

# Multi-Fidelity Optimization of Microwave Structures Using Response Surface Approximation and Space Mapping

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**Abstract** — A computationally efficient method for design optimization of CPU-intensive microwave structures is discussed. The presented technique exploits a response surface approximation surrogate model set up using data from the coarse-mesh EM-based model being a relaxed-accuracy representation of the microwave structure in question. The surrogate model is further subjected to the classical space mapping optimization. It is demonstrated that the new technique is able to provide a satisfactory design with a few electromagnetic simulations of the original structure. Because of using functional approximation, no circuit equivalent coarse model is necessary, which makes the presented approach particularly suitable for structures for which the development of the reliable coarse model is problematic (e.g., antennas).

**Index Terms** — Computer-aided design (CAD), multi-fidelity optimization, response surface approximation, space mapping, electromagnetic simulation, engineering design optimization.

## I. INTRODUCTION

Due to the increasing complexity of contemporary microwave devices and structures as well as the demand for higher accuracy of electromagnetic simulation, the evaluation of microwave structures is becoming more and more time-consuming. Therefore, computer-aided design optimization—a critical part of modern microwave design process—faces fundamental difficulties. Direct optimization involving numerous evaluations of EM-simulation-based objective functions is typically impractical because of its high computational cost, and, in

many cases, because of its infeasibility which is due to poor analytical properties of EM-based objective functions as well as the lack of sensitivity data or sensitivity being too expensive to evaluate. This means, in particular, that the traditional, gradient-based techniques become obsolete. On the other hand, certain modern techniques such as evolutionary algorithms [1] or particle swarm optimizers [2] permit to handle some issues that are problematic for the classical optimization (e.g., objective function discontinuity, lack of derivative information, multiple local optima). However, these methods are even more CPU-intensive because they typically require a huge number of objective function evaluations.

One of the possible approaches to alleviate this problem is decomposition, i.e., breaking down an EM model into smaller parts and combine them in a circuit simulator to reduce the CPU-intensity of the design process [3]-[7]. This is only a partial solution though, because the EM-embedded co-simulation model is still subjected to direct optimization.

Space mapping (SM) is a technique that has been successfully applied to microwave engineering design problems as well as in other engineering fields [8]-[13] and seems to be one of the most efficient approaches to date. SM allows efficient optimization of expensive or “fine” models—usually implemented with a CPU-intensive EM simulator—by means of the iterative optimization and updating of the so-called “coarse” models, less accurate but cheaper to evaluate. The coarse model is supposed to be a physically-based representation of the fine model. In order to take advantage of the space mapping principle, the coarse model should be computationally much cheaper than the fine

model. Therefore, equivalent-circuit models or models exploiting analytical formulas are preferred [8]. Reliable equivalent-circuit models, however, may be difficult to develop for certain types of microwave devices (e.g., antennas, waveguide structures). Moreover, an extra simulator must be involved in the optimization process.

In this paper, another method is proposed that is a combination of a response surface approximation (RSA) approach [14] and space mapping, and does not require a circuit-based coarse model. Therefore, it can be implemented using a single EM-simulator, here, FEKO. The presented method uses a space-mapped RSA-based surrogate established with the coarse-mesh EM-based model and a generic surrogate-based optimization principle [15]. Design optimization examples are provided to demonstrate the robustness of the proposed approach.

## II. OPTIMIZATION APPROACH

### A. Design Optimization Problem Formulation

The goal is to solve the following problem

$$\mathbf{x}_f^* \in \arg \min_{\mathbf{x}} U(\mathbf{R}_f(\mathbf{x})) \quad (1)$$

where  $\mathbf{R}_f \in R^m$  denotes the response vector of a fine model of the device of interest, e.g., the modulus of the reflection coefficient  $|S_{21}|$  evaluated at  $m$  different frequencies.  $U$  is a given scalar merit function, e.g., a minimax function with upper and lower specifications. Vector  $\mathbf{x}_f^*$  is the optimal design to be determined. As mentioned in the introduction,  $\mathbf{R}_f$  is assumed to be computationally expensive so that the direct optimization is usually prohibitive. In this paper,  $\mathbf{R}_f$  is evaluated using FEKO.

### B. Initial Surrogate Model

A basis of the proposed approach is a computationally cheap surrogate model. We assume that the surrogate model is a response surface approximation (RSA) model. Here, we exploit a radial basis function (RBF) interpolation [16]; the surrogate will be denoted as  $R_{RBF}$ . Normally, RSA model would be set up using a sampled fine model data. However, in order to reduce the computational overhead, the surrogate is constructed using a simplified representation  $\mathbf{R}_c$  of the fine model.  $\mathbf{R}_c$  is evaluated in the same EM simulator as the fine model, however, with much coarser mesh. This not only results in a much shorter evaluation time, but also introduces some inaccuracy, which will be dealt with in Section II.C.

Let  $X_B = \{x^1, x^2, \dots, x^N\}$  denote a base set, such that the responses  $R_c(x^j)$  are known for  $j = 1, 2, \dots, N$ . Here, the base set is selected using a modified Latin hypercube sampling algorithm [17] that gives a quite uniform distribution of samples in the design space. Figure 1 shows an example allocation of 50 base points in the unity interval  $[0,1] \times [0,1]$ .

We shall adopt the notation  $\mathbf{R}_c(\mathbf{x}) = [\mathbf{R}_{c,1}(\mathbf{x}) \dots \mathbf{R}_{c,m}(\mathbf{x})]^T$ , where  $\mathbf{R}_{c,k}(\mathbf{x})$  is the  $k$ th component of the response vector  $\mathbf{R}_c(\mathbf{x})$ . The radial basis function model  $R_{RBF}$  is defined as

$$\mathbf{R}_{RBF}(\mathbf{x}) = \begin{bmatrix} \sum_{j=1}^N \lambda_{1,j} \phi(\|\mathbf{x} - \mathbf{x}^j\| / \gamma) \\ \dots \\ \sum_{j=1}^N \lambda_{m,j} \phi(\|\mathbf{x} - \mathbf{x}^j\| / \gamma) \end{bmatrix} \quad (2)$$

where  $\|\cdot\|$  denotes the Euclidean norm. The parameters  $\lambda_{k,j}$  are calculated so that they satisfy

$$\Phi \boldsymbol{\lambda}_k = \mathbf{F}_k, \quad k = 1, 2, \dots, m \quad (3)$$

where  $\boldsymbol{\lambda}_k = [\lambda_{k,1} \lambda_{k,2} \dots \lambda_{k,N}]^T$ ,

$$\mathbf{F}_k = [\mathbf{R}_{c,k}(\mathbf{x}^1) \dots \mathbf{R}_{c,k}(\mathbf{x}^N)]^T \quad (4)$$

and  $\Phi$  is an  $N \times N$  matrix with elements

$$\Phi_{ij} = \phi(\|\mathbf{x}^i - \mathbf{x}^j\| / \gamma) \quad (5)$$

where  $\gamma = (2/(nN^{1/n})) \sum_{k=1}^n \max\{j, k=1, \dots, N : \|\mathbf{x}^j - \mathbf{x}^k\|\}$  is a normalization factor representing an average distance between base points ( $n$  is the number of design variables).

In this paper we use a Gaussian basis function defined as

$$\phi(r) = e^{-cr^2}, \quad r \geq 0, \quad c > 0 \quad (6)$$

Parameter  $c$  is adjusted to minimize the generalization error calculated using cross-validation [15]. Figure 2 shows the example of the scalar  $\mathbf{R}_{RBF}$  model surface. Note that the RBF model has an interpolation property (guaranteed by the condition (3)), i.e., the response surface fits exactly the  $\mathbf{R}_c$  model at all base designs.

### C. Space Mapping Correction of the Response Surface Approximation Model

The surrogate model  $\mathbf{R}_{RBF}$  is computationally cheap but it is not as accurate representation of the microwave structure in question as the fine model  $\mathbf{R}_f$ . This is not only because  $\mathbf{R}_{RBF}$  is set up using a limited number of base points, but, most importantly, because it is constructed using the data from the coarse-mesh model  $\mathbf{R}_c$  instead of the original fine model  $\mathbf{R}_f$ . Therefore, before

optimization, the model  $R_{RBF}$  has to be corrected to improve its (local) accuracy with respect to the fine model.

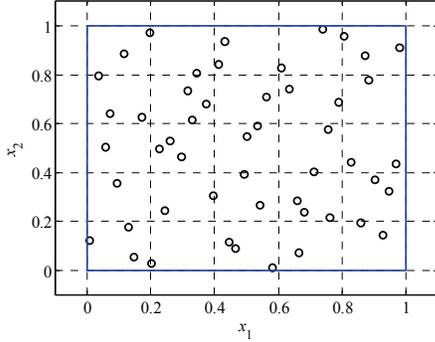


Fig. 1. Example of the base set for the RBF surrogate model  $R_{RBF}$ : 50 base points allocated in the unity interval  $[0,1] \times [0,1]$  using the modified Latin hypercube sampling [17].

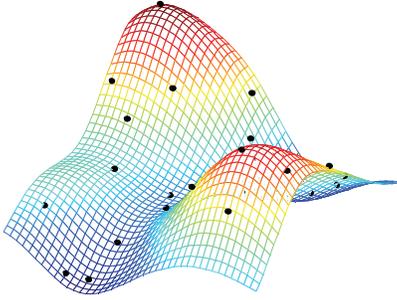


Fig. 2. Example of the (scalar)  $R_{RBF}$  model. Base points denoted using black circles.

In this paper the surrogate model is corrected using a classical space mapping (SM) approach [13]. The corrected surrogate model  $R_{SM}$  is defined as follows

$$R_{SM}(x) = P_L(R_{RBF}(P_R(x, p_R)), p_L) \quad (7)$$

where SM parameters are obtained using a parameter extraction (PE) process

$$(p_L, p_R) = \arg \min_{y, z} \sum_{x \in X_{PE}} \|R_f(x) - P_L(R_{RBF}(P_R(x, y)), z)\| \quad (8)$$

Here,  $P_L$  is an output-SM-like mapping (e.g.,  $P_L(R, p_L) = P_L(R, A, d) = A \cdot R + d$ ) [13],  $P_R$  is an input-SM-like mapping (e.g.,  $P_R(x, p_R) = P_R(x, B, c) = B \cdot x + c$ ) [8], whereas  $X_{PE}$  is the set of points (designs) used in PE.

#### D. Optimization Procedure [18]

The proposed optimization procedure establishes an RSA model  $R_{RBF}$  using sampled data from the coarse-mesh model  $R_c$ . The space-mapping-

corrected RSA model,  $R_{SM}$ , is then created using (7), (8) with the parameter extraction based on a current design at which the fine model response is known. Subsequently, a new design is found by means of optimizing the  $R_{SM}$  model. The surrogate models are set up in a restricted domain, being the neighbourhood of a current design. More specifically, the neighbourhood is defined by a small deviation  $\delta$  from the current design; the value of  $\delta$  is updated after each iteration of the optimization algorithm.

The optimization procedure can be formalized as follows [18]:

- Step 0* Set  $i = 0$ ; Initialize control parameters:  $\delta \in (0, 1)$  and  $N$  (positive integer); optimize the model  $R_c$  to find an initial design  $x^{(0)} = \arg \min \{x : U(R_c(x))\}$ ;
- Step 1* Assign lower bounds  $x_{\min}$  and upper bounds  $x_{\max}$  for the design variables:  $x_{\min} = (1 - \delta) \cdot x^{(i)}$  and  $x_{\max} = (1 + \delta) \cdot x^{(i)}$ ;
- Step 2* Select the base set  $X_B = \{x^1, \dots, x^N\}^\#$  so that  $x_{\min} \leq x^j \leq x_{\max}$  (component-wise),  $j = 1, \dots, N$ ; evaluate  $R_c$  at all designs from  $X_B$ ;
- Step 3* Establish the surrogate model  $R_{RBF}$  according to (2)-(6);
- Step 4* Establish the corrected surrogate model  $R_{SM}$  according to (7) and (8) using  $X_{PE} = \{x^{(i)}\}$ ;
- Step 5* Find a new design  $x^{(i+1)}$  by optimizing  $R_{SM}$ :  $x^{(i+1)} = \arg \min \{x_{\min} \leq x \leq x_{\max} : U(R_{SM}(x))\}$ ;
- Step 6* Update  $\delta$ :  $\delta = \max \{j = 1, \dots, N : |x_j^{(i+1)} - x_j^{(i)}| / |x_j^{(i+1)}|\}^\S$ ; Set  $i = i + 1$ ;
- Step 7* If the termination condition is not satisfied, go to 1; else END;

<sup>#</sup> The base set is selected using a modified Latin hypercube sampling [17].

<sup>\S</sup>  $x_j^{(i)}$  and  $x_j^{(i+1)}$  are the  $j$ th components of  $x^{(i)}$  and  $x^{(i+1)}$ , respectively; the updating procedure assumes positive values for all design variables.

Note that the updating rule for  $\delta$  ensures that the new surrogate model domain is not larger than the previous one. The algorithm is terminated after user-defined maximum number of iterations or if the value of  $\delta$  becomes sufficiently small.

Computational cost of the optimization process is determined by the evaluation time  $t_c$  of the coarse-mesh model  $R_c$  and the evaluation time  $t_f$  of the fine model  $R_f$  (other factors such as the cost of setting up  $R_{RBF}$  and  $R_{SM}$  models can be neglected). The total optimization time can be calculated as

$$t_{opt} = t_c \sum_{i=0}^{n_{iter}} N_i + (n_{iter} + 1)t_f \quad (9)$$

where  $n_{iter}$  is the number of iterations of the optimization algorithm,  $N_0$  is the number of evaluations of  $R_c$  necessary to find  $x^{(0)}$  (cf. *Step 0*), and  $N_i$ ,  $i > 0$ , is the number of new base points at iteration  $i$  (may be smaller than  $N$  because some base points from previous iterations are reused).

To measure the computational efficiency of the proposed algorithm a relative time  $t_{rel}$  is used that is the number of fine model evaluations required to complete the optimization procedure:

$$t_{rel} = n_{iter} + 1 + (t_c / t_f) \sum_{i=0}^{n_{iter}} N_i \quad (10)$$

It should be noted that it is possible to use the coarse-mesh model  $R_c$  directly as a coarse model in the SM optimization algorithm. However, the computational cost of such a process is expected to be much higher than for the technique proposed here because of the larger total number of evaluations of  $R_c$  (both parameter extraction and surrogate model optimization would be performed directly on  $R_c$ ). Also, analytical properties of the coarse-mesh EM model may be poor (the model may be non-differentiable or even discontinuous) in contrast to the RSA-based model which is always smooth.

### III. EXAMPLES

#### A. 2<sup>nd</sup>-Order Tapped-Line Microstrip Filter [19]

Consider a second-order tapped-line microstrip filter [19] shown in Fig. 3. The design parameters are  $\mathbf{x} = [L_1 \ g]^T$ . The fine model  $R_f$  is simulated in FEKO [20]. The number of meshes for the fine model is 360. Simulation time for the fine model is 204 s. The design specifications are  $|S_{21}| \geq -3$  dB for  $4.75 \text{ GHz} \leq \omega \leq 5.25 \text{ GHz}$ , and  $|S_{21}| \leq -20$  dB for  $3.0 \text{ GHz} \leq \omega \leq 4.0 \text{ GHz}$ , and  $6.0 \text{ GHz} \leq \omega \leq 7.0 \text{ GHz}$ .

The coarse-mesh  $R_c$  is the structure in Fig. 3 also simulated in FEKO, however, the number of meshes is only 48. The number of meshes for  $R_f$  and  $R_c$  correspond to  $\mathbf{x}_c = [6.0 \ 0.1]^T$  mm. The simulation time for  $R_{fc}$  is about 8 s. Initial design  $\mathbf{x}^{(0)} = [3.83 \ 0.103]^T$  mm is found by optimizing  $R_c$  and requires 34 model evaluations using  $\mathbf{x}_c$  as a starting point (small number of evaluations is due to using relaxed tolerance requirements). The number of base points to set up model  $R_{RBF}$  is  $N = 25$ . Initial value of  $\delta$  is 0.3. The space-mapping-

corrected model  $R_{SM}$  uses only output SM of the form:  $R_{SM}(\mathbf{x}) = \mathbf{A} \cdot R_{RBF}(\mathbf{x})$  with  $\mathbf{A} = \text{diag}\{a_1, a_2, \dots, a_m\}$  (i.e.,  $P_L(\mathbf{R}) = \mathbf{A} \cdot \mathbf{R}$ , and  $P_R(\mathbf{x}) = \mathbf{x}$ ). This choice comes from the fact that a relatively small surrogate model domain allows us to assume that the misalignment between the surrogate model and the fine model has similar character throughout the domain.

We performed three iterations of the optimization algorithm. Figure 4 shows the responses of models  $R_f$  and  $R_c$  at the initial design (fine model specification error +0.9 dB), the  $R_f$  response at  $\mathbf{x}_c$  (specification error +1.0 dB), as well as response of  $R_f$  at the final design  $\mathbf{x}^{(3)} = [3.92 \ 0.145]^T$  mm (specification error -0.7 dB). Figure 3 shows the surrogate model domains, base sets, and the evolution of the design for all three iterations. The total number of evaluations of model  $R_c$  is 59 and it is smaller than  $3N = 75$ , which is because some of the base points were reused as indicated in Fig. 5. Table 1 indicates that the total optimization time corresponds to only 7.6 evaluations of the fine model.

For the sake of comparison, an SM optimization of the filter was also performed using directly  $R_c$  as a coarse model and the same output SM surrogate. The optimization time was 48 minutes, almost twice as much as for the proposed technique (with the total evaluation time of  $R_c$  being almost three times larger), even though the SM matrix  $\mathbf{A}$  can be, in this case, obtained analytically without performing the parameter extraction process (8). In case of using any kind of input SM [8], the optimization cost would be much higher.

The first example is provided mostly to illustrate the operation of the proposed optimization algorithm (cf. Fig. 5). Other design problems are provided in the next sub-sections.

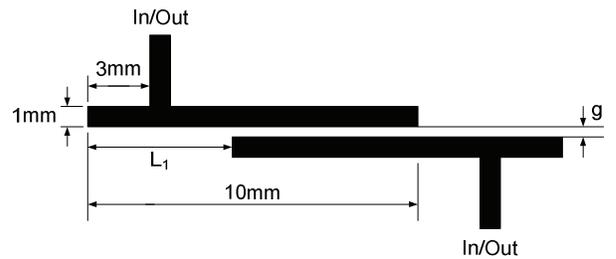


Fig. 3. Geometry of the second-order tapped-line microstrip filter [19].

Table 1: 2<sup>nd</sup>-order tapped line filter: optimization cost

Algorithm Component	Model Involved	Number of Model Evaluations	CPU Cost	
			$t_{opt}$ [min]	$t_{rel}$
Optimization of $\mathbf{R}_c$	$\mathbf{R}_c$	34	4.5	1.3
Setting up base sets for $\mathbf{R}_{RBF}$	$\mathbf{R}_c$	59	7.9	2.3
Evaluation of $\mathbf{R}_f$	$\mathbf{R}_f$	4	13.6	4.0
Total optimization time	N/A	N/A	26.0	7.6

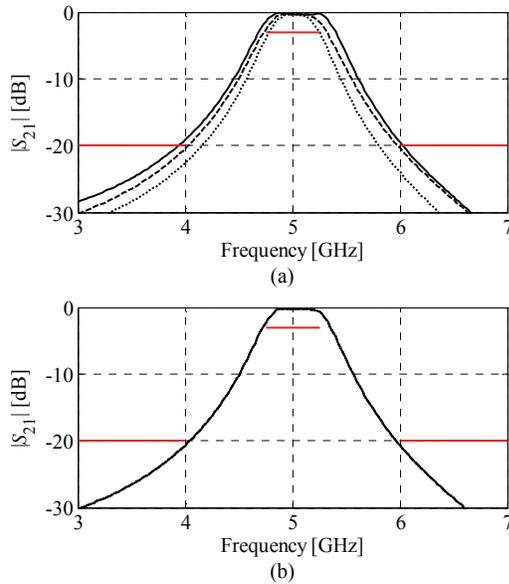


Fig. 4. Second-order tapped-line filter: (a) responses of  $\mathbf{R}_f$  (solid line) and  $\mathbf{R}_c$  (dashed line) at initial design  $\mathbf{x}^{(0)}$  and response of  $\mathbf{R}_f$  at  $\mathbf{x}_c$  (dotted line); (b) response of  $\mathbf{R}_f$  at the final design.

### B. Patch Antenna [21]

Consider the patch antenna [21] shown in Fig. 6. This antenna is printed on a substrate with relative dielectric constant  $\epsilon_r = 2.32$  and height  $h = 1.59$  mm. The design parameters are the patch length and width, i.e.,  $\mathbf{x} = [L \ W]^T$ . The objective is to obtain  $50 \ \Omega$  input impedance at 2 GHz. The fine model  $\mathbf{R}_f$  is simulated in FEKO [20]. The number

of meshes for the fine model is 1024, which ensures mesh convergence for the structure. Simulation time for the fine model is 41s.

The coarse-mesh model  $\mathbf{R}_c$  is the structure in Fig. 6 also simulated in FEKO, however, the number of meshes is only 100. Simulation time for model  $\mathbf{R}_c$  is 0.6s. The number of meshes for  $\mathbf{R}_f$  and  $\mathbf{R}_c$  correspond to  $\mathbf{x}_c = [50 \ 100]^T$  mm.

Initial design  $\mathbf{x}^{(0)} = [50.85 \ 101.86]^T$  mm is found by optimizing  $\mathbf{R}_c$  and requires 39 model evaluations. The number of base points to set up model  $\mathbf{R}_{RBF}$  is  $N = 30$ . Initial value of  $\delta$  is 0.01. As before, the space-mapping-corrected model  $\mathbf{R}_{SM}$  is of the form  $\mathbf{R}_{SM}(\mathbf{x}) = \mathbf{A} \cdot \mathbf{R}_{RBF}(\mathbf{x})$ .

The fine model response at the initial design is  $38.15 \ \Omega$ . The response of  $\mathbf{R}_f$  at the design obtained after four iterations of the proposed optimization procedure,  $\mathbf{x}^{(4)} = [50.25 \ 101.09]^T$  mm, is  $49.94 \ \Omega$ . The total number of evaluations of model  $\mathbf{R}_c$  is 97. For illustration purposes, Fig. 7 shows the response surface of the fine model, the  $\mathbf{R}_{RBF}$  model, and the space-mapping-corrected RBF model  $\mathbf{R}_{SM}$  at the first iteration of the optimization procedure. Table 2 summarizes the computational cost of the optimization: the total optimization time corresponds to only 6.8 evaluations of  $\mathbf{R}_f$ .

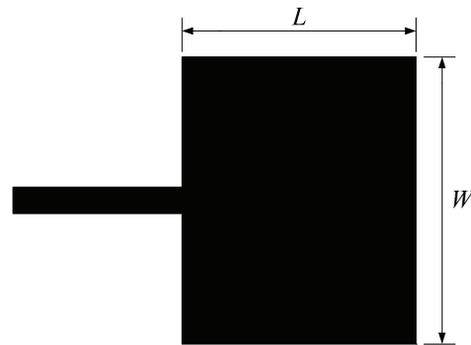


Fig. 6. Geometry of the patch antenna [21].

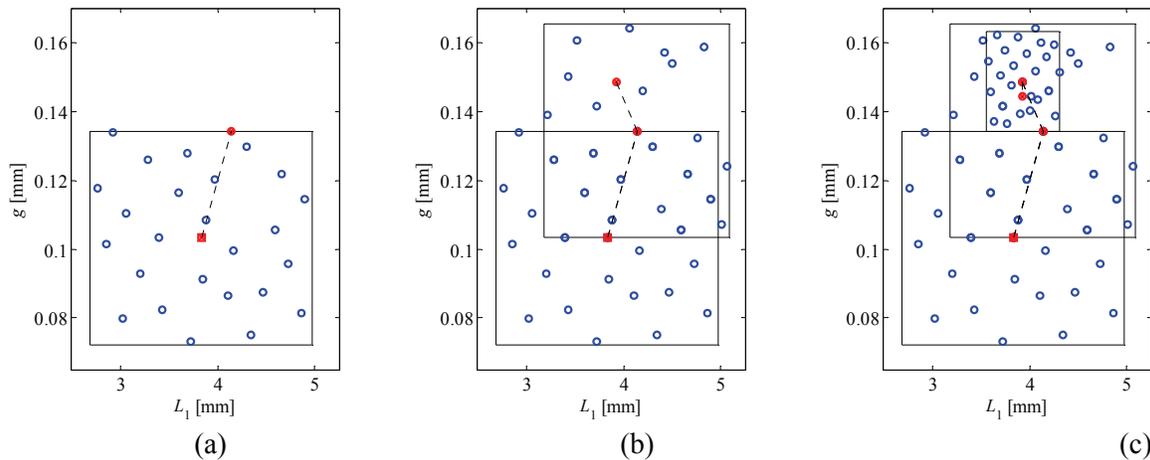


Fig. 5. Surrogate model domains, base sets (circles) and updated designs (filled circles) after: (a) first iteration, (b) second iteration, and (c) third iteration of the optimization procedure. Initial design is marked as a square.

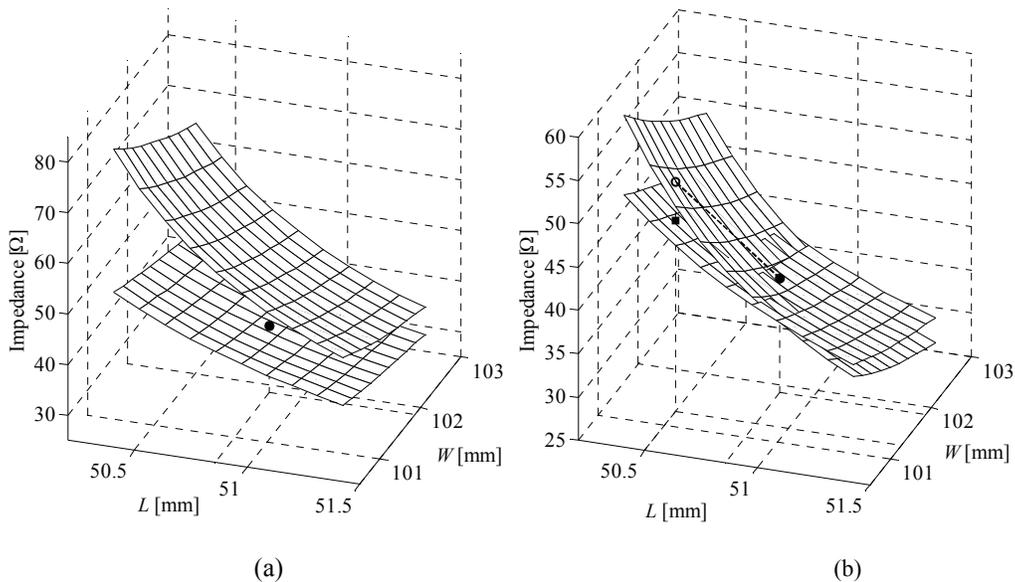


Fig. 7. Patch antenna: (a) fine model response surface (bottom) and the  $\mathbf{R}_{RBF}$  model response surface (top) at the first iteration of the proposed optimization procedure. Initial fine model response is denoted as the filled circle; (b) fine model response and the SM-corrected RBF model  $\mathbf{R}_{SM}$  at the first iteration. Initial fine model response, optimal response of the  $\mathbf{R}_{SM}$  model and the corresponding fine model response denoted as the filled circle, empty circle and the filled rectangle, respectively.

The direct optimization of the fine model using Matlab's *fmincon* routine was performed for comparison purposes using  $x^{(0)}$  as a starting point. Direct optimization required 54 evaluations of  $R_f$  to obtain a comparable design (almost 40 minutes of CPU time compared to less than 5 minutes required by the procedure discussed in this paper).

It should be noted that in case of the patch antenna no circuit equivalent model is available. This is a serious problem for the standard space mapping technique. In [21], the coarse-mesh FEKO model was used as a coarse model for space mapping algorithm to optimize the same patch antenna. Special meshing techniques had to be used to make the coarse model optimizable, and cost-

saving termination conditions were used. Nevertheless, the computational cost of SM optimization was about 50% to over 100% higher than that reported here (depending on the space mapping type used to build the surrogate model).

Table 2: Patch antenna: optimization cost.

Algorithm Component	Model Involved	Number of Model Evaluations	CPU Cost	
			$t_{opt}$ [min]	$t_{rel}$
Optimization of $\mathbf{R}_c$	$\mathbf{R}_c$	39	23	0.6
Setting up base sets for $\mathbf{R}_{RBF}$	$\mathbf{R}_c$	97	57	1.4
Evaluation of $\mathbf{R}_f$	$\mathbf{R}_f$	5	205	5
Total optimization time	N/A	N/A	285	7

### C. 2<sup>nd</sup>-Order Capacitively-Coupled Dual-Behavior Resonator (CCDBR) Microstrip Filter [19]

Consider a second-order capacitively-coupled dual-behavior resonator (CCDBR) microstrip filter [19] shown in Fig. 8. The design variables are  $\mathbf{x} = [L_1 L_2 L_3]^T$ . Parameter  $S$  is set to 0.05 mm. The fine model is simulated in FEKO [20]. The number of meshes for the fine model is 1134. Simulation time for the fine model is 37.7 min. The design specifications are  $|S_{21}| \geq -3$  dB for  $3.8$  GHz  $\leq \omega \leq 4.2$  GHz, and  $|S_{21}| \leq -20$  dB for  $2.0$  GHz  $\leq \omega \leq 3.2$  GHz and  $4.8$  GHz  $\leq \omega \leq 6.0$  GHz.

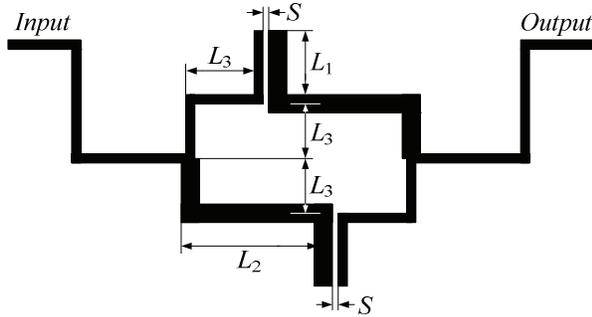


Fig. 8. Geometry of the 2<sup>nd</sup>-order CCDBR filter [19].

The coarse-mesh model  $\mathbf{R}_c$  is the structure in Fig. 8 also simulated in FEKO with the number of meshes equal to 130. The number of meshes for  $\mathbf{R}_f$  and  $\mathbf{R}_c$  correspond to  $\mathbf{x}_c = [2.89 6.24 0.92]^T$  mm (optimal solution of the circuit equivalent ADS model [22]). The simulation time for  $\mathbf{R}_c$  is 37 s.

Initial design  $\mathbf{x}^{(0)} = [2.97 4.69 1.54]^T$  mm is found by optimizing  $\mathbf{R}_c$  and requires 63 model

evaluations (small number of evaluations is due to using relaxed tolerance requirements). The number of base points to set up model  $\mathbf{R}_{RBF}$  is  $N = 50$ . Initial value of  $\delta$  is 0.1. As before, the space-mapping-corrected model  $\mathbf{R}_{SM}$  is of the form  $\mathbf{R}_{SM}(\mathbf{x}) = \mathbf{A} \cdot \mathbf{R}_{RBF}(\mathbf{x})$ .

Figure 9 shows the responses of models  $\mathbf{R}_f$  and  $\mathbf{R}_c$  at the initial design (fine model specification error +0.8 dB), the  $\mathbf{R}_f$  response at  $\mathbf{x}_c$  (specification error +6.7 dB), as well as the fine model response at the final design,  $\mathbf{x}^{(3)} = [3.21 4.63 1.27]^T$  mm, obtained after three iterations (specification error – 1.5 dB). The total number of evaluations of model  $\mathbf{R}_c$  is 112. Table 3 summarizes the computational cost of the optimization: the total optimization time corresponds to only 6.8 evaluations of  $\mathbf{R}_f$ .

For comparison purposes, the direct optimization of the fine model using Matlab's *fminimax* routine was performed using  $\mathbf{x}^{(0)}$  as a starting point. The design obtained after 100 evaluations of  $\mathbf{R}_f$  (over 63 hours of CPU time; the algorithm was terminated without convergence) corresponds to the specification error of –0.6 dB.

On the other hand, SM optimization of the filter using directly  $\mathbf{R}_c$  as a coarse model resulted in the design comparable with the one obtained using the proposed technique, however, the optimization time was 390 minutes, 50% more than for our method (with the total evaluation time of  $\mathbf{R}_c$  being 120% larger). For this example, the SM parameters can be determined analytically; otherwise (e.g., in case of using input SM [8]), the optimization cost would be substantially higher.

Table 3: CCDBR filter: optimization cost

Algorithm Component	Model Involved	Number of Model Evaluations	CPU Cost	
			$t_{opt}$ [min]	$t_{rel}$
Optimization of $\mathbf{R}_c$	$\mathbf{R}_c$	63	39	1.0
Setting up base sets for $\mathbf{R}_{RBF}$	$\mathbf{R}_c$	112	69	1.8
Evaluation of $\mathbf{R}_f$	$\mathbf{R}_f$	4	151	4.0
Total optimization time	N/A	N/A	259	6.8

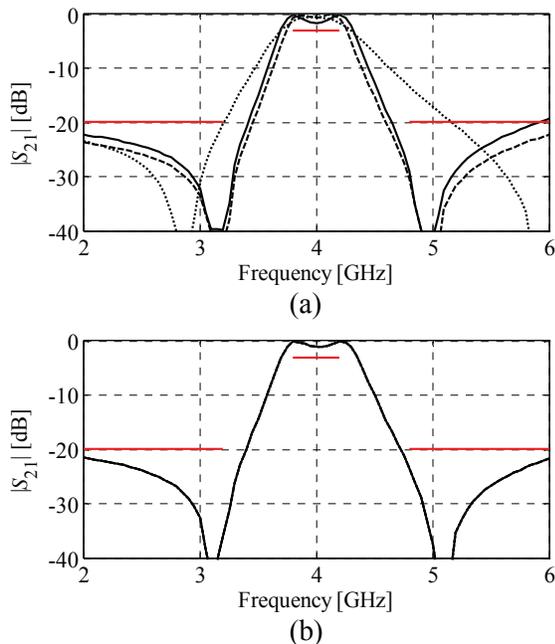


Fig. 9. 2<sup>nd</sup>-order CCDBR filter: (a) responses of  $R_f$  (solid line) and  $R_c$  (dashed line) at initial design  $x^{(0)}$  and response of  $R_f$  at  $x_c$  (dotted line); (b) response of  $R_f$  at the final design.

#### IV. CONCLUSION

An efficient algorithm for microwave design optimization is discussed that combines response-surface-approximation-based surrogate modeling, space mapping and multi-fidelity electromagnetic simulations. Unlike classical space mapping, the proposed technique does not require a circuit-equivalent or analytical coarse model, which makes it particularly suitable for problems where finding such a coarse model may be problematic, e.g., antennas. Although our technique is illustrated using microwave structures evaluated with FEKO, it can be used with any other electromagnetic simulator. It is demonstrated that the presented method is able to yield satisfactory design with the optimization time corresponding to a few evaluations of the fine model.

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