Improvement of Transmission Line Matrix Method Algorithm Frequency Response Based on Modification of Cell Impedance

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Abstract **─** In this paper, an accurate and numerically robust singularity correction technique for the transmission line matrix method (TLM) algorithm is proposed. The impedance of the adjacent cells to the singularity is corrected by a scalar correction factor, which amounts to a quasi-static correction of the electric and magnetic energy stored in the TLM cells at the singularity. The effectiveness of this method in accurate modeling of structures with metallic strips (sharp edges) and 90 degree edge corners has been clearly validated against published measurement and common TLM simulation data.

Index Terms **─** Dispersion error, singularity, TLM.

I. INTRODUCTION

Like all other numerical techniques the transmission line matrix method (TLM) is subject to various sources of error and must be treated with caution to obtain reliable and accurate results. Regarding the discrete nature of the TLM method, there are two error sources, velocity or dispersion error and coarseness error.

In most of the simulations, the wavelength of the TLM network is large enough compared with the cell size; therefore, it can be assumed that the fields propagate with the same frequency-independent velocity in all directions and the TLM network shows the behavior of a continuous medium. However when the frequency or the cell size is increased, the network velocity becomes dispersive and depends on the frequency and the direction of propagation. Hence, the assumption of the continuous medium for the TLM network is not valid and leads to an error that is referred to as velocity or dispersion error.

The coarseness error occurs when the TLM mesh is too coarse to resolve the highly non-uniform fields at corners and edges. On the other hand, due to the singularity of some components of the electromagnetic field at corners and edges, the coarseness error occurs.

Both of the dispersion and coarseness errors cause a shift in the frequency response of the structures under investigation as the shift is usually towards low frequencies. However, positive shift in some particular combinations of dielectric and magnetic materials is possible too.

Since the effect of field singularities is significant, many approaches have been proposed for dealing with the field singularities in the vicinity of metal edges in FDTD and TLM [1] . A possible solution could be to choose a very fine mesh[2,3,4]. Another approach is to use appropriate boundary conditions [5] . The lower memory requirements and computation time are the advantages of this approach. A better approach is to use basis functions at the discontinuities that resemble the singular fields at metallic edges and corners [5] . The idea used in this approach is that the singular field distribution is quasi-static since the time derivative of the fields is insignificant compared with their space derivative. Therefore, the properties of the quasi-static field sub-region can be modified so that the stored energy is correct even though the field itself is poorly resolved. For example, modifying the update equations in a FDTD algorithm or the scattering matrix of cells in the vicinity of singularity in TLM is one possible correction method [6].

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Using an additional modeling element is another approach that is used for the singularity correction [7, 8]. Discontinuity modeling with material properties is another approach used for modeling singularities. The main idea in this approach is based on the quasi-static distribution of the singular field. It means some of the field components become infinite at sharp edges and corners, whereas the stored energy remains finite. Although this technique is used in perfectly matched layer boundary conditions (PML) [9] , the local character of singularity fields implies that they are independent of boundary conditions several mesh cells away.

II. LOCAL MODIFICATION OF TLM CELLS

This method is based on the local modification of the impedance in the TLM cells directly surrounding the discontinuity. It is robust and independent of the type of singularity from the numerical standpoint. Furthermore, it can be easily implemented to function automatically and has a negligible computational cost compared to the other methods. Modification of TLM cell impedance in the vicinity of discontinuities is implemented by changing the dielectric permittivity and magnetic permeability. The constitutive parameters are the only structural parameters presented in the 3D-TLM dispersion equation[10]. In other words, modification of these parameters resulted in modification of the dispersion relation in the 3D-TLM network.

Maxwell's equation for a symmetric anisotropic media are given by

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \qquad \nabla \times H = \varepsilon \frac{\partial E}{\partial t}. \tag{1}$$

The permittivity tensor ε and the permeability tensor μ are

$$\mu = \begin{bmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{bmatrix}, \varepsilon = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}. \tag{2}$$

Following as [10], the dispersion relation is obtained as the combination of wave vector $\vec{k}$ and constitutive parameters μ and ε. A significant simplification of the dispersion relation is obtained for isotropic media leading to

$$\omega^2 \left(k_x^2 + k_y^2 + k_z^2 - \varepsilon\mu\omega^2\right)^2 = 0. \tag{3}$$

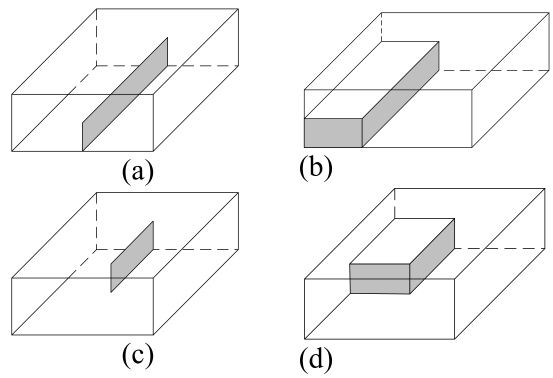

Fig. 1. Four types of singularities (a) knife edge, (b) 90 degree edge, (c) knife edge corner, (d) 90 degree edge corner.

The dispersion analysis for the symmetrical condensed node (SCN) of 3D-TLM yields a direct relationship between the relative permittivity and relative permeability as seen in the equation (3) [10]. In this paper, we used the above relation to compensate the locally structural dispersions created by discontinuities of electromagnetic structures. To further explain the capability of the proposed technique in this paper, some common structural discontinuities such as knife edge, 90 degree edge, knife edge corner, and 90 degree edge corner are investigated.

For modeling these structures, both constitutive parameters of the cells' surrounded by discontinuities are reduced by the same relative amount in order to preserve the local intrinsic wave impedance of the field space. For example in the knife edge structure, the ε and μ of the edge cells are modified such that the resonant frequencies of a resonator that contains the edge singularity become virtually independent of the cell size.

The required change in ε and μ can be computed approximately using the known expressions for the quasi-static fields. This is illustrated in Fig. 2 where the dominant resonant frequency of a cavity with a knife edge singularity has been drawn as a function of mesh parameter (Δl) used to compute that frequency. Note that the mesh parameter is small enough for the dispersion error to be negligible $(\Delta l/\lambda << 1/20)$.

If the structural singularity is not corrected, the frequency varies linearly with the mesh parameter. After the modification, the computed resonant frequency is practically independent of the mesh parameter. The correction region is along the edge but contains only the immediately adjacent cells. Figure 3 shows the location of the modified cells.

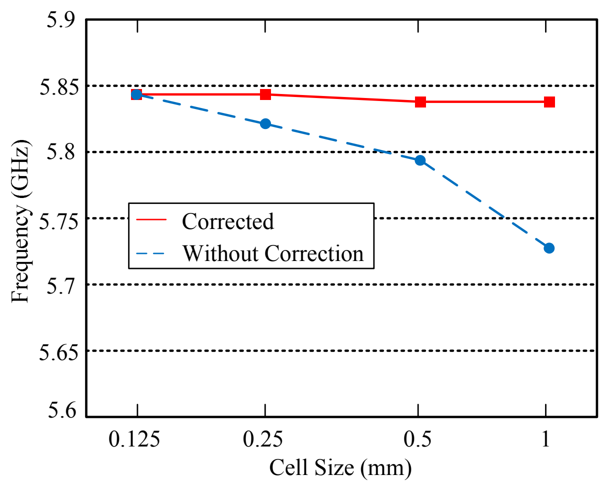

Fig. 2. Resonant frequency of cavity containing a knife edge singularity with a=20 mm, b=15 mm, c=10 mm, and $\varepsilon_r = \mu_r$ =2.

As can be seen in Fig. 2, the frequency shift phenomenon is dropped after modification. The frequency shift between simulation with Δl =1 mm and Δl =0.125 mm before modification is about 135 MHz and after modification this shift is reduced to 5 MHz which means 90% error correction by the corrected algorithm.

This modification can be repeated for other types of singularities like 90 degree edge. Fig. 4 shows the location of cell correction for this type of singularity. The cavity dimensions are a=14 mm, b=10 mm, c=6 mm, d=5 mm, and s=3 mm with parameters of $\varepsilon_r = \mu_r$ =2.

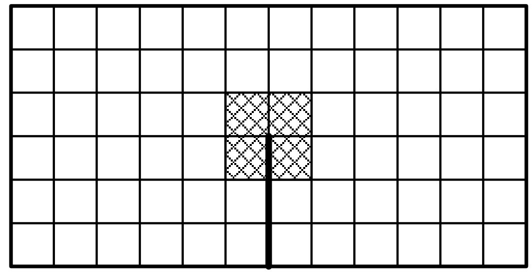

Fig. 3. Location of the cell correction for knife edge singularity.

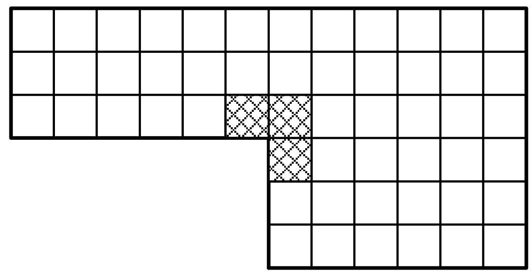

Fig. 4. Location of the cell correction for 90 degree edge singularities.

Figure 5 shows the simulation results of the 90 degree edge singularity. Elimination of the frequency shift is evident which led to independence of the resonant frequency to the mesh parameter. In this type of singularity, the corrected cells are located adjacent to the 90 degree edge singularity. With respect to the above results, it is clear that the physical modeling of discontinuities by changing the cell impedance in the location of singularity is accurate and efficient. In addition, this approach not only increases the accuracy of the applied numerical methods but also decreases the computing cost and the simulation time of modeled structures.

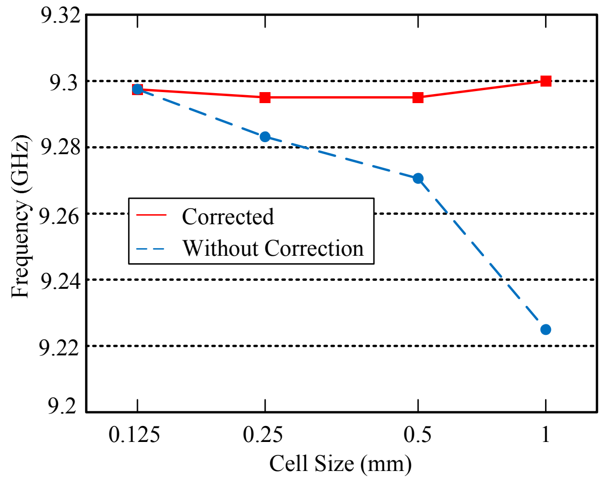

Fig. 5. Resonant frequency of cavity containing 90 degree edge singularities.

In order to experimentally validate the accuracy and efficiency of the correction method proposed in this paper, this approach has been used to analyze microstrip structures such as antenna and filter.

III. SIMULATION

A. Patch antenna

The geometry of the antenna is exactly the same as in [11] so that the results can be compared (Fig. 6). The feed line of the antenna is 50 Ω microstrip with ε_r =2.2 and h=0.794 mm.

In simulation of this antenna, the computational domain with $120\times70\times25$ cells is used and the patch is modeled by $40\,\Delta x\ \times 32\,\Delta y$ where Δx = 0.4 mm, Δy = 0.389 mm, and Δz = 0.265 mm. The width and length of this feed line and the height of substrate are modeled by 6, 50 and 3 nodes, respectively.

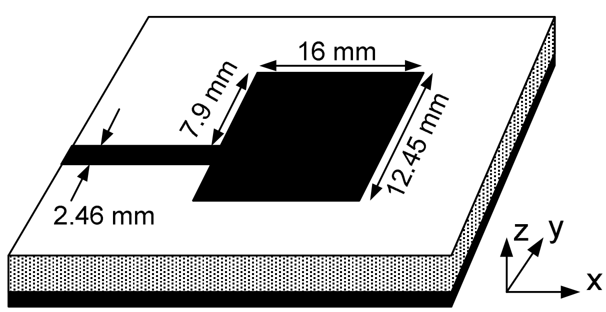

Fig. 6. Patch antenna.

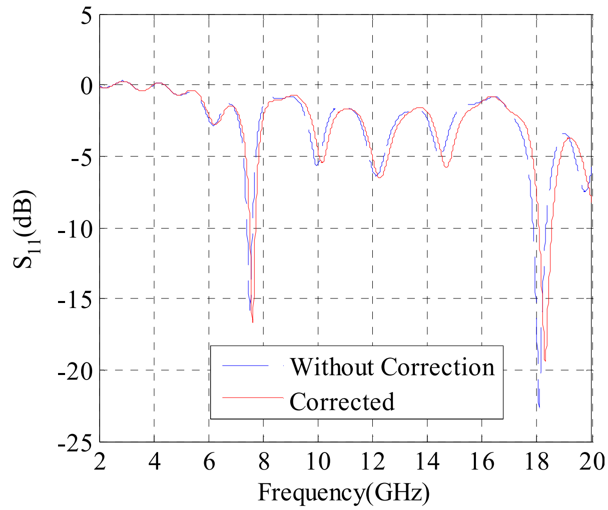

Fig. 7. Result of patch antenna simulation.

The time step is $\Delta t = 0.43$ ps and simulation is performed for 10000 time steps because of the highly resonant behavior of the antenna. Note that this structure has two types of singularities in form of edge and corner and both of them should be corrected.

The results of the simulation by modified algorithm are listed in Table 1. In Fig. 7, the reflection coefficient of the patch antenna is also compared with corrected TLM result.

By considering measurement frequencies, the applied modification is about 80% and the whole of the frequency response is not corrupted by this modification in the frequency range of response.

Table 1: Resonant frequency of the patch antenna

Frequency (GHz)	**1st**	**2nd**
TLM Without correction	7.51	18.07
Corrected TLM	7.61	18.29
Measurement [12]	7.60	18.37
Without correction Error(%)	1.31	1.63
Corrected Error(%)	0.13	0.44

B. Low pass filter

Figure 8 shows the geometry of a low pass microstrip filter which fabricated by RT/Duroid 5880 (ε_r = 2.2, h= 0.794 mm) and the substrate is on the y-z plane. Dimension of the simulation cell is Δx = 0.4233 mm, Δy = 0.1985 mm, and Δz = 0.4064 mm and the computational domain with $20\times74\times86$ cells is used for simulation of this filter.

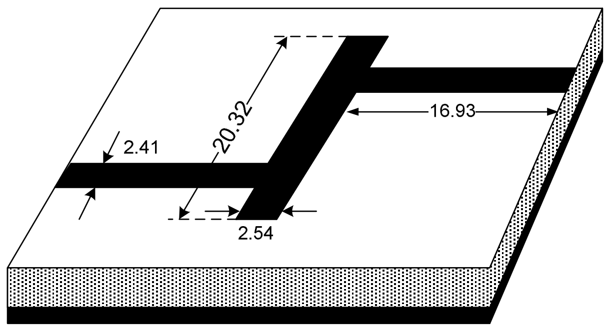

Fig. 8. Geometry of low pass filter.

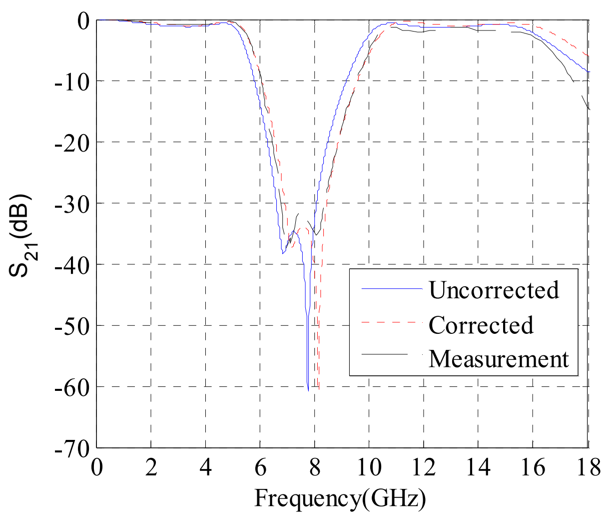

Fig. 9. Insertion loss of low pass filter.

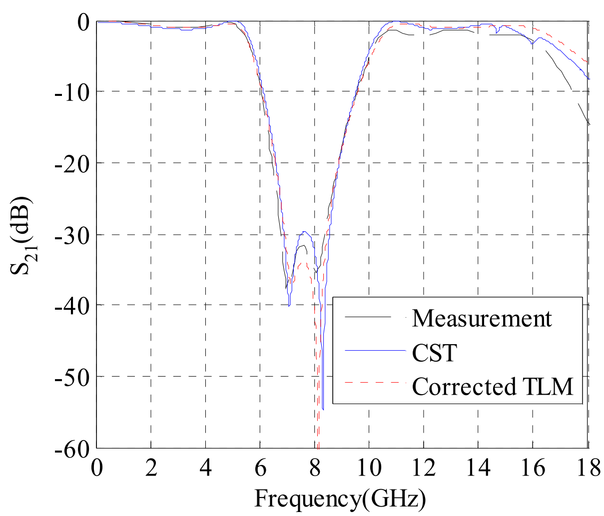

Fig. 10. Corrected TLM filter result compared with CST microwave studio simulation and measurement result.

The distance from the source plane to the edge of the long patch is 30 Δx and the reference planes for ports 1 and 2 are 10 Δx from the edges of the patch. The time step is $\Delta t = 0.3176$ ps. The simulation is performed for 10000 time steps to allow the response on both ports to become nearly zero.

In Fig. 9, the simulation results of the filter are shown in comparison with measurement results [11] . With respect to the filter geometry, the impedance of the edge discontinuities and adjacent cells has been changed in the corrected algorithm.

In comparison with measurement results, the proposed modification in the TLM algorithm could compensate shift in the frequency response of the filter, as the cut-off frequency of the low pass filter improved approximately 90% in comparison with the common TLM algorithm. In general, a good agreement of the corrected algorithm results and CST Microwave Studio simulation result is seen in Fig. 10.

CONCLUSION

In this paper, an accurate and numerically robust singularity correction technique for the TLM algorithm has been proposed. The impedance of the cell adjacent to the singularities are modified by a scalar correction factor, which amounts to a quasi-static correction of the electric and magnetic energy stored in the TLM cells at the singularities. The correction factor also affected the dispersion equation because of the constitutive parameter dependency of this equation. Numerical validation shows that this correction method reduces the coarseness error due to singularity without any penalty in terms of computational burden.

REFRENCES

[1] J. Hesselbarth and R. Vahldieck, “Resonance Frequencies Calculated Efficiently with the Frequency-Domain TLM Method,” *IEEE MWCL*, vol. 13, no. 5, pp. 190-192, May 2003.

[2] J. A. Morente, J. A. Porti, H. Magan, and O. Torres, “Improved Modeling of Sharp Zones in Resonant Problems with the TLM method,” *Applied Computational*

Electromagnetic Society (ACES) Journal, vol. 14, no. 2, pp. 67-71, July 1999.

[3] J. L. Herring and C. Christopolous, "Multigrid Transmission-Line Modeling Method for Solving Electromagnetic Field Problems," *Electronics Letters*, vol. 27, no. 20, pp. 1794-1795, Sep. 1991.

[4] R. E. Collin, *Field Theory of Guided Waves*, 2nd edition, IEEE Press., New York, 1991.

[5] U. Pekel and R. Mittra, "A Finite-Element-Method Frequency-Domain Application of the Perfectly Matched Layer (PML) Concept," *Microwave and Opt. Tech. Lett.*, vol. 9, no. 3, 1995.

[6] K. R. Umashankar, and A. Taflove, "A Novel Method to Analyze Electromagnetic Scattering of Complex Objects," *IEEE Trans. Electromagn. Compat.*, vol. 24, pp. 392-405, Nov. 1982.

[7] C. J. Railton, "The Inclusion of Fringing Capacitance and Inductance in FDTD for the Robust Accurate Treatment of Material Discontinuities," *IEEE MTT-S International Microwave Symposium Digest*, pp. 391-394, 2000.

[8] P. Sewell, J. Wykes, A. Vukovic, D. W. P. Thomas, T. M. Benson, and C. Christopolous, "Multi-Grid Interface in Computational Electromagnetics," *Electronics Letters*, vol. 40, pp. 162-163, Feb. 2004.

[9] J. P. Berenger, "A Perfectly Matched Layer for the Absorption of Electromagnetic Waves," *Journal of Computational Physics*, vol. 114, no. 185, 1994.

[10] C. Huber, M. Krumpholz, and P. Russer, "Dispersion in Anisotropic Media Modeled by Three-Dimensional TLM," *IEEE Transactions on Microwave Theory and Techniques*, vol. 43, no. 8, pp.1923-1934, August 1995.

[11] D. M. Sheen, S. M. Ali, and J. A. Kong, "Application of the Three-Dimensional Finite-Difference Time-Domain Method to the Analysis of Planar Microstrip Circuits," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-38, no. 7, pp. 849-857, July 1990.

[12] T. Shibata, T. Hayashi, and T. Kimura, "Analysis of Microstrip Circuits using Three Dimensional Full Wave Electromagnetic Field Analysis in the Time Domain," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-36, no. 6, pp. 1064-1070, June 1998.

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