Lagrangian BEM-FEM Solution of Electromagnetic-Mechanical Coupled Problems

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***Abstract*—In the framework of the electro-mechanical coupled problems, the coupling between the magnetic diffusion and the elastic deformation equations is treated numerically. A mixed BEM-FEM approach for the electromagnetic part, and a FEM approach for the mechanical part, are proposed. Advantages are taken from a Lagrangian formulation of the problem.**

I. INTRODUCTION

The use of forces and torques of electromagnetic origin is spreading wider and wider in the world of industrial applications. The interactions between electromagnetic fields and mechanical systems could be divided into two categories: *weakly coupled* and *strongly coupled*, depending on the level of interaction between the two subsystems. In the first case the evolution of the mechanical system on which the electromagnetic forces are acting alters only slightly the configuration of the electromagnetic field. It is possible, in this case, to devise "two-step" numerical solution schemes, solving first for the electromagnetic field in the previous mechanical configuration, then evaluating the electromagnetic forces, and finally updating the mechanical configuration. In the second case, instead, it is mandatory to solve for the electromagnetic field *and* the mechanical deformation *together*, in a consistent way. This implies considering the full coupled problem, since the conductors' shape modifies the electromagnetic field map so strongly that we cannot deny this effect, even in a first approximation.

A possible way to undertake these coupled problems is to use a Lagrangian approach, solving them in a reference configuration, and then "projecting" the solution in the actual configuration.

Into the class of strongly coupled problems is included the electromagnetic forming [1]. This is an industrial process in which the magnetic force generated by an excitation coil is exploited to give a certain shape to a piece of conducting material.

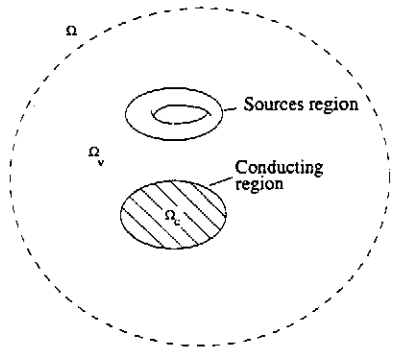

Fig.1 - Problem geometry

In this paper we propose a method to treat electromechanical coupled problems. To show the performance of the method, the effect of an external magnetic field on a piece of linear elastic conductor will be examined (Fig.1). The problem can be considered as an extremely simplified case of magnetic forming, as we neglect the nonlinear mechanical behavior of the material, to focus our attention on the electromagnetic aspect of the phenomenon.

To numerically solve the problem, a Lagrangian mixed finite elements-boundary elements method [2] is used; this allows to discretize the conducting region only, and to treat more effectively the contact with external objects, such as forming devices.

In Section 2 the mathematical formulation of the problem is given, while in Section 3 the numerical implementation is discussed. In Section 4 an example of application of the method is presented, and in Section 5 some conclusions are drawn.

II. MATHEMATICAL FORMULATION

Let Ω be a region of space in which it is possible to identify a conducting region Ω_c (see Fig. 1). Let's assume for the sake of simplicity that no source is present in the region Ω_c. Let then Ω_v be the remaining part of Ω. We aim to study

Manuscript received April 15, 1997
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the deformation of the region Ω_c due to the presence of electromagnetic forces.

The problem can be stated as:

$$\begin{cases} \mathcal{E}(\mathbf{s}_e, t; \mathbf{x}, \mathbf{e}) = 0 \\ \mathcal{M}(\mathbf{s}_m, t; \mathbf{x}, \mathbf{e}) = 0 \end{cases}, \tag{1}$$

where $\mathbf{s}_{(e,m)}$ are vectors describing electromagnetic (e) and mechanical (m) sources, boundary and matching conditions; t represents the time; $\mathbf{x}$ and $\mathbf{e}$ are mechanical and electromagnetic unknowns; $\mathcal{E}$ and $\mathcal{M}$ are operators describing the evolution of the system, comprising the Maxwell equations, the mechanical deformations' laws, and the constitutive relationships.

The Eulerian approach consists of solving (1) in the actual spatial domain Ω, thus using the same reference frame for the whole evolution. This is the typical electromagnetic point of view. In the continuum mechanics context, instead, the most usual point of view is the Lagrangian approach. This consists in introducing a reference configuration $\tilde{\Omega}$ (for example the initial one) and describing the actual configuration by using a placement function u: $\tilde{\Omega} \rightarrow \Omega$. Of course, every spatial quantity, including the electromagnetic field distribution, has to be transformed in a consistent way.

In (1) the mechanical operator $\mathcal{M}$ has to be expressed, as $\mathcal{E}$, in the actual (Eulerian) frame. However, if the Lagrangian approach is used for the mechanical equations, and the electromagnetic equations are also formulated within this approach, a complete Lagrangian form of (1) can be obtained [3]:

$$\begin{cases} \mathcal{E}_L(\tilde{\mathbf{s}}_e, t, \tilde{\Omega}; u, \tilde{\mathbf{e}}) = 0 \\ \mathcal{M}_L(\tilde{\mathbf{s}}_m, t, \tilde{\Omega}; u, \tilde{\mathbf{e}}) = 0 \end{cases}, \tag{1'}$$

where u is the placement function and the symbol $\tilde{}$ identifies variables defined on the reference (Lagrangian) frame. It should be noted that in (1') the placement u substitutes the spatial co-ordinate $\mathbf{x}$. In fact, u completely describes the actual shape of the system, while the force distribution is implicitly contained in $\mathcal{M}_L$.

The Lagrangian approach offers a number of advantages [3]-[5]. In particular, when using this approach, the dependence on the domain geometry appears only via the unknown placement function u, while the definition domain is fixed. In this way, we don't have to worry about the modifications of the material properties of various regions of space, as these properties are defined *on the matter itself*, and transform covariantly with the geometry of their definition domain.

Furthermore, as the Lagrangian definition domain is fixed, no explicit velocity term appears in (1'). Of course, the velocity, defined in the Lagrangian framework as the time derivative of the placement, appears again when considering the *transformed*, Eulerian, quantities.

The mechanical operator $\mathcal{M}$ is usually coupled to the electromagnetic one via the electromagnetic force. In the frame of Lagrangian approaches, this force can be effectively defined as the opposite of the derivative of the electromagnetic energy (or as the derivative of the coenergy), with respect to the placement u. The conditions for the evaluation of the derivative depend on the particular choice for the electromagnetic variable.

For example, by choosing to express the coenergy $\tilde{C}_{em}$ in terms of the magnetic field $\tilde{\mathbf{H}}$ [3], we have:

$$\tilde{\mathbf{F}}_{em} = \left. \frac{\partial \tilde{C}_{em}(\tilde{\mathbf{H}}, u)}{\partial u} \right|_{\substack{\text{Currents} \\ \text{fixed}}}, \tag{2}$$

Note that, in a general coupled problem, each change of a mechanical variable implies a corresponding change in the electromagnetic variables, and vice-versa. In the particular case of weakly coupled problems, the electromagnetic variables $\tilde{\mathbf{e}}$ are not strongly affected by the modification of the mechanical variables; therefore it is possible to state that, considering a variation Δu of the placement (defining the domain geometry):

$$\tilde{\mathbf{e}}(u + \Delta u) \cong \tilde{\mathbf{e}}(u) \ . \tag{3}$$

An analogous relation holds for the mechanical variable u with respect to variation $\Delta\tilde{\mathbf{e}}$ of the electromagnetic quantities. In this case, it is possible to solve the coupled problem in two successive steps, using the preceding value of the electromagnetic variables.

On the other hand, if the approximation (3) does not hold, or holds only for unacceptably small changes in the mechanical variables, we have to solve (1) together.

A. Electromagnetic Equations.

Using the usual symbols, we express the electromagnetic model in the magneto-quasi-static limit as:

$$\begin{cases} \nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} \\ \nabla \times \tilde{\mathbf{E}} = -\dfrac{\partial \tilde{\mathbf{B}}}{\partial t} \end{cases} \quad \text{in } \tilde{\Omega}_c$$

$$\begin{cases} \nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}}_s \end{cases} \quad \text{in } \tilde{\Omega}_v \ , \tag{4}$$

where $\tilde{\mathbf{J}}_s$ is the source current density, and we will suppose that there are no sources in $\tilde{\Omega}_c$. The variables in $\tilde{\Omega}_c$ and $\tilde{\Omega}_v$ are connected by the usual continuity relations, and proper initial and boundary conditions are to be supplied.

Equations (4) are easily seen to be covariant by properly redefining the electromagnetic quantities and the operators acting on them [3]. The whole formulation gains clarity and simplicity from the use of a more suitable mathematical framework, like the differential geometry [6].

As concerning the constitutive relationships in the Lagrangian framework, it should be noted that they are no more simple tensorial products, but more complex

transformation laws between different mathematical objects that depend on the domain deformations [7]. To avoid cumbersome notations, we will hide this process into the usual expressions, in such a way that the constitutive relationships keep the usual form:

$$\begin{cases} \tilde{\mathbf{B}} = \mu \tilde{\mathbf{H}} \\ \tilde{\mathbf{J}} = \sigma \tilde{\mathbf{E}} \end{cases} \quad (5)$$

In (5), σ and μ may be related to the Hodge operator [3]. Anyway, for the sake of simplicity, in the following they are referred to with the usual terms, namely conductivity and permeability of the material considered.

As we adopt a Lagrangian scheme, no velocity term is present in the model. Of course, as told above, the field in the actual configuration is obtained by transforming the Lagrangian quantities by means of the placement function u; when doing this, it can be easily seen that the fields obtained in this way have the same formal expression as those obtained using the Eulerian approach [7].

In the following we will consider the case of a simply connected, axisymmetric conductor with an external driving magnetic field $\mathbf{H}_s$. In this case the most effective variable is the reaction magnetic flux per radiant ψ, defined, in a cylindrical coordinate system, as:

$$\tilde{\psi}(\tilde{r},\tilde{z}) = \int_0^{\tilde{r}} \tilde{B}_z(\tilde{\rho},\tilde{z}) \frac{\partial \rho}{\partial \tilde{\rho}} \tilde{\rho} \, d\tilde{\rho} \,. \quad (6)$$

where $\tilde{B}_{\tilde{z}}$ is the $\tilde{z}$ component of the magnetic flux density due to the induced currents. As the flux is a point-wise quantity, it transforms in a very easy way: $\tilde{\psi}(\tilde{r}) = \psi(u(\tilde{r}))$.

With this choice, in the conducting region the electromagnetic model reduces to the following differential equation:

$$\tilde{L}\tilde{\psi} + \sigma \frac{\dot{\tilde{\psi}}}{\tilde{r}} = -\sigma \frac{\dot{\tilde{\psi}}_s}{\tilde{r}} \,, \quad (7)$$

where the operator $\tilde{L}$ is defined as:

$$\tilde{L} = -\frac{\partial \tilde{r}}{\partial r}\frac{\partial}{\partial \tilde{r}}\left(\frac{1}{\mu \tilde{r}}\frac{\partial \tilde{r}}{\partial r}\frac{\partial}{\partial \tilde{r}}\right) - \frac{\partial \tilde{z}}{\partial z}\frac{\partial}{\partial \tilde{z}}\left(\frac{1}{\mu \tilde{r}}\frac{\partial \tilde{z}}{\partial z}\frac{\partial}{\partial \tilde{z}}\right). \quad (8)$$

and ψ_s is the flux due to the source currents.

In the external vacuum region, an integral formulation can be given:

$$\kappa \tilde{\psi} - \int_{\partial \tilde{\Omega}_v} \frac{1}{\mu \tilde{r}} \tilde{\psi} \frac{\partial G}{\partial \tilde{n}} \frac{\partial \Gamma}{\partial \tilde{\Gamma}} d\tilde{\Gamma} + \int_{\partial \tilde{\Omega}_v} \frac{1}{\mu \tilde{r}} G \frac{\partial \tilde{\psi}}{\partial \tilde{n}} \frac{\partial \Gamma}{\partial \tilde{\Gamma}} d\tilde{\Gamma} = 0 \,, \quad (9)$$

where κ=1 in $\tilde{\Omega}_v$, and κ=0.5 on the boundary $\partial\tilde{\Omega}_v$, supposed to be regular enough; G is the fundamental solution of the Laplace problem in cylindrical coordinate, i.e., the flux in (r, z) generated by a filamentary current in (ρ, ζ):

$$G(r,z;\rho,\zeta) = \frac{\mu}{4\pi} \frac{1}{k} \sqrt{r\rho} \left[\left(1 - \frac{k^2}{2}\right) K(k) - E(k) \right], \quad (10)$$

and $k = \sqrt{\dfrac{4r\rho}{(r+\rho)^2 + (z-\zeta)^2}}$; K and E are the complete elliptic integrals of the first and second kind respectively.

Therefore, the electromagnetic model reduces to (7) and (9), plus initial, boundary and continuity relations. In particular, the flux on the boundary of the conducting region is imposed to be continuous with its normal derivative, and this gives the required link between (7) and (9).

The use of an integral formulation for the external region has two advantages, particularly valuable in the perspective of a numerical solution of the problem. First of all, in this way we have to discretize only the conducting region, thus obtaining a better approximation with the same computational burden. Secondly, when the conductor touches an obstacle, no remeshing is needed in spite of the change of topology of Ω_v.

To seek for an approximated solution of (7) and (9), a Galerkin approach can be applied. Let's assume that the flux and its normal derivative belong to a suitable functional space S. In the hypothesis that the normal to the boundary is directed along z on the rotation axis, this space can be L^2_0, i.e., the space of square integrable functions vanishing on the z axis. Equation (7) becomes:

$$\begin{aligned} &\int_{\tilde{\Omega}_c} \nabla\tilde{\psi} \frac{1}{\mu\tilde{r}} \nabla\alpha \underline{\underline{\mathbf{J}}}^{-1} d\tilde{\Omega} - \int_{\partial\tilde{\Omega}_c} \frac{1}{\mu\tilde{r}} \tilde{\lambda}\alpha \frac{d\Gamma}{d\tilde{\Gamma}} d\tilde{\Gamma} + \\ &\int_{\tilde{\Omega}_c} \frac{\sigma}{\tilde{r}} \dot{\tilde{\psi}} \alpha \underline{\underline{\mathbf{J}}} d\tilde{\Omega} = -\int_{\tilde{\Omega}_c} \frac{\sigma}{\tilde{r}} \dot{\tilde{\psi}}_s \alpha \underline{\underline{\mathbf{J}}} d\tilde{\Omega} \quad \forall \alpha \in S, \end{aligned} \quad (11)$$

and (9) becomes:

$$\begin{aligned} &\frac{1}{2} \int_{\partial\tilde{\Omega}_v} \tilde{\psi}\alpha \frac{\partial\Gamma}{\partial\tilde{\Gamma}} d\tilde{\Gamma} + \int_{\partial\tilde{\Omega}_v} \alpha \left(\int_{\partial\tilde{\Omega}_v} \frac{1}{\mu\tilde{r}} \tilde{\psi} \frac{\partial G}{\partial\tilde{n}} \frac{\partial\Gamma'}{\partial\tilde{\Gamma}'} d\tilde{\Gamma}' \right) \frac{\partial\Gamma}{\partial\tilde{\Gamma}} d\tilde{\Gamma} \\ &- \int_{\partial\tilde{\Omega}_v} \alpha \left(\int_{\partial\tilde{\Omega}_v} \frac{1}{\mu\tilde{r}} G\tilde{\lambda} \frac{\partial\Gamma'}{\partial\tilde{\Gamma}'} d\tilde{\Gamma}' \right) \frac{\partial\Gamma}{\partial\tilde{\Gamma}} d\tilde{\Gamma} = 0 \\ &\forall \alpha \in S \textit{ not vanishing on } \partial\tilde{\Omega}_v \,, \end{aligned} \quad (12)$$

where $\lambda = \partial\psi/\partial n$ is the normal derivative of the flux due to the induced currents, and $\underline{\underline{\mathbf{J}}}$ represents the Jacobian matrix of the placement function u.

In this framework, and supposing that the material is linear, the electromagnetic force density can be expressed as the opposite of the derivative of the magnetic energy density while keeping the flux fixed:

$$\tilde{\mathbf{f}}_{em} = -\frac{\partial}{\partial u}\left(\frac{1}{\tilde{r}^2} (\nabla\tilde{\psi})^2 \right)\Big|_{\substack{\text{Fixed} \\ \text{flux}}} . \quad (13)$$

B. Mechanical Equations

In this paper, for the sake of simplicity the conducting material is assumed to behave as an elastic material. We will further assume that the deformations are small enough for the linear elastic approximation to hold. As told above, this is not reasonable in the magnetic forming process because, by definition, a forming process implies a permanent deformation; anyway, the plastic theory would only introduce a nonlinear mechanical relationship, which does not alter the structure of the electromagnetic model.

In this hypothesis we have [8]:

$$\rho\ddot{\tilde{\mathbf{x}}} + \nabla\cdot\left(\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\nabla\tilde{\mathbf{x}}\right) + \nabla\times\left(\frac{E}{2(1+\nu)(1-2\nu)}\nabla\times\tilde{\mathbf{x}}\right) = \tilde{\mathbf{f}}_{em}, \tag{14}$$

where $\tilde{\mathbf{x}}$ is the deformation of the material, E is the Young modulus, ν is the Poisson ratio, and ρ is the mass density. In this case $u=u_0+\tilde{\mathbf{x}}$, where u_0 maps the underfomed configuration onto itself.

For the mechanical model, a weak formulation can also be obtained by applying the standard Galerkin approach:

$$\int_{\tilde{\Omega}_v}\alpha\rho\ddot{\tilde{\mathbf{x}}}d\tilde{\Omega}_v + \int_{\tilde{\Omega}_v}\nabla\alpha\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\nabla\tilde{\mathbf{x}}d\tilde{\Omega}_v + \int_{\tilde{\Omega}_v}\nabla\times\alpha\frac{E}{2(1+\nu)(1-2\nu)}\nabla\times\tilde{\mathbf{x}}d\tilde{\Omega}_v = \int_{\tilde{\Omega}_v}\alpha\tilde{\mathbf{f}}_{em}d\tilde{\Omega}_v \tag{15}$$

$\forall\alpha\in S\times S$ *vanishing on* $\partial\tilde{\Omega}_v$.

III. Numerical Approach

By using a FEM approach to treat the conducting region, and a BEM approach to treat the vacuum region, we obtain the numerical electromagnetic model:

$$\begin{cases}\dfrac{d(\underline{\underline{L}}\underline{\psi})}{dt} + \underline{\underline{R}}\underline{\psi} + \underline{\underline{C}}\underline{\lambda} = \underline{s},\\ \underline{\underline{A}}\underline{\psi} + \underline{\underline{B}}\underline{\lambda} = 0\end{cases} \tag{16}$$

where $\underline{\psi}$ and $\underline{\lambda}$ are the nodal flux vector and the normal derivatives vector on the boundary, respectively. Both $\underline{\psi}$ and $\underline{\lambda}$ are described in the same finite dimensional base for S [2]. The resulting ODE system is solved with a Cranck-Nicolson scheme.

Numerical advantages are obtained when modifying the formulation in the vacuum region in such a way that a self adjoint operator results, thus leading to symmetric matrices.

The matrices appearing in the discretized Lagrangian model are to be assembled at each time step. They are in general different from the Eulerian corresponding ones, since we have to back project the relations that hold in the actual configuration onto the reference one. Anyway, when assuming a linear behavior for the placement function u with respect to the position vector in the reference frame (which we do), the Lagrangian approach amounts simply to assembling the Eulerian matrices at each time step, as the Jacobian matrix $\underline{\underline{J}}$ becomes constant.

Adopting also a standard FEM approach to discretize (15), we obtain [9]:

$$\underline{\underline{M}}\ddot{\underline{x}} + \underline{\underline{K}}\underline{x} = \underline{f}, \tag{17}$$

where $\underline{\underline{M}}$ is the mass matrix, $\underline{\underline{K}}$ is the stiffness matrix, $\underline{f}$ is the forcing vector, and $\underline{x}$ is the nodal displacement vector.

The resulting ODE system is solved with a Newmark scheme.

The final coupled system,

$$\begin{cases}\dfrac{d(\underline{\underline{L}}(\underline{x})\underline{\psi})}{dt} + \underline{\underline{R}}(\underline{x})\underline{\psi} + \underline{\underline{C}}(\underline{x})\underline{\lambda} - \underline{s}(\underline{x}) = 0\\ \underline{\underline{A}}(\underline{x})\underline{\psi} + \underline{\underline{B}}(\underline{x})\underline{\lambda} = 0\\ \underline{\underline{M}}\ddot{\underline{\tilde{x}}} + \underline{\underline{K}}\underline{\tilde{x}} - \underline{f}(\underline{\psi},\underline{\lambda},\underline{\tilde{x}}) = 0,\end{cases} \tag{18}$$

is, in general, nonlinear, as the electromagnetic matrices depend on the placement of the mesh nodes, while the forcing term in the mechanical subsystem depends, in a nonlinear fashion, on the magnetic flux.

The inversion of (18) is rather cumbersome, as we have to modify the matrices of the system at each time step. In addition, their re-assembling has to be performed at each step of the nonlinear solver. In order to simplify the solution of (18), some approximations can be introduced, depending on the time step amplitude and on the deformation amplitude.

The strongest simplification is well suited for the weakly coupled systems. It consists in solving at each time step alternatively for the electromagnetic variables, using the mechanical deformation found at the preceding time step, and then for the mechanical variables, using the electromagnetic quantities already found.

Less rough approximations consist in substituting into the matrices $\underline{\underline{L}}$, $\underline{\underline{R}}$, $\underline{\underline{C}}$ and the vectors $\underline{s}$ and $\underline{f}$ their polynomial expansion with respect to the variable $\underline{x}$. For example, with reference to the matrix $\underline{\underline{L}}$, it is possible to state that:

$$\underline{\underline{L}}(\underline{x}) \cong \underline{\underline{L}}(\underline{x}_0) + \left.\frac{\partial\underline{\underline{L}}}{\partial\underline{x}}\right|_{\underline{x}=\underline{x}_0}\Delta\underline{x} + \ldots \tag{19}$$

Similar expressions for the other matrices and for the vectors hold. The more terms we use, the better the approximation [10]; if we limit (19) to the first term, we obtain the so-called sensitivity analysis [11].

Of course the simplification to be used should be chosen on the basis of the desired degree of approximation, and on the amplitude of the time step.

TABLE I
MECHANICAL PARAMETERS

Mechanical Parameters	"soft" case	"hard" case	Copper
Young modulus	200 10^6 [Pa]	200 10^7 [Pa]	110 10^9 [Pa]
Poisson ratio	0.26	0.26	0.26
Mass.density	80 [Kg./m^3]	800 [Kg./m^3]	8970 [Kg./m^3]

IV. APPLICATION EXAMPLE

The proposed approach has been implemented into a C code named "ALEPH-2D". It has been used to simulate the behavior of the very simple spheroidal conductor depicted in Fig. 2, considered from the mechanical point of view as a linear elastic material. This hypothesis gives the example no practical application, but the code can easily be generalized to treat plastic deformations, which have on the other hand great practical importance. The conducting material is supposed to have the same conductivity σ as copper. The exciting field $\mathbf{H}_s$ is supposed to be uniform and directed along the z-axis, while its time dependence is reported in Fig. 3.

Two situations are considered, with two sets of mechanical parameters, reported in Table I, together with those of the copper, for reference. The mechanical parameters are modified with respect to the copper to highlight their impact on the level of coupling between the electromagnetic and mechanical equations.

In Fig. 4 and in Fig 6 are reported the magnetic fluxes at the point (1.0, 0.0) for the two sets of mechanical parameters.

The curves are obtained with 50 and 500 time steps, by keeping fixed the electromagnetic subsystem matrices $\underline{\mathbf{L}}$, $\underline{\mathbf{R}}$, $\underline{\mathbf{C}}$ and the vector $\underline{\mathbf{s}}$ (weak coupling limit), or updating them at each time step, and solving the resulting nonlinear system with the Picard algorithm. In Fig. 5 and 7 are reported the variations of the r co-ordinate for the point P in Fig. 2 in the same cases.

It is easy to see that, in the case of 500 time steps, no differences arise between the weak and the strong coupling schemes. When the time discretization is coarser, instead, the weak coupling reveals to be a poor approximation in the simulation with the "soft" material. This lack of accuracy is not serious for the magnetic flux, but becomes important if considering the mechanical displacement.

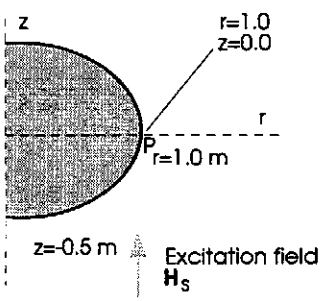

Fig. 2 - Example geometry

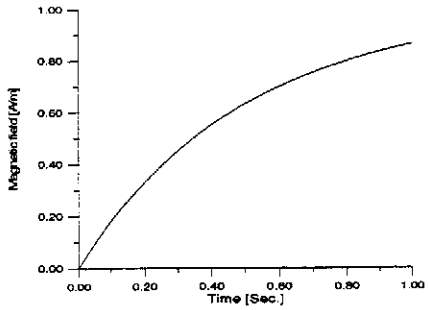

Fig. 3 - Magnetic field time profile

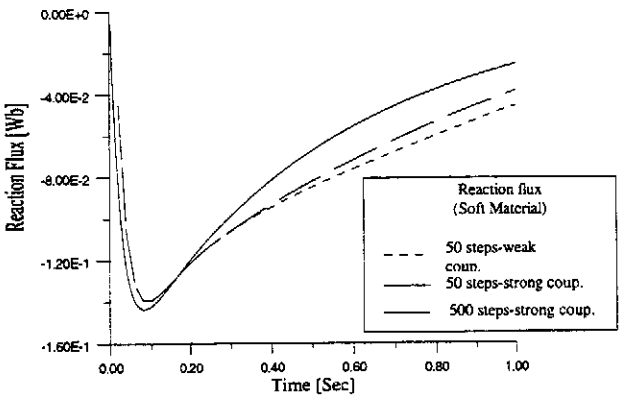

Fig. 4 - Reaction flux in the soft material case

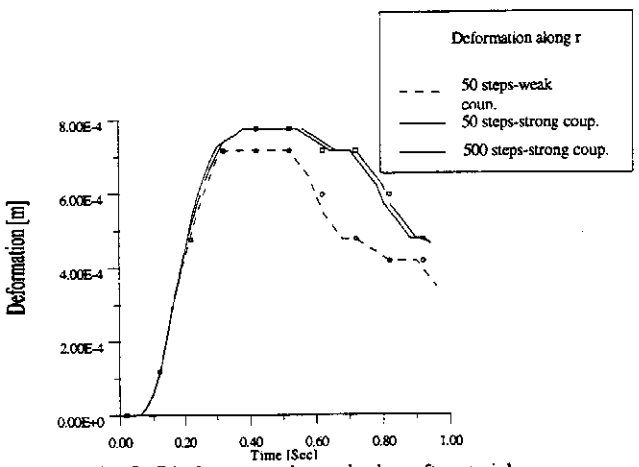

Fig. 5 - Displacement along r in the soft material case

When considering the "hard" material, instead, the mechanical deformation is much lower, and no evident difference occurs in either the variables. This confirms that the validity of the weak coupling should be tested against time step and displacement amplitudes.

V. CONCLUSIONS

In this paper a numerical method for treating the electromechanical coupled problems has been presented. The solution of this class of problems should be undertaken by taking into account that:

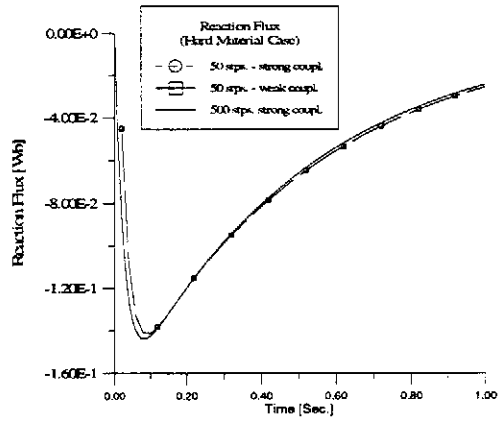

Fig. 6 - Reaction flux in the hard material case

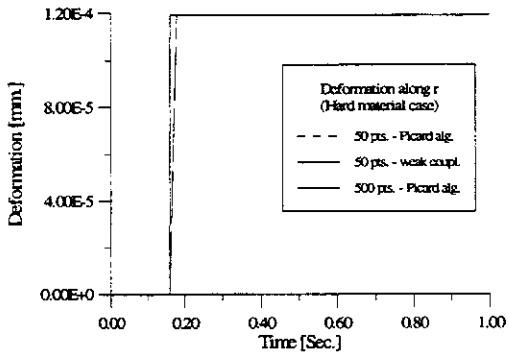

Fig. 7- Displacement along r in the hard material case

A. the coupling between the electromagnetic and mechanical equations prevents in some cases the possibility to face independently the two sets of equations,

B. the resulting numerical model is nonlinear due to the dependence of the electromagnetic matrices on the displacement and to the nonlinear character of the electromagnetic force.

The mechanical equations are local by nature as they hold only where the stressed material is present. Unfortunately, the electromagnetic equations describe nonlocal phenomena, thus typically requiring a model describing the whole space. This can be overcome by adopting an integral approach that allows to consider the external field effect by means of a suitable boundary equation.

Advantages can be taken by introducing a Lagrangian formulation to face the whole model, as usual for the pure continuum mechanics problems. In this way, the integration domain remains fixed, in spite of the mechanical deformations, and no explicit velocity term appears in the equations. Furthermore, if a suitable formulation in the external region is adopted, symmetric electromagnetic matrices are obtained.

In this framework, in order to treat more effectively the resulting numerical model, some useful simplification can easily be applied, as the weak coupling approach, or the sensitivity analysis.

On the basis of the exposed considerations, a Lagrangian FEM-BEM code has been created (ALEPH 2-D), designed to treat electromechanical coupled problems. The formulation leads to nonsymmetric matrices, but this problem can be easily solved.

Its effectiveness has been shown with a simple numerical example, representing a rough schematization for the magnetic forming technique, which is a challenging problem due to its strongly coupled nature.

Work is in progress to treat more complex mechanical constitutive relationships, as nonlinear elasticity and plasticity, as well as magneto-hydro-dynamics.

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