Modelling Dynamic Hysteresis in Ferromagnetic Sheets under Time-periodic Supply Conditions

Oriano Bottauscio
Istituto Elettrotecnico Nazionale "G. Ferraris", Torino, Italy

Mario Chiampi, Maurizio Repetto
Politecnico di Torino, Torino, Italy

[1]*Abstract*—**This paper presents an application of the Dynamic Preisach Model, as proposed by Bertotti, to the analysis of electromagnetic phenomena inside a ferromagnetic lamination subject to imposed time-periodic magnetic flux. The field problem, considered as one dimensional, is formulated in terms of magnetic field and the Fixed Point technique is used for the treatment of the hysteretic nonlinearity. The solution is carried out in the harmonic domain by means of the Finite Element method. The original implementation of the Dynamic Preisach Model is detailed and comparisons with measurements are presented in the case of sinusoidal excitations.**

I. Introduction

The analysis of the electromagnetic field distribution inside a ferromagnetic iron sheet has always been addressed as an important problem in electrical engineering. The evaluation of the phenomena going on inside a ferromagnetic lamination can, in fact, give important indications on the performances (supply currents, losses, efficiency, etc.) of the most common electromechanical devices.

In the past, the problem was solved analytically. Introducing some drastic simplifications on the spatial distribution of magnetic field and induction, formulae were elaborated for the evaluation of eddy current and hysteresis losses. Despite their simplicity, these formulae give loss predictions which are quite correct in the range of flux density and frequency values usually present in classical electrical machines and in the standardised experimental conditions used in an Epstein yoke setup.

The evolution of technology, which tends to increase the stress of the materials in innovative devices, and the impact of electronic supply, that is the use of non-sinusoidal supply waveforms, have, unfortunately, shown that the classical formulae are not always reliable, calling thus for a much deeper study of the phenomenon.

From the previous considerations, there is an evident need for computational procedures able to take into account the complex material behaviour, together with the presence of eddy currents under time-periodic, not necessarily sinusoidal, supply conditions.

The problem is characterised by a simple geometry; in fact, since the usual thickness values of the ferromagnetic sheets, ranging from 0.35 to 1 mm, are much smaller than the other dimensions, the problem can be considered as one-dimensional.

Despite the simple geometric structure, the analysis of the electromagnetic field is made difficult by the material modelling. Nonlinearity, hysteresis and dynamic effects cannot be neglected when a thorough assessment of the phenomenon is needed. All the characteristics of the problem (material modelling, eddy currents, time-periodicity, flux imposed supply) set serious constraints on the possible setup of the field problem and on its numerical solution, so that a special formulation has to be devised. In addition, the possible presence of rate-dependent hysteresis effects compels the use of some material model able to take into account this kind of phenomena. The Classical Preisach Model (CPM), which has been extensively and efficiently used for the simulation of hysteretic phenomena, is, unfortunately, not able to model dynamic effects. An innovative material model, based on the CPM and proposed by Bertotti [1], correctly simulates dynamic hysteresis effects but, as will be explained in the following, it requires a special formulation in order to be used in a computationally efficient way.

The research activity performed on this subject is reported in the following sections: Section II discusses the field formulation used for the finite element solution of the problem; Section III presents the dynamic hysteresis model and its particular implementation. Finally, Section IV deals with the comparison with measurements.

II. Field Formulation

As it has been mentioned above, the mathematical formulation of the problem is significantly constrained by the requirements imposed by the material modelling and supply conditions.

The domain under consideration is represented by a lamination as sketched in Fig. 1. As it is well known, the thickness of the iron sheet (x coordinate) usually ranges from 0.35 to 1 mm, while it extends in the y and z direction to

Manuscript received September 23, 1996.
O. Bottauscio, e-mail botta@ien.it, phone +39 11 3919776, fax +39 11 6509471
M. Chiampi, e-mail chiampi@polel1.polito.it, phone +39 11 5647142, fax +39 11 5647199
M. Repetto, e-mail repetto@polel1.polito.it, phone +39 11 5647140, fax +39 11 5647199

several centimetres. This geometry models well the operating conditions of the iron lamination when used in the core of electrical machines; in addition it is a good representation of the standardised measurement conditions inside an Epstein yoke setup, so that comparisons with experimental values can be performed easily.

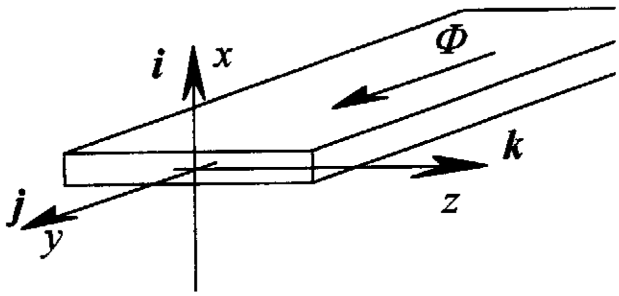

Fig. 1 - Geometric domain of the problem and Cartesian reference axes.

The field equations governing the problem are:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \quad (1)$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} \quad (2)$$

together with the material relationships:

$$\boldsymbol{B} = \chi(\boldsymbol{H}) \text{ or } \boldsymbol{H} = \zeta(\boldsymbol{B}) \text{ and } \boldsymbol{J} = \sigma \boldsymbol{E}$$

Following the reference frame shown in Fig. 1, it can be stated that:

$$\boldsymbol{B} = B_y(x,t)\boldsymbol{j};\quad \boldsymbol{H} = H_y(x,t)\boldsymbol{j};\quad \boldsymbol{J} = J_z(x,t)\boldsymbol{k} \quad (3)$$

Starting from the previous equations, two formulations can be devised: a potential one based on the magnetic vector potential A and a field one based on the magnetic field $\boldsymbol{H}$. Both formulations allow one to take into account eddy currents, but they are different from the point of view of the imposition of external supply sources.

In the potential formulation, the imposed magnetic flux flowing through the lamination is given by the difference between the values of A at the boundary of the sheet, which is the total value of magnetic flux flowing through it. The flux is thus imposed by means of Dirichlet boundary conditions.

On the contrary, it is easy to show that the boundary values of magnetic field $\boldsymbol{H}$ are tied to the supplied current. In this case, the imposed flux has to be inserted in the formulation by means of additional constraints. The imposition of the supply flux is not univocal [2], but one of the possible solutions is to add a supplementary equation to the previous set, that is:

$$\Phi(t)\boldsymbol{j} = \int_L \chi[\boldsymbol{H}(x,t)]dx \quad (4)$$

where Φ is the imposed flux, having y direction with versor $\boldsymbol{j}$, and L is the thickness of the lamination.

Both previous formulations need a special strategy for the solution of the nonlinear problem. The Fixed Point (FP) technique is a well established nonlinear solution scheme [3,4] not only for the usual monodrome nonlinear characteristics, but also for polydrome hysteretic curves [5].

The basic assumption of FP is the splitting of the nonlinear characteristic in two terms: a linear one and a nonlinear residual, that is:

$$\begin{aligned}\chi(\boldsymbol{H}) &= \mu_T \boldsymbol{H} + \boldsymbol{S}(\boldsymbol{H}) \\ \zeta(\boldsymbol{B}) &= \nu_T \boldsymbol{B} + \boldsymbol{R}(\boldsymbol{B})\end{aligned} \quad (5)$$

where μ_T, ν_T are constant parameters, while $\boldsymbol{R}$ and $\boldsymbol{S}$ are the nonlinear residuals which can be iteratively evaluated.

Despite its natural implementation of flux conditions, the potential formulation has a major drawback; in fact, it provides the behaviour of the magnetic flux density, so that the $\boldsymbol{R}(\boldsymbol{B}) = \zeta(\boldsymbol{B}) - \nu_T \boldsymbol{B}$ relationship has to be used in the iterative scheme. This feature is in contrast with the most used hysteresis models which handle the curve $\boldsymbol{B} = \chi(\boldsymbol{H})$. Even if an inversion of these models can be devised, as in the case of CPM [6], this procedure tends to slow down numerical computations. In addition, not all the hysteresis models allow this inversion, as for instance the dynamic Preisach model that has been used. This fact forces the use of an H-based formulation, which, at the moment, seems to be the only one able to take into account both eddy currents and dynamic hysteresis effects.

The linearised field equation inserted in the iterative loop then becomes:

$$\nabla \times \nabla \times \boldsymbol{H}_m = -\sigma \frac{\partial}{\partial t}\left(\mu_T \boldsymbol{H}_m + \boldsymbol{S}_{m-1}(\boldsymbol{H})\right) \quad (6)$$

where m is the iteration index.

Since the phenomenon under analysis is time-periodic and the equation has been linearised, its time behaviour can be treated by means of harmonic decomposition. Equation (6) is thus decomposed into N harmonic equations leading to the final iterative relation:

$$\begin{cases}\nabla \times \nabla \times \underline{\boldsymbol{H}}_m^{(n)} = -j\sigma\omega n \mu_T \underline{\boldsymbol{H}}_m^{(n)} - j\sigma\omega n \underline{\boldsymbol{S}}_{m-1}^{(n)} \\ \underline{\Phi}^{(n)}\boldsymbol{j} = \int_L \left(\mu_T \underline{\boldsymbol{H}}_m^{(n)} + \underline{\boldsymbol{S}}_{m-1}^{(n)}\right)dx\end{cases} \quad (7)$$

where n is the harmonic index.

Equation (7) can then be discretised by means of the Finite Element Method (FEM) using the Galerkin weighted residual method.

At each iteration step, the Finite Element solution of the problems (7) gives the harmonic content of $\boldsymbol{H}$ in each element and then the time behaviour of $\boldsymbol{H}$ is determined. using the inverse Fast Fourier Transform. In order to compute the residual $\boldsymbol{S}_m$, the waveform of the magnetic flux density is evaluated through the Dynamic Preisach Model (DPM), following the approach proposed by Bertotti [1] which gives the evolution of the magnetization starting from a known initial state. Since the periodic conditions do not allow an a-priori knowledge of the system states, an initial demagnetised state is considered and the transient $\boldsymbol{B}$

evolution, under the computed periodic $\boldsymbol{H}$ excitation, is studied until the periodic permanent behaviour is reached (usually after 3-4 periods). Thus, the waveform of $\boldsymbol{S}_m$ in each element is computed and, after a Fast Fourier Transform, the right hand sides of Eqn. (7) in the harmonic domain are updated. The process is iterated until convergence. A flow-chart of the iterative scheme is presented in Fig. 2.

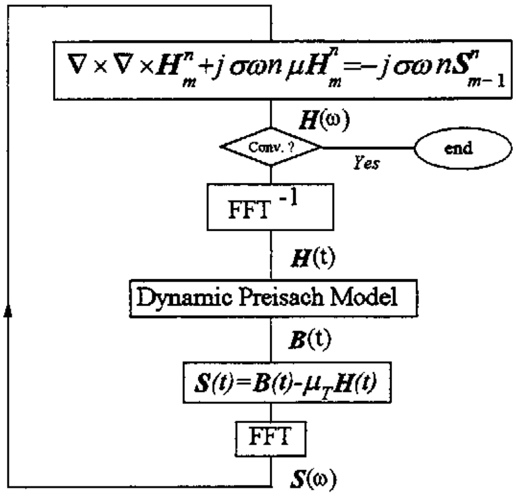

Fig. 2. Schematic flowchart of the procedure used in the Finite Element computations.

III. Dynamic hysteresis model

Simulation of ferromagnetic hysteresis can be performed by means of several models. Among them, the Preisach model [7] is by far the most used and the most reliable.

Even if it was originally devised for modelling ferromagnetism, not all the features of ferromagnetic hysteresis are efficiently modelled by CPM. In fact, since CPM is based on the superposition of the responses of elementary rectangular operators, whose state is dependent only on the actual value of the magnetic field, it cannot model rate-dependent hysteresis effects.

Thus, several Preisach based models, which introduce some modifications to the CPM to follow the material behaviour better, have been proposed. The modification proposed by Bertotti [1] changes the behaviour of the elementary operator. Its state no more follows a rectangular characteristic but is governed by an ordinary differential equation. The elementary operator with up and down switching field values at (α,β), has a normalised magnetisation value ruled by:

$$\begin{cases} \dfrac{dM}{dt} = k_d(H-\alpha) & H \geq \alpha \\ \dfrac{dM}{dt} = k_d(H-\beta) & H \leq \beta \end{cases} \tag{8}$$

where $|M| \leq 1$ and k_d is a material dependent parameter.

The new operator is congruent with the Classical Preisach one for slow varying field, but it changes its output when the rate of the applied field is increased, as sketched in Fig. 3, where elementary loops for different frequencies of the applied field are shown.

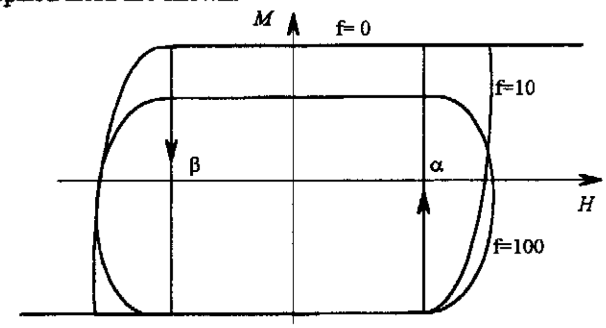

Fig. 3. Magnetisation response of the elementary Bertotti's operator for different normalised frequency values of applied field.

The implementation of DPM within a computational procedure for the magnetic field solution is not a simple task. In fact, this model establishes a time evolution of magnetisation in the Preisach elementary operators and allows them to assume intermediate values between the positive and negative saturation. This means that the state of the material cannot be defined by a staircase boundary separating positive and negative operators, as in CPM, but the magnetisation of any single operator has to be stored. In addition, in order to obtain smooth responses from the hysteresis model, several operators have to be considered, often leading to unacceptable processing times.

For such a purpose, a new approach is proposed reducing the number of operators and associating each of them with a portion of Preisach plane. Thus, for an elementary operator centred in (α_0,β_0) we define, for an increasing excitation H, an interval (α_1,α_2) $(\alpha_1<\alpha_0<\alpha_2)$: the magnetisation begins to increase when $H>\alpha_1$ (even if $H<\alpha_0$); for $\alpha_1<H<\alpha_2$ the operator is partially activated, while for $H\geq\alpha_2$ the behaviour is the one of the operator concentrated in α_0. Similar considerations can be repeated for the negative switching operation. According to this approach, the differential equation governing the evolution of magnetisation for the positive switching is modified to the form:

$$\begin{aligned} &\frac{dM}{dt} = 0 && \text{for } H < \alpha_1 \\ &\frac{dM}{dt} = \int_{\alpha_1}^{H} k_d(H-h')w(h')dh' && \text{for } \alpha_1 \leq H \leq \alpha_2 \\ &\frac{dM}{dt} = k_d(H-\alpha_0) && \text{for } H > \alpha_2 \end{aligned} \tag{9}$$

where H is the excitation and w is a weighting function, which is assumed to be piecewise linear in the interval $\alpha_1-\alpha_2$ and zero elsewhere, as shown in Fig. 4. The amplitude of this function is computed, imposing, for $H\geq\alpha_2$, the same behaviour as the operator concentrated in α_0. A similar relation is adopted for decreasing excitations.

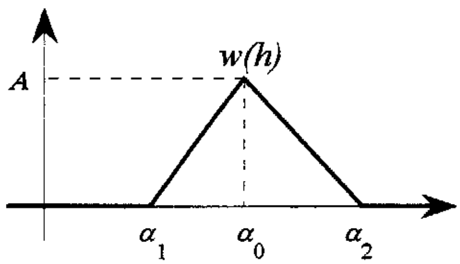

Fig. 4. Weighting function used for the switching of the elementary operator.

The saturation of the distributed operator is a function of the point, since it simulates the behaviour of several elementary dipoles. This characteristic can be modelled by a saturation function which rules the maximum value of magnetisation that the operator centred in α_0 can reach at a given field value. The piecewise linear function sketched in Fig. 5 is the most obvious choice.

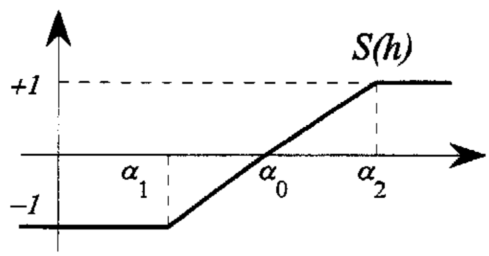

Fig. 5. Saturation function of the distributed operator.

The proposed generalisation, which is associated with each elementary operator a portion of Preisach plane, is very efficient for modelling the dynamic behaviour, but it is not completely compatible with the static limit. In fact, the ordinary differential equation governing the operator behaviour imposes an increase of the magnetisation everytime h is greater than α, even if h is decreasing. Obviously, this behaviour disagrees with the static Preisach model, where the magnetisation is decreasing, or constant, when the applied field is going down. In order to allow the use of the proposed model both for static and dynamic cases, a modification has to be introduced. This purpose is achieved by means of a function which constrains the magnetisation to follow a static behaviour, that is $B_{new} \le B_{old}$ if $H_{new} \le H_{old}$, when the frequency of the phenomenon is under a given limit specified by the user.

The distribution of the operators in the Preisach plane can be obtained from the Preisach distribution function $\varphi(\alpha, \beta)$, by a double integration of φ over the Preisach triangle. Due to the symmetry of the distribution function, the integration can be performed over one half of the Preisach plane. The double integration is performed in two steps. First, an integration of $\varphi(\alpha, \beta)$ is performed over domains like the ones shown in Fig. 6, in order to compute a function $F(\alpha)$ defined as:

$$F(\alpha) = \int_0^{\alpha} \int_{-\alpha}^{\alpha} \varphi(\alpha', \beta')\, d\alpha'\, d\beta' \qquad (10)$$

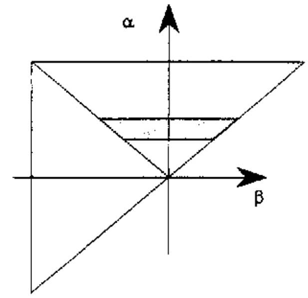

Fig. 6. Domain of integration of function $F(\alpha)$.

Given a total number N_T of operators, they are distributed in N_α lumps over the α axis according to the function $F(\alpha)$. The N_β operators for each lump ($N_\beta = N_T/N_\alpha$) are then placed at a constant value of $\alpha = \alpha_i$ according to the function G defined as:

$$G(\beta) = \int_{-\alpha_i}^{\alpha_i} \varphi(\alpha_i, \beta')\, d\beta' \qquad (11)$$

In such way, it is easy to define the boundary of the interval $\alpha_1 - \alpha_2$ as the preceding and following level of α_i.

In this way the operators are concentrated in the regions where the gradient of the function φ is greater so that accurate and smooth predictions can be obtained also using a reduced number of operators. Nevertheless, experience has shown that their number cannot be reduced below a certain limit if an accurate prediction of losses has to be obtained. In fact, due to the concentration of the operators in the vicinity of the coercive field values, they are quite coarse near the saturation level. Due to the linear saturation function used, this fact can lead to small discrepancies in magnetisation values in the proximity of saturation. Unfortunately, even if these discrepancies have a small absolute value, their contribution to the area cycle evaluation is not negligible due to the large excursion of magnetic field in saturation. In the practical cases a minimum number of 800 operators has been used with satisfactory results.

Figures 7 and 8 report the placement of 200 operators in the case of a FeSi 2% material; the vertices of the quadrangles reported in the figures represent the limits of the area associated to each operator ($\alpha_1 \div \alpha_2$; $\beta_1 \div \beta_2$).

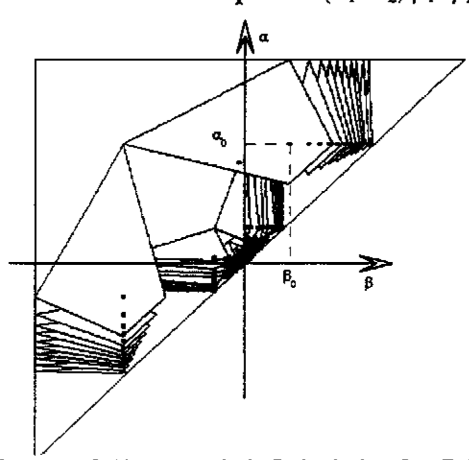

Fig. 7. Placement of 200 operators in the Preisach plane for a FeSi 2% material.

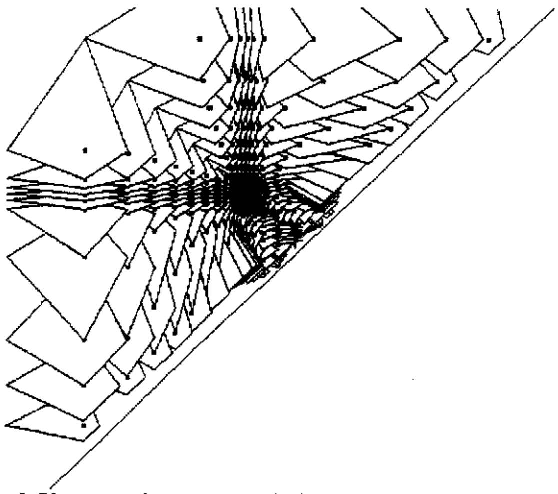

Fig. 8. Placement of 200 operators in the Preisach plane for a FeSi 2% material, zoom in the coercive field zone.

IV. COMPARISON WITH MEASUREMENTS

The proposed Finite Element scheme and the implementation of the DPM have been coupled in a computational procedure for the evaluation of dynamic hysteresis effects in ferromagnetic laminations. This procedure has been tested first versus other similar procedures based on the Classical Preisach Model, showing good performances both in terms of field and loss prediction accuracy and in computational burden. It has then been validated by means of comparisons with measurements performed on an Epstein yoke setup.

A FeSi 3.5% material 0.35 mm thick was selected for these tests. This material was selected because, due to its microscopic structure, its excess loss contribution was not negligible.

A series of measurements was performed by supplying a controlled sinusoidal flux waveform and changing both the maximum value of flux and its frequency.

Comparisons are performed on the basis of the dynamic cycle. From the experimental side this cycle is the locus of the flux-current measured throughout the period. It is easy to show that the computed boundary value of the magnetic field H is related to the absorbed currents so that it can be easily simulated.

Figure 9 shows the locus of the maximum values of magnetic flux density inside the lamination, comparing the distribution with another one obtained by means of CPM. This behaviour is shown at supply frequencies of 50 and 1600 Hz. Both these figures highlight that, even in such relatively thin sheets like the ones taken into consideration, it is not possible to neglect the skin effect caused by eddy currents.

Figures 10 and 11 show two dynamic cycles, both computed and measured on an Epstein yoke, at 50 and 400 Hz for an average maximum value of flux density of 1 T.

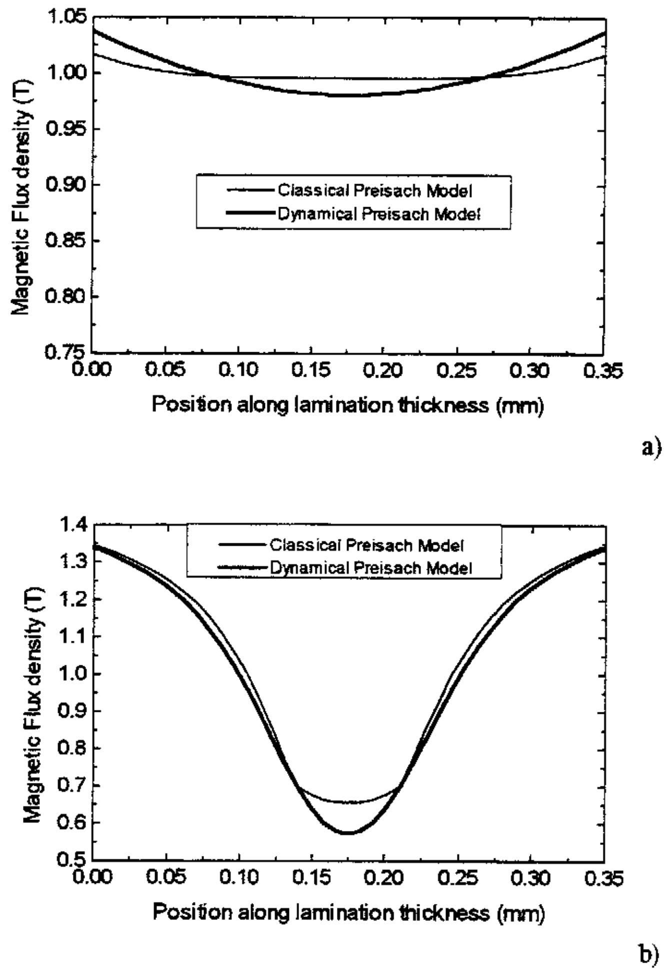

Fig. 9. Locus of peak values of magnetic flux density with an imposed flux corresponding to a maximum average value of 1 T, a) f=50 Hz, b) f=1600 Hz.

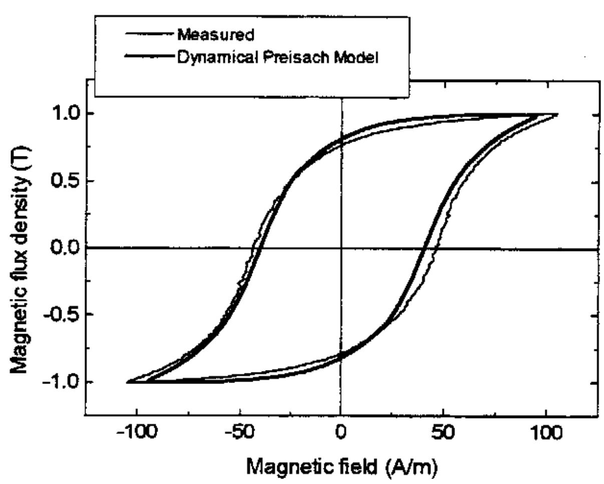

Fig. 10. Experimental and computed dynamic cycles at 50 Hz with an imposed flux corresponding to a maximum average value of 1 T.

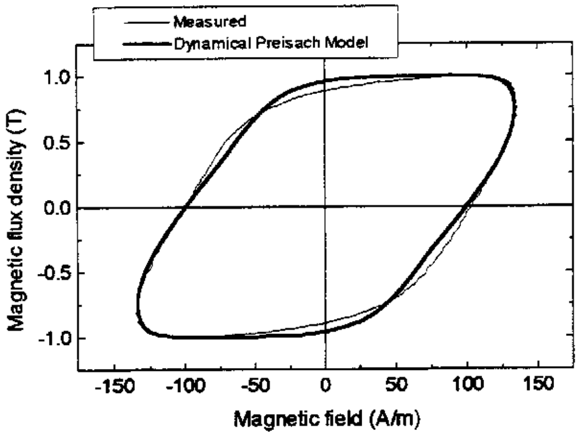

Fig. 11. Experimental and computed dynamic cycles at 400 Hz with an imposed flux corresponding to a maximum average value of 1 T.

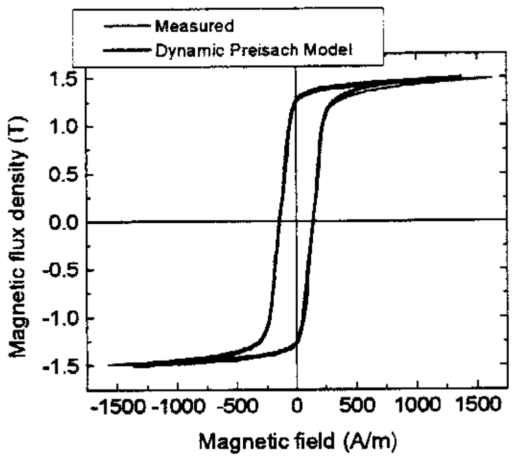

Fig. 12. Experimental and computed dynamic cycles at 400 Hz with an imposed flux corresponding to a maximum average value of 1.5 T.

Similar computations have been performed for higher magnetic flux density values; Fig. 12 presents the computed and experimental dynamic cycles for an average maximum value of flux density of 1.5 T at 400 Hz.

As can be seen, the agreement between computations and measurements is satisfactory both at low and high saturation levels, even if at high frequencies some discrepancies can be found. These discrepancies can be attributed to an imprecise estimation of the dynamic parameter k_d which is here tuned on the basis of the area of the cycle disregarding its shape and has not been computed according to the microstructure of the material.

V. Conclusions

The work performed has shown the possibility of using a Dynamic Preisach Model inside a Finite Element computational scheme. In order to use Bertotti dynamic hysteresis model, a particular implementation has been developed allowing one to reduce the number of magnetic entities describing the magnetisation process.

The proposed approach has shown a good performance both in terms of accuracy of results and computational times, even if some discrepancies are still experienced.

In the future, the work will continue to increase the accuracy of the estimation of the field by means of a more precise identification of the dynamic coefficient k_d.

References

[1] G. Bertotti, "Dynamic generalization of the scalar Preisach model of hysteresis", *IEEE. Trans. Mag.*, Vol.28, pp.2599, 1992.

[2] D.A. Philips "Microscopic fields and fluxes in ferromagnetic laminations taking into account hysteresis and eddy current effects", *J. Magn. Magnetic Materials*, Vol. 160, July 1996, pp. 5-10.

[3] M. Chiampi, D. Chiarabaglio, M. Repetto, "An improved technique for nonlinear magnetic problems", *IEEE Trans. Mag.*, Vol. 30, n. 6, Nov. 1994, pp. 4332-4334.

[4] I.D.Mayergoyz, "Iteration methods for the calculation of steady magnetic fields in nonlinear ferromagnetic media", *COMPEL*, Vol. 1, n. 2, pp. 89-110, 1982.

[5] M. Chiampi, D. Chiarabaglio, M. Repetto, "A Jiles-Atherton and Fixed Point combined technique for time periodic magnetic field problems with hysteresis", *IEEE Trans. Mag.*, Vol. 31, n. 6, Nov. 1995, pp. 4306-4311.

[6] O. Bottauscio, M. Chiampi, D. Chiarabaglio, M. Repetto, "A hysteretic periodic field solution using Preisach model and fixed point technique", *IEEE Trans. Mag.*, Vol. 31, Nov. 1995, pp. 3548-3550.

[7] I.D.Mayergoyz, *Mathematical models of hysteresis*, New York, Springler-Verlag, 1991