surface sources. The IMGF "propagates" the fields of these sources throughout the problem volume of interest, and satisfies as its only "boundary condition" the

Sommerfeld radiation condition as r--->∞. The required boundary conditions on the object(s) being modeled are satisfied through the MoM solution of the formal integral equation.

One analytical approach for developing more efficient IE models is use of the specialized Green's functions which satisfy additional boundary conditions. Although such Green's functions are available for only a relatively small set of separable geometries, they none-the-less provide a useful alternative for some problems of practical interest. These include planes, cylinders, spheres, ellipsoids, etc. for which delta-source Green's functions provide formally a means of modeling wires or other objects located in their vicinity. The tradeoff in using such Green's functions is that of reducing the number of unknowns needed in the model at the expenses of dealing with a significantly more complex integral-equation kernel.

For example, modeling an object near the infinite, planar interface between two half spaces could include the interface tangential fields as unknowns as well as those associated with the object itself [Miller (1981)]. Alternatively, a Green's function which satisfies the interface boundary conditions can be used instead, one possibility being inclusion of the Sommerfeld integrals as part of the integral-equation kernel [Miller, et. al. (1972a), (1972b)]. Although this is advantageous in that the fields over a (necessarily) truncated area of the infinite interface do not need explicit solution, the complexity of numerically evaluating the Sommerfeld integrals must then be dealt with. Even so, the advantages of using the specialized Green's function make it an attractive alternative. Examples are given by: Lucke (1951) and Lytle (1971) for a cylinder; by Einziger and Felsen (1983) for a cylindrical sector; by Tesche (1972) for a planar region bounded by perfectly conducting, parallel plates; by Tesche and Neureuther (1970) for a sphere; and by Pathak (1983) for an eigen-function expansion of dyadic Green's functions. In Table 16, we summarize several of the types of Green's functions that have been or might be used in IE modeling.

2. Hybrid approaches

One example of a hybrid approach is that of combining MoM with the geometrical theory of diffraction (GTD), as first reported by Thiele and Newhouse (1969) and more recently as summarized by Burnside and Pathak (1980). GTD has the attractive property that the computation time is essentially independent of increasing problem size or frequency, in contrast with the moment method. On the other hand, increasing complexity can result in the need to trace more rays and compute more diffraction coefficients, which can ultimately limit the applicability of GTD models. As an alternative to modeling a given problem using only the MoM or GTD alone, they can be used in combination to exploit their complementary advantages, thereby obtaining a model which is significantly better than either one used separately. Such "hybrid" techniques appear to be one of the more promising means for developing the models needed for large, complex problems.

Other hybrid techniques have been developed, one example being the combined use of a DE model and a modal expansion, termed the unimoment method by Mei (1974). As implemented by Mei, this technique involves solving a bounded, three-dimensional, body of revolution using a DE model and matching this solution across a spherical enclosing surface to an external, infinite medium where the solution is given by a modal expansion. A similar solution to a two-dimensional problem was presented by Miller and

Olte (1966) for an inhomogeneously sheathed, infinite cylinder. Combination of DE and IE modeling in the time domain is described by Taflove and Umashankar (1980), where the IE model provides the short-circuit currents on a body whose interior response is subsequently obtained using the DE model. Several of these hybrid treatments are summarized in Table 17.

3. Other

Other analytically based methods for simplifying the formulation and/or the subsequent computation include the impedance boundary condition (IBC) and the physical optics (PO) approximation. The IBC [Senior (1981)] involves an assumption that the tangential electric and magnetic fields on the surface of a penetrable object can be related through a surface impedance Z_{surf}, e.g., $\mathbf{E}_{tan} = Z_{surf}\mathbf{n}\times\mathbf{H}_{tan}$, where $\mathbf{n}$ is a surface normal and $Z_{surf} = (1 + j)\omega\mu\delta/2$ with δ the skin depth. This assumption halves the number of unknowns needed for modeling the object since the magnetic field can replace the electric field throughout the IE model, as is usually done, or vice versa. The benefit of the IBC occurs primarily in the computational effort needed in obtaining the solution matrix, as computing the direct matrix is essentially unchanged because the interaction fields of the replaced unknown must still be accounted for.

For acceptable results, the radius of curvature of the body must be large with respect to skin depth δ. A modified form which is applicable to bodies having a smaller radius of curvature is given by $E_u = (1 - p)Z_{surf}H_v$ and $E_v = (1 + p)Z_{surf}H_u$ where $p = (1 - j)\delta(C_v - C_u)/4$ and C_v and C_u are the principal curvatures [Mitzner (1967)]. Note that the object need not be solid, as the IBC can also be used to develop a sheet-impedance model for dielectric shells [Harrington and Matuz (1975)].

Just as the IBC can be used to eliminate one-half of the unknown surface fields involved in modeling a penetrable body, the PO approximation can eliminate the remaining unknowns. It involves making the further assumption that the surface current can be expressed directly in terms of the incident field. For a perfect conductor, the substitution $\mathbf{J}_S = 2\mathbf{n}\times\mathbf{H}^{inc}$ is made, although more generally we might use $\mathbf{J}_S = (1\text{-}R)\mathbf{n}\times\mathbf{H}^{inc}$ and $\mathbf{K}_S = -(1+R)\mathbf{n}\times\mathbf{E}^{inc}$, where R is the plane-wave reflection coefficient for the object. Using the PO approximation thus circumvents the need to compute a direct matrix or to obtain its solution, and the fields resulting from the incident excitation can be computed directly by evaluating the appropriate source integrals.

A further logical extension of IBC and PO is typified by the reflection- coefficient approximation (RCA) or modified-image theory that has been used in modeling interface problems [Miller, et. al. (1972a), (1972b), Burke and Miller (1984)]. In applying the RCA, not only are the interface sources not solved for, but their integration over the interface is circumvented by approximating their effects with image fields multiplied by Fresnel plane-wave reflection coefficients. This is an exact procedure for perfectly conducting, planar interfaces, but is otherwise an approximation although the RCA has a reasonably wide scope of applicability. The RCA is one way of avoiding the rigor, but computational complexity, of the Sommerfeld Green's function for the interface problem as discussed above. By extending the concept of an image treatment, an approach of rigor comparable to the Sommerfeld treatment can be developed [Lindell, et. al. (1985)], but also of comparable computational complexity [Burke and Miller (1986)].

B. Numerical

There are a number of numerical possibilities by which the cost of a moment-method model might be reduced. These include using alternate solution techniques for solving the linear system of equations and fully exploiting problem symmetries, the goal of either being to reduce the N^2 and N^3 dependencies of the matrix fill and solution terms in Eq. (3). Another possibility is provided by the so-called near-neighbor approximation (NNA) which involves neglecting interactions between source-field points separated by more than some specified distance when using a sub-domain basis [Ferguson, Lehman, and Balestri (1976)], or between modes separated by more than some wavenumber difference when using an entire-domain basis [Medgeyesi-Mitschang and Putnam (1983), (1985)]. The fill time for the direct matrix and its subsequent solution time can then also be reduced, although achieving the latter depends on being able to exploit the sparseness of a matrix whose non-zero coefficients may in general be widely distributed throughout it. Some of these numerical means of reducing computer time are summarized in Table 18.

1. Alternate Solution Techniques

The number of multiplies/divides (operations) needed in applying direct solution techniques to linear systems is proportional to N_x^3. For this computational investment there is obtained a solution matrix which is independent of the RHS forcing function or excitation. Solutions are then available for an additional computational cost per solution proportional to N_x^2 (multiplication of an N_x'th order matrix by an N_x'th order vector or the equivalent).

If a RHS-dependent solution is acceptable, then techniques which have a lower-order dependence on N_x can be considered. This might be the case for example where an antenna or single-source problem is being modeled or where the fields scattered from an object for a limited number of incidence angle or sources are needed. In such situations, iterative, rather than direct solution techniques provide logical alternatives. While the details can vary since a variety of iteration strategies are available, the basic goal is to achieve an acceptably accurate solution after a number of iterations N_i such that $N_i \ll N_x$. Since each iteration requires on the order of N_x^2 operations, this means that a solution for one RHS might be achieved after $N_x^2 N_i$ operations rather than the N_x^3 required by a direct solution.

Some of the better-known iteration techniques are Jacobi, Gauss-Seidel, and Gauss-Seidel with simultaneous over relaxation [Press, et. al. (1986)], with the first being of interest only in a comparative sense. They vary basically in how the updating is performed at each iteration step.

A quite different kind of iteration procedure that has received much attention recently is the conjugate-gradient technique (CGT) [Sarkar (1986)]. This approach actually can be found in two forms. One is where CGT is applied to an already-computed direct matrix, and in that sense is conceptually similar to other iteration methods, in which form it is called the conjugate-direction method. The other is where CGT is incorporated into the process of generating the original matrix itself, in which case the effect is to adaptively vary the basis and weighting functions being used for the MoM model. An especially

attractive property of CGT is that convergence is guaranteed after a sufficient number of iterations N_I, although because N_I can be in principle as large as N_X that outcome may not be very beneficial since up to N_X^3 operations could possibly be needed. The convergence rate of CGT, as is the case of other iterative techniques, is determined by the eigen-value spectrum of the operator matrix, with the best situation being all eigen values nearly equal. In this respect, CGT "prefers" a well-conditioned matrix, as one measure of matrix conditioning is a small ratio of the maximum to minimum eigen value. A distinct advantage of CGT is that a quantitative error measure is provided at each iteration step so that there is no numerical uncertainty about when the iterations can be concluded.

2. Exploiting Problem Symmetries

Problem symmetries provide one of the more effective means of reducing both computer time and storage. The three basic types of symmetry are reflection, rotation, and translation symmetry. The first arises when an object is reflected about 1, 2 or 3 planes, as demonstrated by a straight wire, rectangular plate and right-angle quadrilateral. Rotation (or circulant) symmetry occurs when an object reproduces itself upon rotation through $2\pi/n$ radians where the number of rotational sectors is denoted by the integer n. When n is finite, as for a regular polygon having n sides, the rotation symmetry is discrete. But n can also be effectively infinite, as for a circular loop which exhibits continuous rotation symmetry. The relationship between discrete and continuous rotational symmetry is equivalent to that between the discrete and continuous Fourier transform. Translation symmetry results when an object is created by rectilinear translation of some fixed shape, for which a straight wire again serves as the most basic example [Medgeysi-Mitschang and Putnam (1983)].

Such symmetries can occur singly or in combination and can involve one or more unknowns per symmetric sector. The right circular cylinder provides an example that exhibits all three types (3 reflection planes, rotation about its axis, and translation along its axis). The computational benefits of symmetry accrue both in reducing computer storage and solution time because object symmetry creates a repetitive pattern in the direct matrix, the solution matrix, and the resulting solution, thus reducing the number of needed operations. We further discuss each symmetry type briefly below. Exploiting object symmetry requires that the excitation (RHS) be decomposed in the same manner as the object itself, in a fashion analogous to how a plane wave is expanded in circular harmonics when incident on an infinite, circular cylinder.

a. Reflection symmetry

Consider the example of one-plane reflection symmetry provided by a single horizontal wire located over a perfectly conducting, infinite plane. In this case, the direct matrix takes the form

$$Z = \begin{matrix} Z_{ss} & Z_{si} \\ Z_{is} & Z_{ii} \end{matrix}$$

where the subscripts "i" and "s" refer to the image fields and the self fields respectively and we observe that $Z_{ii} = Z_{ss}$, and $Z_{si} = Z_{is}$. But since the current I_i on the image will be the negative of the current I_s on the actual wire for any excitation applied to the latter, the direct matrix simplifies to

$$Z = Z_{ss} - Z_{si}$$

Similarly, if the wire were to be parallel to a perfect magnetic interface, then the image current would be equal to the actual current and the direct matrix simplifies to

$$Z = Z_{ss} + Z_{si}$$

These two cases correspond respectively to odd- and even-mode excitation. Compared with an unsymmetric problem having the same number of unknowns, the storage is reduced by one-half and the solution time by one-fourth. We note that this particular problem exhibits not only object and hence direct-matrix symmetry, but solution symmetry as well due to the constraints imposed by the image plane.

In general, the computer time for reflection symmetry can be expressed by

$$T_{\omega,ref} \approx A_{fill}(N_x/2^p)^2 + A_{solve}m(N_x/2^p)^3 \qquad (4a)$$

where p = 1,2, or 3 is the number of reflection planes, and m = 1,...,2^p is the number of excitation modes for which a solution is required.

b. Rotation Symmetry

In the case of rotation symmetry, the direct matrix has the general form

$$Z = \begin{matrix} Z_1 Z_2 Z_3 \cdots Z_n \\ Z_n Z_1 Z_2 \cdots Z_{n-1} \\ \cdot \\ \cdot \\ Z_2 Z_3 \cdots Z_n Z_1 \end{matrix}$$

where each Z_i can be a single coefficient or a matrix of order n_x where there are $n_x = N_x/n$ unknowns per rotational sector. For the circulant matrix which results from rotation symmetry, $Z_{ij} = Z_{i+k-\alpha n, j+k-\beta n}$, where $\alpha, \beta = 1$ if $i+k>n$, $j+k>n$ respectively and are otherwise zero. Rotation symmetry thus produces a special Toeplitz matrix, in that it is not only diagonal but has only n independent coefficients or blocks rather than the 2n-1 of the usual Toeplitz form.

Because of its structure, a circulant matrix yields solutions that are given by a Fourier series of discrete, orthogonal modes. Solution of the original matrix thus can be transformed to solution of m =1 to n reduced or modal matrices, the actual number m needing solution depending on the kind of excitation being used. Each of these reduced matrices yields a solution for one of the m modes out of which the general, overall solution is constructed. Reduction of the original direct matrix into these modal matrices can occur because the angular variation of each solution mode is known. This is analogous to how knowledge that a horizontal wire over a perfectly conducting ground plane has oppositely directed currents is used in the case of reflection symmetry. In the case of rotation symmetry and mode number m for example, the solution in sector r is related to that in sector s by exp[im(r-s)]. Therefore, knowledge of the solution in sector s=1 is sufficient to construct the modal solution for any mode m.

The solution time for a problem having rotation symmetry is given for the general case by

$$T_{\omega,rot} \approx A_{fill}N_x^2/n + A_{solve}n_m(N_x/n)^3 \qquad (4b)$$

where the fill time is thus reduced by a factor of n and the solution time by a factor of at

least n^2 relative to a problem without symmetry. Similarly, the storage is reduced by a factor of n as well. It is worth noting that when $n = N_x$ the solution time is proportional to $1 \le m \le N_x$.

c. Translation Symmetry

An object having translation symmetry produces a matrix having the structure

$$Z = \begin{matrix} Z_1 Z_2 Z_3 \cdots Z_n \\ Z_2 Z_1 Z_2 \cdots Z_{n-1} \\ \cdot \\ \cdot \\ Z_n Z_{n-1} \cdots Z_1 \end{matrix}$$

which is similar in appearance to a circulant matrix and where again each Z_i can be a single coefficient or a matrix of order n_x where there are $n_x = N_x/t$ unknowns per t cross sections in the direction of translation. While the matrices produced by rotation and translation symmetry are both of Toeplitz form, they exhibit subtle though significant differences. For rotation symmetry, the inverse (solution) matrix is circulant as well, but for translation symmetry the inverse (solution) matrix is instead the sum of two products of two triangular Toeplitz matrices [Bitmead and Anderson (1980)]. One consequence of this difference is that the computer time needed for translation symmetry is not reduced as much as for rotation symmetry. In particular, rotation symmetry results in a solution time which varies between N_x^0 and N_x^1, depending on the number of excitation modes, while that for translation symmetry can be as high as N_x^2 depending on the algorithm used. The overall computer time associated with translation symmetry is approximated by

$$T_{\omega,tran} \approx A_{fill} N_x + A_{solve} N_x [\log N_x]^2 \qquad (4c)$$

where the latter term arises from transforming the Toeplitz matrix of order N_x into a circulant matrix of order $2N_x$ [Bitmead and Anderson (1980)].

d. Combinatorial Symmetry

When a problem possess two or more kinds of symmetry it can be possible to exploit their combination as a factored sequence to obtain very significant further reduction in solution time. The possibility for doing so depends on the extent to which the modes which characterize each kind of symmetry remain orthogonal or separable. When mode coupling occurs between separately symmetric objects because they are not symmetrically oriented, as demonstrated by co-planar, identical, polygonal loops, then although the self-interaction of each object remains separable their interaction is not. If the loops are co-axial with their sides aligned however, then both the rotation and reflection symmetry can be fully exploited. One such example is given below to demonstrate the effects of combinatorial symmetry.

e. Example of Symmetry Application

Because the full impact of symmetry may be best appreciated by using a numerical example, let us briefly consider modeling a coaxial array of N_P identical, parallel, regular polygons having N_S sides so that $N_x = N_S N_P$. This problem leads to an overall direct matrix of N_x'th order which can be put in a block circulant array

of $N_S x N_S$ translation matrices each of order N_P. Since the sum of Toeplitz matrices is a Toeplitz matrix, the direct, block circulant matrix can be reduced to N_S Toeplitz matrices each of order N_P. Each of these N_S matrices can then be solved a time proportional to N_P^2.

For purposes of illustration, let $N_P = N_S = 1{,}000$ for a total of 1,000,000 unknowns, and assume that NEC [Numerical Electromagnetics Code, Burke and Poggio (1980)] is being run on a CDC-7600 computer for which $A_{fill} \approx 1.4 \times 10^{-7}$ hour and $B_{solve} \approx 5.6 \times 10^{-10}$ hour. The overall computer time can thus be concluded to change from (ignoring the fill time which is insignificant relative to the solution time)

$T \approx 5.6 \times 10^{8}$ hours ($\approx$64,000 years) without symmetry;

to $T \approx 5.6 \times 10^{2}$ hours with rotation symmetry utilized;

to $T \approx 5.6 \times 10^{-1}$ hours ($\approx$34 min) with both symmetries exploited.

A time reduction by a factor of $\sim N_x^{3/2} = 10^9$ is achieved in this case. Assuming that roundoff or other errors do not invalidate the numerical results for problems having this number of unknowns, the possibility of solving extremely large problems having high-order symmetry seems feasible. This is worth considering not only for those problems of practical interest that do have exploitable symmetries, but for testing other techniques for reducing computer time such as the near-neighbor approximation.

3. Near-neighbor Approximations

Motivation for the near-neighbor approximation (NNA) comes from the possibility that a window can be defined outside of which interactions are small enough to be ignored while still retaining acceptable accuracy in the solution. The NNA can be applied spatially in which case the rationale for ignoring interactions comes from the geometric attenuation of the fields with increasing distance from the source [Ferguson, et. al. (1976)]. A modal NNA can also be implemented for which the rationale is provided by the observation that coupling between modes tends to decrease as the difference between their mode numbers increases [Medgyesi-Mitschgang and Putnam (1983), (1985)]. The benefit in either case is that a direct matrix is produced having a smaller proportion of non-zero coefficients as object size increases, with a consequent potential reduction in both computer storage and solution time.

If the average number of non-zero coefficients in each row of the sparse, direct matrix which results from the NNA is W, then the matrix fill time increases only as WN_x rather than N_x^2. The subsequent solution time however depends on a number of factors, the most important one being the pattern of non-zeroes in the direct matrix. For the one-dimensional problem of a straight wire for example, the NNA direct matrix is banded of width W about the main diagonal and can be solved without further approximation in a time proportional to $N_x W^2$ and a savings relative to a full-matrix solution $\sim N_x^2$.

For two- and three-dimensional problems it is not generally possible to exploit matrix sparseness as easily and fully, as a banded matrix having many zero coefficients within the band is produced. The result for a square plate and a penetrable cube is a solution time proportional to N_x^2 and $N_x^{7/3}$ respectively with a corresponding savings over a

full-matrix solution of order N_x and $N_x^{2/3}$. While these may not be insignificant reductions in solution time for large-enough N_x, realizing the fullest benefits of the NNA requires some other approach to exploit matrix sparseness.

4. Other

Other numerical techniques for reducing computer-time requirements might be considered. Two we discuss here are "borrowed" from the area of signal processing, adaptive modeling for reducing the number of unknowns needed to achieve a desired accuracy, and model-based parameter estimation for increasing the utilization efficiency of what has been computed.

a. Adaptive Modeling

Most CEM seems to involve sampling of both the unknowns to be solved and the equations to be matched in a way that is determined more by problem geometry than electromagnetic requirements. Perhaps the simplest example is that of a straight wire for which equal-length segments are most often employed when using a sub-domain basis, resulting in equally spaced equation-match points as well when using collocation. This modeling strategy is used more for convenience than from knowledge that it provides the best accuracy for a given number of unknowns. The use of adaptive sampling could lead to improved modeling performance by adjusting the sampling to the requirements of a particular problem.

There are at least two ways by which adaptive sampling might be implemented. The simpler approach which we denote as static adaptation, would develop pre-determined sampling strategies for various problem classes which would then be used when problems from among them are being modeled. The other, more complex but potentially of better performance, would involve dynamic adaptation to increase the sampling during the course of the modeling computation. The latter approach would require quantitative assessment of the modeling error at a given stage in the computation together with a strategy for changing the sampling to improve the modeling accuracy. One version of the conjugate gradient technique is adaptive in this sense. A desirable attribute of dynamic adaptation is to utilize as fully as possible all previous computations when increasing the sampling at a given stage in the process, so that information already computed is not discarded.

b. Model-based parameter estimation

Identifying trends in data by sequentially connecting adjacent points with straight lines or using smoothed approximations thereto is an exercise familiar to any engineering student, a procedure typically described as curve fitting. The basic idea is to see whether curves which "fit" the data exhibit any sort of explainable behavior which reveals some underlying physics and/or mathematics. A classical example of curve fitting is polynomial interpolation and extrapolation as previously discussed in connection with selecting basis and testing functions for using the moment method.

When the curve-fitting procedure employs an analytical description based on the physics of the process which produced the data, the exercise may be more accurately described as one of model-based parameter estimation (MBPE). The analytical description provides the model whose parameters are adjustable constants needing numerical determination before the model is quantifiable. These constants are estimated by fitting

the model to the data, usually in some least-mean-square sense. In electromagnetics, and in the context of our previous discussion that source-field relationships establish a transfer-function representation, we observe that MBPE is a tool for solving the inverse problem. One example is that of finding the complex resonances or poles of objects from their temporal or spectral responses using an exponential- or pole-series model.

Besides being a tool for treating inverse problems, MBPE also provides a means for developing more efficient tools for the direct or analysis problem. The principle in the latter case comes from the observation that the mathematical complexity of a formal, analytical solution can obscure the physical simplicity of what that solution describes. As a specific example we can cite the Sommerfeld solution of the interface problem whose treatment has drawn much attention. While the Sommerfeld integrals can be complicated to evaluate either analytically or numerically, the fields they describe exhibit reasonably simple spatial variation. It was this simplicity that was exploited by Brittingham et. al. (1977) in linearly interpolating in a spatial mesh of pre-computed Sommerfeld fields to develop the direct matrix for wires near an interface. By replacing direct Sommerfeld-integral evaluation with this simple curve-fitting procedure, the computer time required for using such models was reduced by a factor of 100 or more. Latter, an analytically more accurate model than linear interpolation was developed based on asymptotic approximations to the Sommerfeld integrals with even better results [Burke and Miller (1984)].

Modeling interfaces appears to be just one of many direct problems in electromagnetics that are suited to MBPE for increasing their numerical efficiency. Other examples of MBPE include developing more efficient algorithms for computing the radiation pattern of a parabolic reflector [Bucci et. al. (1983)], and synthesizing and imaging aperture and antenna source distributions [Miller and Lager (1978), Inagake and Garbacz (1982), and Miller (1983)].

c. Numerical Green's function

The solution matrix that is obtained for a given object or structure in essence is the numerical equivalent of a generalized Green's function in that the boundary conditions prescribed in obtaining the matrix are satisfied over the structure's surface whatever the excitation to which it is exposed. Whether that excitation is caused by an incident plane wave to model its scattering properties, or a localized field to model its radiation characteristics, the solution matrix represents a self-contained numerical analog of the structure it approximates.

In many applications however, for example when antenna placement on the structure is being evaluated, the basic solution matrix is incomplete without including the antenna(s) in the model as well. Although their inclusion will not significantly increase the size of the direct matrix that must be solved when these added antennas are small relative to the overall structure, each new position they occupy changes that part of the direct matrix to which they contribute. In such cases it is computationally advantageous to partition the direct matrix into two parts, one for the basic structure and a remainder that accounts for the antenna-structure interaction. We thus have

$$Z = \begin{matrix} Z_{ss} & Z_{sa} \\ Z_{as} & Z_{aa} \end{matrix}$$

where "s" and "a" subscripts denote the structure and antenna respectively. A solution to

the moment-method problem can then be expressed as

$$I_s = Y_{ss}[E_s - Z_{sa}I_a] = I_s + /_s$$

$$I_a = Y_{aa}[E_a - Z_{as}I_s] = I_a + /_a$$

where I and $/$ are the self- and mutual-interaction currents respectively. By substituting I_s into the second equation, we obtain

$$I_a = (1 - Y_{aa}Z_{as}Z_{sa})^{-1}(I_a - Y_{aa}Z_{as}Y_{ss}E_s)$$

$$I_s = I_s - Y_{ss}Z_{sa}I_a$$

so that a matrix of only order "a•a" rather than "s•s" needs to be solved for each new antenna location. The role played by Y_{ss} in this case is analogous to that of an analytic Green's function for various special geometries as discussed above. The reduction in computer time is of order "s" relative to solving the combined problem.

VII. VALIDATION, ERROR CHECKING, AND ERROR ANALYSIS

A. Modeling Uncertainties

The process of proceeding from an original physical problem to computed results is one which is subject to numerous uncertainties caused by a variety of factors. Perhaps foremost among these factors is the degree of arbitrariness associated with many of the choices that are made by the code developer and/or modeler in the course of eventually obtaining numerical results. Whereas the numerical evaluation of classical boundary-value problems such as scattering from a sphere is numerically robust in the sense that different workers using different computers and different software can obtain results in agreement to essentially as many significant figures as they wish, the same observation cannot be made for moment-method modeling.

Modeling uncertainties can be assigned to two basic error categories, a physical modeling error ϵ_P, and a numerical modeling error ϵ_N as outlined in Table 19. The former is due to the fact that for most problems of practical interest varying degrees of approximation are needed in developing a simplified or idealized problem representation that will be compatible with the computer code to be used for the modeling computations. The latter is due to the fact that the numerical results obtained are almost invariably only approximate solutions to that idealized representation. We note that although an analytical expression may in principle represent a formally exact solution, the process of obtaining numerical results in that case is still one which inevitably involves finite-precision evaluation of the formal solution.

By its very nature, the physical modeling error requires some kind of measurement for its determination, except for those few problems whose analytical solution in principle involves no physical idealization nor subsequent numerical approximation. One example of such problems is that of determining the scattering or radiating properties of the perfectly conducting or dielectric sphere.

The numerical modeling error is itself comprised of two components in general, determination of which would normally involve one or more kinds of computation. The first and generally more important of these components is the solution error which arises because the computer model used, even if solved exactly, would not provide an exact solution for the idealized problem representation. The solution error arises essentially due to the fact that the computer model is solved using a finite number of unknowns. The

other, generally less important contributor to the numerical modeling error is the equation error which arises because the numerical results obtained from the computer model used may not numerically satisfy the modeling equations. The equation error may be caused both by round-off due to the computer word size as well as the solution algorithm employed, as in the case of iteration, for example. The impact of equation error can be expected to increase with increasing condition number of the direct matrix. As an indicator of the kinds of issues that must be kept in mind when developing a numerical model, a brief list of generic modeling guidelines is presented in Table 20.

B. Validation and Error Checking

One of the most time consuming and long lasting of the tasks associated with any model development is that of validation. Long after work on the model has been completed questions will continue to arise about whether a given result is valid or whether the model can be applied to a given problem. There are essentially two kinds of validation procedures that can be considered to answer such questions, which are:

1) Internal Validation, a check that can be made concerning solution validity within the model itself as summarized in Table 21.
2) External Validation, a check that utilizes information from other sources which could be analytical, experimental or numerical as summarized in Table 22.

Existing computer models often do not perform internal checks on the results they produce, but instead leave that as an exercise to the user. For example, NEC (Numerical Electromagnetics Code) one the more widely used models, could provide and indeed has been exercised to give various kinds of checks relating to power balance, reciprocity and boundary-condition matching. But the software to do this is not a integral part of the code, generally being "patched in" by the user for a particular problem and check. It would seem to be of extremely great potential value if a variety of such checks could be built into the code and exercised as desired by the modeler.

1. Internal Checks

As a particular example of the use to which internal checks could be put, consider the case when a problem new to the modeler is being implemented and the initial results are obtained. Present practice usually involved "eye-balling" the data to see if it feels right, perhaps having first run some documented test cases to verify code performance. Since these test cases would not be likely to closely resemble the new problem, their successful solution would not provide much insight concerning the new results. If however, a series of checks built into the code could then be exercised at the modeler's discretion to verify that conditions necessary for a valid solution of Maxwell's Equations are satisfied, confidence in the model's reliability could be established. These checks might range from being as exhaustive as boundary-condition matching would be, to being fairly simple, such as reciprocity and power conservation. They could only be viewed as necessary but not sufficient conditions for solution validity, and could only involve such behavioral aspects as are not implicit in the model already (e.g., some formulations produce symmetric matrices so that bi-static scattering and transmit-receive reciprocity are assured). It would seem feasible to develop a figure-of-merit from the results of such checks that would provide in a single number a "quality factor" for the solution.

2. External Checks

The second kind of check involves use of independent data from other sources. Perhaps the most convincing overall is experimental data, but analytical or numerical results should be comparably useful. Indeed, one of the most convenient computational checks would be provided by a code that permits two different models to be developed for the same problem, for example by incorporating user-selectable basis and weight functions. For greatest utility, such checks ideally should not be of single-point nature, for example to compare results for input impedance at a single frequency. Experience shows that computer models produce results that exhibit slight frequency shifts, angle shifts or spatial shifts in field quantities with respect to "exact" solutions, or even other computer models. Consequently, global comparisons are usually more meaningful, but even then may not be straightforward. If the shifts mentioned are observed, it would seem appropriate to develop a correlation measure to establish the minimum squared difference between the two results as they are shifted along the axis of the common variable. For other models and applications, the results may be even less directly comparable, as is the case for IE and DE modeling approaches. Some work is needed in the general area of how results from two different representations of the same problem can be most meaningfully compared.

VIII. CONCLUDING REMARKS

In the preceding discussion, we have presented a selective survey of computational electromagnetics. Attention has been directed to radiation and scattering problems formulated as integral equations and solved using the Method of Moments. Beginning from the viewpoint of electromagnetics as a transfer-function process, we concluded that the basic problem is one of developing source-field relationships, or field propagators. Of the various ways by which these propagators might be expressed, we briefly discussed the Maxwell curl equations and Green's-function source integrals as providing the analytical basis for moment-method computer models. We then considered at more length some of the numerical issues involved in developing a computer model, including the idea of sampling functions used both to represent the unknowns to be solved for and to approximate the equations that they must satisfy. Some of the factors involved in choosing these sampling functions and their influence on the computational requirements were examined. Next, we discussed some ways of decreasing the needed computer time based on either analytical or numerical approaches. Some closing comments were directed to the important problem of validation, error checking and error analysis. Throughout our discussion, emphasis has been given to implementation issues involved in developing and using computer models as opposed to exploring analytical details.

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LIST OF TABLES

21. ***INTERNAL CHECKS USEFUL AS MEASURES OF SOLUTION VALIDITY***

22. ***EXTERNAL CHECKS USEFUL AS MEASURES OF SOLUTION VALIDITY***

Table 1

CLASSIFICATION OF MODEL TYPES IN CEM

FIELD PROPAGATOR	**DESCRIPTION BASED ON:**
Integral operator	Green's function for infinite medium or special boundaries.
Differential operator	Maxwell Curl Equations or their integral counterparts.
Modal expansions	Solutions of Maxwell's Equations in particular coordinate system and expansion.
Optical description	Rays and diffraction coefficients.
APPLICATION	**REQUIRES:**
Radiation	Determining the originating sources of a field.
Propagation	Obtaining the fields distant from a known source.
Scattering	Determining the perturbing effects of medium inhomogeneities.
PROBLEM TYPE	**CHARACTERIZED BY:**
Solution domain	Time or frequency.
Solution space	Configuration or wavenumber.
Dimensionality	1D, 2D, 3D.
Electrical properties of medium and/or boundary	Dielectric; lossy; perfectly conducting; anisotropic; inhomogeneous; non-linear.
Boundary geometry	Linear; curved; segmented; compound; arbitrary

Table 2

STEPS IN DEVELOPING A COMPUTER MODEL

STEP	ACTIVITY
Conceptualization	Encapsulating observation and analysis in terms of elementary physical principals and their mathematical description.
Formulation	"Fleshing out" of the elementary description into a more complete, formally solved, mathematical representation.
Numerical implementation	Transforming into a computer algorithm using various numerical techniques.
Computation	Obtaining quantitative results.
Validation	Determining the numerical and physical credibility of the computed results.

Table 3

DESIRABLE ATTRIBUTES IN A COMPUTER MODEL

Attribute	Description
Accuracy	The quantitative degree to which the computed results conform to the mathematical and physical reality being modeled. Accuracy, perferably of known and better yet, selectable, value is the single most important model attribute. It is determined by the physical modeling error (ϵ_P) and numerical modeling error (ϵ_N) [see Section VII].
Efficiency	The relative cost of obtaining the **needed** results. It is determined by the human effort required to develop the computer input and interpret the output, and by the associated computer cost.
Utility	The applicability of the computer model in terms of problem size and complexity. Utility also relates to ease of use, reliability of results obtained, etc.

Table 4

REPRESENTATIVE APPROXIMATIONS THAT ARISE IN MODEL DEVELOPMENT

APPROXIMATION	IMPLEMENTATION/IMPLICATIONS
Conceptualization	
Physical Optics	Surface sources given by tangential components of incident field, with fields subsequently propagated via a Green's function. Best for backscatter and main-lobe region of reflector antennas, from resonance region (ka>1) and up in frequency.
Geometrical Theory of Diffraction	Fields propagated via a divergence factor with amplitude obtained from diffraction coefficient. Generally applicable for ka>2-5. Can involve complicated ray tracing.
Geometrical Optics	Ray tracing without diffraction. Improves with increasing frequency.
Compensation Theorem	Solution obtained in terms of perturbation from a reference, known solution.
Born-Rytov	Approach used for low-contrast, penetrable objects where sources are estimated from incident field.
Rayleigh	Fields at surface of object represented in terms of only outward propagating components in a modal expansion.
Formulation	
Surface Impedance	Reduces number of field quantities by assuming an impedance relation between tangential E and H at surface of penetrable object. May be used in connection with physical optics.
Thin-Wire	Reduces surface integral on thin, wire-like object to a line integral by ignoring circumferential current and circumferential variation of longitudinal current which is represented as a filament. Generally limited to ka<1 where a is the wire radius.
Numerical Implementation	
$\partial f/\partial x \rightarrow (f_+ - f_-)/(x_+ - x_-)$	Differentiation and integration of continuous functions represented in terms of analytic operations on sampled approximations, for which polynomial or trigonometric functions are often used. Inherently a discretizing operation, for which typcially $\Delta x < \lambda/2\pi$ for acceptable accuracy.
$\int f(x)dx \rightarrow \Sigma f(x_i)\Delta x_i$	
Computation	
Deviation of numerical model from physical reality	Affects solution accuracy and relatability to physical problem in ways that are difficult to predict and quantify.
Non-converged solution	Discretized solutions usually converge globaly in proportion to $\exp(-AN_x)$ with A determined by the problem. At least two solutions using different numbers of unknowns N_x are needed to estimate A.

Table 5

COMPARISON OF STEPS IN TIME- AND FREQUENCY-DOMAIN MODELING

Frequency Domain		**Time Domain**
$\partial/\partial t = i\omega$	{Maxwell's equations}	Time dependent
$L(\omega,\lvert\mathbf{r}-\mathbf{r}'\rvert)f(\omega,\mathbf{r}') = g(\omega,\mathbf{r})$	{Plus boundary conditions, etc.}	$L(\lvert\mathbf{r}-\mathbf{r}'\rvert)f(\tau,\mathbf{r}') = g(t,\mathbf{r})$ $\tau = t-\lvert\mathbf{r}-\mathbf{r}'\rvert/c$
where **r** is observation point, **r'** is source point, c is speed of light in medium		
$L(\omega,\lvert\mathbf{r}-\mathbf{r}'\rvert)$ depends on ω and $\lvert\mathbf{r}-\mathbf{r}'\rvert$	{Note that}	$L(\lvert\mathbf{r}-\mathbf{r}'\rvert)$ depends on $\lvert\mathbf{r}-\mathbf{r}'\rvert$ only
$\sum Z_{ij}f_j = g_i$ $i,j = 1,\ldots,N_x$ where N_x is number of space samples	{Apply MoM to get Nth order system}	$\sum Z_{ij}f_{jk} = g_{ik}$ $i,j = 1,\ldots,N_x$; $k = 1,\ldots,N_t$ where N_t is number of time samples
$f_i = \sum Y_{ij}g_j$ where $Y = Z^{-1}$	{Matrix manipulation yields}	$f_{ik} = \sum Y_{ij}g_{jk}$ where $Y = Z^{-1}$ (Inversion needed only once and only for implicit solution)
Solution obtained for many sources but single frequency	{Observe}	Solution obtained for single source but many frequencies
Do for $M = 1,\ldots,N_f$ frequencies to get f_{im}	f_{im} and f_{ik} are related by Fourier transform	--

Table 6

SOME INTEGRAL EQUATIONS WIDELY USED IN ELECTROMAGNETICS

FREQUENCY DOMAIN	**COMMENTS**
Magnetic-Field Integral Equation	
$\mathbf{J}(\mathbf{r}) - 2\mathbf{n}\times\oint_S \mathbf{J}(\mathbf{r}')\times\nabla' g(\mathbf{r},\mathbf{r}')]da'$	Suited for smooth closed bodies.
$= 2\mathbf{n}\times\mathbf{H}^{inc}(\mathbf{r})$	Note source term is physical
$= L_H(\mathbf{J})$	optics current [Oshiro (1965)].
Electric-Field Integral Equation	
$\mathbf{t}\cdot\oint_S \{i\omega\mu[\mathbf{J}_S(\mathbf{r}')]g(\mathbf{r},\mathbf{r}') - [\mathbf{n}\cdot\mathbf{E}(\mathbf{r}')]\nabla' g(\mathbf{r},\mathbf{r}')\}da'$	Applicable to general bodies
$= \mathbf{t}\cdot\mathbf{E}^{inc}(\mathbf{r})$	such as wires, plates, and shells.
$= L_E(\mathbf{J})$	[Andreason (1965)].
Combined-Field Integral Equation (CFIE)	
$L_H(\mathbf{J}) + \alpha L_E(\mathbf{J})/\eta = 2\mathbf{n}\times\mathbf{H}^{inc}(\mathbf{r}) + \alpha\mathbf{t}\cdot\mathbf{E}^{inc}(\mathbf{r})/\eta$	Eliminates the spurious solutions that occur in either the EFIE or MFIE at interior resonances [Mautz and Harrington (1978)].
TIME DOMAIN	
Magnetic-Field Integral Equation	
$\mathbf{J}_S(\mathbf{r},t) - (1/2\pi)\mathbf{n}\times\oint_S \{1/R + c^{-1}\partial/\partial t'\}\{[\mathbf{J}_S(\mathbf{r}',t')]\times\mathbf{R}\}/R^2 ds'$	Limitations similar to FD MFIE
$= 2\mathbf{n}\times\mathbf{H}^{inc}(\mathbf{r},t)$	[Bennett and Weeks (1970)].
Electric-Field Integral Equation	
$(1/4\pi)\mathbf{n}\times\oint_S \{-\partial\mathbf{J}_S(\mathbf{r}',t')/\partial t' + [1/R + c^{-1}\partial/\partial t']$	Applicability comparable to FD EFIE
$[\mathbf{n}'\cdot\mathbf{E}(\mathbf{r}',t')\mathbf{R}/R]\}/R ds' = \mathbf{n}\times\mathbf{E}^{inc}(\mathbf{r},t)$	[Mieras and Bennett (1982)].

where: $\mathbf{r}$ and t are the observation position and time
$\mathbf{r}'$ and t' are the source position and time
$t' = t - R/c$ is the retarded time
$R = |\mathbf{r}-\mathbf{r}'|$ with $\mathbf{r}$ and $\mathbf{r}'$ on the surface S

$\oint$ denotes a principle-value integral

$g(\mathbf{r},\mathbf{r}') = e^{-ikR}/(4\pi R)$

$\mathbf{n}$, $\mathbf{t}$ are unit normal and tangent vectors to the surface

α is an adjustable parameter to determine relative weighting of MFIE and EFIE contributions, with best results occurring for $0.2 \le \alpha \le 1$

η is free-space impedance

Table 7

COMPARISON OF IE- AND DE-FIELD PROPAGATORS AND THEIR NUMERICAL TREATMENT

	DIFFERENTIAL FORM	**INTEGRAL FORM**
Field Propagator	Maxwell Curl Equations	Green's function
Boundary Treatment		
*At infinity (radiation condition)	Local or global" lookback" to approximate outward propagating wave	Green's function
*On object	Appropriate field values specified on mesh boundaries to obtain stairstep or piecewise linear approximation to the boundary	Appropriate field values specified on object contour which can in principal be a general, curvilinear surface, although this possibility seems to be seldom used
Sampling Requirements		
*No. of space samples	$N_x \propto (L/\Delta L)^D$	$N_x \propto (L/\Delta L)^{D-1}$
*No. of time steps	$N_t \propto (L/\Delta L) \approx cT/\delta t$	$N_t \propto (L/\Delta L) \approx cT/\delta t$
*Linear system L is problem size D is no. of problem dimensions (1,2,3) T is observation time ΔL is spatial resolution δt is time resolution	Sparse, but larger	Dense, but smaller. In this comparison, note that we assume the IE permits a sampling of order one less the problem dimension, i.e., inhomogeneous problems are excluded.
Solution time		
*Frequency domain	$T_\omega \propto N_x^p = (L/\Delta L)^{Dp}; 2 \le p \le 3$	$T_\omega \propto N_x^p = (L/\Delta L)^{(D-1)p}; 2 \le p \le 3$
*Time domain	$T_t \propto N_x N_t = (L/\Delta L)^{D+1}$	$T_t \propto N_x^2 N_t = (L/\Delta L)^{2D-1}$

Table 8

RELATIVE APPLICABILITY OF IE- AND DE-BASED COMPUTER MODELS

TIME DOMAIN DE	TIME DOMAIN IE	ISSUE	FREQUENCY DOMAIN DE	FREQUENCY DOMAIN IE
		MEDIUM		
√	√	Linear	√	√
x	x	Dispersive	√	√
√	x	Lossy	√	√
√	~	Anisotropic	√	√
√	x	Inhomogeneous	√	x
√	x	Nonlinear	x	x
√	x	Time-varying	x	x
		OBJECT		
~	√	Wire	~	√
√	√	Closed Surface	√	√
√	√	Penetrable Volume	√	√
~	√	Open Surface	~	√
		BOUNDARY CONDITIONS		
√	√	Interior Problem	√	√
~	√	Exterior Problem	~	√
√	√	Linear	√	√
√	√	Non-linear	x	x
√	√	Time-varying	x	x
~	x	Half-space	~	√
		OTHER ASPECTS		
~	~	Symmetry Exploitation	√	√
~	√	Far-field Evaluation	~	√
x	~	Number of Unknowns	~	√
√	~	Length of Code	~	x
		SUITABILITY FOR HYBRIDING WITH OTHER:		
~	√	Numerical Procedures	√	√
x	~	Analytical Procedures	~	√
x	~	GTD	x	√

where:

√ signifies highly suited or most advantageous
~ signifies moderately suited or neutral
x signifies unsuited or least advantageous

Table 9

SAMPLING OPERATIONS INVOLVED IN MoM MODELING

	DE MODEL	IE MODEL
EQUATION	$L(s')f(s') = g(s')$	$L(s,s')f(s') = g(s)$
Sampling of:		
Unknown via basis-functions $b_j(s')$ using $f(s') \approx \Sigma a_j b_j(s')$.	Sub-domain bases usually of low order are used. Known as finite-difference procedure when pulse basis is used, and as finite-element approach when bases are linear.	Can use either sub-domain or entire-domain bases. Use of latter is generally confined to bodies of rotation. Former is usually of low order, with piece-wise linear or sinusoidal being the maximum variation employed.
Equation via weight functions $w_i(s)$ $\langle w_i(s), L(s,s')\Sigma a_j b_j(s')\rangle = \langle w_i(s), g(s)\rangle$ to get $Z_{ij}a_j = g_i$.	Point-wise matching is commonly employed, using a delta-function. Pulse and linear matching are also used.	Point-wise matching is commonly employed, using a delta function. For wires, pulse, linear, and sinusoidal testing is also used. Linear and sinusoidal testing is also used for surfaces.
Operator.	Operator sampling for DE models is entwined with sampling the unknown in terms of the difference operators employed.	Sampling needed depends on the nature of the integral operator $L(s,s')$. An important consideration whenever the field integrals cannot be evaluated in closed form.
Solution of:		
$Z_{ij}a_j = g_i$ for the a_j.	Interaction matrix is sparse. Time-domain approach may be explicit or implicit. In frequency domain, banded-matrix technique usually employed.	Interaction matrix is full. Solution via factorization or or iteration.

Table 10

EXAMPLES OF GENERIC BASIS/WEIGHT-FUNCTION COMBINATIONS

Method	jth Term of Basis	ith Term of Weight
Galerkin	$a_j b_j(\mathbf{r}')$	$w_i(\mathbf{r}) = b_j(\mathbf{r})$
Least square	$a_j b_j(\mathbf{r}')$	$Q(\mathbf{r})\partial\epsilon(\mathbf{r})/\partial a_i$
Point matching	$a_j\, \delta(\mathbf{r}-\mathbf{r}_j)$	$\delta(\mathbf{r}-\mathbf{r}_i)$
General collocation	$a_j b_j(\mathbf{r}')$	$\delta(\mathbf{r}-\mathbf{r}_i)$
Subsectional collocation	$U(\mathbf{r}_j)\Sigma a_{jk} b_k(\mathbf{r}')$	$\delta(\mathbf{r}-\mathbf{r}_i)$
Subsectional Galerkin	$U(\mathbf{r}_j)\Sigma a_{jk} b_k(\mathbf{r}')$	$U(\mathbf{r}_i)\Sigma b_i(\mathbf{r})$

where:

$\mathbf{r}'$ and $\mathbf{r}$ denote source and observation points respectively

a_j, a_{jk} are unknown constants associated with the j'th basis function (entire domain) or the k'th basis function of the j'th subsection (sub-domain)

$U(\mathbf{r}_k)$ is the unit sampling function which equals one on the k'th sub-domain and is zero elsewhere

$b_j(\mathbf{r}')$ is the j'th basis function

$w_i(\mathbf{r})$ is the i'th testing function

$\delta(\mathbf{r}-\mathbf{r}_j)$ is the Dirac delta function

$Q(\mathbf{r})$ is a positive-definite function of position

$\epsilon(\mathbf{r})$ is the residual or equation error

Table 11
EXAMPLES OF SPECIFIC BASIS/WEIGHT-FUNCTION COMBINATIONS

Application	jth Term of Basis	ith Term of Weight
1D/Wires [Richmond (1965) (1966)]	Constant--$a_j U_j(s)$	Delta function--$\delta(s-s_i)$
1D/Wires [Chao and Strait (1971)	Piecewise linear--$a_{j1}(s-s_j-\delta_j/2)$ + $a_{j2}(s-s_j+\delta_j/2)$	Piecewise linear--$(s-s_j-\delta_j/2)$ + $(s-s_j+\delta_j/2)$
1D/Wires [Mei (1965)	3-term sinusoidal--$a_{j1} + a_{j2}\sin[k(s-s_j)] + a_{j3}\cos[k(s-s_j)]$	Delta function--$\delta(s-s_i)$
1D/Wires [Richmond 1974)] Piecewise sinusoidal--	Piecewise sinusoidal--$a_{j1}\sin[k(s-s_j-\delta_j/2)]$ + $\sin[k(s-s_j-\delta_j/2)]$ + $\sin[k(s-s_j+\delta_j/2)]$	$a_{j2}\sin[k($
2D/Surfaces [Oshiro(1965)]	Weighted Delta function--$a_j\delta(s-s_j)\Delta_j$	Delta function--$\delta(s-s_i)$
2D/ Rotational Surfaces [Mautz & Harrington(1969)]	Piecewise linear axially, and exp(inø) azimuthally	Same (Galerkin's method)
2D/Surfaces [Glisson & Wilton (1980)] (Galerkin's method)	Piecewise linear	Same
2D/Surfaces [Putnam and Medgeysi-Mitschang (1986)] (Galerkin's method)	Piecewise linear subdomain/Fourier series entire domain	Same
3D/Volumes [Schaubert et. al. (1984)] (Galerkin's method)	Piecewise linear	Same

Table 12

SUMMARY OF OPERATION COUNT FOR SOLUTION OF GENERAL DIRECT MATRIX HAVING N_x UNKNOWNS

Method	To Obtain Solution Matrix	To Obtain Solution	Comments
Cramer's Rule	Expand in co-factors leading to -->	$\sim N_x!$	Not an advisable procedure, but useful to illustrate just how bad the problem could be!
Inversion	N_x^3	N_x^2	Provides RHS-independent solution matrix
Factorization	$N_x^3/3$	N_x^2	RHS independent solution matrix
Iteration	--	N_x^2-N_x^3	Each RHS requires separate solution
Symmetry			
Reflection	$(1to2^p)x(N_x/2^p)^3$	$N_x^2/2^p$	For p=1to3 reflection planes
Translation (Toeplitz)	$f(n_x)N_x^2$	N_x^2	For n_x unknowns per section of translation, with $f(n_x)$ weakly dependent on n_x
Rotation (Circulant)	$mx(N_x/n)^3$	N_x	For n rotation sectors and m = 1 to n modes
Banded	N_xW^2	N_xW	For a bandwidth of W coefficients

Table 13

NOMINAL SAMPLING REQUIREMENTS FOR VARIOUS FIELD QUANTITIES

Quantity	Value
N_x , total number of spatial samples (per field quantity)	$\sim d(L/\Delta L)^d = d(2\pi L/\lambda)^d$
N_t , number of time steps for time-domain model	$\sim (L/\Delta L) = (2\pi L/\lambda)$
N_f , number of frequency steps to characterize spectral response from frequency-domain model	$\sim (L/2\Delta L) = N_t/2$
$N_{sources}$, number of incident sources for monostatic radar cross section in one plane*	$\sim (4L/3\Delta L) = 8\pi L/\lambda$
N_{fields}, number of far fields needed for bistatic pattern in one observation plane*	$\sim N_{source} = (4L/3\Delta L$

where:

λ is the wavelength at the highest frequency of interest.

ΔL is the spatial resolution being sought.

L is object maximum object dimension or dimension in observation plane.

d is the number of spatial dimensions being sampled, and is not necessarily the problem dimensionality D. The distinction is important because when an appropriate Green's function is available, the source integrals are usually one dimension less than the problem dimension, i.e., d = D-1. An exception is an inhomogeneous, penetrable body where d = D when using an integral equation.

* Assuming ~6 samples per lobe of the scattering pattern are needed.

Table 14

HIGHEST-ORDER TERMS IN SOLUTION TIME APPROXIMATIONS FOR DE AND IE FREQUENCY- AND TIME-DOMAIN MODELS

PROPAGATOR ---> / DOMAIN ↓		Differential Eq.	Integral Eq.
Frequency		$T_{DE,\omega} \approx \propto (L/\Delta L)^{3d-2}$	$T_{IE,\omega} \approx \propto (L/\Delta L)^{3d}$
Time	explicit	$T_{DE,t} \approx \propto (L/\Delta L)^{d+1}$	$T_{IE,t} \approx \propto (L/\Delta L)^{2d+1}$
	implicit	$T_{DE,t} \approx \propto (L/\Delta L)^{2d+1}$	

where a single calcaluation is performed (one frequency for a FD solution or one source for a TD calculation).

Table 15

SOME ANALYTICAL MEANS OF REDUCING COMPUTER TIME

Method	Implementation Requirements	Motivation
Specialized Green's functions	Delta-source fields for geometry of interest	Eliminate need to solve unknowns over the specific surface
Hybrid models	Interactions between parts of problem modeled using different formulations	Model self interaction using most efficient formulation
Impedance boundary conditions	Boundary smooth enough that ratio of local fields determined by constitutive parameters	Reduce number of unknowns by a factor of two
Physical optics	Local surface field determined by constitutive parameters and incident field	Eliminate all unknowns, but excitation dependent

Table 16

A SUMMARY OF SOME SPECIALIZED GREEN'S FUNCTIONS

Problem	Green's Function	Characteristics
Planar, penetrable interface.	Continuous radial spectrum [Banos (1966)].	Infinite integrals whose numerical evaluation is time consuming and having analytical approximations of limited applicability.
Infinite, circular cylinder.	Continuous axial spectrum and discrete azimuthal (cosine/ sine) series [Lucke (1951)].	Adds to difficulty of integral evaluation needed to sum azimuth series.
Sphere.	Discrete spectrum in elevation (Legendre polynomials) and azimuth (cosine/sine series) [Tesche & Neureuther (1970)].	Double series can be computationally demanding. Simplifies for single, radial monopole.
Infinite, parallel plates.	Discrete spectrum (infinite set of images) in transverse direction [Tesche (1972)].	Series is slowly convergent, and is poorly behaved at resonant separations.
Rectangular waveguide.	Extension of parallel plate treatment to two transverse image sets and double series.	Double series increases the convergence problem from ~N to ~N^2.
Rectangular cavity.	Extension of rectangular waveguide to closed cavity, with three image sets and triple series [Wu and Chang (1986)].	Triple series convergence ~N^3, but transformed to single series by Wu and Chang.

Table 17

SOME HYBRID MODELS USED IN COMPUTATIONAL ELECTROMAGNETICS

Approach	Implementation	Motivation
GTD/IE [Thiele and Newhouse (1969)].	Addition to MoM impedance matrix of fields due to diffracted rays.	Reduce number of MoM unknowns when modeling large objects.
Modal/DE [Mei (1974)].	Develop solution for penetrable object using DE and match to modal outward propagating fields.	To include radiation condition in frequency-domain, DE model.
Modal/Experimental [Wacker (1981)].	Measure antenna near fields over plane, cylinder, or sphere and transform to far field using modal expansion.	To permit measurement in near field of antennas too large for practical, direct far-field measurement.
IE/DE [Taflove and Umashankar (1980)].	Apply IE model over enclosing surface within which fields are modeled using DE.	To exploit exterior-region advantage of IE and interior-region generality of DE.

Table 18

SOME NUMERICAL MEANS OF REDUCING COMPUTER TIME

Method	Implementation Requirements	Motivation
Iterative technique to solve direct matrix	Various special-purpose iteration routines which include convergence checks	Reduce solution time from N_x^3 to ~N_x^2
Exploiting problem symmetries.	Special programs for filling and solving symmetric systems of various kinds.	Save varying amounts of computer time/storage, but limited in applicability.
Near-neighbor approximations.	Strategy for setting interaction-window width and exploiting sparseness of direct matrix.	Save factor of N_x or more in time/storage, but produces approximate solution.
Adaptive modeling.	Variation of numerical model as computation proceeds to achieve a specified local accuracy.	Fewer unknowns and controllable error, but increased programming complexity.
Model-based parameter estimation.	Development of approximate, simpler expressions to replace rigorous, but computationally time-consuming, rigorous ones.	Reduce time needed to obtain direct matrix, or number of samples needed, e.g., to estimate transfer function.
Numerical Green's function	Development and storing of solution matrix for primary object of interest.	Need only to solve interaction terms to model effect of other nearby objects.

Table 19

ERROR TYPES THAT OCCUR IN COMPUTATIONAL ELECTROMAGNETICS

CATEGORY	DEFINITION
Physical Modeling Error, ϵ_P	Arises because the numerical model used is normally an idealized mathematical representation of the actual physical reality.
Numerical Modeling Error, ϵ_N	Arises because the numerical results obtained are only approximate solutions to that idealized representation, and is comprised of two components: 1) Solution error--The difference that can exist between the computed results and an exact solution even were the linear system of equations to be solved exactly, due to using a finite number of unknowns; and 2) Equation error--The equation mis-match that car occur in the numerical solution because of round-off due to finite-precision computations, or when using an iterative technique because of limited solution convergence.

Table 20

GENERIC GUIDELINES FOR IE MODELING

MODELING PARAMETER OR ISSUE	NOMINAL RANGE OR VALUE	REASON
Wire Length, L	$L>10d$	Neglect of end caps in thin-wire treatment.
Wire diameter, d	$\lambda>\pi d$	Neglect of circumferential effects.
Wire segment length, Δ		
As related to diamter	$\Delta>d$	Use of thin-wire kernel in integral equation. Can be relaxed by use of extended kernel [Burke and Poggio (1980)].
As related to wavelength	$\Delta<\lambda/2\pi$	Necessity of sampling current densely enough in wavelengths.
Step change in wire radius, δa	$\Delta>10\delta a$	Neglect of sources on stepped surface (similar to end-cap problem).
Source location	Do not place on open-ended segment	Avoids non-physical situation of driving wire at open end.
Angle of wire bend, α	$\alpha>2\pi a/\Delta$	Keeps adjacent wires from occupying too large a common volume.
Axial separation of parallel wires, r		
With match points aligned	$r>3a$	Neglect of circumferential current variation.
Otherwise	$r>10a$	Avoids placing one match point in error field of a junction.
Wire mesh model of solid surface		
Mesh size $\Delta x\Delta$	$\Delta<\lambda/10$	To reduce field "leakage" to acceptable level.
Wire radius	$a=\Delta/2\pi$	To have wire area equal to surface area of solid.
Surface-patch area, Δ_s		
Frequency domain	$\lambda>2\pi\sqrt{\Delta_s}$	Need to sample currents densely enough in wavelengths.
Time domain	$\lambda_{min}>2\pi\sqrt{\Delta_s}$	
Piecewise model of curved wire or surface of radius of curvature R	$R>\Delta$ $R>\sqrt{\Delta_s}$	Necessity of sampling a circular arc at least 6 times per 2π radians.
Starting time in time-domain	$E^{inc}(t)\|_{max}\geq$	To achieve numerically smooth

solution, t_{st}	$10^x E^{inc}(t_{st})$	buildup of exciting field and 10^x accuracy.
Stopping time in time-domain solution, t_{sp}.	$E^{inc}(t)\|_{max} \geq 10^x \|E^{inc}(t->\infty) - E^{inc}(s_{sp})\|$ or I(t) reaches steady state	Stabilize final response.
Time step in TD solution, δ	$c\delta \leq \Delta$	Satisfies Courant stability condition. Required for explicite solution.
Maximum frequency of transient source in TD solution, using Gaussian excitation, i.e. $E^{inc}(t) = \exp(-g^2t^2)$	$g \sim 2f_{max}$	Ensures source spectrum does not exceed upper frequency for model validity.

Table 21

NTERNAL CHECKS USEFUL AS MEASURES OF SOLUTION VALIDITY

CONVERGENCE MEASURES

MEASURE	EXAMPLE	TESTS	PROPERTIES
Local	$\lim[I(s)], \lim[\mathbf{E}(\mathbf{r})]$ as $N \rightarrow N_{max}$	Convergence of input impedance, current, fields, etc.	Reasonable measure of solution behavior, but can yield non-monotonic result.
Global	$\int I(s)I^*(s)ds$ or $\int \mathbf{E}(\mathbf{r})\mathbf{E}^*(\mathbf{r})dx^n$	Convergence over entire object of current or convergence of field in n=1,2, or 3 dimensions.	A more complete measure of convergence.
Random (local or global)	$\Sigma F(\mathbf{r}_n)$, with $F(\mathbf{r}_n)$ a field quantity which is function of a random variable $\mathbf{r}_n$.	Convergence of any field quantity measured by a random observation variable.	Permits estimation of convergence and uncertainty of convergence estimates.
		OTHER MEASURES	
Power Balance	$P_{in} + P_{loss} = P_{radiated}$	Whether supplied power equals sum of radiated plus dissipated power.	Provides good check on antenna source model for radiation resistance. A necessary, but not sufficient condition.
Boundary Condition Matching	$E_{tan}(\mathbf{r}') = 0$, $\mathbf{r}'$ on object modeled	Degree to which specified conditions on the boundary are satisfied.	Most fundamental check on solution. Consistency requires use of same weight function as for model itself. Can be computationally expensive. Necessary and sufficient condition.
Reciprocity	$E(\emptyset_1^{inc}, \emptyset_2^{scat}) = E(\emptyset_2^{inc}, \emptyset_1^{scat})$	Whether interchanging observation and source locations yields identical results.	Useful check for antenna and bistatic scattering patterns. Necessary but not sufficient condition.
"Non-physical Behavior" of Solution	---	Computed results to exhibit physically behavior.	Can be a subjective check. One example is provided by spatial oscillation in current when thin-wire approximation is violated.

Table 22

EXTERNAL CHECKS USEFUL AS MEASURES OF SOLUTION VALIDITY

MEASURE	EXAMPLE	TESTS	PROPERTIES
		ANALYTICAL	
Any observable provided by a formally exact solution.	Sources, near and far fields.	Any observable provided by the computer model.	Provides a necessary and sufficient condition for solution validity. Available for only special geometries, but gives canonical benchmarks.
		COMPUTATIONAL	
Far fields.	Radiation pattern, bi-static and mono-static scattering pattern.	Consistency of the quantity least sensitive to solution errors.	A useful test, but one which is often subject to angle shifts between results from two models.
Near fields and sources.	Near-field cuts, current and charge distributions.	Quantities most often directly computed by model.	A more demanding test for comparison, but one which often exhibits spatial shifts between models.
Input impedance/ susceptance.	---	Source models and single-port input characteristics.	Especially sensitive measure in terms of input susceptance. Highly advisable to examine over a range of frequencies because shifts in frequency also occur.
		EXPERIMENTAL	
Same observables as used for computational checks.	---	Physical modeling error and relative correlation of actual problem with numerical model.	Perhaps the most reassuring check to make, but also often the most difficult.

GEMACS - AN EXECUTIVE SUMMARY

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ABSTRACT

The General Electromagnetic Model for the Analysis of Complex Systems (GEMACS) is a continually evolving computer program developed by the Air Force to accomplish a sophisticated analysis of the electromagnetic field phenomena associated with a given radiating or scattering Air Force system.

Originally conceived to be a tool for use in electromagnetic compatibility (EMC) analysis, it is growing in power and capability into a system of modules that can be applied to the investigation of almost any electromagnetic phenomenon associated with a physical radiating or scattering system.

This paper is meant to be an informational tutorial on GEMACS, without equations, whose sole objective is to acquaint the reader with this powerful, general- purpose, user-friendly electromagnetic fields analysis system.

This paper consists of two separate parts, each dealing with the same material but presented in a different format to accommodate two broad general classes of reader. The first major division contains little explanatory text and numerous figures and is meant to give a quick overview of the system of GEMACS for those without the time and/or the motivation to get into very much detail. Be forewarned, however, that this first division is meant to give the reader the motivation to find the time and the inclination to read and digest all of the more detailed material in the second part of the paper.

PART I. THE QUICK LOOK

The following figure describes the philosophy that underlies the initial and continuing development of the GEMACS computer code. It is meant to be all things to an electromagnetic fields analyst, and to be so in an efficient and accurate manner.

==========

PHILOSOPHY

We Are Incorporating into GEMACS All
Solution Techniques Which Are Required
to Give Engineering Answers to Practical
EM Problems that Occur in the Design,
Development, Production, and Analysis
of Major Systems.

==========

A sampling of the general classes of problems that can be analyzed on arbitrary platforms such as aircraft, spacecraft, huts, and weapons systems is shown in the following figure. The output of GEMACS is in the form of tabular data giving the current distribution on wires and surfaces, coupling between pairs of antennas, near- and far-field electric field strengths, antenna terminal parameters, and the listing of the data contained in any one of the intermediate data sets generated in the analysis of a system.

==========

APPLICATIONS

Antenna Performance
EM Radiation and Scattering
EMP
EMC
Jamming Susceptibility
ECM and ECCM
Radar Cross Section

==========

A number of algorithms and formulations is built into the GEMACS system of modules. This is what gives the analyst the ability to analyze many different classes of problems efficiently and accurately. Some of these can be classified as physics models and some are the mathematical solution techniques that solve the resulting system of simultaneous linear equations that is the result of the application of the physics models to the structure being studied.

==========

PHYSICS

Method of Moments (MOM)
Geometrical Theory of Diffraction (GTD)
Finite Difference (FD)
MOM/GTD/FD Hybridization
Loaded Wire Scatterers
Imperfect Ground

MATHEMATICS

Model Order Reduction
Banded Matrix Iteration
Lower/Upper Matrix Decomposition

==========

There are at the present time three widely used and fairly general physics formulations that are resident in the GEMACS system. They apply to structures that are small compared to the wavelength, or at most a little larger than the resonant frequency region, (Method of Moments), to structures that are very large compared to the wavelength (Geometrical Theory of Diffraction), and to the interior of the structure (Finite Difference). The main characteristics of each of these three modeling techniques are shown in the following figure.

==========

METHOD OF MOMENTS

Electrically Small Objects
Fine Resolution
Wire Grid or Patch Models
Impedance Interactions
Exterior Environment

GEOMETRICAL THEORY OF DIFFRACTION

Electrically Large Objects
Coarse Resolution
Plates and Cylinders
Ray Tracing
Multiple Reflection/Diffraction
Exterior Environment

FINITE DIFFERENCE

Electrically Moderate Size
Fine Resolution
Cell Gridding
Wires Within Cells
Electric Fields
Interior Environment

==========

In order to perform a complete analysis of the structure under consideration, one must take into account all regions of the problem (i.e., interior cavities, apertures, and exterior environment) even when one is focusing on a specific part of the structure. This is the case for which the built-in hybridization capability within GEMACS comes to the fore. This hybridization process is totally invisible to the analyst once he has defined the types of interactions in which he is interested and has specified the quantities to be output by GEMACS. The use, coupling, and interaction of the various modules, as well as the transfer of data among the modules in the proper dimensions, are automatically handled by the overhead structure of the GEMACS system. It is this modularity that also allows for the relative ease with which new modules can be added to provide increased capabilities within the GEMACS system, or the relative ease with which the existing modules can be modified to increase their accuracy or applicability.

==========

HYBRIDIZATION METHODOLOGY

Exterior Problem Interaction
Matrix Is Calculated Using MOM/GTD
Interior Problem Interaction Matrix
Is Calculated Using FD
Total Problem Solution Is Found by
Using Householder Method of Modified
Matrices to Link the Separate
Solutions at Their Common Interfaces.

==========

This then completes the broad overview of the GEMACS system. It is more than a computer code in the strict sense of the word. It is a number of modules each of which are designed to perform a specific set of procedures or functions, and each of which is tied to the whole through a well defined data structure and transfer discipline. GEMACS is therefore a system in the broad sense of the word.

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AVAILABILITY

No charge
FORTRAN Source Code
Contact Author
315-330-2465

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It is hoped that the reader has made it thus far and that he is intrigued enough to make/take the time to read the following detailed summary (still without equations) regarding the GEMACS system.

PART II. THE DETAILED SUMMARY

Early in the decade of the 70s the Air Force recognized the need for a set of computerized mathematical models that would provide system, equipment, and circuit designers with an analysis capability to focus in on those system components and frequency ranges for which an electromagnetic interference condition could exist once system integration had been completed. There would be one or more electromagnetic compatibility analysis computer programs that would be applicable to ground, airborne, and space platforms, any of which could have an arbitrary collection of transmitting and receiving antennas. The locations of these antennas were allowed to be also totally arbitrary. The presence of apertures (filled or unfilled) served to further confuse the situation. A further complication was presented by the fact that the exterior skin of the structure could be a highly conductive metal or paint, or it could be made of some advanced composite material (sometimes painted), or more usually a combination of both. The basic frequency range of the equipments was 3 MHz to 40 GHz. This should give some idea of the broad generalities within which the problem could be defined.

A system variable that was recognized as very troublesome and hard to define a priori was the amount of electromagnetic coupling that could be expected between any two points in space, exterior and/or interior to the external geometry of the system being considered. An abundance of somewhat specific models was available, depending on structure geometry, electrical size, observable being quantified and desired accuracy of its calculation, etc. There were also available some very general, very sophisticated techniques which could be used to model complex structures across broad ranges of frequency. These latter techniques required large amounts of computer resources to perform the analysis. Coincidentally, this was also the time frame in which the computer industry was blossoming out with powerful versions of hardware at relatively modest costs compared to a decade earlier.

In addition to the requirements levied on the physics models for use in electromagnetic fields analysis, several constraints were placed on the design and coding of their respective implementing computer programs. Since it was envisioned that each computer program would be distributed to many government, industrial, and educational agencies and institutions thereby involving a multiplicity of computer systems, it was deemed necessary to put a strong emphasis on making the computer programs as transportable as possible. Therefore, all coding was to be in FORTRAN (originally ANSI standard 1966 and now 1977), utilizing no assembly language,

including no non-ANSI standard utilities or functions, no overlaying, no graphics packages, and other such options that could restrict the transfer of the computer program from one machine to another.

The types of output data that were to be provided were not much better defined than the physical characteristics of the problem. Broadly speaking the Air Force needed data regarding the near- and far-field patterns of antennas as they were located on the structure, the input impedance of the antennas (again in situ), the coupling between pairs of antennas considering their locations and the locations of all scattering and reflecting centers, the electric field distribution present in apertures of arbitrary dimensions and arbitrarily filled, the value of the current on the skin of the structure, and the radar cross section (monostatic and bistatic). To make matters even worse, the Air Force wanted to have these data anywhere in the frequency spectrum, both within the design band of the individual equipments as well as out of that design band.

In April 1974 RADC initiated a contract to develop an electromagnetic fields analysis program using the Method of Moments (MOM) technique to model the system and perform the analysis. Briefly explained, the system is modeled by a number of electrically small straight-wire subsections and/or by a number of electrically small subareas (or patches). Each of these elements is then conceived to be a point source radiating to, and interacting with, all of the other elements making up the system geometry. These various interactions are represented by a set of simultaneous equations, which is represented in a matrix notation and solved by a suitable solution process. Given the boundary conditions and the external electromagnetic environment (the right-hand-side of the equation), the currents on the subsections and/or the current densities on the subareas are calculated for a particular frequency. Once these currents and current densities are known, it is then a simple matter of matrix multiplication with a suitable Green's function to obtain the field at any point (or series of points) in the near or far field of the structure. Coupling between the terminals of antennas and the input impedance of any antenna fall out of the solution with ultimate ease. Proper positioning and incrementing of the source with respect to the structure

under consideration will allow one to calculate the monostatic and/or bistatic cross section of the object.

The advantages to the use of the Method of Moments technique are many. The technique requires very little electromagnetic fields expertise to properly and completely model the system and perform an analysis. Knowing the current density everywhere on the surface of the system gives the analyst quite a bit of flexibility and capability to further calculate and quantify many other electromagnetic phenomena associated with the system. Due to the fact that all of the computations, the geometrical description, and the output are stored on a checkpoint file (magnetic tape or disk pack) the analyst has a database for the system, which can be modified at some future date to reflect a proposed modification to the physical system. It is then a relatively simple and inexpensive process to then determine the new electromagnetic posture of the modified system.

A serious drawback to the indiscriminate use of the Method of Moments to analyze any structure for any problem type and observable is the fact that when the geometry becomes electrically large (i.e., exceeding what could be considered a resonance region problem) the amount of computer resources required to obtain the answer becomes impractical for most budgets. It is for this reason that a second general technique was implemented within the GEMACS computer program. This is the Geometrical Theory of Diffraction (GTD), which is a high-frequency (elecrically large structure) asymptotic formulation whose accuracy increases as the frequency of the analysis increases.

When using the GTD formulation for the analysis of a structure, one models the system geometry as a set of plates with or without a cylinder and its endcaps. The electric field at one or more points of interest is then computed using ray tracing techniques, following the ray from the source through a series of reflections and/or diffractions from plates, the cylinder, plate edges, and endcap rims. Each such scattering center is treated as a local source of electromagnetic energy, and the contributions from all such localized sources are summed at the field point. In this formulation the current distribution on the structure is not calculated. However, the code can calculate such observables as the electric field distribution in the near

and far field of the object, the coupling between antennas, the radar cross section, etc.

These two formulations have been totally hybridized within the GEMACS computer program. It should be noted though that the algorithms are physically separate in their own modules even though the computations are mathematically combined without any intervention by the analyst once the problem has been defined by him. Such total hybridization even extends to the description of the geometry to be analyzed. One single consistent command language and one coherent geometry modeling language are imbedded within GEMACS, both extending to the use of both physical/mathematical formulations. It is extremely convenient to now determine, for example, the antenna terminal characteristics even when that antenna is located on an electrically large structure. One simply models the antenna and its near physical environment with wire subsections and small patch subareas, and then places this localized geometrical description within the context of a GTD representation of the overall geometry. This procedure can be followed for as many specialized areas of interest as necessary. Such a modeling scheme will therefore allow one to calculate the current density in the vicinity of an antenna or an aperture. One can also determine the terminal parameters of antennas located on a large and complex structure without incurring the cost associated with modeling the entire geometry using MOM modeling elements.

Thus, we have in GEMACS an extremely powerful tool that allows one to efficiently and completely model and analyze the complex external electromagnetic environment associated with a system of arbitrary electrical size, taking very particular care in areas of critical concern. Furthermore, there is a very high degree of transportability associated with this particular computer program, thus allowing for a relatively straightforward installation of this program on a wide range of commonly used computer systems. In addition, maintenance, updates, and distribution are available in one location (RADC), providing easy access to user intercommunication, continuing guidance and education, and current information regarding modifications and corrections.

However, the Air Force problem also extends to the interior of the structure. What happens once the external electromagnetic field penetrates the surface of the

structure? What is the distribution of the interior electric field in the presence of wires, racks, bulkheads, dielectrics, and all else? Where does the energy penetrate, and how much energy is present in each of the apertures? What is needed is another technique or set of techniques that can be hybridized with the MOM and GTD techniques already present. We need a way to analyze the aperture coupling as a function of frequency, incidence angle, and polarization of the incoming wave. We need a method to predict the electric field present at any point in space within the geometry, taking into consideration all that exists within the system. RADC contracted with the BDM Corporation/Albuquerque to address these technological needs.

They have designed an upgrade to GEMACS which hybridizes a Finite Difference (FD) algorithm in the frequency domain with the MOM and GTD formulations to solve both the exterior and interior problems in the presence of each other, coupling them by a finite difference model of any connecting apertures. The apertures themselves will be treated as interior cavities (filled or empty, thick or thin) such that every geometry will be represented by at least three regions--the exterior, the aperture, and the interior. A second aperture would then result in a four-region problem, and so on. Solving all regions simultaneously will then assure the analyst that all interactions are being properly considered in relationship to each other.

Much consideration was given to reduce the order of the interaction matrix equation so that the analysis will not be totally impractical in terms of required computer resources and turn-around time, as well as the resolution and round-off limitations of computing machines. The implementation of the Model Order Reduction Formulation and the CONECT command into the GEMACS vocabulary not only accomplishes this goal of minimizing the resources needed for the FD solution, but can also be useful in reducing the magnitude of the analysis even if only an exterior problem is being analyzed.

The result is a logical extension of the vocabulary and syntax within GEMACS. The exterior-only interaction matrix equation is logically expanded to include the new coupling phenomena introduced by the presence of the aperture and the interior of the structure. This is totally invisible to the nalyst. He is only required to specify the interactions

that he will want to be considered. The classes of problems that are able to be treated include external field-to-internal wire coupling, aperture coupling, antenna coupling to internal wiring, bundle-to-bundle coupling for bundles of wires, aperture loading and shielding design, cavity resonance phenomena, to name just a few. The observables that are calculated include the currents on wires and in wire bundles, the power delivered to a known load, and the shielding effectiveness of bulkheads with holes for cable feedthroughs.

There is of course one very obvious problem that must be faced if one desires to accomplish a very thorough analysis of a system. Who is going to input all the geometry data into the front end of the GEMACS system of analysis tools? An auxiliary, but also important, question is concerned with the accuracy and resolution of such input data. How does one avoid the normal typographical errors on input? How closely can one read system drawings for locations, distances, etc.? How finely can one model a structure, given the finite size of the computer word?

These problems are being seriously considered at RADC, and some alternative solutions are being studied. Top priority is being given to the question of inputting tremendous amounts of geometrical data with a minimum of effort on the part of the user and a minimum of possibility of error occurring at the time of input. Some work on the development of a graphics input processor for MOM and GTD is now being pursued in another division at RADC. A concentrated effort to expand the present work to include the graphics capability for the FD geometry is planned for FY 86. Graphics packages to a certain extent can be expected to limit the transportability characteristic of GEMACS, but what will have been accomplished at RADC will be included as a separate module to be sent to those agencies possessing a compatible computer graphics system. The methodology for the implementation of a graphics input processing capability will of course be totally transportable and fully documented as the RADC effort progresses.

The same line of reasoning can also be applied to post-processing the tables and data that can be obtained as a result of the GEMACS analysis. More conclusions -- and more meaningful conclusions -- can be reached when the tabular

data are cast in a form that is more easily read, interpreted, and presented either orally or in a written report. This capability of generating graphical output appears to be more straightforward to implement than the input graphics, but the limiting of transportability will still result.

Many other lines of development are also possible and can be pursued if the requirement for the added capability is sufficient within the community of the Air Force and its contractors. Examples of such extensions include the explicit characterization of Radar Absorbent Materials (RAM) and advanced composite materials, calculation of monostatic and bistatic radar cross section, and analysis in the time domain.

In summary, what is available is a veritable tool box of techniques that can be used by an electromagnetic fields analyst in a variety of scenarios on arbitrary physical geometries to study in detail and with a fair degree of accuracy almost any conceivable observable of interest. In addition, the range of applicability to new and novel scenarios, geometries, and observables is still increasing. Furthermore, a central agency that is available for distribution, debugging, and information dissemination gives the community a certain amount of confidence that the latest and best data regarding GEMACS are available quickly and accurately.

FOR FURTHER READING

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New Version of ESP

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The purpose of this note is to announce that Version III of the "Electromagnetic Surface Patch Code" (ESP) and user's manual is ready for distribution. ESP is a user oriented computer code, using the method of moments (MM), for the analysis of the radiation and scattering from 3D geometries consisting of an interconnection of thin wires and perfectly conducting polygonal plates. Since the method is based upon an MM solution of an electric field integral equation, it is applicable to open and closed surfaces.

The ESP code can model an almost arbitrary thin wire antenna as an interconnection of straight segments. An arbitrary perfectly conducting surface is modeled as an interconnection of perfectly conducting polygonal plates. The code uses a special attachment mode which enforces continuity of current at wire/plate junctions, provided that the junction is at least 0.1λ from a plate edge. The code can also treat several plates which intersect along a common edge.

The user defines the plates by specifying the (x,y,z) coordinates of the corners of each plate, as-well-as the maximum segment size in wavelengths for the MM modes. The code then segments each polygonal plate into overlapping, piecewise sinusoidal, quadrilateral, surface patch, dipole modes. Similar overlap modes are automatically inserted to enforce continuity of the normal component of current at plate/plate junctions. As the frequency is changed, the density of the modes is automatically adjusted so that the size of the quadrilateral cells do not exceed the specified maximum segment size in wavelengths. Thus, from the user's standpoint, the plate model is frequency independent. The main advantage of polygonal plate modeling is that a user can define a shape as complex as an aircraft with about a dozen polygonal plates, and not be directly concerned with the possibly hundreds of surface patch modes into which the plates are segmented. Our experience is that this represents a practical method for specifying complex shapes without the need for a special computer assisted graphics or geometry package interfaced to the code.

In summary, the ESP can treat geometries consisting of:

1. thin wires with finite conductivity and lumped loads

2. perfectly conducting polygonal plates

3. wire/plate junctions (at least 0.1λ from the edge of a plate)

4. plate/plate junctions, including several plates of different size which intersect along a common edge.

5. excitation by either a voltage generator or a plane wave.

ESP can compute most of the quantities of engineering interest including:

1. current distributions

2. antenna input impedance, radiation efficiency, and mutual coupling

3. far zone radiation patterns (both polarizations)

4. back, bistatic, and forward scattering patterns (full scattering matrix).

There are three main improvements in Version III of the ESP code. First, it is written in *standard* FORTRAN 77. Second, many errors or problems, uncovered by the author and users of the code in the last few years, have been corrected. The author would especially like to thank Prof. Ray Luebbers at Pennsylvania State Univ. and the antenna group at Lockheed, Sunnyvale for their in depth and valuable suggestions. Finally, all graphics or plotting has been removed from the basic ESP code. Instead, ESP writes data files which can then be read by separate codes to produce the geometry and pattern plots. Two auxiliary plotting codes, written in the "Graphical Kernel System" (GKS) language are provided for this purpose.

We follow export controls in the distribution of the ESP codes. Our policy is to only send to U.S. government agencies and U.S. companies with U.S. government contacts. In these cases, the ESP Version III code plus user's manual can be obtained (for a nominal fee to cover material and handling costs) by writing to:

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Requests for the ESP code from non U.S. companies must be done on a government to government basis.

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