Using Adaptive Cross Approximation for Efficient Calculation of Monostatic Scattering with Multiple Incident Angles

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Abstract ─ An adaptive cross approximation (ACA) based method is proposed for the fast analysis of the monostatic radar cross-section (RCS). Using the low-rank property, several largest eigenvalues and corresponding eigenvectors of the multiple right hand sides can be computed and saved efficiently by the ACA algorithm. The iterative solution of linear equations is required at these principle eigenvectors. Compared with solving linear equations at each angle repeatedly, the proposed method is able to greatly reduce the computation time. In order to efficiently solve the linear equations, the flexible general minimal residual (FGMRES) iterative solver is applied to compute the coefficients of Rao-Wilton-Glisson (RWG) basis functions. Numerical results demonstrate that the proposed method is efficient for monostatic RCS calculation with high accuracy.

Index Terms ─ Adaptive cross approximation (ACA), low-rank property, monostatic RCS, and surface integral equation (SIE).

I. INTRODUCTION

Electromagnetic wave scattering problems address the physical issue of detecting the diffraction pattern of the electromagnetic radiation scattered from a large and complex body when illuminated by an incident incoming wave. A good understanding of these phenomena is crucial to radar cross section (RCS) calculation, antenna design, electromagnetic compatibility, and so on. All these simulations are very demanding in terms of computer resources, and require efficient numerical methods to compute an approximate solution of Maxwell's equations. Using the equivalence principle, Maxwell's equations can be recast in the form of integral equations that relate the electric and magnetic fields to the equivalent electric and magnetic currents on the surface of the object. Amongst integral formulations, the surface field integral equation (SIE) is widely used for electromagnetic wave scattering problems as it can handle the most general geometries. The matrix associated with the resulting linear systems is large, dense, complex, and non-Hermitian [1, 21]. It is basically impractical to solve matrix equations using direct methods because they have a memory requirement of $0(N^2)$, where N refers to the number of unknowns. This difficulty can be circumvented by use of iterative methods, and the required matrix-vector product operation can be efficiently evaluated by the multilevel fast multipole algorithm (MLFMA) [2, 3]. The use of MLFMA reduces the memory requirement to $0(NlogN)$ and the computational complexity of per-iteration to $0(NlogN)$. However, it is still time- and memory- consuming for calculation of monostatic RCS since it requires repeated solution of SIE at each incident direction.

Conventional interpolation methods, such as AWE and the cubic-spline (CS) interpolation method, can easily approximate the monostatic RCS. AWE [4, 5] is a kind of classical method

which is widely used in computational electromagnetics. It utilizes the high-order derivatives of the incident current vector at the interval center to extrapolate the value of nearby points, and is considered to be the AWE extrapolation method in this paper. AWE interpolation method [6] has been introduced by Wei. Piecewise interpolation technique is used and the first-order derivative of samples is required. The cubic spline interpolation method [7, 8] is another popular numerical approximate method. It utilizes the information of C^1-continuous to evaluate the first derivative of the incident current vector instead of solving the large linear equations.

How to select the sampling points is a difficult problem for interpolation and extrapolation methods. The active learning method is proposed by Zhao to obtain the optimal samples over wide frequency band [9, 10]. In [7, 8], Liu proposed an adaptive sampling method to obtain the optimal samples for monostatic RCS calculation with wide angular band. However, the performance of adaptive sampling is very sensitive with the initial samples. Furthermore, the sampling nodes are not the exact optimal samples but the approximately optimal samples.

In monostatic RCS computation [18], the main computation cost is the solution of the linear system with multiple right hand sides. Inspired by [11, 20, 22], the multiple right hand sides can be approximately described by a low-rank form. SVD can evaluate several large eigenvalues and corresponding eigenvectors of the multiple right hand sides. Moreover, the SVD process can be efficiently performed by the adaptive cross approximation (ACA) algorithm [17]. The process of solving linear equations is only required at these principle eigenvectors. Compared with the repeated solution at each angle, the proposed method is able to greatly reduce the times of linear equations solution. Furthermore, this non-adaptive method leads to a more robust algorithm than the adaptive sampling method. Accordingly, the ACA-based method is used for fast computation of monostatic RCS in this paper.

The remainder of this paper is organized as follows. Section II demonstrates the basic theory and formulations of surface integral equations. Section III describes the low-rank decomposition of multiple right hand sides in monostatic RCS computation. The application of the ACA algorithm is also discussed in this section. Numerical experiments of several geometries are presented to demonstrate the efficiency of this proposed method in Section IV. The conclusion is provided in Section V.

II. CFIE FORMULATINONS

For electromagnetic scattering from the perfect electrical conductor (PEC), the combined field integral equation (CFIE) is widely used for closed structure [12]. The CFIE formulation of electromagnetic wave scattering problems using planar RWG basis functions for surface modeling is presented in [13]. The resulting linear systems from CFIE formulation after Galerkin's testing are briefly outlined as follows:

$$\sum_{n=1}^{N} Z_{mn} a_n = V_m \,,\ m = 1,2,\ldots,N \tag{1}$$

where $Z_{mn} = \dfrac{\alpha}{\eta} Z_{mn}^{EFIE} + (1-\alpha) Z_{mn}^{MFIE}$ and

$$Z_{mn}^{EFIE} = jk\eta \int_{S_m} \mathbf{\Lambda}_m \int_{S_n} \left(\overline{\overline{\mathbf{I}}} + \frac{\nabla\nabla}{k^2} \right) G(\mathbf{r}, \mathbf{r}') \mathbf{\Lambda}_n d\mathbf{r}' d\mathbf{r}$$

$$Z_{mn}^{MFIE} = \frac{1}{2} \int_{S_m} \mathbf{\Lambda}_m \mathbf{\Lambda}_n d\mathbf{r} - \int_{S_m} \mathbf{\Lambda}_m \cdot \hat{\mathbf{n}} \times \nabla \times \int_{S_n} G(\mathbf{r}, \mathbf{r}') \mathbf{\Lambda}_n d\mathbf{r}' d\mathbf{r}$$

$$V_m = \int_{S_m} \mathbf{\Lambda}_m \cdot \left[\alpha \frac{\mathbf{E}^{inc}}{\eta} + (1-\alpha) \hat{\mathbf{n}} \times \mathbf{H}^{inc} \right] d\mathbf{r}.$$

Here, G($\mathbf{r}$, $\mathbf{r}'$) refers to the Green's function in free space and $\{a_n\}$ is the column vector containing the unknown coefficients of the surface current expansion with RWG basis functions. Also, $\mathbf{r}$ and $\mathbf{r}'$ denote the observation and source point locations. η and k denote the free space impendence and wave number, respectively. Once the matrix equation (1) is solved by numerical matrix equation solvers, the expansion coefficients $\{a_n\}$ can be used to calculate the scattered field and RCS. In the following, we use A to denote the coefficient matrix in equation (1), $\mathbf{x} = \{a_n\}$, and $\mathbf{b} = \{V_m\}$ for simplicity. Then, the CFIE matrix equation (1) can be symbolically rewritten as:

$$\mathbf{Ax} = \mathbf{b}. \tag{2}$$

To solve the above matrix equation by an iterative method, the matrix-vector products are needed at each iteration step. Physically, a matrix-vector product corresponds to one cycle of iterations between the basis functions. The basic idea of the fast multipole method (FMM) is to

convert the interaction of element-to-element to the interaction of group-to-group. Here, a group includes the elements residing in a spatial box. The mathematical foundation of the FMM is the addition theorem for the scalar Green's function in free space. Using the FMM, the matrix-vector product Ax can be written as:

$$\mathbf{Ax} = \mathbf{A}_N\mathbf{x} + \mathbf{A}_F\mathbf{x}. \quad (3)$$

Here, A_N is the near part of A and A_F is the far part of A. In the FMM, the calculation of matrix elements in A_N remains the same as in the MoM procedure. However, those elements in A_F are not explicitly computed and stored. Hence, they are not numerically available in the FMM. It has been shown that the operation complexity of FMM to perform Ax is $0(N^{1.5})$. If the FMM is implemented in multilevel, the total cost can be reduced further to $0(NlogN)$ [2, 3].

III. LOW-RANK DECOMPOSITION OF MULTIPLE RIGHT HAND SIDES

The monostatic RCS requires repeated calculations at each angle over the band of interest. This process can be computationally prohibitive for computation of monostatic RCS of an electrically large object. In order to accelerate the computation of monostatic RCS, an ACA based method is proposed and the methodology on how the efficient calculation of monostatic scattering is discussed in this section.

The computation of monostatic RCS can be considered as linear equations with multiple right hand sides

$$\mathbf{A}\cdot\mathbf{X} = \mathbf{B}, \quad (4)$$

where **A** is the impedance matrix, **X** is the multiple complex coefficient vector of RWG basis, and **B** is the multiple right hand side generated by the incident wave. And

$$\mathbf{X} = [\mathbf{x}(\theta_1), \ldots, \mathbf{x}(\theta_n)], \quad \mathbf{B} = [\mathbf{b}(\theta_1), \ldots, \mathbf{b}(\theta_n)], \quad (5)$$

where θ_i is the i^{th} incident angle. Using the traditional singular value decomposition (SVD), the matrix **B** can be described as the form of eigenvalue and eigenvector.

$$\mathbf{B} = \mathbf{U}\cdot\mathbf{\Sigma}\cdot\mathbf{V}^{H}. \quad (6)$$

If the dimension of **B** is $N \times M$, the dimension of matrices **U**, **Σ**, and **V** are $N \times M$, $M \times M$, $M \times M$, respectively. N is the number of unknowns. **Σ** is a diagonal matrix including all the eigenvalues of **B** while **U** and **V** contain all the eigenvectors of **B**. When **B** is the multiple right hand sides in the linear system connecting with the SIE used for monostatic RCS, matrix **B** is low-rank and can be approximately described as a low-rank SVD form.

$$\mathbf{B} \approx \mathbf{U}_k\cdot\mathbf{\Sigma}_k\cdot\mathbf{V}_k^{H}, \quad (7)$$

where the dimension of matrices $\mathbf{U}_k$、$\mathbf{\Sigma}_k$ and $\mathbf{V}_k$ are $N \times k$, $k \times k$, $M \times k$, respectively. Only k largest eigenvalues and corresponding eigenvectors are reserved in (7). Substitude (7) to (4), the linear equations can be rewritten as

$$\mathbf{X} \approx (\mathbf{A}^{-1}\cdot\mathbf{U}_k)\cdot\mathbf{\Sigma}_k\cdot\mathbf{V}_k^{H}. \quad (8)$$

Here, $\mathbf{A}^{-1}\cdot\mathbf{U}_k$ can be computed by any iterative solver. If using direct solver to compute the inversion of matrix **A** [19], the proposed method will become useless. Therefore, the times for solution of equation (2) is only k when using the SVD method while that of traditional direct solution is M for monostatic RCS, where M is the number of multiple right hand sides. Generally, k is much smaller than M which leads to an efficient method for computation of monostatic RCS over a wide angular band.

The key problem for this proposed method is how to obtain the decomposition form of multiple right hand sides. As well known, the traditional SVD method is a good analytical solution for this problem. However, SVD requires the computation of the matrix including all right hand sides and the complexity of the computation time of SVD is $O(nm^2 + mn^2)$, where m and n respect to the number of rows and columns. When the number of unknowns or right hand vectors is large, this analytical solution is not practical. In order to alleviate this difficulty, an adaptive cross approximation (ACA) algorithm is applied and performs a more efficient property than the traditional SVD method.

Using the adaptive cross approximation approach, a low-rank matrix can be decomposed into two matrices $\mathbf{U}_{ACA}$ and $\mathbf{V}_{ACA}$. The RHS matrix **B**, which is low-rank, can also be decomposed into $\mathbf{U}_{ACA}$ and $\mathbf{V}_{ACA}$ matrices. The formula of the decomposition is listed below

$$\mathbf{B} \approx \mathbf{U}_{ACA}\cdot\mathbf{V}_{ACA}^{H}, \quad (9)$$

where the dimension of matrices $\mathbf{U}_{ACA}$ and $\mathbf{V}_{ACA}$ are $N \times k$, $M \times k$, respectively. Substitute (9) to (4), the linear equations can be rewritten as

$$\mathbf{X} \approx (\mathbf{A}^{-1}\cdot\mathbf{U}_{ACA})\cdot\mathbf{V}_{ACA}^{H}. \quad (10)$$

k is much smaller than M which leads to an efficient method for computation of monostatic

RCS over a wide angular band. Here, $\mathbf{A}^{-1}\cdot\mathbf{U}_{ACA}$ is computed by an iterative solver.

The algorithm of ACA is presented as follows, which allows it to generate only a few rows and columns of the matrix and approximates the rest of the matrix using only this information.

Adaptive Cross Approximation Algorithm

Let $\mathbf{U}_{ACA}$ = NULL which is used to save the selected columns and $\mathbf{V}_{ACA}$ = NULL which is used to save the selected rows.

Step1: Choose the first column $\mathbf{u}_1$ randomly and let $\mathbf{U}_{ACA} = \mathbf{U}_{ACA} \cup \{\mathbf{u}_1\}$. Find the max value $\mathbf{u}_{1k}$ in $\mathbf{u}_1$. Then choose the first row $\mathbf{v}_1$ which located at kth row in the matrix. Let $\mathbf{V}_{ACA} = \mathbf{V}_{ACA} \cup \{\mathbf{v}_1\}$.

Step 2: Find the max value $\mathbf{v}_{ik}$ in $\mathbf{v}_i$. Then choose the $(i + 1)$th column u_{i+1} which located at kth column in the matrix.

Step 3: $\mathbf{u}_{i+1} = \mathbf{u}_{i+1} - \sum_{j=1}^{i} v_{j,i}\mathbf{u}_j$, and let $\mathbf{U}_{ACA} = \mathbf{U}_{ACA} \cup \{\mathbf{u}_{i+1}\}$.

Step 4: Find the max value $\mathbf{u}_{i+1,k}$ in $\mathbf{u}_{i+1}$. Then choose the $(i + 1)$th row $\mathbf{v}_{i+1}$ which located at kth row in the matrix.

Step 5: $\mathbf{v}_{i+1} = \mathbf{v}_{i+1} - \sum_{j=1}^{i} u_{j,i}\mathbf{v}_j$, and let $\mathbf{V}_{ACA} = \mathbf{V}_{ACA} \cup \{\mathbf{v}_{i+1}\}$.

Step 6: If $\dfrac{\langle \mathbf{u}_{i+1}, \mathbf{u}_{i+1}\rangle \langle \mathbf{v}_{i+1}, \mathbf{v}_{i+1}\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle \langle \mathbf{v}_1, \mathbf{v}_1\rangle} < \varepsilon$, the algorithm will stop, otherwise, go to Step 2. The low-rank decomposition form of RHS matrix is $\mathbf{B} \approx \mathbf{U}_{ACA}\cdot\mathbf{V}_{ACA}^{\mathrm{H}}$.

This algorithm produces a sequence of decompositions of a matrix into a sum of low-rank matrix and error matrix. Neither the original matrix nor the error matrix will be computed completely. How to decide the tolerance error ε is the most important thing in ACA. In order to avoid the numerical error, the ε is required to be small enough. In this paper, the ε satisfies $\varepsilon = 10^{-3}$.

IV. NUMERICAL RESULTS

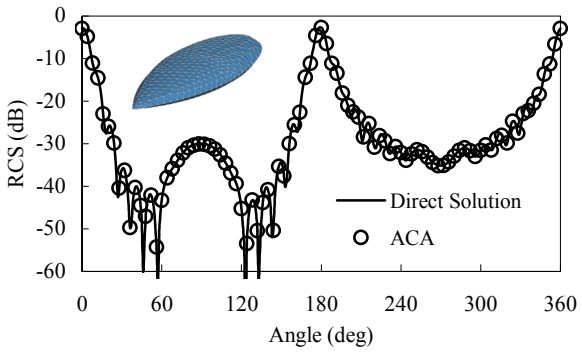
In this section, a number of numerical results are presented to demonstrate the accuracy and efficiency of the ACA based method for fast calculation of monostatic RCS over wide angular band. The flexible general minimal residual (FGMRES) [14, 15] algorithm is applied to solve linear systems. The dimension size of the Krylov subspace is set to be 30 for the outer iteration and the dimension is set to be 10 for the inner iteration. The tolerance of the inner iteration is 0.1 in this paper. All experiments are conducted on an Intel Core(TM) II Duo with 3.45 GB local memory and run at 2.40 GHz in single precision. The iteration process is terminated when the 2-norm residual error is reduced by 10^{-3}, and the limit of the maximum number of iterations is set as 1000.

Three geometries are applied to illustrate the performance of our proposed method. They consist of a NASA almond with 1815 unknowns [16], a PEC double-ogive with 4635 unknowns, and the VFY-218 model with 40725 unknowns. Since the number of right hand vectors is small for 1-D angular sweep in this paper, it is feasible to apply the traditional SVD for computing the eigenspace of multiple right hand sides. As shown in Figs.1-3, the monostatic RCS curve of NASA almond, double-ogive, and VFY-218 which computed by ACA method is compared with the curve computed by direct solution repeatedly. It is obvious that the ACA method is accurate since there is no significant difference between the RCS result obtained by the direct solution and the ACA method. As shown in Tab.1, when compared with the traditional SVD method, the ACA based method provides little advantage on total computation time since the number of right hand side is small.

For monostatic RCS simultaneous theta and phi sweep, the number of right hand sides is 32761 (181×181) for almond and double-ogive in this paper. Due to the time complexity of traditional SVD is $O(nm^2 + mn^2)$, it is not suitable to obtain the eigenvalue of the multiple right hand sides directly. Thus, the ACA is applied in the last two examples. From these results, the same conclusion as that in 1-D monostatic RCS examples can be obtained, which shows the ACA based method is accurate.

The relative error of the last two examples is demonstrated by Figs. 4(c), 5(c), 6(c), and 7(c). In order to demonstrate the relative error of the proposed method, the formulation of relative error is defined as

$$error = \frac{\left|\mathbf{E}_{svd} - \mathbf{E}_{direct}\right|}{\left|\mathbf{E}_{direct}\right|} \quad . \qquad (9)$$



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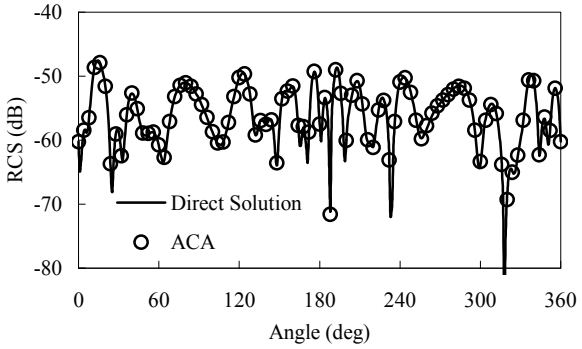
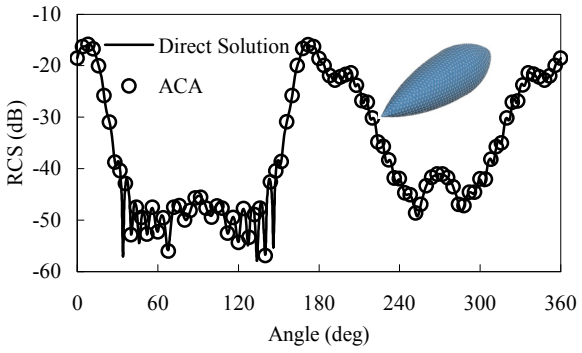


(b)

Fig. 1. monostatic RCS of NASA almond for simultaneous theta sweep: (a) HH-pol; (b) HV-pol.



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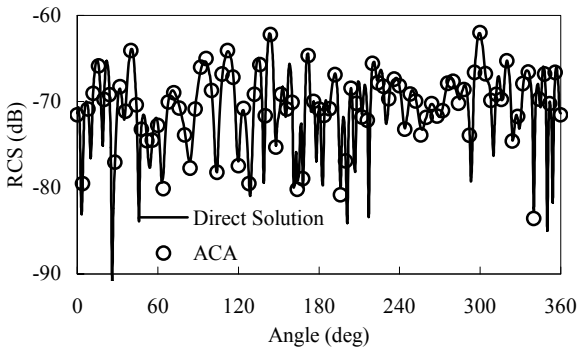
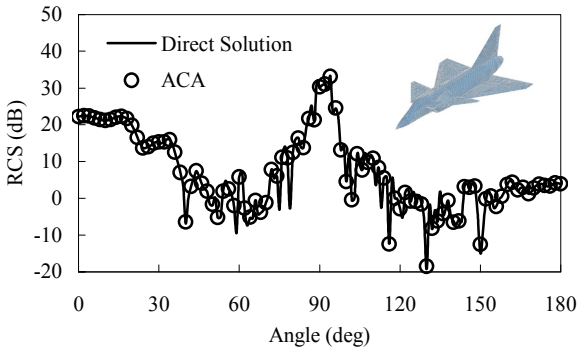


(b)

Fig. 2. monostatic RCS of double-ogive for simultaneous theta sweep: (a) HH-pol; (b) HV-pol.



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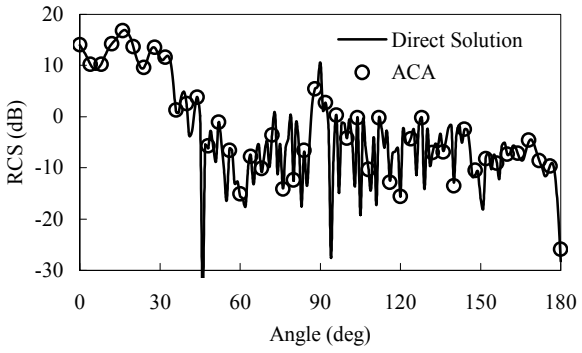
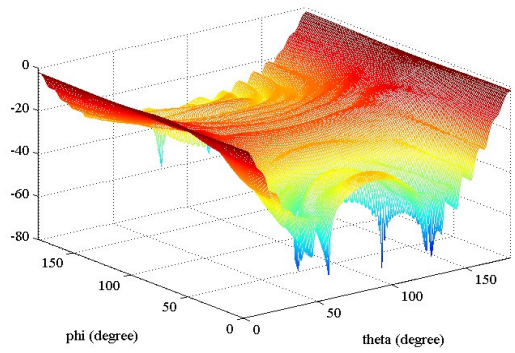


(b)

Fig. 3. monostatic RCS of VFY-218 for simultaneous theta sweep: (a) HH-pol; (b) HV-pol.



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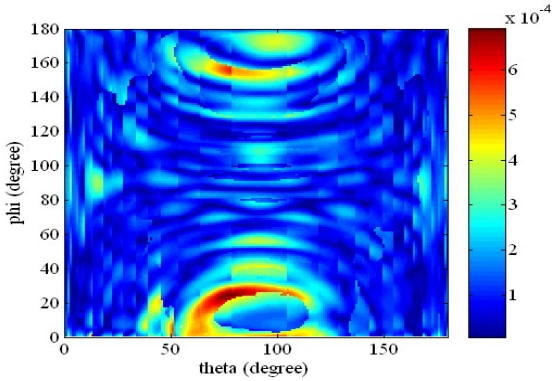
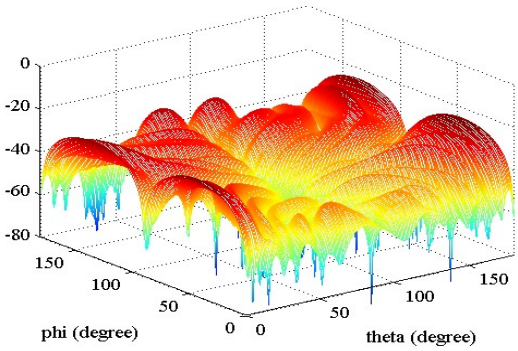


(b)

Fig. 4. Monostatic RCS of NASA almond for simultaneous theta and phi sweep, HH-pol: (a) RCS; (b) relative error.



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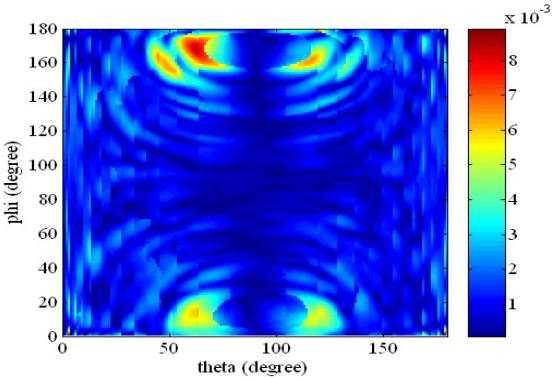
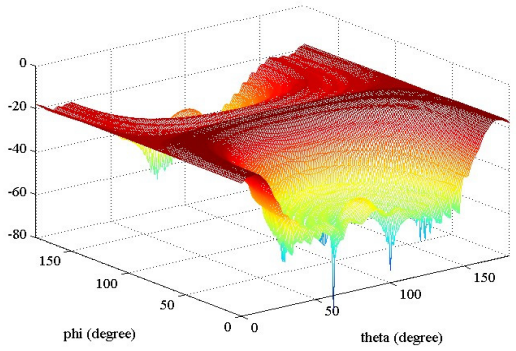


(b)

Fig. 5. Monostatic RCS of NASA almond for simultaneous theta and phi sweep, HV-pol: (a) RCS; (b) relative error.



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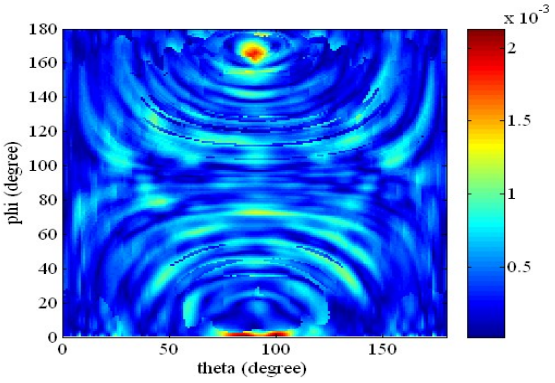


(b)

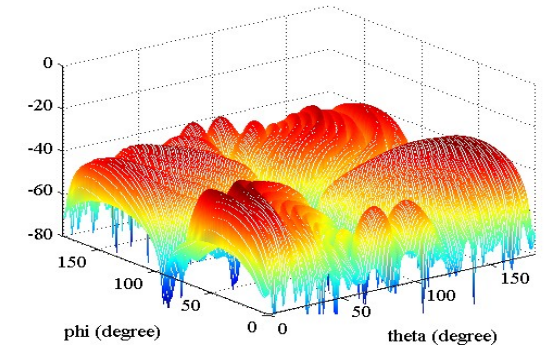
Fig. 6. Monostatic RCS of double-ogive for simultaneous theta and phi sweep, HH-pol: (a) RCS; (b) relative error.

Table 1: Computation time of monostatic RCS with 1-D angular sweep

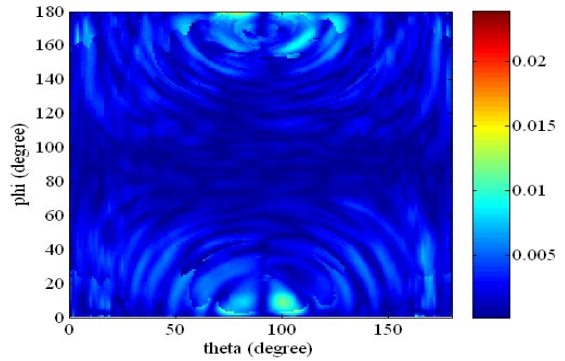
Object	f (Hz)	N	Pol	Angular Sweep		Time (Second) / Number of Solutions		
				Theta (deg)	Phi (deg)	Direct Solution	Traditional SVD	ACA
Almond	5G	1815	HH	0~360	0	222 / 361	38 / 22	26 / 25
			HV	0~360	0	234 / 361	39 / 26	26 / 26
D-Ogive	12G	2571	HH	0~360	0	652 / 361	92 / 45	89 / 45
			HV	0~360	0	647 / 361	92 / 41	88 / 45
VFY-218	0.5G	40725	HH	90	0~180	106618 / 361	29855 / 73	28732 / 73
			HV	90	0~180	106545 / 361	29734 / 73	28744 / 73

Table 2: Computation time of monostatic RCS for simultaneous theta and phi sweep

Object	f (Hz)	N	Pol	Angular Sweep		Time (Second) / Number of Solutions		
				Theta (deg)	Phi (deg)	Direct Solution	Traditional SVD	ACA
Almond	5G	1815	HH	0~180	0~180	20315 / 32761	2340 / 158	570 / 159
			HV	0~180	0~180	20322 / 32761	2287 / 161	566 / 161
D-Ogive	12G	2571	HH	0~180	0~180	60832 / 32761	8714 / 293	2208 / 295
			HV	0~180	0~180	60813 / 32761	8862 / 295	2187 / 295



(a)



(b)

Fig. 7. Monostatic RCS of double-ogive for simultaneous theta and phi sweep, HV-pol: (a) RCS; (b) relative error.

From the error shown in Figs. 4(c), 5(c), 6(c), and 7(c), it is obvious that the error of cross-polarization is larger than co-polarization. However, the error of both polarizations is small enough to obtain the accurate result. Thus, the ACA based method can be considered as an accurate method.

Tables 1 and 2 show the CPU time of direct solution, traditional SVD based method, and ACA based method. It is shown that plenty of time can be saved in traditional SVD and ACA based method since the times of equation solutions is much fewer than solving the linear equations (4) repeatedly. Comparing the CPU time between the traditional SVD method and the ACA method, it is concluded that the ACA method is more efficient when the number of right hand side vectors is large enough.

V. CONCLUSION

In this paper, an ACA based method is proposed for efficient analysis of the monostatic scattering from electrically large objects over a wide angular band. After obtaining the eigenvalues and eigenvectors of the multiple right hand sides by the adaptive cross approximation algorithm, the total solution time can be saved since the iterative solution of linear system is only needed at several largest eigenvalues. In order to fast solution of the linear system, the MLFMA,

and Krylov subspace iterative solver are used to efficiently solve the linear equations. Numerical experiments demonstrate that our proposed method is more efficient when compared with the repeated solution at each angle for electromagnetic scattering from the electrically large objects.

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