Analysis of Mutual Coupling in Interconnect Lines using Finite Difference Time Domain Method

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Abstract **–** In this paper, we present a simple technique for analyzing the mutual coupling effects in interconnects using the Finite Difference Time Domain (FDTD) method. The interconnect lines are divided into a set of uniform segments of parallel lines with short lengths. Next, the mutual capacitances and inductances of each of these segments are extracted by incorporating the FDTD solution into the telegrapher's equations. Two examples of coplanar lines and microstrip lines on different dielectric substrates are studied.

I. INTRODUCTION

The Finite Difference Time Domain (FDTD) [1] has been previously used to analyze interconnect lines and extract their equivalent circuit parameters, *viz.*, the series inductance L and the shunt capacitance C [2, 3]. In this paper we modify the algorithm in [3] in order to model the mutual coupling effects between the interconnect lines.

This modification allows one to consider a system of n coupled lines and compute the self and mutual capacitances and inductances between each 2 lines. By increasing the number of unknowns the number of excitations (simulations) is increased to generate enough equations.

Since this technique is a FDTD based method therefore a single run can provide the frequency response in the frequency band of operation by adjusting the amplitude of the Gaussian pulse spectrum into the band. In addition the staggered location of computing the electric and magnetic fields, and in turn computing voltages and currents along the transmission line, in FDTD mesh allows one to avoid the numerical differentiation in the telegrapher's equations and maintain the simplicity of the method. The numerical results show that this approach is accurate enough to obtain the parameters of several practical circuits.

This method is particularly useful for non-uniform transmission lines and it can readily provide the self and mutual capacitance and inductances along the line. We have shown the usefulness of the technique by considering the microstrip step discontinuity and variation of its main parameters along the line as well as in close proximity of the step discontinuity itself.

Two examples of interconnect lines of practical interest are considered: (i) coplanar striplines on different dielectric substrates interconnecting a 12 channel fiber-optic module to IC driver; (ii) and a uniform microstrip line in parallel with a step discontinuity in its width.

II. THEORY

We start with a 2n-port network comprising of n parallel transmission lines. The lines may be approximated by a number of uniform parallel line sections, each of which may be represented with a combination of L, C, R and G and mutual inductance L^m_{ij} and capacitance C^m_{ij}, as shown in Fig. 1. Next, we write the telegrapher's equations in the matrix form as,

$$\hat{V}_1 - \hat{V}_2 = \hat{R}\,\hat{I}_1 + j\omega\,\hat{L}\,\hat{I}_1 \tag{1-a}$$

$$\hat{I}_1 - \hat{I}_2 = \hat{G}\hat{V}_1 + j\omega\,\hat{C}\hat{V}_1, \tag{1-b}$$

where,

$$\hat{V}_i = \begin{pmatrix} V_i^{\,1} \\ V_i^{\,2} \\ \cdots \\ V_i^{\,n} \end{pmatrix}, \quad \hat{I}_i = \begin{pmatrix} I_i^1 \\ I_i^2 \\ \cdots \\ I_i^n \end{pmatrix}, \quad i = 1, 2 \tag{2}$$

$$\hat{R} = \begin{pmatrix} R_1 0 0 \ldots\ldots 0 \\ 0 R_2 0 \ldots\ldots 0 \\ \ldots\ldots\ldots\ldots \\ 0 \ldots\ldots\ldots R_n \end{pmatrix}, \; \hat{G} = \begin{pmatrix} G_1 0 0 \ldots\ldots 0 \\ 0 G_2 0 \ldots\ldots 0 \\ \ldots\ldots\ldots\ldots \\ 0 \ldots\ldots\ldots G_n \end{pmatrix}, \tag{3}$$

$$\hat{L} = \begin{pmatrix} L_{11} L_{12} \ldots L_{1n} \\ L_{21} L_{22} \ldots L_{2n} \\ \ldots \\ L_{n1} L_{n2} \ldots L_{nn} \end{pmatrix}, \; \hat{C} = \begin{pmatrix} C_{11} C_{12} \ldots C_{1n} \\ C_{21} C_{22} \ldots C_{2n} \\ \ldots \\ C_{n1} C_{n2} \ldots C_{nn} \end{pmatrix}. \quad (4)$$

The relationships between the elements of the C matrix and the physical mutual capacitances in Fig. 1, for example for ports 1 and 2, are given by,

$$C_{11} = C_1 + C_{12}^m + C_{13}^m + \ldots + C_{1n}^m \quad (5)$$

$$C_{12} = C_{21} = -C_{12}^m . \quad (6)$$

To determine the vectors in equation (2) from the FDTD simulations, we discretize the computational domain in a way such that the segmentations coincide with the FDTD meshing, both having rectangular boundaries. After the time-domain simulation is over and the level of the pulse is negligible in the entire computational domain, the time domain signals of the components of electric and magnetic fields in any location in the computational domain can be obtained and analyzed. We particularly compute the vertical component of the electric field underneath the transmission line obtained from the FDTD simulation and add them between all the cells from the ground plate to the conductor (computing the line integral) to derive the voltage V for each cell.

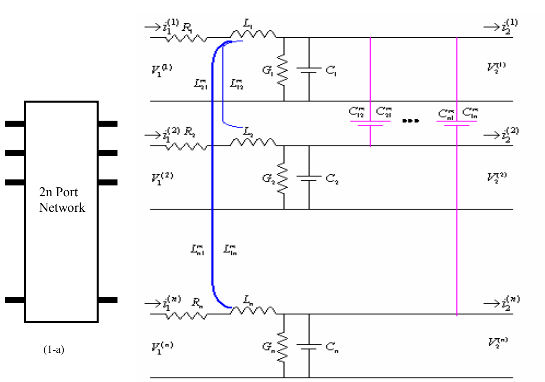

Fig. 1. (a) The schematic and (b) equivalent circuit of a 2n port network comprising of parallel transmission lines.

The same procedure is repeated for the time domain magnetic field signals around the conductor to carry out the closed-path line integral and following the Ampere's law to obtain the current I, which is computed at a half-cell away from the point where V is derived. At the final step we transform the above voltages and currents by using Fast Fourier Transform routine in Matlab in order to derive their frequency domain counterparts.

Next, in order to determine elements of the matrices in equations (3) and (4), we substitute the V and I vectors in equation (2), that are obtained from the excitations of each of the n ports—one part at a time—in equations (1-a) and (1-b). Having n equations for the n unknowns we are able to solve the equations for the R, L, G, C matrices in equations (3) and (4). The above procedure is repeated for each segment of the line and for all frequencies of interest. It should be noted that the staggered sampling of the V and I, which occurs naturally in the FDTD algorithm, enables us to circumvent the need to numerically differentiate the data derived from the FDTD simulation.

III. NUMERICAL EXAMPLES

For the first example we consider the geometry in Fig. 2 comprising of 6 pairs of coplanar striplines (50 microns wide and separated by 125 microns gap) on the dielectric substrate (0.4 mm thickness). This geometry is one half of the actual 12 channel interconnect that is inserted between IC driver and the VCSEL package in a fiber-optic module. Since the differential lines or coplanar striplines provide separate return current path for each signal their use significantly reduces the crosstalk [4-6]. In order to model the matched loads terminating the two ends (the driver IC and the VCSEL package) the lines are truncated by using perfectly matched layers (PML). Furthermore, the coplanar stripline 3 is fed by a voltage source (Gaussian pulse with 25 GHz 3dB cutoff frequency) connected between the traces. Next, the voltages and currents at the reference planes 1 and 2 (rp1, rp2), located in the center of the line 3 (active) and 4 (passive) are observed in the time domain. The same procedure is repeated when the line 4 is excited. The data from the above simulations are transferred into frequency domain and used to calculate self and mutual (per unit length) inductance L and capacitance C of lines 3 and 4 (in the presence of the other lines).The computational domain divided by 40 cells in wavelength inside the dielectric in all three directions. The simulation is carried out until the level of the time signal in the load end of the line is insignificant (20000 time steps). This number of time steps gives a sufficient resolution in the frequency domain.

The self and mutual capacitance and inductance per unit length of line 3 and 4 vs. frequency are shown in Figs. 3 and 4. In addition, the change in mutual capacitance due to different substrates, Du Pont's LTCC (ε_r=7.8), Alumina (ε_r=9.6) and the case of epoxy-added (same thickness as substrate) on top of the traces are shown in Fig. 5.

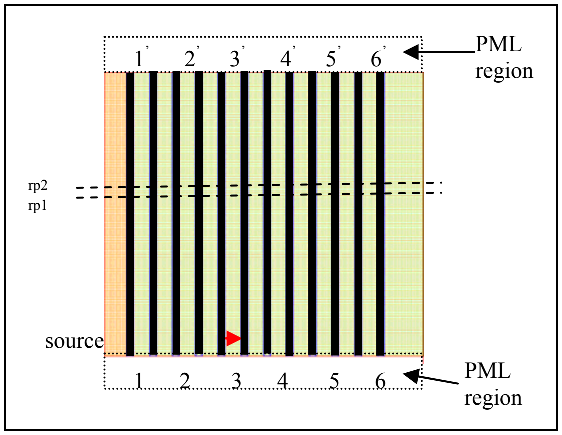

Fig. 2. The top view of the interconnect geometry omprising of 6 pairs coplanar striplines.

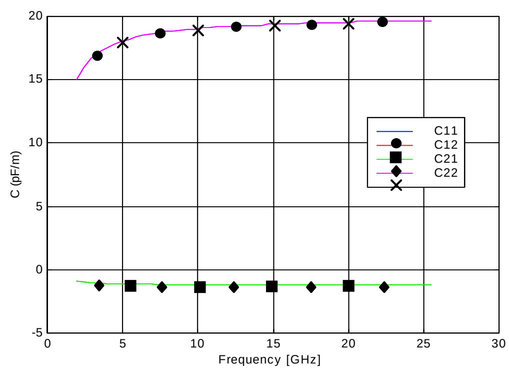

Fig. 3. The self and mutual capacitance C per unit length of two adjacent lines vs. frequency.

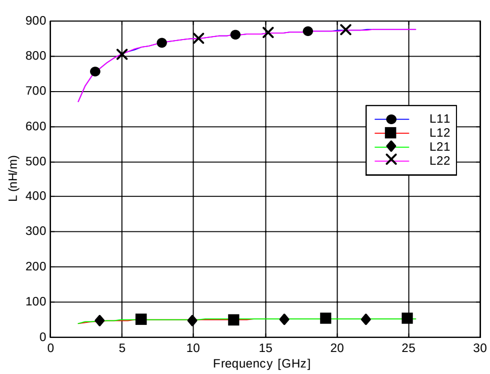

Fig. 4 . The self and mutual inductance L per unit ength of two adjacent lines vs. frequency.

For the second example, we simulate the uniform microstrip line located parallel to a step-in-width sitting on dielectric substrate backed by perfect electric conductor (Fig. 6). The computational domain in this example also is divided by 40 cells in wavelength inside the dielectric in all three directions. The simulation is carried out until the level of the time signal everywhere in the domain is insignificant (10000 time steps). Following the same procedure as in the previous example, and using 20 segments (FDTD cells) in left and right of the step discontinuity, we can derive the self and mutual inductance along the line at a frequency of 10 GHz. The inductance values for the left side of the geometry, comprising of two parallel microstrip lines with the same dimensions as in [2], are in good agreement with the reference data. However, the step discontinuity found to be at the 20th cell causes these values to change, as may be seen in Fig. 7.

It must be noted, however, that the lossless dielectric and perfect conductors have been used in the examples, which lead to zero resistance and conductance in the computations.

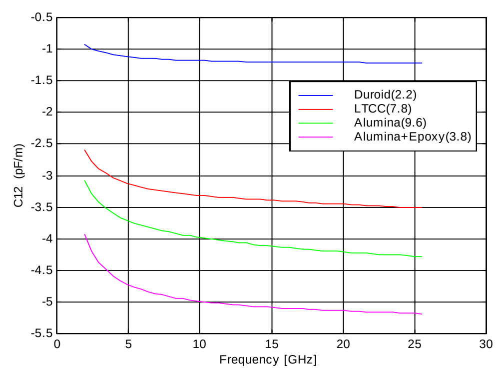

Fig. 5. The mutual capacitance per unit length of two adjacent lines for different substrates vs. frequency.

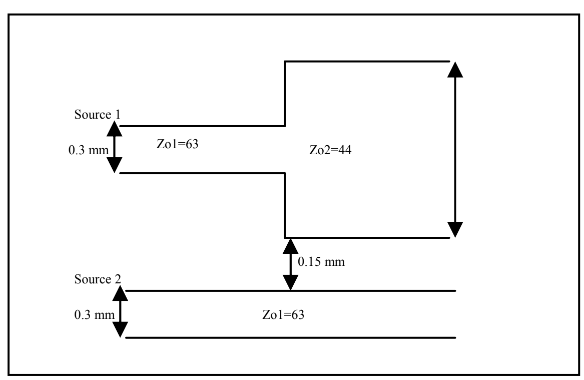

Fig. 6. Top view of an interconnect structure comprising of a uniform microstrip line parallel to a step-in-width.

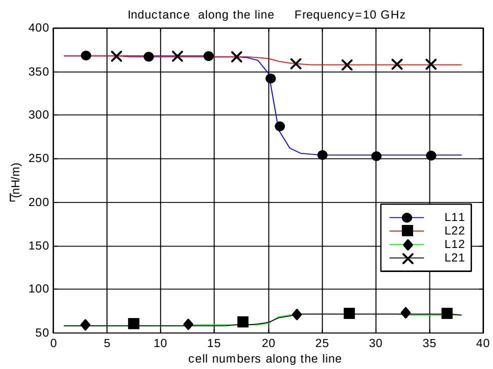

Fig. 7. The self and mutual inductance per unit length of uniform microstrip line in parallel with step-in-width vs. cell numbers along the line for frequency of 10 GHz.

IV. CONCLUSION

A simple technique to analyze the mutual coupling effect in interconnects using the Finite Difference Time Domain (FDTD) method is presented. It was shown that the staggered sampling of the voltage and current, which occurs naturally in the FDTD algorithm, enables us to circumvent the need to numerically differentiate the data derived from the FDTD simulation. The validity of technique is shown through several examples in extraction of self and mutual capacitances of the coupled lines.

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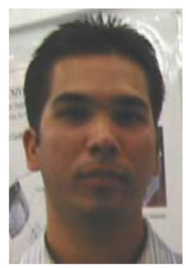

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