Generalized Formulation for the Scattering from a Ferromagnetic Microwire

Ahmed M. Attiya and Majeed A. Alkanhal

Electrical Engineering Dept.
King Saud University, Riyadh, 11421, P.O.Box 800, Saudi Arabia
aattiya@ksu.edu.sa, majeed@ksu.edu.sa

Abstract **─** In this paper, we present a full wave analysis for the scattering of an obliquely incident plane wave of an arbitrary polarization angle due to an infinite magnetized ferromagnetic microwire. The analysis is based on expanding the incident and scattered waves as infinite sets of cylindrical waves and matching the corresponding modes on the surface of the microwire to obtain the unknown amplitudes of the scattered fields. The gyromagnetic properties of the ferromagnetic material introduce a coupling between the electric and magnetic field components inside the microwire. This coupling is the reason of cross polarization in the scattered field.

Index Terms: Analytical techniques, ferrites, ferromagnetic microwires, scattering.

I. INTRODUCTION

Ferromagnetic microwires based on amorphous ferromagnetic alloy compositions have gained a significant interest in RF and microwave applications due to their unique electromagnetic properties and the feasibility of simple fabrication and manufacturing process [1-2]. These ferromagnetic materials are characterized by high electrical conductivity, ferromagnetic resonance, gyromagnetic properties, and tunable characteristics. These properties make ferromagnetic microwires to be good candidates for different applications in microwave range including synthesizing artificial double negative metamaterials, shielding materials, and absorbing materials [3-5]. Another similar configuration for synthesizing double negative metamaterial is based on an array of conducting wires embedded inside a ferromagnetic host [6]. Tuning properties of ferromagnetic microwires make them suitable to obtain tunable synthetic materials [7-11]. This can also be used to design microwave sensing tools like stress sensors [10]. In addition, it may be also expected to obtain polarization rotation from these ferromagnetic materials due to their gyromagnetic properties. This polarization rotation can introduce an additional feature which may be useful to other applications like electromagnetic polarizer or radar cross section reduction.

These unique properties of ferromagnetic microwires were the motivation for different authors to introduce different techniques for modeling the electromagnetic wave interaction with these microwires. Effective medium properties and magneto-impedance are used to obtain simple analytical expressions for the equivalent effective parameters of ferromagnetic microwire composites [11]. Other numerical techniques like TLM and FDTD can also be used to present a numerical analysis for the scattering from general gyrotropic media [12-13]. However, the small diameter of the ferro-magnetic microwire represents a significant computational challenge in these numerical methods. More recently, Liberal *et al* [14-15] introduced a full wave analysis for the scattering of an infinite ferromagnetic microwire and an infinite array of these microwires. Their analysis is based on expanding the incident and scattered waves as superpositions of cylindrical waves. The unknown amplitudes of the scattered fields are obtained by applying the boundary conditions on the surface of the microwire. They also extended their analysis to study the scattering of a ferromagnetic

Submitted On: Jan. 8, 2012
Accepted On: April 20, 2012

microwire inside a rectangular waveguide and verified their analysis with experimental results. The advantage of this analysis is that it is full wave analysis which includes all the parameters of the ferromagnetic microwire without introducing approximation. Liberal *et al* [14-15] showed that this analysis cannot be obtained by other numerical techniques like method of moments or finite difference time domain due to the complicated properties of the ferromagnetic materials combined with the very small radius of the microwire.

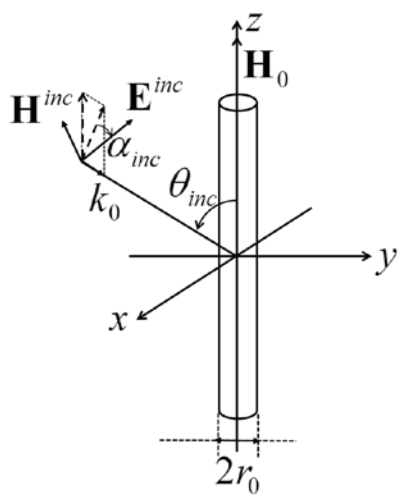


Fig. 1. Geometry of the problem.

However, the analysis of Liberal *et al* [14-15] was limited only for the case of normal incidence on the microwire with an electric field polarization parallel to its axis. For the mentioned applications of ferromagnetic microwires, it may be required to have a general insight of the electromagnetic wave interaction with ferromagnetic microwire at different angles of incidence and different polarization. This was the motivation here to extend their analysis for the case of the scattering from a ferromagnetic microwire due to an obliquely incident plane wave of an arbitrary polarization as shown in Fig. 1. The main difference in this case is coupling between the electric and magnetic field components in the ferromagnetic material due to its gyromagnetic properties. This coupling vanishes completely for normal incidence. Thus, this property is not included in the analysis of Liberal *et al* [14-15]. The resulting wave equation inside the ferromagnetic microwire in this case is represented as a fourth order differential equation as shown in the analysis in the following section. This is similar to the problem of electromagnetic wave interaction with ferrite material in closed and open waveguides which was studied by different authors [16-19].

In the following section, we present the analysis of the problem. In Section 3, sample results for the scattering of two different ferromagnetic microwires are presented for different angle of incidence and different polarization angles. Then concluding remarks are discussed.

II. THEORY AND ANALYSIS

In the region $\rho \geq a_0$ outside the microwire, the incident plane wave and scattered fields along the longitudinal direction of the wire can be represented as a combination of inward and outward cylindrical functions as follows:

$$E_z^{inc} = \left(E_0 \sin\theta_{inc} \cos\alpha_{inc}\right) \sum_{n=-\infty}^{\infty} j^n J_n(\beta_{\rho 0}\rho) e^{-j\beta_z z} e^{-jn\phi} \text{ ,} \tag{1-a}$$

$$H_z^{inc} = \left(\frac{E_0}{Z_0} \sin\theta_{inc} \sin\alpha_{inc}\right) \sum_{n=-\infty}^{\infty} j^n J_n(\beta_{\rho 0}\rho) e^{-j\beta_z z} e^{-jn\phi} \text{ ,} \tag{1-b}$$

$$E_z^{s} = \left(E_0 \sin\theta_{inc}\right) \sum_{n=-\infty}^{\infty} C_n H_n^{(2)}(\beta_{\rho 0}\rho) e^{-j\beta_z z} e^{-jn\phi} \text{ ,} \tag{2-a}$$

$$H_z^{s} = \left(\frac{E_0}{Z_0} \sin\theta_{inc}\right) \sum_{n=-\infty}^{\infty} D_n H_n^{(2)}(\beta_{\rho 0}\rho) e^{-j\beta_z z} e^{-jn\phi} \text{ ,} \tag{2-b}$$

where $\beta_z = k_0 \cos\theta_{inc}$ is the longitudinal propagation constant and $\beta_{\rho 0} = k_0 \sin\theta_{inc}$ is the radial propagation constant. The amplitude of the incident field is assumed to be unity such that $E_0 = 1V/m$. It should be noted that the polarization angle of the scattered is included in the unknown amplitudes of the scattered electric and magnetic fields, C_n and D_n, respectively.

The transverse field components can be obtained in terms of the longitudinal field components. For the present case, we are interested in the ϕ components of the electric and magnetic fields to be applied in the tangential boundary conditions on the microwire. These ϕ components can be presented in terms of the longitudinal field components as follows:

$$E_\phi = \left(-\frac{j\beta_z}{\beta_{\rho 0}^2}\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu_0}{\beta_{\rho 0}^2}\frac{\partial H_z}{\partial \rho} \right), \quad (3\text{-a})$$

$$H_\phi = \left(-\frac{j\omega\varepsilon_0}{\beta_{\rho 0}^2}\frac{\partial E_z}{\partial \rho} - \frac{j\beta_z}{\beta_{\rho 0}^2}\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} \right). \quad (3\text{-b})$$

By applying (3) into (1) and (2), the incident and scattered ϕ field components are obtained as:

$$E_\phi^{inc} = -E_0 \frac{n\cos\theta_{inc}\cos\alpha_{inc}}{k_0\rho\sin\theta_{inc}} \sum_{n=-\infty}^{\infty} j^n J_n(\beta_{\rho 0}\rho)e^{-j\beta_z z}e^{-jn\phi} + jE_0 \sin\alpha_{inc} \sum_{n=-\infty}^{\infty} j^n J_n'(\beta_{\rho 0}\rho)e^{-j\beta_z z}e^{-jn\phi} , \quad (4\text{-a})$$

$$H_\phi^{inc} = -j\frac{E_0}{Z_0}\cos\alpha_{inc} \sum_{n=-\infty}^{\infty} j^n J_n'(\beta_{\rho 0}\rho)e^{-j\beta_z z}e^{-jn\phi} - \frac{E_0}{Z_0}\frac{n\cos\theta_{inc}\sin\alpha_{inc}}{k_0\rho\sin\theta_{inc}} \sum_{n=-\infty}^{\infty} j^n J_n(\beta_{\rho 0}\rho)e^{-j\beta_z z}e^{-jn\phi} , \quad (4\text{-b})$$

$$E_\phi^{s} = -E_0 \frac{n\cos\theta_{inc}}{k_0\rho\sin\theta_{inc}} \sum_{n=-\infty}^{\infty} C_n H_n^{(2)}(\beta_{\rho 0}\rho)e^{-j\beta_z z}e^{-jn\phi} + jE_0 \sum_{n=-\infty}^{\infty} D_n H_n'^{(2)}(\beta_{\rho 0}\rho)e^{-j\beta_z z}e^{-jn\phi} , \quad (4\text{-c})$$

$$H_\phi^{s} = -j\frac{E_0}{Z_0} \sum_{n=-\infty}^{\infty} C_n H_n'^{(2)}(\beta_{\rho 0}\rho)e^{-j\beta_z z}e^{-jn\phi} - \frac{E_0}{Z_0}\frac{n\cos\theta_{inc}}{k_0\rho\sin\theta_{inc}} \sum_{n=-\infty}^{\infty} D_n H_n^{(2)}(\beta_{\rho 0}\rho)e^{-j\beta_z z}e^{-jn\phi} . \quad (4\text{-d})$$

On the other hand, the fields inside the ferromagnetic microwire biased by a dc magnetic field along its axis at $\rho \le a_0$ are related as:

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -j\omega\mu H_\rho + \omega\kappa H_\phi , \quad (5\text{-a})$$

$$\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho} = -j\omega\mu H_\phi - \omega\kappa H_\rho , \quad (5\text{-b})$$

$$\frac{1}{\rho}\frac{\partial(\rho E_\phi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial E_\rho}{\partial \phi} = -j\omega\mu_0 H_z , \quad (5\text{-c})$$

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = j\omega\left(\varepsilon_0 - j\frac{\sigma}{\omega}\right)E_\rho , \quad (5\text{-d})$$

$$\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} = j\omega\left(\varepsilon_0 - j\frac{\sigma}{\omega}\right)E_\phi , \quad (5\text{-e})$$

$$\frac{1}{\rho}\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \phi} = j\omega\left(\varepsilon_0 - j\frac{\sigma}{\omega}\right)E_z , \quad (5\text{-f})$$

where μ and k are the elements of the permeability tensor of the ferromagnetic material and σ is its electrical conductivity. The parameters of ferromagnetic materials are given by [14-15]:

$$\mu = \mu_0 + \mu_0\left(\frac{\omega_0\omega_m\left(\omega_0^2 - \omega^2(1-\alpha^2)\right) - j\alpha\omega\omega_m\left(\omega_0^2 + \omega^2(1+\alpha^2)\right)}{\left[\omega_0^2 - \omega^2(1+\alpha^2)\right]^2 + 4\omega_0^2\omega^2\alpha^2} \right) \quad (6\text{-a})$$

$$k = \mu_0\left(\frac{\omega\omega_m\left(\omega_0^2 - \omega^2(1+\alpha^2)\right) - j2\omega_0\omega_m\omega^2\alpha}{\left[\omega_0^2 - \omega^2(1+\alpha^2)\right]^2 + 4\omega_0^2\omega^2\alpha^2} \right), \quad (6\text{-b})$$

where $f_0 = \omega_0 / 2\pi$ is the Larmor frequency, $f_m = \omega_m / 2\pi$ is the resonance frequency at the saturation limit and α is a dimensionless damping factor.

It should be noted that the longitudinal dependence of the field components inside the microwire is the same as outside the microwire to satisfy the tangential propagation boundary condition. Thus, the field dependence inside the microwire is also $e^{-j\beta_z z}$ where $\beta_z = k_0\cos\theta_{inc}$ is the same as in the free space surrounding the microwire.

The transverse field components inside the ferromagnetic microwire can be obtained in terms of the longitudinal field components as follows:

$$E_\rho = \frac{1}{D}\left(a\frac{\partial E_z}{\partial \rho} + b\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} - e\frac{\partial H_z}{\partial \rho} - d\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} \right), \quad (7\text{-a})$$

$$E_\phi = \frac{1}{D}\left(-b\frac{\partial E_z}{\partial \rho} + a\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} + d\frac{\partial H_z}{\partial \rho} - e\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} \right), \quad (7\text{-b})$$

$$H_\rho = \frac{1}{D}\left(b\frac{\omega\varepsilon}{\beta_z}\frac{\partial E_z}{\partial \rho} - a\frac{\omega\varepsilon}{\beta_z}\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} + a\frac{\partial H_z}{\partial \rho} + b\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} \right) \quad (7\text{-c})$$

$$H_\phi = \frac{1}{D}\left(a\frac{\omega\varepsilon}{\beta_z}\frac{\partial E_z}{\partial \rho} + b\frac{\omega\varepsilon}{\beta_z}\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} - b\frac{\partial H_z}{\partial \rho} + a\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} \right) \quad (7\text{-d})$$

where

$$a = j\beta_z\beta_\rho^2 , \quad (8\text{-a})$$

$$b = \omega^2\kappa\beta_z\left(\varepsilon_0 - j\frac{\sigma}{\omega}\right), \quad (8\text{-b})$$

$$d = -j\omega\mu\left(\beta_\rho^2 - \frac{\omega^2\kappa^2}{\mu}\left(\varepsilon_0 - j\frac{\sigma}{\omega}\right) \right) , \quad (8\text{-c})$$

$$e = \omega\kappa\beta_z^2 , \quad (8\text{-d})$$

$$D = \left(\omega^2\kappa\varepsilon\right)^2 - \beta_\rho^4 , \quad (8\text{-e})$$

$$\beta_\rho = \sqrt{\omega^2\mu\left(\varepsilon_0 - j\frac{\sigma}{\omega}\right) - \beta_z^2} . \quad (8\text{-f})$$

By inserting Eq. (7) with the definitions of Eq. (8) into Eq. (5), one can obtain two coupled wave equations of the longitudinal field components as follows [16-17]:

$$\nabla_t^2 E_z + c_1 E_z - j d_1 H_z = 0, \tag{9-a}$$

$$\nabla_t^2 H_z + f_1 H_z + j g_1 E_z = 0, \tag{9-b}$$

where

$$c_1 = \left(\beta_{\rho 1}^2 - \frac{\omega^2 \kappa^2}{\mu}\left(\varepsilon_0 - j\frac{\sigma}{\omega}\right)\right), \tag{10-a}$$

$$d_1 = \frac{\mu_0 \omega \kappa \beta_z}{\mu}, \tag{10-b}$$

$$f_1 = \frac{\mu_0 \beta_\rho^2}{\mu}, \tag{10-c}$$

$$g_1 = \frac{\omega \kappa \beta_z}{\mu}\left(\varepsilon_0 - j\frac{\sigma}{\omega}\right). \tag{10-d}$$

It should be noted that the coupling between the longitudinal electric and magnetic field components vanishes for the case of normal incidence where $\beta_z = 0$. These two coupled wave equations can be represented as two decoupled fourth order differential equation as follows:

$$\left(\nabla_t^2 \nabla_t^2 + (f_1 + c_1)\nabla_t^2 + f_1 c_1 - d_1 g_1\right)\Psi_z = 0, \tag{11}$$

where Ψ_z is either E_z or H_z. This fourth order differential equation can be represented as a multiplication of two wave equations of different propagation constants as follows:

$$\left(\nabla_t^2 + \gamma_{\rho 1}^2\right)\left(\nabla_t^2 + \gamma_{\rho 2}^2\right)\Psi_z = 0, \tag{12-a}$$

where the two propagation constants are:

$$\gamma_{\rho 1/2} = \sqrt{\frac{1}{2}\left((f_1 + c_1) \pm \sqrt{(f_1 - c_1)^2 + 4 d_1 g_1}\right)}. \tag{12-b}$$

The general solution of Eq. (12) for the longitudinal electric field inside the microwire in terms of cylindrical waves is [16-17]:

$$E_z^t = \sum_{n=-\infty}^{\infty}\left(A_n J_n(\gamma_{\rho 1}\rho) + B_n J_n(\gamma_{\rho 2}\rho)\right) e^{-jn\phi} e^{-j\beta_z z}. \tag{13-a}$$

A similar solution can be obtained for the longitudinal magnetic field. The unknown amplitudes of the longitudinal magnetic field can be related to the corresponding ones of the longitudinal electric field by inserting Eq. (13-a) into Eq. (9-b). Thus, the general solution of the longitudinal magnetic field component in terms of the unknown amplitudes of the longitudinal electric field component is given by:

$$H_z^t = \sum_{n=-\infty}^{\infty}\left(\eta_1 A_n J_n(\gamma_{\rho 1}\rho) + \eta_2 B_n J_n(\gamma_{\rho 2}\rho)\right) e^{-jn\phi} e^{-j\beta_z z}, \tag{13-b}$$

where the coupling coefficients between the longitudinal electric and magnetic field components are given by:

$$\eta_{1/2} = \frac{j g_1}{\gamma_{\rho 1/2}^2 - f_1}. \tag{14}$$

By inserting Eq. (13) into Eqs. (7-b) and (7-d), one can obtain the ϕ components of the electric and magnetic fields inside the microwire as:

$$E_\phi^t = \sum_{n=-\infty}^{\infty}\left[A_n \mathrm{X}_{1n}(\rho) + B_n \mathrm{X}_{2n}(\rho)\right] e^{-jn\phi} e^{-j\beta_z z}, \tag{15-a}$$

$$H_\phi^t = \sum_{n=-\infty}^{\infty}\left[A_n \Lambda_{1n}(\rho) + B_n \Lambda_{2n}(\rho)\right] e^{-jn\phi} e^{-j\beta_z z}, \tag{15-b}$$

where

$$\mathrm{X}_{in}(\rho) = \frac{1}{D}\left(d\eta_i \gamma_{\rho i} - b\gamma_{\rho i}\right) J_n'(\gamma_{\rho i}\rho) + \frac{1}{D}\frac{jn(e\eta_i - a)}{\rho} J_n(\gamma_{\rho i}\rho), \tag{16-a}$$

$$\Lambda_{in}(\rho) = \frac{1}{D}\left(a\gamma_{\rho i}\frac{\omega}{\beta_z}\left(\varepsilon_0 - j\frac{\sigma}{\omega}\right) - b\gamma_{\rho i}\eta_i\right) J_n'(\gamma_{\rho i}\rho) - j\frac{1}{D}\frac{n}{\rho}\left(a\eta_i + b\frac{\omega}{\beta_z}\left(\varepsilon_0 - j\frac{\sigma}{\omega}\right)\right) J_n(\gamma_{\rho i}\rho). \tag{16-b}$$

"*i*" here stands for either 1 or 2. By applying the boundary conditions of the tangential field components at the boundary of the microwire one can obtain a linear system of equations for each cylindrical wave mode "*n*" to obtain the corresponding unknown field amplitudes as shown in Eq. 17 below.

By solving the above system of equation one can obtain the unknown amplitudes of the scattered and penetrated field components. For a very thin wire compared with the free space wave length as in the present case of the microwire, the zero-order cylindrical wave is the dominant mode. The amplitudes of the other modes are much smaller such that they can be ignored compared with the amplitudes of the zero-order mode [14].

$$\begin{bmatrix} J_n(\gamma_{\rho1}a_0) & J_n(\gamma_{\rho2}a_0) & -\sin\theta_0 H_n^{(2)}(\beta_{\rho0}a_0) & 0 \\ Z_0\eta_1 J_n(\gamma_{\rho1}a_0) & Z_0\eta_2 J_n(\gamma_{\rho1}a_0) & 0 & -\sin\theta_0 H_n^{(2)}(\beta_{\rho0}a_0) \\ k_0 a_0 \sin\theta_0 \mathrm{X}_{1n}(a_0) & k_0 a_0 \sin\theta_0 \mathrm{X}_{2n}(a_0) & n\cos\theta_0 H_n^{(2)}(\beta_{\rho0}a_0) & -jk_0 a_0 \sin\theta_0 H_n'^{(2)}(\beta_{\rho0}a_0) \\ Z_0 k_0 a_0 \sin\theta_0 \Lambda_{1n}(a_0) & Z_0 k_0 a_0 \sin\theta_0 \Lambda_{2n}(a_0) & jk_0 a_0 \sin\theta_0 H_n'^{(2)}(\beta_{\rho0}a_0) & n\cos\theta_0 H_n^{(2)}(\beta_{\rho0}a_0) \end{bmatrix} \begin{bmatrix} A_n \\ B_n \\ C_n \\ D_n \end{bmatrix}$$
$$= \begin{bmatrix} j^n \sin\theta_{inc}\cos\alpha_{inc} J_n(\beta_{\rho0}a_0) \\ j^n \sin\theta_{inc}\sin\alpha_{inc} J_n(\beta_{\rho0}a_0) \\ -n\cos\theta_{inc}\cos\alpha_{inc} j^n J_n(\beta_{\rho0}a_0) + k_0 a_0 \sin\theta_{inc}\sin\alpha_{inc} j^{n+1} J_n'(\beta_{\rho0}a_0) \\ -j^{n+1}k_0 a_0 \sin\theta_{inc}\cos\alpha_{inc} J_n'(\beta_{\rho0}a_0) - j^n n\cos\theta_{inc}\sin\alpha_{inc} J_n(\beta_{\rho0}\rho) \end{bmatrix}. \qquad (17)$$

Thus, we are concentrating only on C_0 and D_0 in the results shown in the following section.

III. RESULTS AND DISCUSSIONS

In this section, we present sample results for the scattering of two ferromagnetic microwires at different angles of incidence and different polarization angles. The ferromagnetic material is assumed to be $(\mathrm{Co}_{0.94}\mathrm{Fe}_{0.06})_{75}\mathrm{Si}_{12.5}\mathrm{B}_{12.5}$. The parameters of this ferromagnetic material are [14-15] $f_0 = \omega_0 / 2\pi = 5.666\,\mathrm{GHz}$, $\alpha = 0.02$ and $f_m = \omega_m / 2\pi = 11.642\,\mathrm{GHz}$.

For ferromagnetic materials, there are two main mechanisms for electromagnetic wave propagation, namely along the dc magnetization biasing or perpendicular to it [20]. For the former mechanism, the plane wave can be presented a superposition of two oppositely circular polarized waves. One of them has an equivalent permeability $\mu + k$ while the other has an equivalent permeability $\mu - k$. The first circular polarization has a resonance at Larmor frequency while the second circular polarization does not have any resonance effect. On the other hand, for the case of electromagnetic wave propagation which is perpendicular to the dc magnetic biasing while its electric field is parallel to the dc magnetic biasing, the equivalent permeability is $\mu_e = \left(\mu^2 - k^2\right)/\mu$. This case is known as an extra ordinary wave [20]. The equivalent permeability in this case has a resonance frequency at 9.9 GHz for the present ferromagnetic material. The final case, where both the propagation direction and the electric field polarization are perpendicular to the dc magnetic biasing, the equivalent permeability is the permeability of free space. Thus, for the present material we have two resonance frequencies, Larmor resonance frequency at 5.666 GHz and extra ordinary wave resonance at 9.9 GHz.

The following results shown in Figures 2 to 5 present the amplitudes of the zero-order modes of the scattered fields as functions of frequency f and angle of incidence θ_{inc} for different polarization angles α_{inc}. The radius of the microwire in Figs. 2 and 3 is 45 μm while the corresponding one in Figs. 4 and 5 is 4 μm. It can be noted that only the resonance effect of extra ordinary wave is detected in the amplitudes of the scattered fields. For the case of $\alpha_{inc} = 0^\circ$, the incident field is pure TM wave. Thus, D_0 in this case corresponds to the amplitude of a cross polarized component of the scattered field. The resonance behavior in this case appears as a minimum in the amplitude of the co-polarized scattered field and as a maximum in the amplitude of the cross-polarized field component. It can be noted that the cross polarized component vanishes completely for normal incidence while it has a peak at θ_{inc} in the range from 40° to 60° depending on the frequency and the radius of the microwire.

For the case of a polarization angle $\alpha_{inc} = 45^\circ$, the main characteristics of the reflected E_z component is the same as in the case of the pure TM wave. However, the amplitude of C_0 is decreased in this case by nearly a factor of $\cos\alpha_{inc}$. On the other hand, the amplitude of D_0 is monotonically increased with frequency and angle of incidence for the case of the thick microwire while it is nearly not affected for the thin microwire.

Finally, for the case of a polarization angle $\alpha_{inc} = 90^{\circ}$ where the incident wave represents pure TE wave, the amplitude of D_0 is the co-polarized component of the scattered field while C_0 is the cross-polarized component. It can be noted that in this case the co-polarized component is not characterized by any resonance behavior. However, for the thick microwire, the results show a resonance behavior at a cross-polarized component around the resonance frequency of extra ordinary wave. This resonance behavior is nearly negligible in the case of the thin microwire. It can also be noted that the cross polarized component vanishes completely in normal incidence as in the case of TM incident wave.

From these results, it can be concluded that the scattering of magnetized ferromagnetic microwire introduces a cross polarized component for oblique incidence and the frequency response of the scattered field is mainly characterized by a resonance effect around the resonance frequency of extra ordinary waves in most cases for both co-polarized and cross-polarized components.

IV. CONCLUSION

In this paper, we presented a full wave analysis for the scattering of an obliquely incident plane wave of an arbitrary polarization angle due to an infinite magnetized ferromagnetic microwire. The present analysis represents a generalization for a previously published analysis based on only normal incidence TM wave. The main difference in the present case lies in the coupling between the electric and magnetic field components due to the gyromagnetic properties of the ferromagnetic microwire. This field coupling introduces cross polarized component in the scattered field. This cross polarized component vanishes completely for the case of normal incidence. The scattered field components are mainly characterized by resonance effects close to the resonance of extra ordinary wave in most cases for both co-polarized and cross-polarized components.

ACKNOWLEDGMENT

This work was supported by The Research Center – College of Engineering – King Saud University.

REFERENCES

[1] D. Y.-Jiang, J. J.-Jun, D. Gang, T. Bin, B. S.-Wei, and H. H.-Hui, “Magnetic and Microwave Properties of Glass-Coated Amorphous Ferromagnetic Microwires”, *Trans. Nonferrous Met. Soc. China*, vol. 17, pp. 1352-1357, 2007.

[2] F. X. Qin, H. X. Peng, L. V. Panina, M. Ipatov, V. Zhukova, A. Zhukov, and J. Gonzalez, "Smart Composites with Short Ferromagnetic Microwires for Microwave Applications," *IEEE Transactions on Magnetics,* vol. 47, no. 10, pp. 4481-4484, Oct. 2011.

[3] F. X. Qin, H. X. Peng, N. Pankratov, M. H. Phan, L. V. Panina, M. Ipatov, V. Zhukova, A. Zhukov, and J. Gonzalez, “Exceptional Electromagnetic Interference Shielding Properties of Ferromagnetic Microwires Enabled Polymer Composites”, *J. Appl. Phys.* vol. 108, pp. 044510, 2010.

[4] L. Liu, Z. H. Yang, L. B. Kong, and P. Li, "Nanocrystalline Ferromagnetic Microwires Silicone Flexible Composite with Optical Transparency," *IEEE Transactions on Electromagnetic Compatibility*, (In Press).

[5] J. Carbonell, H. G.-Miqule, and J. S.-Dehsa, “Double Negative Metamaterials Based on Ferromagnetic Microwires”, *Physical Review* B, vol. 81, pp. 024401, 2010.

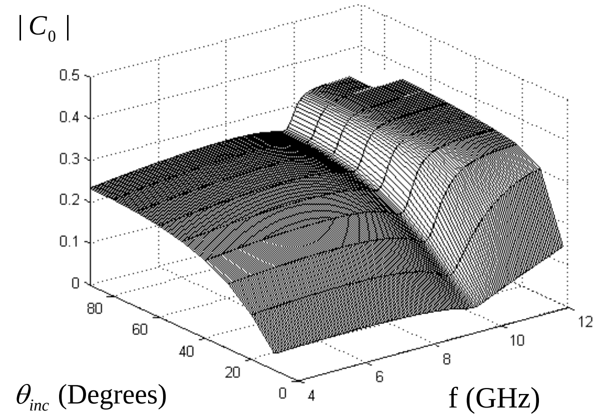
[6] Y. Huang, G. Wen, T. Li, and K. Xie, “Positive-Negative-Positive Metamaterial Consisting of Ferrimagnetic Host and Wire Array,” *Applied Computational Electromagnetics Society* (ACES) *Journal*, vol. 25, no. 8, pp. 696-702, Aug. 2010.

[7] H. G.-Miquel, J. Carbonell, and J. S.-Dehesa, “Left Handed Material Based on Amorphous Ferromagnetic Microwires Tunable by Dc Current”, *Applied Physics Letters*, vol. 97, pp. 094102, 2010.

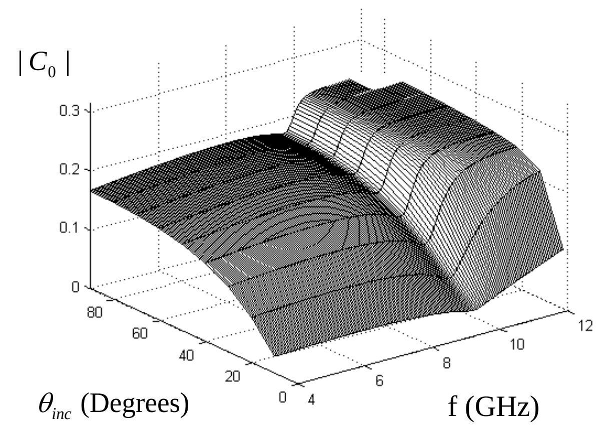
[8] A. Labrador, C. G.-Polo, J. I. P.-Landazábal, V. Zablotskii, I. Ederra, R. Gonzalo, G. B.-Confalonieri, and M. Vázquez, "Magnetotunable Left-Handed Fesib Ferromagnetic Microwires," *Opt. Lett.* vol. 35, pp. 2161-2163, 2010.

[9] M. Ipatov, V. Zhukova, L. V. Panina, and A. Zhukov, “Ferromagnetic Microwires Composite Metamaterials with Tuneable Microwave Electromagnetic Parameters”, *Progress in Electromagnetics Research Symposium* PIERS, vol. 5, no. 6, pp. 586-590, 2009.

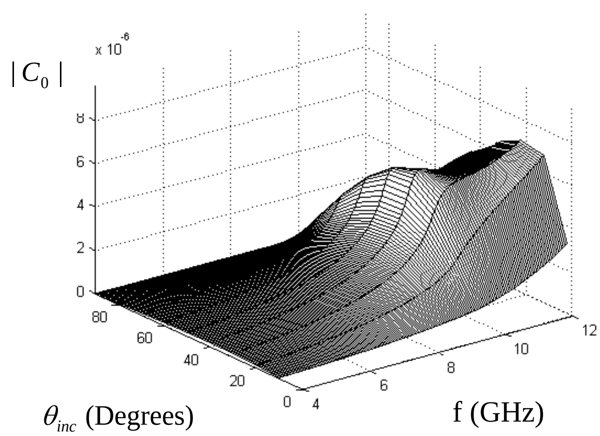
[10] D. Makhnovskiy, A. Zhukov, V. Zhukova, and J. Gonzalez, “Tunable and Self-Sensing Microwave Composite Materials Incorporating Ferromagnetic Microwires”, *Advances in Science and Technology*, vol. 54, pp. 201-210, 2008.



(a) $\alpha_{inc} = 0^\circ$.

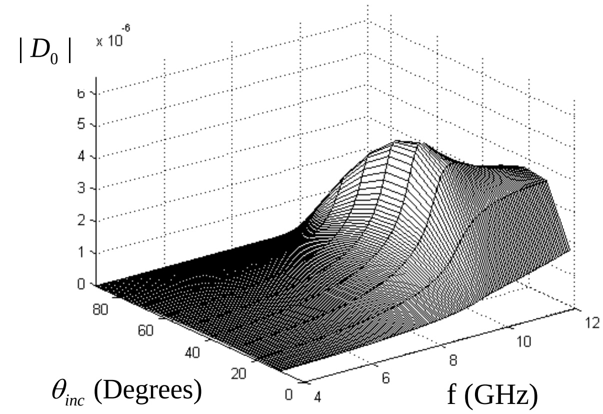


(b) $\alpha_{inc} = 45^\circ$.

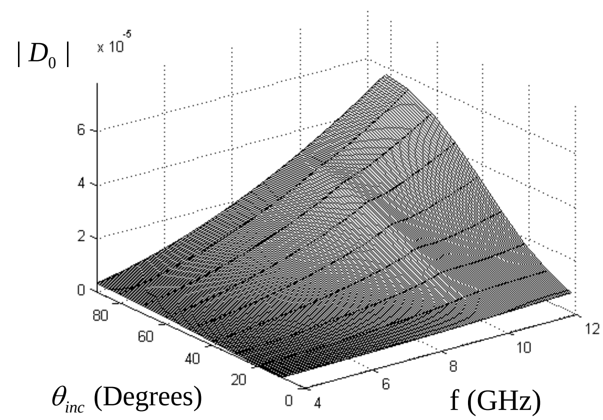


(c) $\alpha_{inc} = 90^\circ$.

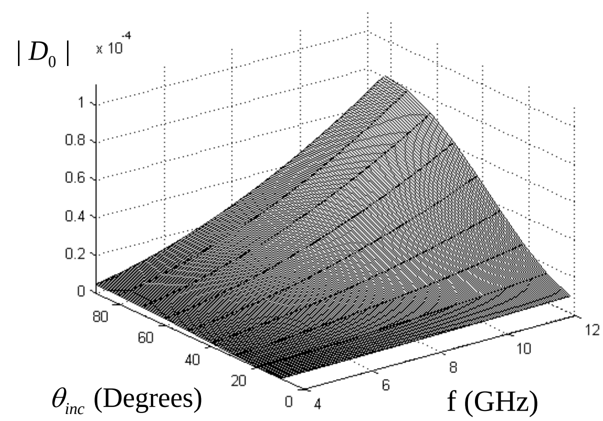
Fig. 2. C_0 scattering coefficient for a ferromagnetic microwire of a radius $r_0 = 45\mu m$ as functions of frequency and angle of incidence (θ_{inc}) at different polarization angles (α_{inc}).



(a) $\alpha_{inc} = 0^\circ$.

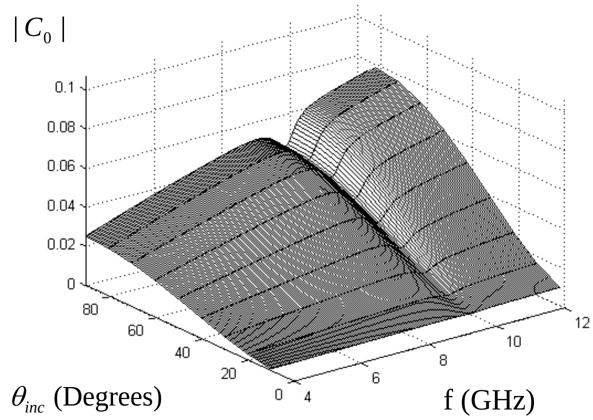


(b) $\alpha_{inc} = 45^\circ$.

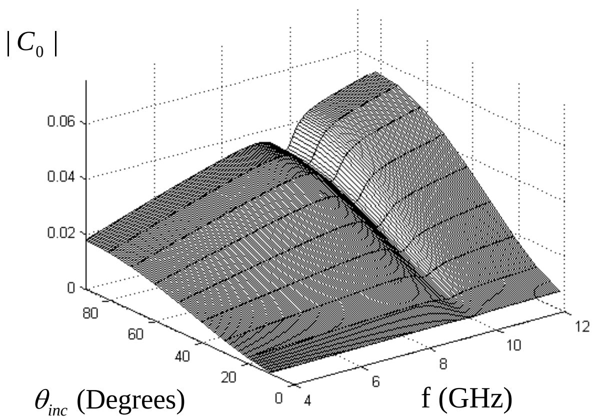


(c) $\alpha_{inc} = 90^\circ$.

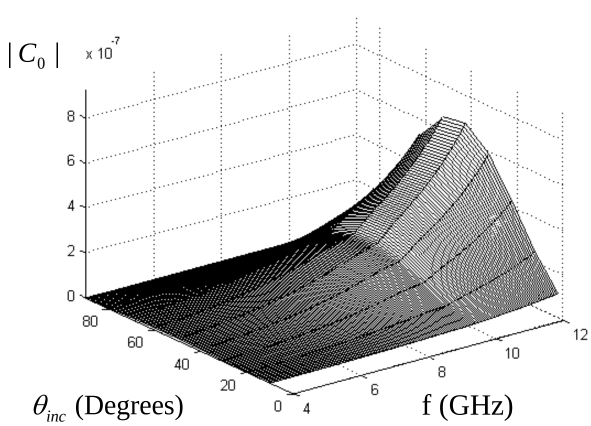
Fig. 3. D_0 scattering coefficient for a ferromagnetic microwire of a radius $r_0 = 45\mu m$ as functions of frequency and angle of incidence (θ_{inc}) at different polarization angles (α_{inc}).



(a) $\alpha_{inc} = 0^\circ$

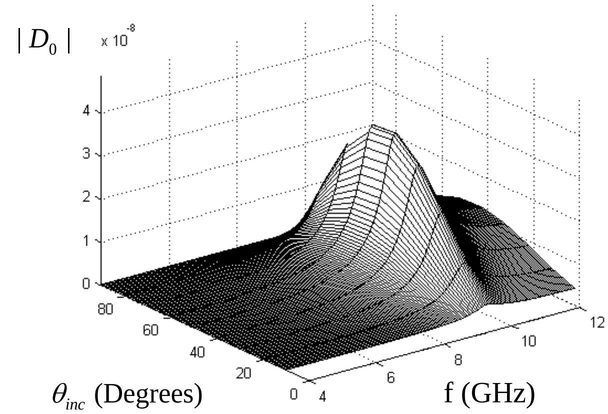


(b) $\alpha_{inc} = 45^\circ$

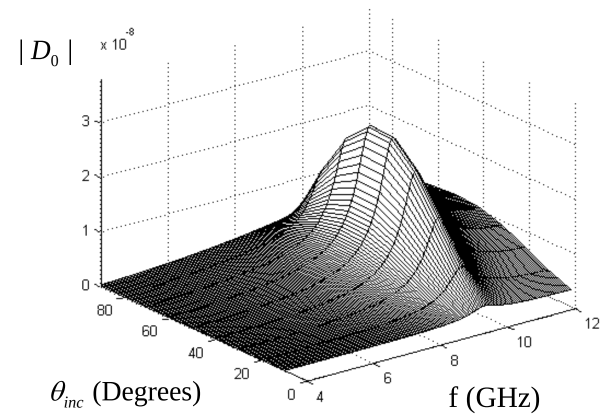


(c) $\alpha_{inc} = 90^\circ$

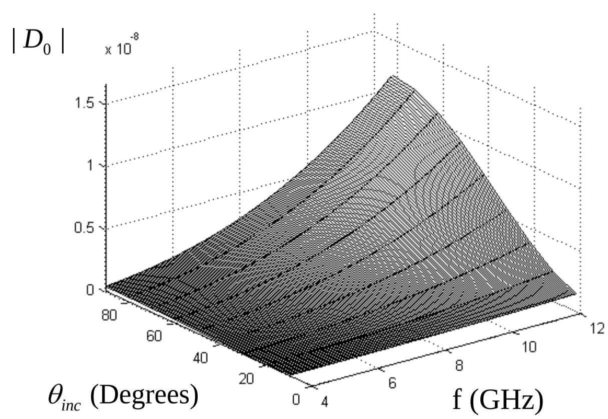
Fig. 4. C_0 scattering coefficient for a ferromagnetic microwire of a radius $r_0 = 4\mu m$ as functions of frequency and angle of incidence (θ_{inc}) at different polarization angles (α_{inc}).



(a) $\alpha_{inc} = 0^\circ$



(b) $\alpha_{inc} = 45^\circ$



(c) $\alpha_{inc} = 90^\circ$

Fig. 5. D_0 scattering coefficient for a ferromagnetic microwire of a radius $r_0 = 4\mu m$ as functions of frequency and angle of incidence (θ_{inc}) at different polarization angles (α_{inc}).

[11] L. V. Panina, M. Ipatov, V. Zhukova, A. Zhukov, and J. Gonzalez, “Microwave Metamaterials with Ferromagnetic Microwires”, *Applied Physics* A, vol. 103, pp. 653-657, 2011.

[12] A. F. Yagli, J. K. Lee, and E. Arvas, “Monochromatic Scattering from Three-Dimensional Gyrotropic Bodies using the TLM Method”, *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 22, no. 1, pp. 155-163, March 2007.

[13] F. Hunsberger, R. Luebbers, and K. Kunz, “Finite-Difference Time-Domain Analysis of Gyrotropic Media - I: Magnetized Plasma," *IEEE Trans. on Antennas and Propagation*, vol. 40, no. 12, pp. 1489-1495, 1992.

[14] I. Liberal, I. Ederra, C. G.-Polo, A. Labrador, J. L. P.-Landazábal, R. Gonzalo , "Theoretical Modeling and Experimental Verification of the Scattering from a Ferromagnetic Microwire," *IEEE Trans. on Microwave Theory and Techniques* , vol. MTT-59, no. 3, pp. 517-526, March 2011.

[15] I. Liberal, I. S. Nefedov, I. Ederra, R. Gonzalo, and S. A. Tretyakov, “Electromagnetic Response and Homogenization of Grids of Ferromagnetic Microwires”, *J. Appl. Phys*. vol. 110, pp. 064909, 2011

[16] R. A. Waldron, "Electromagnetic Wave Propagation in Cylindrical Waveguides Containing Gyromagnetic Media," *Journal of the British Institution of Radio Engineers*, vol.18, no.10, pp. 597-612, Oct. 1958.

[17] R. A. Waldron, "Theory of the Mode Spectra of Cylindrical Waveguides Containing Gyromagnetic Media," *Journal of the British Institution of Radio Engineers*, vol. 19, no. 6, pp. 347-356, 1959.

[18] J. Helszajn and A. A. P. Gibson, “Mode Nomenclature of Circular Gyromagnetic and Anisotropic Waveguides with Magnetic and Open Walls”, *IEE Proc.*, vol. 134, pt. H, no. 6, pp. 488-496, 1987.

[19] Y. Rong, W.-B. Dou, and S. -F. Li, “Generalized Field Theory of Irregular Waveguide Partially Filled with Longitudinally Magnetized Ferrite”, *IEE Proc.*, vol. 138, pt. H, no. 5, pp. 412-416, 1991.

[20] D. Pozar, *Microwave Engineering*. 3rd Ed. John Wiley & Sons, Inc., Ch. 9, 2005.

Ahmed M. Attiya graduated from Zagazig University of Egypt in 1991 and received his M.Sc. and Ph.D. in Electronics and Electrical Communications from Cairo University in 1996 and 2001, respectively. Attiya is a full Professor in Microwave Engineering Dept. Electronics Research Institute (Egypt). He is currently a visiting teaching member in Electrical Engineering Dept., King Saud University. In 2002-2004, he was a research fellow in Virginia Polytechnic Inst. and State University. In 2004-2005, he was a visiting scholar in University of Mississippi. His research interest includes antennas, electromagnetic wave propagation, scattering and differaction problems, and numerical techniques in electromagnetics.

Majeed A. Alkanhal graduated from King Saud University (Saudi Arabia) in 1984 and received his M.Sc. from the same university in 1986. He received Ph.D. in Electronics and Electrical Communications from Electrical Engineering, Syracuse University, Syracuse, New York in 1994. Alkanhal is currently a Full Professor in the Electrical Engineering Dept. Faculty of Engineering, King Saud University. In 2001, he was a Visiting Assistant Professor at the University of Manitoba, Canada. His research interest includes RF and microwave communications and radar systems, radar cross section of complex bodies and materials, electromagnetic scattering from chiral, bi-istropic and bi-anistropic material objects, antenna design and optimization for communication and radar systems, application of modern estimation and optimization techniques to signal processing and digital communications problems, radar and SAR (Synthetic Aperture Radar) systems and measurements, and numerical solutions to electromagnetics and Electronic/ Optical device problems.