# PEC Condition Implementation for the Novel Weakly Conditionally Stable Finite-Difference Time-Domain Method

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***Abstract*** **—** The perfect-electric-conductor (PEC) condition implementation for the novel weakly conditionally stable finite-difference time-domain (NWCS-FDTD) method is discussed in this paper. It shows that the method that the PEC condition is directly incorporated within the tri-diagonal matrix is more accurate and the burden on the computation efficiency is trivial. The theory proposed in this article is validated through numerical example.

***Index Terms*** **—** FDTD method, perfect-electric-conductor (PEC) condition, weakly conditionally stable FDTD method.

## I. INTRODUCTION

To overcome the Courant limit on the time step size of the FDTD method, unconditionally stable methods such as the alternating-direction implicit FDTD (ADI-FDTD) scheme [1-6] have been studied extensively. Although the time step size in the ADI-FDTD simulation is no longer bounded by the Courant–Friedrich–Levy (CFL) criterion, the method exhibits a splitting error [7, 8] that is proportional to the square of the time step size and the spatial derivatives of the field. When field variation and/or the time step size are large, the splitting error becomes pronounced. The accuracy of the ADI-FDTD method is limited.

Based on the theory of the ADI-FDTD method, a novel weakly conditionally stable finite-difference time-domain (NWCS-FDTD) method has been developed recently [9]. In this method, the CFL condition is not removed totally, but being weaker than that of the conventional FDTD method. The time step in this scheme is only determined by one discretization, which is extremely useful for a problem where a very fine mesh is needed in one or two directions. Compared with the ADI-FDTD scheme, the NWCS-FDTD method is with trivial split error, so the accuracy of the NWCS-FDTD technique is better than that of the ADI-FDTD scheme. By defining the field components at only two time steps, the NWCS-FDTD method requires the solution of four tri-diagonal matrices and four explicit updates at each time step. While maintaining the same time step size, the CPU time for the NWCS-FDTD method can be reduced to about 2/3 of that for the ADI-FDTD scheme.

Compared with the conventional weakly conditionally stable finite-difference time-domain (WCS-FDTD) method [10, 11], the NWCS-FDTD method has less split error, so the accuracy of the NWCS-FDTD method is also better than that of the conventional WCS-FDTD method. The detailed comparison between these two methods has been presented in reference [12].

In the NWCS-FDTD method, updating of $H_y$ component needs the unknown $E_x$ and $E_z$ components at the same time step, thus, the perfect-electric-conductor condition implementation for the $E_x$ and $E_z$ components must be incorporated within the solving of the $H_y$ component. This paper gives a simple strategy of the PEC condition implementation of the $E_x$ and $E_z$ components. It shows that the method that the PEC condition is directly incorporated within the tri-diagonal matrix is more accurate and the computation burden of this method is trivial. The

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theory proposed in this article is validated through numerical example.

## II. PEC CONDITION IMPLEMENTATION FOR THE NWCS-FDTD METHOD

Considering the PEC condition implementation for the $E_x$ and $E_z$ components, the updating of the $H_y$ component become,

$$\begin{aligned} H_y^{n+1} &= H_y^n + \frac{\Delta t\partial}{2\mu\partial x} l_z \times \left(E_z^{n+1}+E_z^n\right) \\ &- \frac{\Delta t\partial}{2\mu\partial z} l_x \times \left(E_x^{n+1}+E_x^n\right) \end{aligned} \quad (1)$$

where, $\mu$ is permeability of the medium; $n$ and $\Delta t$ are the index and size of time-step; $l_x$ and $l_z$ are the length factors. At the surface of the perfect-electric-conductor, the values of the length factors are zeros; at other mesh points, the values of $l_x$ and $l_z$ are equal to 1.

The updating equations of the $E_x$ and $E_z$ components are same as those in the reference [9],

$$E_x^{n+1} = E_x^n + \frac{\Delta t\partial}{\varepsilon\partial y} H_z^{n+1/2} - \frac{\Delta t\partial}{2\varepsilon\partial z}\left(H_y^{n+1}+H_y^n\right) \quad (2)$$

$$E_z^{n+1} = E_z^n - \frac{\Delta t\partial}{\varepsilon\partial y} H_x^{n+1/2} + \frac{\Delta t\partial}{2\varepsilon\partial x}\left(H_y^{n+1}+H_y^n\right) \quad (3)$$

Substituting equations (2) and (3) into equation (1), and approximating each derivative in space by centered second-order finite differences, the updating equation for $H_y$ field is given as,

$$\begin{aligned}
&H_y^{n+1}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right) \\
&- \frac{b'}{\Delta x^2}\left[\begin{aligned} &l_z\left(i+1,j,k+\tfrac{1}{2}\right)\times\left[H_y^{n+1}\left(i+\tfrac{3}{2},j,k+\tfrac{1}{2}\right)-H_y^{n+1}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right)\right] \\ &-l_z\left(i,j,k+\tfrac{1}{2}\right)\times\left[H_y^{n+1}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right)-H_y^{n+1}\left(i-\tfrac{1}{2},j,k+\tfrac{1}{2}\right)\right]\end{aligned}\right] \\
&- \frac{b'}{\Delta z^2}\left[\begin{aligned} &l_x\left(i+\tfrac{1}{2},j,k+1\right)\times\left[H_y^{n+1}\left(i+\tfrac{1}{2},j,k+\tfrac{3}{2}\right)-H_y^{n+1}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right)\right] \\ &-l_x\left(i+\tfrac{1}{2},j,k\right)\times\left[H_y^{n+1}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right)-H_y^{n+1}\left(i+\tfrac{1}{2},j,k-\tfrac{1}{2}\right)\right]\end{aligned}\right] \\
&= H_y^{n}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right) \\
&- \frac{b'}{\Delta x^2}\left[\begin{aligned} &l_z\left(i+1,j,k+\tfrac{1}{2}\right)\times\left[H_y^{n}\left(i+\tfrac{3}{2},j,k+\tfrac{1}{2}\right)-H_y^{n}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right)\right] \\ &-l_z\left(i,j,k+\tfrac{1}{2}\right)\times\left[H_y^{n}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right)-H_y^{n}\left(i-\tfrac{1}{2},j,k+\tfrac{1}{2}\right)\right]\end{aligned}\right] \\
&- \frac{b'}{\Delta z^2}\left[\begin{aligned} &l_x\left(i+\tfrac{1}{2},j,k+1\right)\times\left[H_y^{n}\left(i+\tfrac{1}{2},j,k+\tfrac{3}{2}\right)-H_y^{n}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right)\right] \\ &-l_x\left(i+\tfrac{1}{2},j,k\right)\times\left[H_y^{n}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right)-H_y^{n}\left(i+\tfrac{1}{2},j,k-\tfrac{1}{2}\right)\right]\end{aligned}\right] \\
&- S_{xy}\left\{\begin{aligned} &l_z\left(i+1,j,k+\tfrac{1}{2}\right)\times\left[H_x^{n+\frac{1}{2}}\left(i+1,j+\tfrac{1}{2},k+\tfrac{1}{2}\right)-H_x^{n+\frac{1}{2}}\left(i+1,j-\tfrac{1}{2},k+\tfrac{1}{2}\right)\right] \\ &-l_z\left(i,j,k+\tfrac{1}{2}\right)\times\left[H_x^{n+\frac{1}{2}}\left(i,j+\tfrac{1}{2},k+\tfrac{1}{2}\right)-H_x^{n+\frac{1}{2}}\left(i,j-\tfrac{1}{2},k+\tfrac{1}{2}\right)\right]\end{aligned}\right\} \\
&- S_{xz}\left\{\begin{aligned} &l_x\left(i+\tfrac{1}{2},j,k+1\right)\times\left[H_z^{n+\frac{1}{2}}\left(i+\tfrac{1}{2},j+\tfrac{1}{2},k+1\right)-H_z^{n+\frac{1}{2}}\left(i+\tfrac{1}{2},j-\tfrac{1}{2},k+1\right)\right] \\ &-l_x\left(i+\tfrac{1}{2},j,k\right)\times\left[H_z^{n+\frac{1}{2}}\left(i+\tfrac{1}{2},j+\tfrac{1}{2},k\right)-H_z^{n+\frac{1}{2}}\left(i+\tfrac{1}{2},j-\tfrac{1}{2},k\right)\right]\end{aligned}\right\} \\
&- \frac{\Delta t}{\mu\Delta z}\left[l_x\left(i+\tfrac{1}{2},j,k+1\right)\times E_x^n\left(i+\tfrac{1}{2},j,k+1\right)-l_x\left(i+\tfrac{1}{2},j,k\right)\times E_x^n\left(i+\tfrac{1}{2},j,k\right)\right] \\
&+ \frac{\Delta t}{\mu\Delta x}\left[l_z\left(i+1,j,k+\tfrac{1}{2}\right)\times E_z^n\left(i+1,j,k+\tfrac{1}{2}\right)-l_z\left(i,j,k+\tfrac{1}{2}\right)\times E_z^n\left(i,j,k+\tfrac{1}{2}\right)\right]
\end{aligned} \quad (4)$$

where, $b' = \dfrac{\Delta t^2}{4\varepsilon\mu}$, $S_{xy} = \dfrac{\Delta t^2}{2\varepsilon\mu\Delta x\Delta y}$, and $S_{xz} = \dfrac{\Delta t^2}{2\varepsilon\mu\Delta x\Delta z}$.

The other updating of the $E_y$ , $H_x$ and $H_z$ components are according to the scheme shown in reference [9] and the implementation of the PEC condition for the $E_y$ component is according to the strategy presented in reference [3].

## III. NUMERICAL VALIDATION

To demonstrate the accuracy and efficiency of the proposed theory, a numerical example is presented here. A metal plate with dimension 60mm $\times$ 60mm is shown in Fig. 1. Twenty five apertures of 2 mm length and 2 mm width are cut on the plate. All the distances between the apertures are 10 mm. A uniform plane wave polarized along the *z*-direction is normally incident on the aperture, and the time dependence of the excitation function is as follows,

$$E_z(t) = \exp[-\frac{4\pi}{T^2}(t-t_0)^2] \quad (5)$$

where $T$ and $t_0$ are constants, and both equal to $2\times10^{-9}$ s. In such a case, the highest frequency of interest is 1 GHz and the smallest wavelength is 0.3m. The observation point is set at the front of the plate and is 50mm far from the plate.

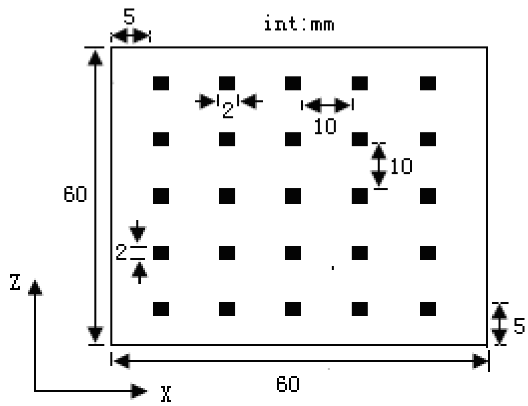


Fig. 1. Geometric configuration of the metal plate.

Applying the FDTD method to compute the time domain electric field component $E_z$ at the observation point, to simulate the apertures precisely, the cell size around the aperture must be small. We choose $\Delta x = \Delta z = 0.5$ mm around the apertures. The cell size $\Delta y$ is set to be 25mm which is 1/12 of the smallest wavelength of interest. The computational region is $60\times30\times60$ meshes. To satisfy the stability condition of the FDTD algorithm, the time-step size for the conventional FDTD is $\Delta t \leq$ 1.17ps. For the NWCS-DTD scheme, the maximum time increment is only related to the space increments $\Delta y$, that is, $\Delta t \leq 83.33$ ps. The outer boundary of computational region is truncated by using the 5 layers CPML absorbing boundary condition.

There are two methods to implement the PEC boundary condition for the $E_x$ and $E_z$ components in the NWCS-FDTD scheme, referred as NWCS-FDTD-1 and NWCS-FDTD-2, respectively. In NWCS-FDTD-1 scheme, it solves the $H_y$ component by using equations (13) and (14) in reference [9], then set the tangential electric field values at the boundary to be zeros directly after the $E_x$ and $E_z$ components are updated by using equations (2) and (3). The NWCS-FDTD-2 method is also to set the tangential electric field values at the boundary to be zeros directly, but the component $H_y$ is implicitly updated by using equation (4).

To demonstrate the high computational efficiency and accuracy of the NWCS-FDTD method, we perform the numerical simulations for a 5 ns time history by using the conventional FDTD, NWCS-FDTD-1 and NWCS-FDTD-2 methods, and compare the computation times and accuracy of these methods. In the conventional FDTD method, the time-step size keeps a constant of 1.17 ps. While in the NWCS-FDTD method, we use time-step size 83.33 ps.

Figure 2 shows the electric field component $E_z$ at observation point calculated by using the conventional FDTD, NWCS-FDTD-1 and NWCS-FDTD-2 methods. It can be seen from this figure that the result calculated by the NWCS-FDTD-2 method agrees well with the result calculated by the conventional FDTD method, but a large deviation of the NWCS-FDTD-1 method from the conventional FDTD method is observed. It is apparent that the NWCS-FDTD-2 method has higher accuracy than the NWCS-FDTD-1 method with same time step size, which is due to that in the NWCS-FDTD-2 method, the implementation of the PEC boundary condition for the $E_x$ and $E_z$ components is incorporated within the solving of the $H_y$ component by using equation (4).

To complete this simulation, the computation times for the conventional FDTD method, NWCS-FDTD-1 method and NWC-FDTD-2 method are 761.58, 30.08, and 31.12 seconds, respectively. Due to the large time step size applied, the CPU time for the NWC-FDTD-2 and NWC-FDTD-1 methods are almost 1/25 of that for the conventional FDTD method. The computation time of the NWC-FDTD-2 method is a little more than that of the NWC-FDTD-1 method, because in the NWC-FDTD-2 method, the updating of the $H_y$ component need to multiply the length factors, but the burden is trivial in comparison with the computation time of the conventional FDTD method.

It should be noted that the PEC modeling accuracy is inversely proportional to the CFLN value which is defined as the ratio of the time step size of WCS-FDTD and FDTD methods. In [9], the CFLN value is only 16. The PEC modeling inaccuracy is not obvious. So in that reference, there was little evidence of PEC modeling inaccuracy. While, in this paper, CFLN=71, the PEC modeling inaccuracy of WCS-FDTD-1 method becomes significant. In such case, the component $H_y$ must be updated by using equation (4).

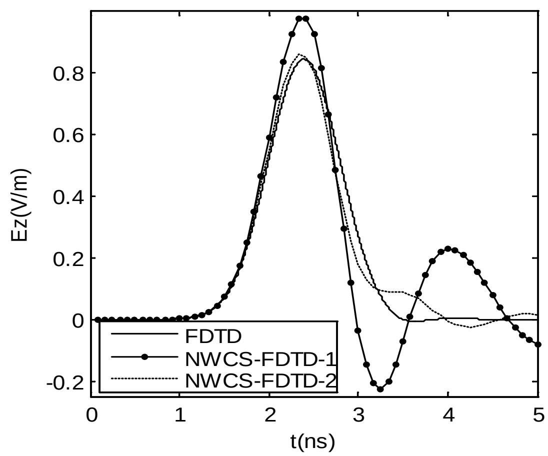


Fig. 2. The comparison of the results calculated by using the conventional FDTD, NWCS-FDTD-1, and NWCS-FDTD-2 methods.

## IV. CONCLUSION

Two strategies for the PEC condition implementation for the NWCS-FDTD method are compared in this paper. It shows that only the method that the PEC condition is directly incorporated within the solution of the $H_y$ component is accurate. The computational complexity of this method and the burden on the computation efficiency are trivial, which is validated by numerical example.

The NWCS-FDTD-2 method is useful for all problems where a very fine mesh is needed in one or two directions, regardless of whether the PEC condition exists. For the PMC condition, the implementation method is similar to the scheme described in reference [3].

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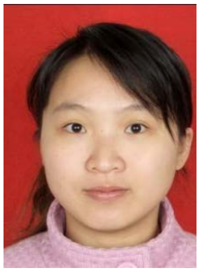

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