A Discrete Random Medium Model for Electromagnetic Wave Interactions with Sea Spray

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Abstract — Understanding the electromagnetic interactions with sea spray is of interest for many applications such satellite or terrestrial communications, remote sensing systems for surveillance and meteorology. The water droplets in spray are a key factor in the energy transfer between the atmosphere and the ocean, which can help the wind speed retrieval algorithms for the global climate models. Furthermore, the presence of these droplets along the propagation path interferes with both land-based and satellite based remote sensing of the ocean surface. This paper investigates the backscatter response from sea spray, which is modeled as a layer of water droplets over a rough ocean surface. The distorted Born approximation technique is used in conjunction with the analytical wave theory to compute backscattering from the medium.

Keywords: remote sensing, scattering, random medium, maritime, ocean, and sea spray.

I. INTRODUCTION

The presence of water droplets along the propagation path interferes with both land-based and satellite-based remote sensing systems of ocean surface. This can cause adverse effects for weather forecasting, wireless communication and military surveillance systems. Understanding the electromagnetic interactions with sea spray is also important as it plays a critical role in the energy exchange between the surface of the ocean and the atmosphere. This energy transfer helps define the boundary condition for atmospheric and oceanic models and is a key factor for the global and regional climate models. The simulation of the complex interaction of the weather system with the sea is often a major challenge in meteorology.

This paper investigates the backscatter response from a layer of water droplets over a rough ocean surface as shown in Fig. 1. The results of this model have been presented before, [1]. This paper extends the earlier findings, and provides an investigation of cluster effects when the particles get in close proximity to each other.

The water droplets in sea spray are modeled as perfectly conducting spheres of various sizes. The droplets are randomly distributed inside the layer, that extends from z = -d to z = 0. Different probability distribution functions (pdf) are used to generate different scenarios for the size distribution of the droplets inside the medium. It is assumed that the layer extends indefinitely in the x-y plane. A Lambertian rough surface, with a dielectric constant equal to that of water is assumed in the background. Sea spray is more complicated than the assumptions made in this analysis. Factors such as ligaments from which the droplets are torn, air bubbles and foam at the water-air interface are currently neglected in the backscatter calculations.

II. THE BACKSCATTER CROSS SECTION

The backscattered fields from the medium are calculated by using the analytical wave theory in conjunction with the distorted Born Approximation. Therefore, the approach is field based and provides more accuracy than power based approaches, such as the radiative transfer. The calculations are valid for a sparse medium; i.e. the fractional volume of the particles is less than the total volume of the medium.

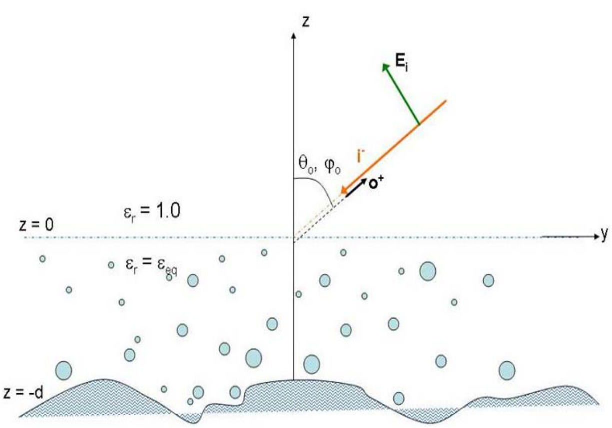

Fig. 1. Sea spray as a layer of discrete random medium.

The Distorted Born Approximation (DBA) is used to calculate the backscattering from the layer of sea spray for both horizontal and vertical polarizations. The approach is similar to the standard Born approximation, which is a single scatter approximation with the exception of two fundamental assumptions: i) mean field propagating inside the medium is incident on each particle. ii) the scattered fields due to the mean wave are computed using a Green's function for the mean wave.

The scattering from a discrete random medium when illuminated by an electromagnetic pulse has been studied before and applied to vegetation or a layer of rain, [2-4]. The assumptions require the solution for the mean field before the scattered fields can be calculated. The Foldy-Lax technique [5] is used to determine an approximate expression for the mean field inside the medium. This approach assumes a sparse medium; i.e. the fractional volume of the scatterers is very small. The solution for the mean wave suggests that the medium can be characterized by an equivalent dielectric constant [6]. The DBA method allows the mean field to attenuate as it propagates inside the medium, as opposed to the Born Approximation, where the mean field is assumed to be identical to the incident field. As a first step, an equivalent propagation constant in the medium is determined based on the statistical properties and scatterer types in the medium. This equivalent propagation constant defines an equivalent medium that accounts for attenuation as the fields propagate inside. The particles are then embedded inside this equivalent medium for scattering calculations. An application of this approach to vegetation can be found in [7], where all particles inside the medium are assumed identical, and a 2-dimensional problem is studied.

The backscattering coefficient from the medium is given by,

$$\sigma_{pq} = \frac{4\pi r^2 \left\langle P_{r_p} \right\rangle}{A P_{i_q}}, \quad p \in \{h_s, v_s\}, \quad q \in \{h_i, v_i\} \tag{1}$$

where P_{i_q} is the power of the incident plane wave with polarization, $\hat{q}$ and $\left\langle P_{r_p} \right\rangle$ is the average received power of the $\hat{p}$ polarized scattered wave, A is the illuminated area on the surface, r is the distance of the radar to this area. The far field is assumed in the calculations; i.e. $k_o r >> 1$.

The backscattered term for a first order approximation is the sum of three distinct contributions referred to as: direct, direct-reflected, and reflected terms.

$$\sigma^o_{pq} = \sigma^o_{pq_d} + \sigma^o_{pq_{dr}} + \sigma^o_{pq_r} \,. \tag{2}$$

The direct term involves volume scattering; i.e., that portion of the backscattered power directly from the particles illuminated.

The direct-reflected term involves a particle and a single bounce from the background before the incident wave is scattered back to the radar. It consists of four components due to the coupling of two possible fields that can arrive back through a single reflection from the ground. The first field directly illuminates the particle and is scattered toward the ground and reaches back through a reflection from the ground. The second field illuminates the particle after being reflected from the ground and is scattered directly back. Both of these fields are in phase and can couple with each other in four different ways in the power calculations. This is an advantage of using a field based approach used in this model, as power based approaches would neglect this coupling effect.

The reflected term arrives back after a double bounce from the background. Figure 2 depicts the different characteristics of these components. The expressions for these terms are given as follows,

Direct Term: $\sigma^o_{pq_d}$

$$\sigma^o_{pq_d} = \left[\sum_{j=1}^{Ntype} \rho^{(j)} 4\pi \left| f^{(j)}_{pq}(-\hat{i}^-, \hat{i}^-) \right|^2 \right] * \left[\frac{1 - e^{-2\,\mathrm{Im}(\kappa_q + \kappa_p)d}}{2\,\mathrm{Im}(\kappa_q + \kappa_p)} \right] . \tag{3}$$

Direct-Reflected Term: $\sigma^o_{pq_{dr}}$

$$\sigma^o_{pq_{dr}} = \sum_{j=1}^{Ntype} \rho^{(j)} \sigma^{(j)}_{qq_{dr}} d \left| \Gamma_{sq} \right|^2 , \tag{4a}$$

$$\sigma^{(j)}_{qq_{dr}} = \sigma^{(j)}_{pq_{dr(1)}} + \sigma^{(j)}_{pq_{dr(2)}} + \sigma^{(j)}_{pq_{dr(3)}} , \tag{4b}$$

$$\sigma^{(j)}_{pq_{dr(1)}} = 4\pi \left| f^{(j)}_{qq}(-\hat{i}^+, \hat{i}^-) \right|^2 , \tag{4c}$$

$$\sigma^{(j)}_{pq_{dr(2)}} = 4\pi \left| f^{(j)}_{qq}(-\hat{i}^-, \hat{i}^+) \right|^2 , \tag{4d}$$

$$\sigma^{(j)}_{pq_{dr(3)}} = 8\pi\,\mathrm{Re}\left\{ f^{(j)}_{qq}(-\hat{i}^+, \hat{i}^-)\, f^{(j)^*}_{qq}(-\hat{i}^-, \hat{i}^+) \right\} . \tag{4e}$$

Reflected Term: $\sigma^o_{pq_r}$

$$\sigma^o_{pq_r} = \left[\sum_{j=1}^{Ntype} \rho^{(j)} 4\pi \left| f^{(j)}_{pq}(-\hat{i}^+, \hat{i}^+) \right|^2 \right] * \left| \Gamma_{g_p} \right|^2 \left| \Gamma_{g_q} \right|^2 e^{-2\,\mathrm{Im}(\kappa_q + \kappa_p)d} * \left[\frac{1 - e^{-2\,\mathrm{Im}(\kappa_q + \kappa_p)d}}{2\,\mathrm{Im}(\kappa_q + \kappa_p)} \right] . \tag{5}$$

In the expressions above, the total number of different spheres inside the medium is denoted by Ntype, and j is an index over the different types. The density of type j in the medium is denoted by $\rho^{(j)}$. The scattering amplitudes for a q-polarized incident wave that is scattered back with p-polarization are denoted by $f_{pq}(\hat{o},\hat{i})$, where the first unit vector in the argument ($\hat{o}$) refers to the outgoing (i.e. scattered) field direction and the second unit vector ($\hat{i}$) corresponds to the incoming (or incident) field direction for a particle. The superscripts + and – over the unit vectors describe the direction of the vector along the z-axis, i.e. the axis normal to the medium boundary. Thus, i^- indicates incidence along a downward direction (i.e., with a negative component along the z-axis), and corresponds to the direction of the incident mean wave. The mean wave can also be incident on a particle after a reflection from the background, and i^+ refers to that direction, which is upward (i.e. with a positive component along the z-axis). Similarly $-i^-$ and $-i^+$ correspond to direction of the scattered fields from the particle for the backscatter case. Figure 3 depicts these vectors.

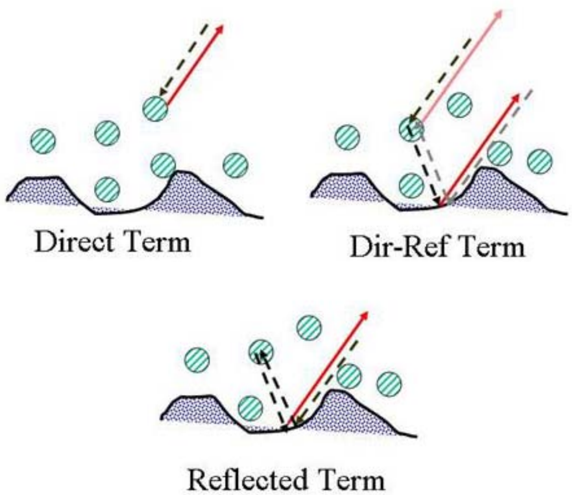

Fig. 2. Backscattering components.

The effective propagation constant inside the medium for different polarization states is represented by κ_p and κ_q, where p and q denote polarization states for the scattered and incident fields, respectively. Both κ_p and κ_q are functions of the equivalent medium characteristics. For an azimuthally symmetric medium, given $k_z = k_o \cos(\theta_o)$, the expressions can be written as,

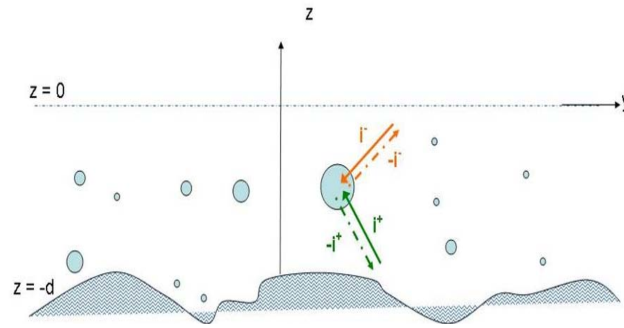

Fig. 3. Unit vector definitions.

$$\kappa_q = k_z + \sum_{j=1}^{Ntype} \frac{2\pi\rho^{(j)}}{k_z} f_{qq}(\hat{i}^-,\hat{i}^-), \quad q \in \{\hat{h}_i, \hat{v}_i\} \tag{6a}$$

$$\kappa_p = k_z + \sum_{j=1}^{Ntype} \frac{2\pi\rho^{(j)}}{k_z} f_{pp}(-\hat{i}^-,-\hat{i}^-). \quad p \in \{\hat{h}_s, \hat{v}_s\} \tag{6b}$$

III. MODEL RESULTS

Modeling the electromagnetic interactions with sea spray requires knowledge on the characteristics of the droplets. Many approaches for generating a spray generation function exist in literature, [8-10]. The sea spray generation function, commonly denoted as dF/dr_0, [9] where r_0 is the radius of a droplet at its formation, has units of number of droplets produced per square meter of surface per second per micrometer increment in droplet radius.

There is not a real consensus on how to model this function, which can be affected by various external parameters such as the wind speed, liquid surface tension, etc. However, a noticeable feature observed of natural sprays is that the droplet size distribution is very broad, and demonstrates a skewed distribution, [11] [12]. A comprehensive discussion on the formation of the droplets is given in [13], and demonstrations of the breakup regimes of liquid jets from ligaments when subjected to flowing gas are provided. The droplet distribution studied in this paper is based on [13] where the pdf for droplet sizes is calculated based on the pdf of ligaments of different lengths. This results in an exponential function as follows,

$$p(d) = e^{-nd/\langle d_0 \rangle} \tag{7}$$

where d is the droplet diameter in millimeters, $<d_0>$ is the average ligament size, and the parameter n is approximately given as $n \approx 3.5$. The distribution function is normalized in the calculations such that the integral over the possible sizes is one. Therefore equation (7) is modified as follows,

$$f(d) = \frac{n}{\langle d_0 \rangle} e^{-nd/<d_0>}. \tag{8}$$

IV. SIMULATION PARAMETERS AND TEST CASES

Simulations for backscattering are based on a 1 m^3 water amount distributed in a medium with 2.5 m height. The possible droplet sizes in the medium are discretized to four values; i.e., d = [0.05, 0.25, 0.50, 1.00] mm, as shown in Fig. 4 where the solid line describes the pdf in equation (8), and the square markers denote the sampled sizes for the simulations.

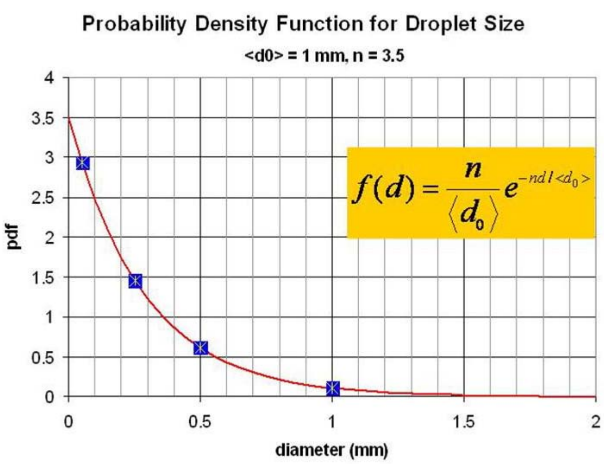

Fig. 4. Droplet size distribution in the simulation based on spray probability density function.

Three different test cases were run using the spray model. Each test case has different scatterer characteristics. The first case represents the $1m^3$ water using the four types of spheres as identified in Fig. 4. The second case generates the same amount of water by using only the smallest size droplets; i.e., diameter is 0.025 mm. The third case generates the same amount of water by using only the largest size droplets, i.e. diameter is 1.00 mm. The total number for each droplet size for the three cases is then computed such that they add up to the total volume of water; i.e., $1m^3$, in the medium. A Lambertian rough surface is assumed in the background.

The scattering calculations from the spheres are based on expressions in [14], which provide approximations to Mie Theory for different values of $k_o a$. A plane wave at 30 GHz is assumed to be incident on the medium, along a 60 degree incidence angle with respect to the z-axis. Only hh-polarization is presented in the table, as the results for vv-polarization is identical due to the assumption of perfect spheres for the droplets. There is no cross-polarized component for the same reason. Only the direct and direct-reflected components are calculated for the total return as the reflected term is expected to be smaller than these terms due to the double bounce from the background and longer path traveled inside the medium. The total backscattering cross-section and its different components from the medium are shown in the table in Fig. 5 for the three test cases.

CASE 1	Diam (mm)	# spheres	Sdd (dB)	Sdr (dB)
Type 1	0.05	2.7389×10^{10}	-75.3	-172.4
Type 2	0.25	1.3601×10^{10}	-36.4	-133.5
Type 3	0.50	0.5670×10^{10}	-22.2	-119.3
Type 4	1.00	9.8526×10^{8}	-11.7	-108.8
Sdd = -11.3 dB	Sdr = -108.4 dB	Stot = -11.3 dB	Atten = -108.6 dB	

CASE 2	Diam (mm)	# spheres	Sdd (dB)	Sdr (dB)
Type 1	0.05	1.5279×10^{13}	-33.9	-36.3
Sdd = -33.9 dB	Sdr = -36.3 dB	Stot = -31.9 dB	Atten = 0. dB	

CASE 3	Diam (mm)	# spheres	Sdd (dB)	Sdr (dB)
Type 1	1.0	1.9099×10^{9}	-11.7	-207.9
Sdd = -11.7 dB	Sdr = -207.9 dB	Stot = -11.7 dB	Atten = -210.6 dB	

Fig. 5. Backscattering cross-section from the medium for different cases - contribution from different terms. The total water content is $1m^3$ for all cases.

It is observed from the simulation results that:

(1) In case 1, strongest contribution to backscatter is from the larger size droplets for the direct term. The dir-ref term is much weaker, as the attenuation is very high (108 db) for this case. Therefore, the total backscattering for this case is due to the direct backscatter from largest particles.

(2) In case 2, the total backscattering is significantly lower than case 1, although the attenuation is negligible for this case. The smaller particles do not have a high return, although the amount of water is exactly the same as in case 1. The direct-reflected term is comparable to the direct term, but the sum of both direct and dir-ref is still lower than case 1. This is because the particle size is much smaller than the wavelength (λ = 10 mm and d = 0.05 mm), and there is not strong backscattering as the particles behave like point sources.

(3) Case 3 results are almost identical to case 1, although case 3 has almost twice the number of spheres in case 1. The reason for almost identical backscattering in

both cases although the particle density is doubled in Case 3 can be explained by the increased attenuation due to the doubling of the particle density inside the medium. The incident field penetrates less into the medium for the denser case, resulting in an effectively equal number of particles contributing to backscattering.

V. CLUSTER EFFECTS

When the number of particles inside a medium increases, the particles are located much closer to each other. The simulation results for the test cases above do not consider such effects as single scattering is assumed in the calculations for scattered fields. As a matter of fact, increasing the number of spheres in the model further will not increase the backscatter response with this approach. However, multiple scattering or clustering effects becomes important under these circumstances, and can yield higher returns than the predictions based on single scattering.

In order to investigate clustering effects, two cases with identical number and size of water droplets were run. The droplets were paired in one simulation and treated as a scatterer type. The other simulation treated the spheres separately using single scattering as before. For the test case with pairs, the scattering from the pair is calculated using HFSS, which is a full wave analysis tool that uses the finite element method. The full wave approach incorporates multiple scattering effects between the spheres in a pair. The bistatic scattering characteristics of the droplet pair as calculated by HFSS is shown in Fig. 6.

The bistatic scattering result generated by HFSS is incorporated in the model, so that the droplet pair is treated as a scatterer in the medium. The equivalent propagation constant of the medium, the attenuation coefficient and the backscattering terms are computed using the HFSS generated results. All spheres were assumed to have diameters of 1 mm, and the pair test case assumed a center to center separation of 1.2 mm between the droplets. The results of the two cases are shown in Fig. 7. Case 3 corresponds to the single scattering case as before, and case 4 shows the effects of pairing the spheres in the medium. Although we have identical particles and particle densities in both cases, accounting for the close proximity of particles, i.e., cluster effects, enhances the backscatter by almost 4 dB. Two factors contribute to this enhancement effect: (i) the backscattering is more than double for pairs versus single sphere. (ii) The attenuation in the medium is a function of the imaginary part of forward scattering amplitude. HFSS results suggest that there is only 25% increase in the imaginary part of the forward scattering amplitude for a pair of spheres versus a single sphere. Therefore, the medium with droplet pairs have less attenuation compares to the medium with single spheres.

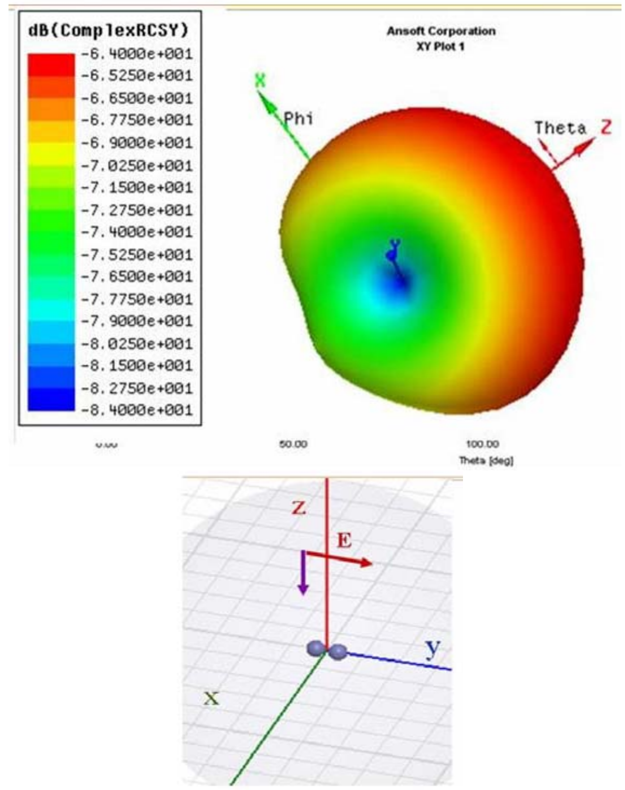

Fig. 6. Bistatic scattering characteristics of the droplet pair - HFSS simulation, 1.2 mm separation between centers of droplets, diameter = 1mm.

CASE 3 (single)	Diam (mm)	# spheres	Sdd (dB)	Sdr (dB)
Type 1	1.0	1.9099×10^9	-11.7	-207.9
Sdd = -11.7 dB	Sdr = -207.9 dB	Stot = -11.7 dB	Atten = -210.6 dB	

CASE 4 (pair)	Diam (mm)	# sphere pairs	Sdd (dB)	Sdr (dB)
Type 1	1.0	0.95496×10^9	-7.2	-123.6
Sdd = -7.2 dB	Sdr = -128.7 dB	Stot = -7.2 dB	Atten = -128.7 dB	

Fig. 7. Comparison of clustering effects for identical medium, pair versus single droplet.

VI. CONCLUSIONS

The interaction of electromagnetic waves with sea spray is a complex phenomenon. Particle size, shape, density and distribution in the medium all play a significant role in modeling these interactions. Furthermore, numerous environmental factors, such as the sea state, wind, and man made influences affect the characteristics of such medium. A statistical electromagnetic model is developed to model this phenomenon as a discrete random medium using single scattering approach and distorted Born approximation. The water droplets are modeled as perfectly conducting spheres. This assumption simplifies the calculations and provides a first order estimate in understanding the characteristics of backscattering from sea spray. Effects of ligaments which are elongated water particles from which droplets are torn are neglected in the current model. The presence of foam and air bubbles at the interface between air and sea are not included either.

Calculations based on this model demonstrate that the particle size and density determine the attenuation levels in the medium. For a low albedo; i.e., lossy, medium the most significant contribution to backscattering is due to volume scattering; i.e., direct term. The strength of the direct-reflected contribution depends on the attenuation levels in the medium, and can be comparable to volume scattering for lossless media. It is also shown that the close proximity of particles can have a significant effect in terms of backscattering and attenuation. This effect was demonstrated by comparing two cases with identical particle size and density. This implies that cluster effects (i.e., groups of two or more droplets) should be investigated to avoid underestimation of backscattering.

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