A MAGNETIC FIELD ITERATIVE TECHNIQUE FOR IMPROVING HIGH FREQUENCY PREDICTION METHODS

Daniel D. Reuster and Gary A. Thiele
Department of Electrical Engineering
University of Dayton
Dayton, OH 45469-0226

Paul W. Eloe
Department of Mathematics
University of Dayton
Dayton, OH 45469-2316

Abstract

A Magnetic Field Iterative Technique (MFIT) is developed as a technique for improving the results from high frequency based prediction methods. The technique combines the accuracy of low frequency methods with the speed of high frequency methods to develop a contraction mapping based, iterative solver, which is directly parallelizable on current massively parallel processing machines. In this paper, the MFIT is developed from a Magnetic Field Integral Equation (MFIE) and its contraction mapping properties are discussed. Results obtained using the technique and illustrations of the technique's convergence properties are presented for both two and three dimensional perfect electrically conducting (PEC) targets.

Introduction

High frequency based numerical prediction methods such as Physical Optics - Physical Theory of Diffraction (PO-PTD) [1] and Geometrical Optics - Geometrical Theory of Diffraction (GO-GTD) [2] have been shown to be successful at approximating the induced current distributions on electrically large targets (e.g., three dimensional bodies with at least one dimension greater than approximately ten wavelengths). For these electrically large bodies, the high frequency series expansion associated with their scattering properties converges quickly and scattering centers located about the bodies can be easily identified. However, as the electrical size of these perfect electrically conducting (PEC) bodies begin to decrease, the ability to clearly define the required scattering centers becomes difficult and the convergence rate of the associated series expansions decreases rapidly. Most often, more exact numerical methods such as the Method of Moments (MoM) [3,4] and the Finite Element Method (FEM) [5] have been utilized for these particular situations. While these methods (MoM and FEM) have been proven to be both accurate and reliable for PEC bodies which are less than approximately three wavelengths on a side, these methods become difficult to implement for much larger bodies. Large memory requirements coupled with long execution times, make the utilization of these methods difficult for large bodies.

The Magnetic Field Iterative Technique (MFIT) is developed here for those PEC bodies whose electrical size falls into the awkward gap which exists between the two groups of numerical methods. Currently, there are few options that exist for efficiently calculating the induced current distributions on these particular PEC bodies. The following method is a hybrid numerical technique which takes advantage of the desired features from both the high frequency and nearly exact solution methods. The method is an extension of previous research performed by Thiele et al. [6-8] on hybrid and iterative methods for solving electrically large problems.

Mathematical Development

For closed volumetric PEC bodies, a Magnetic Field Integral Equation (MFIE) may be stated in terms of the total magnetic field as [3]:

$$\overline{H}^T(\overline{r}) = \frac{1}{4\pi}\int_S (\hat{n}\times\overline{H}^T(\overline{r}'))\times\nabla' G(\overline{r},\overline{r}')ds + \overline{H}^I(\overline{r}) \qquad (1)$$

where $$\lim_{\overline{r}\to\overline{r}'}\frac{1}{4\pi}\int_S (\hat{n}\times\overline{H}^T(\overline{r}'))\times\nabla' G(\overline{r},\overline{r}')ds$$

$$= \frac{1}{2}\left(\overline{H}^T(\overline{r}) - \hat{n}(\hat{n}\cdot\overline{H}^T(\overline{r}))\right) \qquad (2)$$

S is the closed volumetric PEC surface, G is the standard free space Green's function, T is the total field, and I is the incident field. Equations 1 and 2 are in the form of a contraction mapping [9,10], and it may be shown that direct

incident field. Equations 1 and 2 are in the form of a contraction mapping [9,10], and it may be shown that direct iteration of these equations will yield monotonic mean-square convergence provided that the integral operator is bounded by unity. If the integral operator does satisfy this bound, then the above convergence is guaranteed independent of the initial guess. While this unity bound may be shown for explicit geometries, a general proof does not yet exist. Extensive discussion regarding the guarantee of convergence may be found in references [11-14].

Since equations 1 and 2 are normally implemented in discretized form, it is best to discuss the issue of convergence in terms of the discretized equations. Applying a pulse-basis point-matching expansion to the above MFIE results in the following discretized equation where delta is the area of the discretized facets (see Figure 1):

$$\overline{H}_m^T = \sum_{n \neq m} \Delta_n G(\overline{R}_{mn}) \left[\frac{1}{|\overline{R}_{mn}|} + jk \right] \overline{J}_n + \frac{1}{2} \overline{J}_m + \overline{H}_m^I \qquad (3)$$

where $$\overline{J}_n = \frac{\overline{H}_n^T (\overline{R}_{mn} \cdot \hat{n}) - \hat{n}(\overline{R}_{mn} \cdot \overline{H}_n^T)}{|\overline{R}_{mn}|} \qquad (4)$$

and $$\overline{J}_m = \overline{H}_m^T - \hat{n}(\overline{H}_m^T \cdot \hat{n}) \qquad (5)$$

Expressing the discretized MFIE in matrix form transforms the problem of establishing convergence from bounding the integral operator to determining the spectral radius of the reaction matrix $[Z_{mn}]$ (Equation 6). If the spectral radius of the reaction matrix is less than unity, then the discretized MFIE is a true contraction mapping and direct iteration of the discretized equation will guarantee monotonic mean square convergence.

$$\left[H_m^T \right] = \left[\quad Z_{mn} \quad \right] \left[H_n^T \right] + \left[H_m^I \right] \qquad (6)$$

For cases where the spectral radius is not less than unity, it is possible to shift the spectral radius of the matrix $[Z_{mn}]$ by using a relaxation parameter [11-14]. The implementation of the relaxation parameter α is given in Equation 7 and its numerical value is a function of the largest eigenvalue of the $[Z_{mn}]$ matrix. Thus, for all given reaction matrices, it is possible to obtain the desired solution through direct iteration with the guarantee of monotonic mean-square convergence if α is properly chosen as required by the following equation where L is the linear operator used in Equation 6:

$$H^{k+1} = \omega L(H^k) + (1-\omega) H^k \qquad \omega < 1 \qquad (7)$$

While the guarantee of convergence is independent of the initial guess, the rate of convergence is not; and thus, for the MFIT to serve as an efficient method to obtain the induced current distributions on volumetric PEC bodies, the iterative procedure must be initiated with the best guess available such that the computation time associated with the calculation of the initial guess is small compared to the computation time associated with the iterative procedure. For this reason, high frequency techniques such as GO-GTD and PO-PTD are used to generate the initial guess. Both of these techniques have negligible computer run times and are capable of producing approximations for the induced current distributions which are in the realm of rapid convergence. It is the use of the high frequency prediction methods' results, as an initial guess to the iterative process, which creates the hybrid method. By initiating the iterative process with these results, the large amount of computational work associated with calculating the principle current distributions using the more exact methods is removed from the solution process, leaving the iterative process to fine tune the results rather than regenerating the first order solutions which constantly reoccur. In essence, a high frequency prediction method is used to quickly obtain the general portion of the solution and the iterative solution method is used to fine tune the general solution to the particular PEC body.

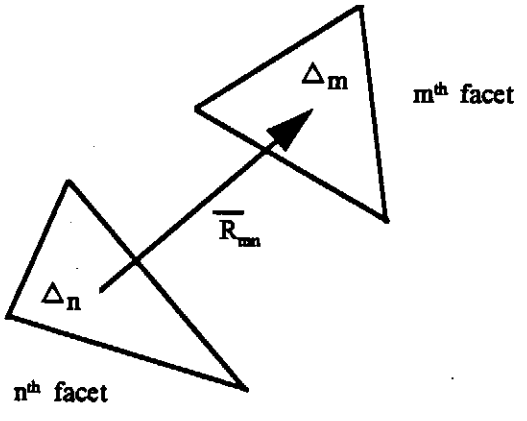


Figure 1 Discretized Geometry

2-Dimensional Example

For the 2-dimensional transverse electric (TE) case, the previous vector MFIE (Equations 1 and 2) simplifies to the following scalar integral equation [6].

$$H_x^T(\bar{r}) = -\frac{jk}{4}\int_C H_z^T(\bar{r}')G(\bar{r},\bar{r}')\cos(\theta)d\ell + H_z^I(\bar{r}) \quad (8)$$

Applying a pulse-basis point-matching expansion to the above integral operation results in the following series expansion [3].

$$H_m^T = -\frac{jk}{4}\sum_{m\neq n}\Delta_n H_n^T G(|\bar{R}_{mn}|)\cos(\theta_{mn}) + \frac{1}{2}H_m^T + H_m^I \quad (9)$$

This 2-dimensional series expansion was applied to a 1 wavelength diameter PEC cylinder (see Figure 2). The 2-dimensional contour was discretized at twenty points per wavelength, and the iterative procedure was initiated with its high frequency Physical Optics approximation [1]. Figure 4 shows the converging currents for the circular cylinder starting with the initial guess, then the current after 1, 5, and 10 iterations ending with the "converged" current. For this example, convergence was based upon a tight mean-square error norm of 0.001. The converged solution was obtained after 19 iterations and the vast improvement after only 5 iterations should be noted. As can be seen, the monotonic mean square convergence associated with contraction mappings is present in the converging iterative solution.

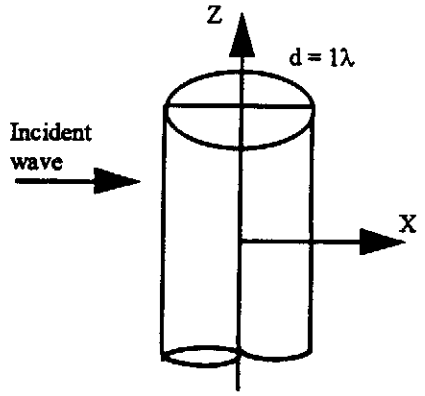


Figure 2 One Wavelength PEC Cylinder Geometry

3-Dimensional Example

A 1 wavelength PEC cube, with a normally incident transverse electric (TE) plane wave is shown as a 3-dimensional example (see Figure 3). The cube geometry was selected because of the non-uniform and singular current distributions that exist about its sharp edged structure. These particular current distributions have been traditionally difficult to calculate and would serve as a rigorous test on the contraction mapping properties of the MFIE formulation. The cube surface was facetized with approximately 1,300 triangular facets and Xpatch [15], a high frequency ray based prediction code, was used to provide the initial current distribution (see Figure 5a). As can be seen from Figure 5a, while the high frequency prediction does a good job of predicting the current distribution on the front face of the cube, it fails to predict any current distributions on the remaining five faces of the cube.

Performing a single iteration on the high frequency predicted current distribution (Figure 5b) results in a non-zero current distribution on all faces of the cube surface. Further iterations (Figures 5c and 5d) again illustrate the contraction like mapping properties of the MFIE formulation and the convergence of the induced current distributions along the surfaces of the PEC cube. Comparing the high frequency predicted current distribution (Figure 5a) with the converged current distribution obtained after 20 iterations (Figure 5d) shows a substantial difference in all but the front face of the cube. Finally, it should be noted that all of the non-uniform and singular current distributions are present and observable in the converged current distribution, and that these difficult current distributions had no detrimental effect on the convergence of the iterative procedure.

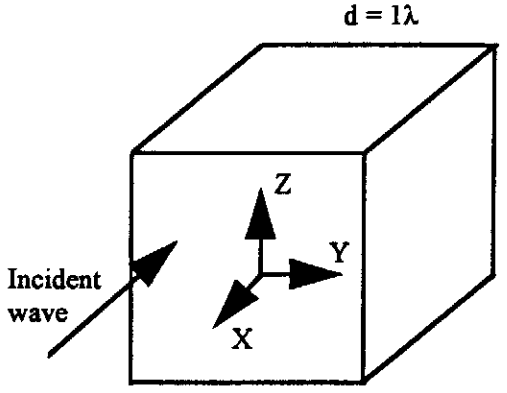


Figure 3 One Wavelength PEC Cube Geometry

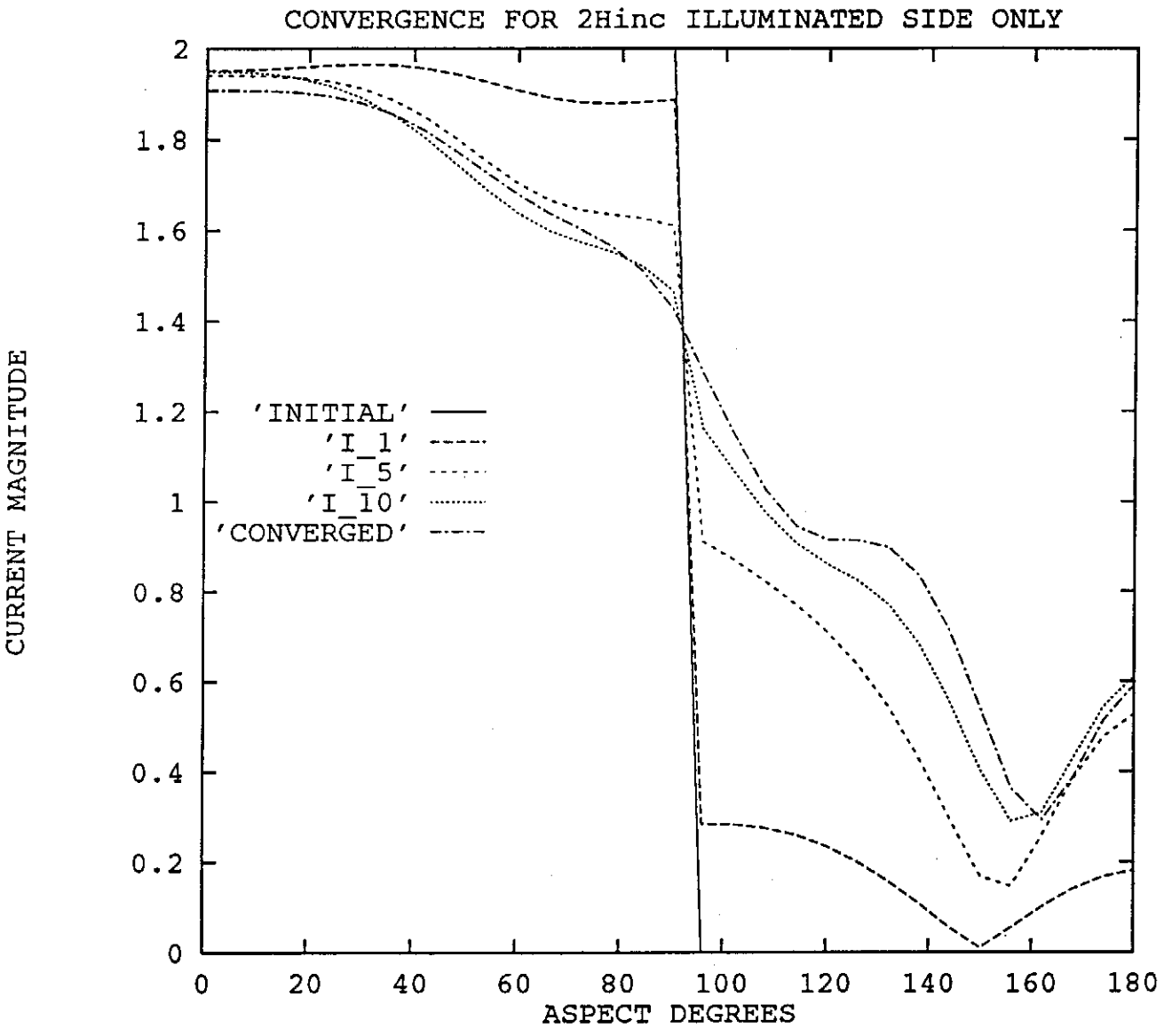
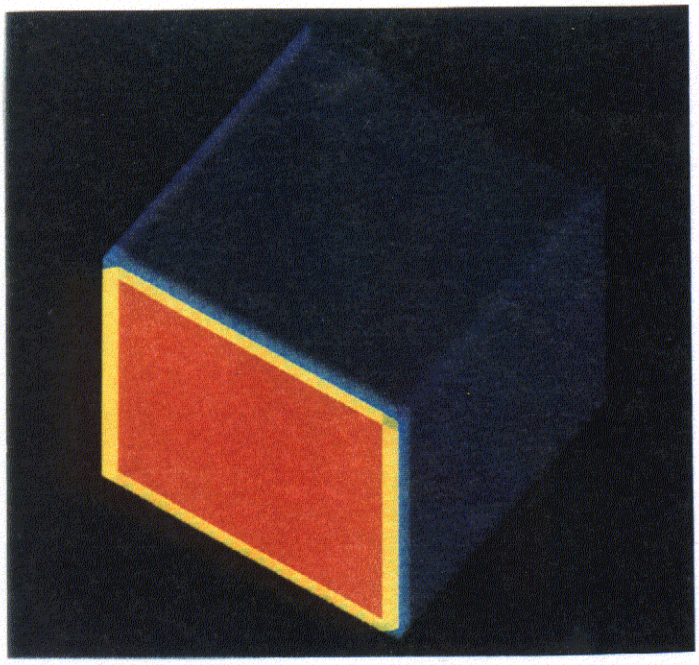
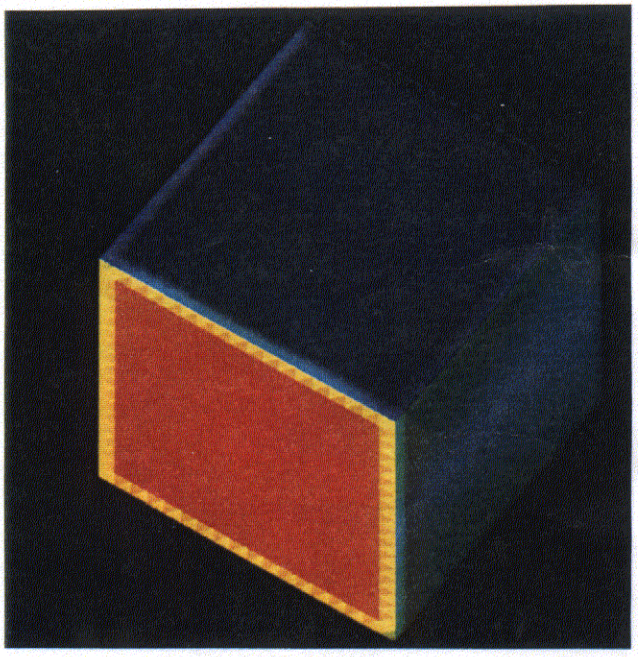


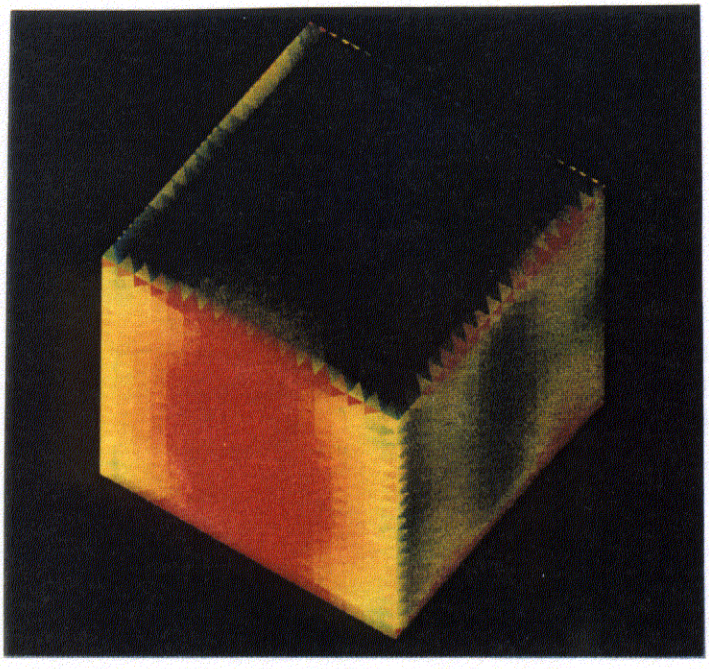
Figure 4 The current magnitudes after 1, 5, 10, and 19 iterations starting with the Physical Optics current distribution on a circular cylinder of 1 wavelength in diameter.



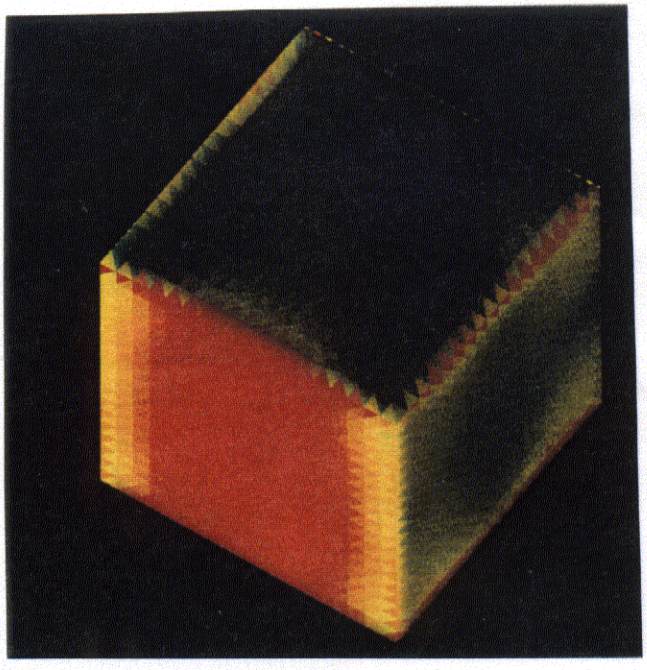
(a) Xpatch Only



(b) 1 Iteration



(c) 10 Iterations



(c) 20 Iterations

Figures 5a-d Current Magnitudes on a 1 Wavelength Cube

Conclusions and Comments

Based upon the results presented, the MFIT shows promise as an efficient method for bridging the gap between traditional high frequency and low frequency numerical techniques. Although this hybrid method is designed for bodies which are 3 to 10 wavelengths in size, the 1 wavelength bodies selected in the examples were chosen because of their ability to compactly illustrate the fundamental principles underlying the MFIT. The use of high frequency methods to provide an initial guess to an iterative solution formulation of low frequency methods allows the combination of the best of both techniques. The information readily obtained using high frequency methods greatly reduces the run time associated with the low frequency methods. Hence, the low frequency portion of this hybrid method is used solely to obtain the desired solution accuracy and valuable computer time is not wasted on obtaining portions of the solution which are well known or more readily obtainable. Furthermore, iterative solvers for systems of linear equations are highly parallelizable on current massively parallel processing machines, thus further reducing the run times associated with the low frequency portion of the hybrid technique.

The contraction mapping principle, associated with the formulation of the MFIT presented, removes many of the dangers commonly encountered when working with iterative solvers [9-10]. The use of the relaxation parameter α allows generic reaction matrices to be quickly reformulated into contraction mapping based systems. For more information on the use of relaxation parameters to ensure contraction mapping properties, the authors recommend references [11-14] and the references therein.

In summary, a procedure has been presented which uses a magnetic field integral equation in an iterative process. The MFIE is discretized and a relaxation parameter is used to ensure the spectral radius of the reaction matrix is less than unity. The discretized MFIE has contraction mapping properties such as guaranteed monotonic mean square convergence. The iterative process starts with a first order approximation to the current by using a high frequency technique such as physical optics or geometrical optics (shooting and bouncing rays) rather than starting at zero current. The use of a high frequency technique as a starting point in the iterative process significantly speeds up the iterative convergence but is not necessary to obtain a converged result since that is guaranteed by the contraction mapping. A future paper will deal with two MFIE formulations in the MFIT and explore their convergence properties in more detail.

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