Scattering by Chiral Lossy Metamaterial Elliptic Cylinders

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Abstract **─** Electromagnetic fields are expressed in terms of appropriate complex angular and a rigorous solution to the problem of scattering of a plane wave from a chiral lossy metamaterial circular or elliptic cylinder is presented in this paper using the method of separation of variables. The incident scattered as well as the transmitted radial Mathieu functions with expansion coefficients. The incident field expansion coefficients are known, but the scattered and transmitted field expansion coefficients are to be determined. Imposing the boundary conditions at the surface of the chiral lossy metamaterial elliptic cylinder enables the determination of the unknown expansion coefficients. Results are presented as normalized scattering widths for circular and elliptic cylinders of different sizes and chiral lossy metamaterial materials, and for both TE and TM polarizations of the incident wave, to show the effects of these on scattering cross widths.

Index Terms **-** Chiral lossy, circular and elliptic cylinders, Mathieu functions, metamaterial.

I. INTRODUCTION

A chiral medium is a reciprocal and isotropic medium characterized by different phase velocities for both right and left circularly polarized waves. In a lossless chiral medium, a linearly polarized wave undergoes a rotation of its polarization as it propagates. For chiral cylinders, these properties result in a coupling of the TM and TE polarizations. Analytic and numerical solutions describing chiral media are given by [1, 19]. Recent work on the scattering and radiation by elliptical structures is given for lossy and lossless dielectric material [20-24].

In this paper, we present the analysis corresponding to the scattering of a plane wave of arbitrary polarization and angle of incidence from a chiral lossy metamaterial elliptic cylinder of arbitrary axial ratio. Such a solution is valuable, since it can be used as a benchmark for validating solutions obtained using approximate or numerical methods. The analysis and the software used for obtaining the results have been validated, by calculating the normalized backscattering widths for an elliptic cylinder of axial ratio approximately 1, and showing that these results are in excellent agreement with the same obtained for a circular cylinder [12, 15].

II. FORMULATION

Consider a linearly polarized monochromatic uniform plane electromagnetic wave incident on an infinitely long chiral lossy metamaterial elliptic cylinder of major axis *a* and minor axis *b*, at an incident angle φ_i with respect to the positive *x* axis of a Cartesian co-ordinate system located at the centre of the cylinder as shown in Fig. 1. The axis of the cylinder is assumed to be along the z axis. Since the scatterer under consideration is bounded by elliptical surface, it is convenient to use Mathieu functions to satisfy the boundary conditions. Let's define the x and y coordinates of the Cartesian coordinate system in terms of *u* and *v* coordinates of an elliptical coordinate system also located at the centre of the cylinder in the form $x = F\cosh u \cos v$, $y = F\sinh u \sin v$, with *F* being the semi-focal length of the ellipse. A time dependence of $e^{j\omega t}$ is assumed throughout the analysis, but suppressed for convenience where ω is the angular velocity.

For a TE polarized plane wave of amplitude H_0, the axial component of the incident magnetic field can be written as

$$H_z^i = H_0 e^{jk\rho\cos(\varphi-\varphi_i)}. \qquad (1)$$

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where $k = 2\pi/\lambda$, λ being the wavelength in the region exterior to the cylinder, and ρ, φ are the polar coordinates.

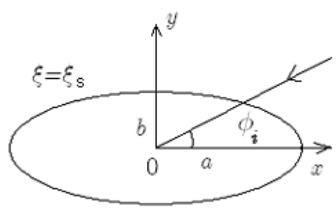

Fig. 1. Geometry of the problem.

Let the elliptic cylindrical vector wave functions $\mathbf{N}$ and $\mathbf{M} = k^{-1}(\nabla \times \mathbf{N})$ be defined as [24]

$$\mathbf{N}_{qm}^{(i)}(c,\xi,\eta) = \hat{z}\, R_{qm}^{(i)}(c,\xi) S_{qm}(c,\eta). \qquad (2)$$

$$\mathbf{M}_{qm}^{(i)}(c,\xi,\eta) = \frac{1}{kh}\begin{bmatrix} \hat{u} R_{qm}^{(i)}(c,\xi) S_{qm}^{'}(c,\eta) \\ -\hat{v}\, R_{qm}^{(i)'}(c,\xi) S_{qm}(c,\eta) \end{bmatrix}. \qquad (3)$$

where q=e,o, S_{qm} and $R_{qm}^{(i)}$ are the even and odd complex angular and radial Mathieu functions of the i[th] kind, both of m order, respectively, $\xi = \cosh u$, $\eta = \cos v$, $c = kF$, $\hat{k}$ denotes a unit vector in the positive k direction, the prime on *S* and *R* denotes their respective derivative with respect to *v* and *u*, while $h = F\sqrt{\xi^2 - \eta^2}$.

The incident magnetic field can be expanded in terms of angular and radial Mathieu functions as

$$\mathbf{H}^i = \sum_m^{\infty} A_{qm} \mathbf{N}_{qm}^{(1)}(k). \qquad (4)$$

in which

$$A_{qm} = H_o\, j^m \frac{\sqrt{8\pi}}{N_{qm}(c)} S_{qm}(c, \cos\varphi_i). \qquad (5)$$

$$N_{qm}(c) = \int_0^{2\pi} [S_{qm}(c,\eta)]^2 dv\,. \qquad (6)$$

The summation over m starts from 0 for even Mathieu functions while 1 for odd Mathieu functions.

Using Maxwell's equations, the electric field can be expanded in terms of angular and radial Mathieu functions as

$$\mathbf{E}^i = j\,Z \sum_m^{\infty} A_{qm} \mathbf{M}_{qm}^{(1)}(k). \qquad (7)$$

where Z is the wave impedance in the region exterior to the cylinder.

Since the elliptic cylinder is made up of chiral metamaterial, the scattered electromagnetic field will have a cross-polarized component in addition to the co-polarized component in contrast to that for a dielectric elliptic cylinder, which would only have a co-polarized component. These co- and cross polarized scattered field components can also be expressed in terms of Mathieu functions as

$$\mathbf{H}^s = \sum_m^{\infty}\left[B_{qm} \mathbf{N}_{qm}^{(4)}(k) + C_{qm} \mathbf{M}_{qm}^{(4)}(k) \right]. \qquad (8)$$

$$\mathbf{E}^s = j\,Z \sum_m^{\infty}\left[B_{qm} \mathbf{M}_{qm}^{(4)}(k) + C_{qm} \mathbf{N}_{qm}^{(4)}(k) \right]. \qquad (9)$$

where B_{qm} and C_{qm} are the unknown expansion coefficients of the co- and cross-polarized scattered field components, respectively, while $\mathbf{N}_{qm}^{(4)}$ and $\mathbf{M}_{qm}^{(4)}$ are the elliptic cylindrical vector wave functions of the fourth kind. The transmitted fields inside the chiral material may be written in terms of left and right circularly polarized waves as

$$\begin{aligned} \mathbf{H}^c &= D_{qm} \sum_m^{\infty}\left[\mathbf{N}_{qm}^{(1)}(k_R) + \mathbf{M}_{qm}^{(1)}(k_R) \right] \\ &+ F_{qm} \sum_m^{\infty}\left[\mathbf{N}_{qm}^{(1)}(k_L) + \mathbf{M}_{qm}^{(1)}(k_L) \right]. \end{aligned} \qquad (10)$$

$$\mathbf{E}^c = jZ_c \begin{pmatrix} \sum_m^{\infty} D_{qm}\left[\mathbf{N}_{qm}^{(1)}(k_R) + \mathbf{M}_{qm}^{(1)}(k_R) \right] \\ -F_{qm} \sum_m^{\infty}\left[\mathbf{N}_{qm}^{(1)}(k_L) - \mathbf{M}_{qm}^{(1)}(k_L) \right] \end{pmatrix}. \qquad (11)$$

where Z_c is the wave impedance of the chiral medium, D_{qm} and F_{qm} are the unknown transmitted expansion coefficients while the left and right circularly polarized wave numbers k_R and k_L are

$$k_R = \frac{\omega\sqrt{\mu_c \varepsilon_c}}{1 + \gamma\omega\sqrt{\mu_c \varepsilon_c}}, \quad k_R = \frac{\omega\sqrt{\mu_c \varepsilon_c}}{1 - \gamma\omega\sqrt{\mu_c \varepsilon_c}}. \qquad (12)$$

μ_c is the permeability of the chiral medium, ε_c is the permittivity of the chiral medium, and γ is the chiral admittance of the chiral medium. The unknown expansion coefficients can be obtained by imposing the tangential boundary conditions at the surface $\xi = \xi_s$ of the chiral elliptic cylinder, which can be expressed mathematically as

$$H_z^i + H_z^s = H_z^c. \quad (13)$$

$$H_\eta^s = H_\eta^c. \quad (14)$$

$$E_z^i + E_z^s = E_z^c. \quad (15)$$

$$E_\eta^s = E_\eta^c. \quad (16)$$

Substituting the appropriate expressions in (13)-(16), and applying the orthogonal property of the angular Mathieu functions yields

$$\left[A_{qm} R_{qm}^{(1)}(c,\xi_s) - B_{qm} R_{qm}^{(4)}(c,\xi_s)\right] N_{qm}(c) = \sum_m D_{qm} R_{qm}^{(1)}(c_R,\xi_s) M_{qm}(c,c_R) + \sum_m F_{qm} R_{qm}^{(1)}(c_R,\xi_s) M_{qm}(c,c_R), \quad (17)$$

$$C_{qm} R_{qm}^{(4)'}(c,\xi_s) N_{qm}(c) = \frac{k}{k_R} \sum_m D_{qm} R_{qm}^{(1)'}(c_R,\xi_s) M_{qm}(c,c_R) - \frac{k}{k_R} \sum_m F_{qm} R_{qm}^{(1)'}(c_R,\xi_s) M_{qm}(c,c_R), \quad (18)$$

$$C_{qm} R_{qm}^{(4)}(c,\xi_s) N_{qm}(c) = \frac{Z_c}{Z} \sum_m D_{qm} R_{qm}^{(1)'}(c_R,\xi_s) M_{qm}(c,c_R) - \frac{Z_c}{Z} \sum_m F_{qm} R_{qm}^{(1)'}(c_R,\xi_s) M_{qm}(c,c_R), \quad (19)$$

$$\left[A_{qm} R_{qm}^{(1)'}(c,\xi_s) - B_{qm} R_{qm}^{(4)'}(c,\xi_s)\right] N_{qm}(c) = \frac{Z_c}{Z}\frac{k}{k_c} \sum_m D_{qm} R_{qm}^{(1)}(c_R,\xi_s) M_{qm}(c,c_R) + \frac{Z_c}{Z}\frac{k}{k_c} \sum_m F_{qm} R_{qm}^{(1)}(c_R,\xi_s) M_{qm}(c,c_R). \quad (20)$$

where $c_R = k_R F$ and $c_L = k_L F$. To solve for the unknown scattered field coefficients, the system of equations (17-20) may be written in matrix form and the unknown coefficients can be obtained by matrix inversion. The bistatic scattering width is defined as [24]

$$\sigma = \lim_{\rho\to\infty} 2\pi\rho \frac{\mathrm{Re}[(\mathbf{E}^s \times \mathbf{H}^{s*})\cdot\hat{\rho}]}{\mathrm{Re}[(\mathbf{E}^i \times \mathbf{H}^{i*})\cdot\hat{\rho}]} \quad (21)$$

with the asterisk denoting the complex conjugate $\hat{\rho}$ denoting the unit vector in the direction of increasing ρ and Re[] denoting the real part of a complex number.

For TE polarization we can write an expression for the normalized bistatic scattering width as

$$\frac{\sigma}{\lambda} = \frac{\sigma_{\text{co-polar}}}{\lambda} + \frac{\sigma_{\text{cross-polar}}}{\lambda}. \quad (22)$$

where

$$\sigma_{\text{co-polar}} = \lambda \left| \sum_{m=0}^{\infty} j^m B_{em} S_{em}(c,\cos\phi) + \sum_{m=1}^{\infty} j^m B_{om} S_{om}(c,\cos\phi) \right|^2. \quad (23)$$

and

$$\sigma_{\text{cross-polar}} = \lambda \left| \sum_{m=0}^{\infty} j^m C_{em} S_{em}(c,\cos\phi) + \sum_{m=1}^{\infty} j^m C_{om} S_{om}(c,\cos\phi) \right|^2. \quad (24)$$

The expressions for the incident, scattered and transmitted electromagnetic fields for TM case can be obtained using the duality principle of the TE case.

III. NUMERICAL RESULTS

The obtained numerical results are presented as normalized echo pattern widths for chiral lossy and lossless metamaterial circular and elliptic cylinders of different sizes, axial ratios, incident angles, permittivities and permeabilities for both TE and TM polarizations of the incident wave. We have selected $ka = 0.5$, $\varepsilon_{rc} = \pm 4.0$, $\mu_{rc} = \pm 2.0$, $k\gamma = 0.15$ and $\varphi_i = 180^o$ to compare the obtained data with published results. To validate the analysis and the software used for calculating the results, we have computed the normalized echo pattern widths for chiral circular cylinder of axial ratio 1.001, as shown in Fig. 2 and in the TM case. The results are in full agreement, verifying the accuracy of the analysis as well as the software used for obtaining the results [12, 15]. Also Fig. 2 shows the echo pattern for chiral lossy and lossless metamaterial circular cylinder with various values of ε_{rc}. The results show for this particular parameters the co polarized echo pattern increases while the cross polarized decreases by varying the lossy metamaterial when compared with the conventional chiral case (solid lines). Figure 3 is similar to Fig. 2 except for the TE case. The TE case behaves similar to the TM case.

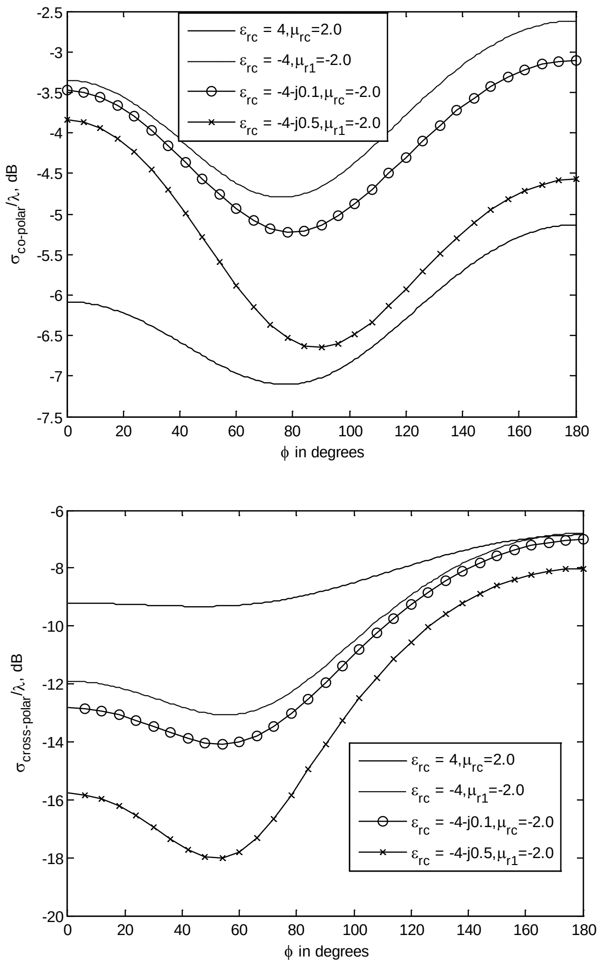

Fig. 2. Echo pattern width versus scattering angle for co and cross polarized chiral lossy metamaterial circular cylinder of ka=0.5, axial ratio 1.001, $k\gamma = 0.15$ and $\varphi_i = 180^o$, TM case.

Figure 4 shows the echo pattern for conventional chiral, lossy and lossless metamaterial elliptic cylinder with ka=0.5, kb=0.25 and TM case. The numerical results show that the co polarized echo pattern decreases for the scattering angle less than 150^o while the cross polarized decreases at all scattering angles when compared to the conventional chiral case (solid lines). Figure 5 is similar to Fig. 4 except for TE case where it behaves differently for the co polarized case.

Figure 6 shows the echo pattern width for chiral lossy metamaterial elliptic cylinder of different axial ratios, $\varepsilon_{rc} = -4.0 - j\,0.5$ and $\varphi_i = 90^o$ and for TE case. It can be seen by increasing the axial ratio will decrease the echo pattern width for some particular co polarized cases. Figure 7 shows the echo pattern for different chiral materials and for the same parameters as in Fig. 6.

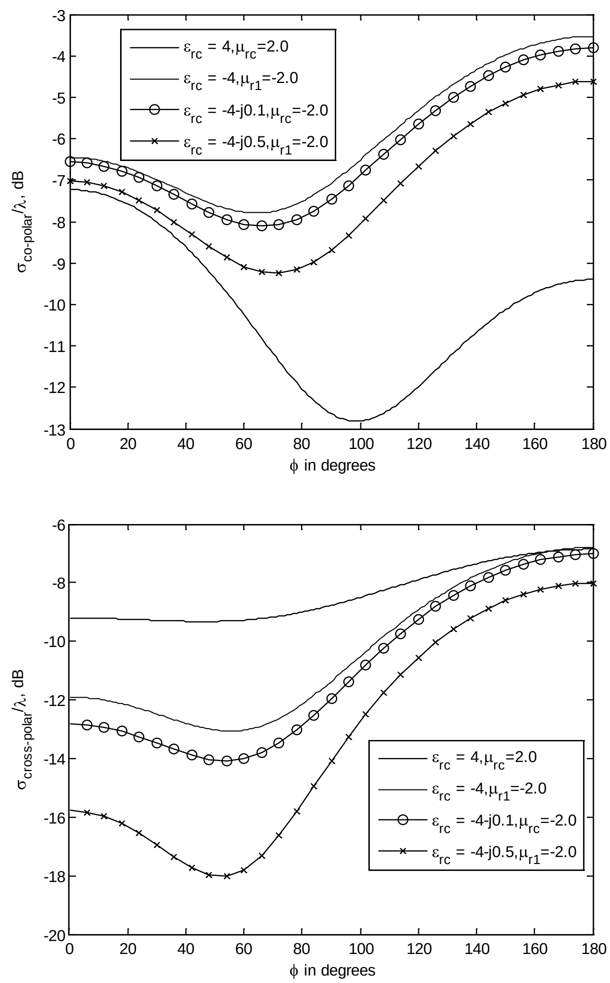

Fig. 3. Echo pattern width versus scattering angle for co and cross polarized chiral lossy metamaterial circular cylinder of ka=0.5, axial ratio 1.001, $k\gamma = 0.15$ and $\varphi_i = 180^o$, TE case.

IV. CONCLUSION

A rigorous solution to the problem of scattering of a plane electromagnetic wave by a chiral lossy and lossless metamaterial circular and elliptic cylinder have been presented using the method of separation of variables. Results have been presented in the form of normalized echo pattern width for circular and elliptic cylinders of different sizes, axial ratios and chiral and lossy metamaterials, for both TE and TM polarization of the incident wave, to show the effects of the above parameters on scattering from circular and elliptic cylinders. The results obtained in this paper are important, since they can be used as benchmarks to validate similar results obtained using other approximate or numerical methods [15] and also to get an insight into how the changing of various parameters associated with the chiral lossy metamaterial cylinder changes the scattering widths that could be obtained from it.

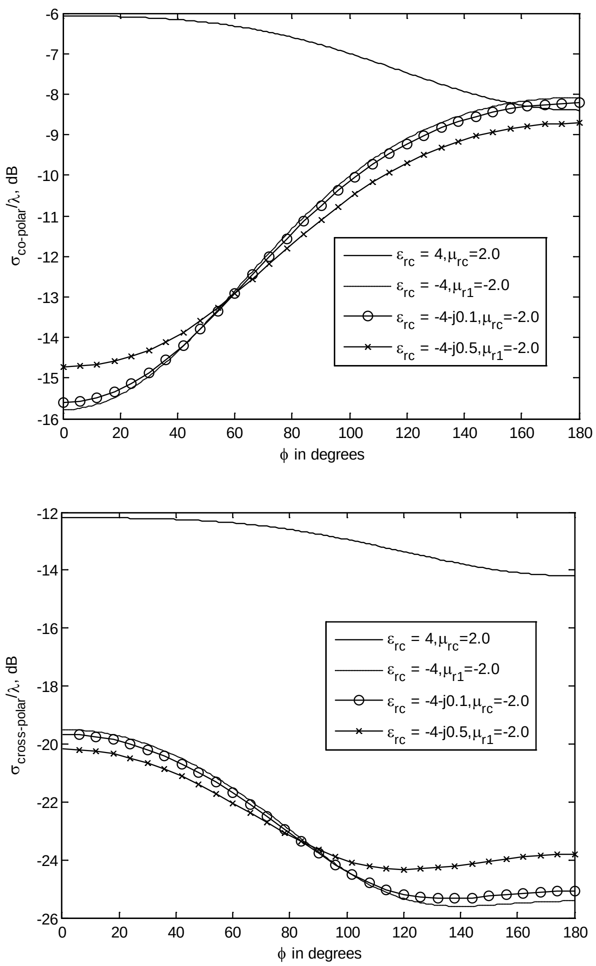

Fig. 4. Echo pattern width versus scattering angle for co and cross polarized chiral lossy metamaterial elliptic cylinder of ka=0.5, kb=0.25, $k\gamma = 0.15$ and $\varphi_i = 180^o$, TM case.

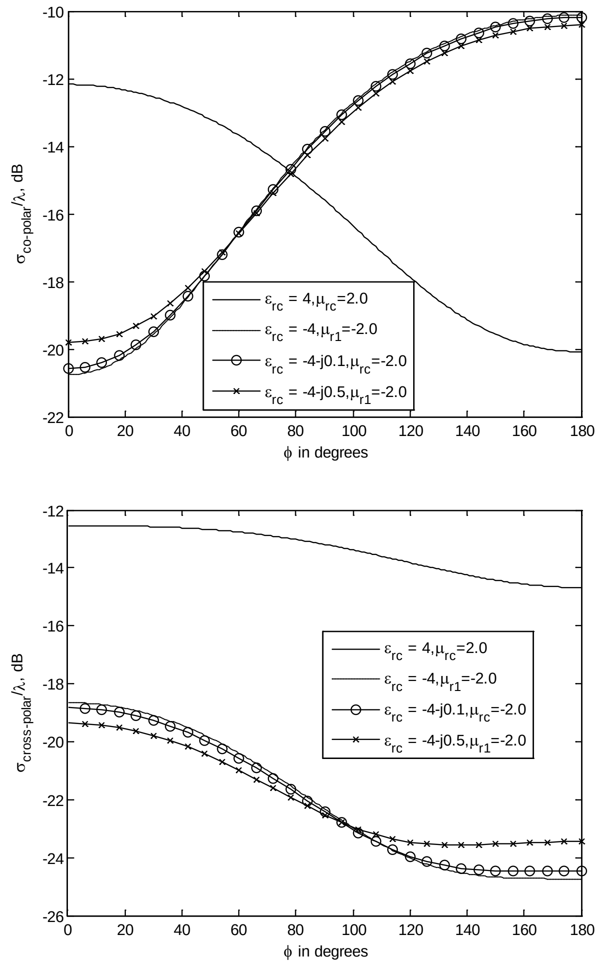

Fig. 5. Echo pattern width versus scattering angle for co and cross polarized chiral lossy metamaterial elliptic cylinder of ka=0.5, kb=0.25, $k\gamma = 0.15$ and $\varphi_i = 180^o$, TE case.

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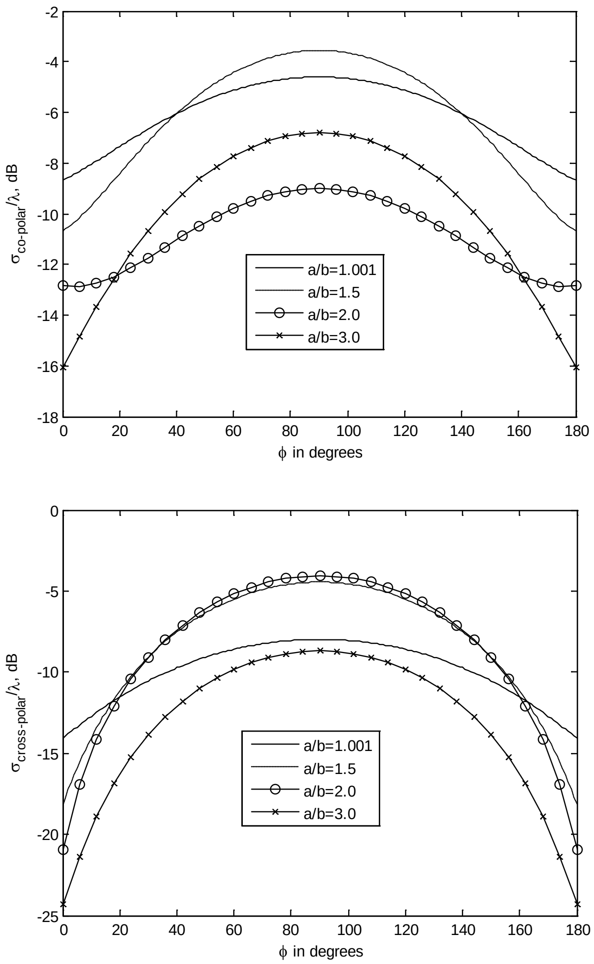

Fig. 6. Echo pattern width versus scattering angle for co and cross polarized chiral lossy metamaterial elliptic cylinder of different axial ratio, ka=0.5, $k\gamma = 0.15$, $\mu_{rc} = -2.0$, $\varepsilon_{rc} = -4.0 - j\,0.5$ and $\varphi_i = 90^o$, TE case.

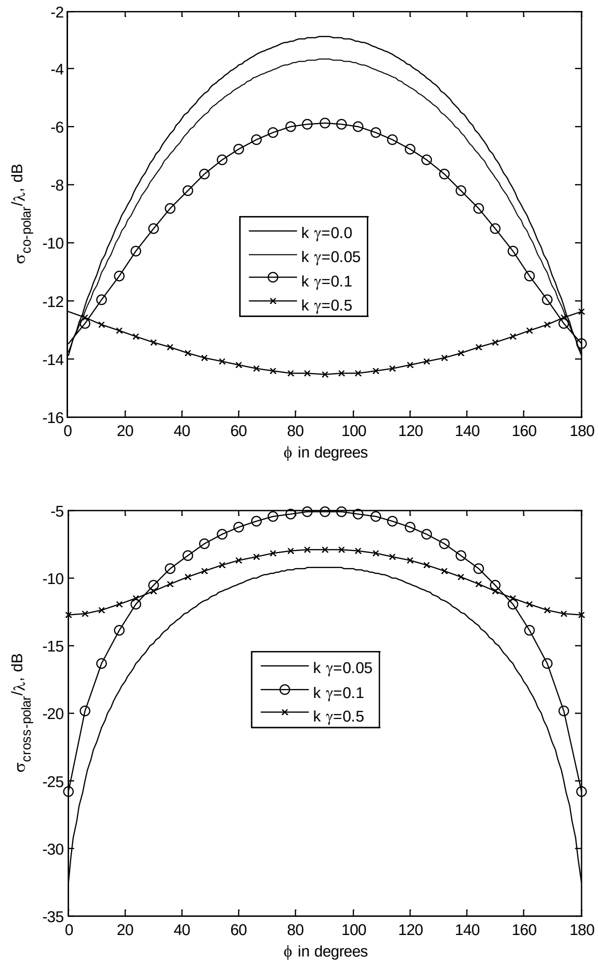

Fig. 7. Echo pattern width versus scattering angle for co and cross polarized chiral lossy metamaterial elliptic cylinder of different $k\gamma$, ka=0.5, kb=0.25, $\mu_{rc} = -2.0$, $\varepsilon_{rc} = -4.0 - j\,0.5$ and $\varphi_i = 90^o$, TE case.

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