An Adaptive Preconditioning Technique using Fuzzy Controller for Efficient Solution of Electric Field Integral Equations

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Abstract ─ For efficiently solving large dense complex linear systems that arise in the electric field integral equation (EFIE) formulation of electromagnetic scattering problems, a new adaptive preconditioning technique using fuzzy controller (FC) is introduced and used in the context of the generalized minimal residual iterative method (GMRES) accelerated with the multilevel fast multipole method (MLFMM). The key idea is to control the choice of the preconditioner to be used in an iterative solver by using fuzzy controller. This approach allows the expert knowledge to be taken into account on the controller design and utilizes feedback to tune the cores of the fuzzy set. Numerical results show that the best preconditioner can be selected while maintaining low cost for adaptive procedures.

Index Terms ─ Adaptive preconditioning technique, electric field integral equation, fuzzy controller, multilevel fast multipole method.

I. INTRODUCTION

In electromagnetic wave scattering calculations, a classic problem is to compute the induced currents on the surface of an object illuminated by a given incident plane wave. Such calculations, relying on Maxwell's equations, are crucial to the simulations of many industrial processes ranging from electromagnetic compatibility, antenna design, calculation of radar cross section (RCS), and so on. All of these simulations are very demanding in terms of computer resources, and require fast and efficient numerical methods and approximate solution of Maxwell's equations. Using the equivalence principle, Maxwell's equations can be recast in the form of integral equations that relate the electric and magnetic fields to the equivalent electric and magnetic currents on the surface of the object.

The integral formulation considered in this paper is the electric integral equation (EFIE) [1]. It is widely used for electromagnetic wave scattering problems as it can handle the most general geometries without any assumption. However, the matrix associated with the resulting linear systems is large and dense for electrically large targets in electromagnetic scattering. It is basically impractical to solve EFIE matrix equations using direct methods because they have a memory requirement of $O(N^2)$ and computational complexity of $O(N^3)$, where N refers to the number of unknowns. This difficulty can be circumvented by using Krylov iterative methods, and the required matrix-vector product operation can be efficiently evaluated by multilevel fast multipole mehthod (MLFMM) [2]. The use of MLFMM accelerated Krylov methods reduce the memory requirement to $O(N)$ and the computational complexity to $O(NlogN)$.

It is well-known that EFIE provides a first-kind integral equation which is ill-conditioned and gives rise to linear systems that are challenging to solve by the Krylov methods. Therefore, a variety of preconditioning techniques have been used to improve the conditioning of the system before the iterative solution. Simple preconditioners like the diagonal or diagonal blocks of the coefficient matrix can be effective only when the matrix has some degree of diagonal dominance [3]. Symmetric successive over-relaxation (SSOR) preconditioner shows good performance in conjugate gradient (CG) iterative method [4], but becomes poor for nonsysmmetric systems. Incomplete LU (ILU) decomposed preconditioners have been successfully used on nonsysmmetric dense systems in [5], but the factors of the ILU preconditioner may become very ill-conditioned. Approximate inverse methods are generally less prone to instabilities on indefinite systems [6], and several preconditioners of this type have been proposed in electromagnetism. It has been shown in [7] that this technique outperforms more classical approaches like incomplete factorizations.

In this paper, we consider the performance of different predonditioners used in different problems. The choice of preconditioning methods suitable for one problem may not be the best for another one [13, 14]. Arbitrary selection in some cases lead to numerical problems like loss of convergence due to those initial choices. As an attempt for a possible remedy, a good choice of the preconditioner is made adaptively by a fuzzy controller after several iterations while maintaining low requirements for computer resources [8]. As a result, the idea of this work is to develop a general framework to dynamically change the parameters by taking into account the modeler knowledge. And the choices related to those preconditioning methods are considered as a control problem.

This paper is organized as follows. Section II gives a brief introduction to the EFIE formulation and MLFMM. Section III describes the construction and implementation of the fuzzy controller in more details. Numerical experiments with a few electromagnetic scattering problems are presented to show the efficiency of the adaptive preconditioner by FC in Section IV. Section V gives some conclusions.

II. EFIE Formulation and MLFMM

The EFIE formulation of electromagnetic wave scattering problems using planar Rao-Wilton-Glisson (RWG) basis functions for surface modeling is presented in [1]. The resulting linear systems from EFIE formulation after Galerkin's testing are briefly outlined as follows:

$$\sum_{n=1}^{N} Z_{mn} a_n = V_m, \qquad \mathrm{m} = 1, 2, ..., N, \quad (1)$$

where

$$Z_{mn} = jk\int_s \mathbf{f}_m(\mathbf{r})\bullet\int_{s'}(\bar{\mathbf{I}} + \frac{1}{k^2}\nabla\nabla\bullet)[G(\mathbf{r},\mathbf{r}')\mathbf{f}_n(\mathbf{r}')]dsds'$$

$$V_m = \frac{1}{\eta}\int_s \mathbf{f}_m(\mathbf{r})\bullet\mathbf{E}^i(\mathbf{r})ds\,.$$

Here $G(\mathbf{r}, \mathbf{r}')$ refers to the Green's function in free space and $\{\alpha_n\}$ is the column vector containing the unknown coefficients of the surface current expansion with RWG basis functions. Also, as usual, $\mathbf{r}$ and $\mathbf{r}'$denote the observation and source point locations. $\mathbf{E}^i(\mathbf{r})$ is the incident excitation plane wave, and η and k denote the free space impendence and wave number, respectively. Once the matrix equation (1) is solved by the numerical matrix equation solvers, the expansion coefficients $\{\alpha_n\}$ can be used to calculate the scattered field and RCS. In the following, we use $\mathbf{A}$ to denote the coefficient matrix in equation (1), $\mathbf{x} = \{\alpha_n\}$, and $\mathbf{b} = \{V_m\}$ for simplicity. Then, the EFIE matrix equation (1) can be symbolically rewritten as:

$$\mathbf{Ax = b}. \quad (2)$$

To solve the above matrix equation by an iterative method, the matrix-vector products are needed at each iteration. Physically, a matrix-vector product corresponds to one cycle of iteractions between the basis functions. The basic idea of the fast multipole method (FMM) is to convert the interaction of element-to-element to the interaction of group-to-group. Here a group includes the elements residing in a spatial box. The mathematical foundation of the FMM is the addition theorem for the scalar Green's function in free space. Using the FMM, the matrix-vector product $\mathbf{Ax}$ can be written as:

$$\mathbf{Ax} = \mathbf{A}_N\mathbf{x} + \mathbf{A}_F\mathbf{x}. \quad (3)$$

Here $\mathbf{A}_N$ is the near part of $\mathbf{A}$ and $\mathbf{A}_F$ is the far part of $\mathbf{A}$.

In the FMM, the calculation of matrix elements in $\mathbf{A}_N$ remains the same as in the MoM procedure. However, those elements in $\mathbf{A}_F$ are not explicitly computed and stored. Hence, they are not numerically available in the FMM. It has been shown that the operation complexity of FMM to perform $\mathbf{Ax}$ is $0(N^{1.5})$. If the FMM is implemented in multilevel, the total cost can be reduced further to $0(N\log N)$ [2].

III. ADAPTIVE PRECONDITIONER USING FUZZY CONTROLLER

In this section, we show how fuzzy logic provides a methodology for representing and implementing the expert knowledge about how to control the process. In particular, we apply this methodology to control the process of the preconditioner of an iterative solver. We first underline the main components and characteristic mechanisms of a FC. Afterwards, we present how to control the choice of the preconditioner using FC.

First of all, the "early steps" is defined as the first several steps of the iterative solver. If the convergence rate of the iterative solver can be evaluated approximately by the early steps. Using this information, we could decide which preconditioner is the most suitable one to accelerate the solution of the linear system. The key problem is how to evaluate the convergence rate from the early steps. In this paper, the residual of the iterative solver and the difference of the residual between two steps are used to evaluate this information.

Generally, a preconditioner corresponding to the smallest residual at the first step can be considered as the best preconditioner. However, the largest difference of the residual between the first two steps can be considered as the best precondioner. High order difference can also be used to describe the property of a preconditioner. Therefore, a fuzzy controller is used and shown in Figure 1. The *process* block is the object to be controlled. $\boldsymbol{u}(t)$ is the process input and $\boldsymbol{y}(t)$ is the process output. $\boldsymbol{r}(t)$ represents the desired target for the output of the process. The *controller* block is for changing the value of $\boldsymbol{u}(t)$ based on the controller input $\boldsymbol{y}(t)$ and the target $\boldsymbol{r}(t)$. The error as well as the rate of change-in-error defined as

$$\boldsymbol{e}(t) = \boldsymbol{r}(t) - \boldsymbol{y}(t), \tag{4}$$

$$\frac{\partial}{\partial t}\boldsymbol{e}(t) = \frac{\boldsymbol{e}(t) - \boldsymbol{e}(t-\Delta t)}{\Delta t}, \tag{5}$$

where Δt is the time between two consecutive data captured by the controller. In particular, Δt is set equal to one in an iterative solver.

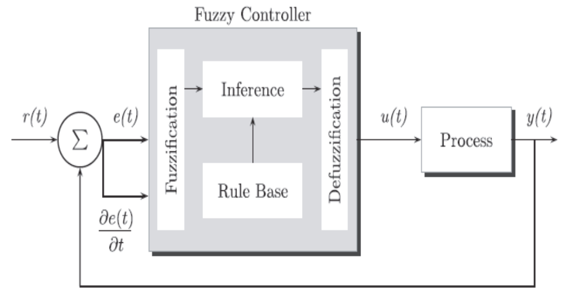

Fig. 1. Block diagram of a feedback fuzzy control.

As a controller for the choice of the preconditioner when solving equation (2), the feedback fuzzy control system takes advantage of residual at each iterative step. $\boldsymbol{u}(t)$ is the preconditioner selected by controller, $\boldsymbol{y}(t)$ is the approximate solution, and $\boldsymbol{r}(t)$ represents the right-hand-side of the equation (2). As a result, $\boldsymbol{e}(t)$ is the residual defined by

$$\boldsymbol{e}(t) = \boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}(t). \tag{6}$$

Therefore, $\partial\boldsymbol{e}(t)/\partial t$ is the rate of change-in-residual which means the difference of residual between two iterative steps.

This fuzzy-logic-based approach allows expert knowledge to be taken into account on the controller design. A preconditioning method is selected by the controller with the principle that the best preconditioner performs highest convergence rate for a given problem. After several iterations, the approximate convergence rate can be defined by using the high order difference of residual which is shown as

$$\boldsymbol{rate} = \boldsymbol{e} + \partial\boldsymbol{e}/\partial t + \frac{1}{2}\partial^2\boldsymbol{e}/\partial t^2 + \cdots \tag{7}$$

Obviously, if the order equal to the total number of iterations, the rate can describe the convergence exactly. Due to the finite computer resource, we often use two or three iterative steps to compute the approximate rate. The formulations can be defined by

$$\boldsymbol{rate} = \boldsymbol{e} + \partial\boldsymbol{e}/\partial t, \tag{8}$$

$$\boldsymbol{rate} = \boldsymbol{e} + \partial\boldsymbol{e}/\partial t + \frac{1}{2}\partial^2\boldsymbol{e}/\partial t^2. \tag{9}$$

As a result, we choose the preconditoner with the largest convergence rate as a suitable preconditioning method.

Assume that three preconditioning methods are available ranging from Jacobi, SSOR, and SAI (sparse approximate inverse). The main steps of this preconditioning method are described as follows:

Step 1: Construct the preconditioners by those three methods separately.

Step 2: Do several iterations by Krylov iterative methods and note the residual and change-in-residual at each step. In this paper, the number of iterations is set to be 3.

Step 3: Apply the FC to choose the best preconditioner.

Step 4: Use the best preconditonier to complete the iteration.

IV. NUMERICAL RESULTS

In this section, we show some numerical results that illustrate the effectiveness of the proposed adaptive preconditioning method for the solution of large dense linear systems arising from the discretization of EFIE formulation in electromagnetic scattering problems. In our experiments, the restarted version of GMRES(m) [9] algorithm is used as an iterative method, where m is the dimension size of Krylov subspace for GMRES. Additional details and comments on the implementation are given below:

(1) Zero vector is taken as initial approximate solution for all examples.

(2) The maximum number of iterations is limited to be 2000.

(3) The iteration process is terminated when the normwise backward error is reduced by 10^{-3} for all examples.

We investigate the performance of the adaptive preconditioner using fuzzy controller on four examples, which are shown in figures 2-5. They consist of an almond with 1815 unknowns at 3GHz, a double ogive with 2574 unknowns at 5GHz, a cube with 3366 unknowns at 350MHz, and a sphere with 3972 at 200MHz. The first two geometries come from [10], the side length of the cube is 1m and the radius of the sphere is also 1m. The numerical results of bistatic RCS for horizontal polarization are also displayed in figures 2-5 for these four geometries. All experiments are performed on a Pentium 4 with 2.66 GHz CPU and 960MB RAM in single precision.

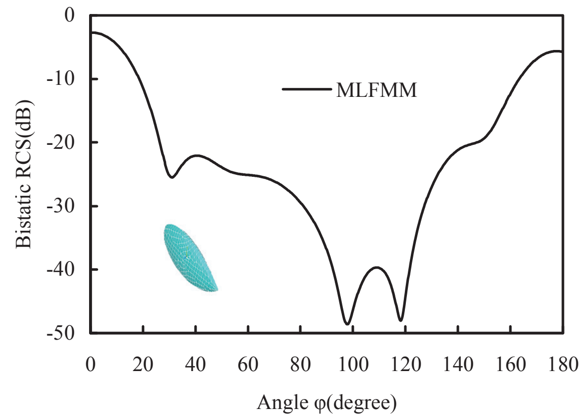

Fig. 2. Bistatic RCS for horizontal polarization at 3GHz for NASA Almond.

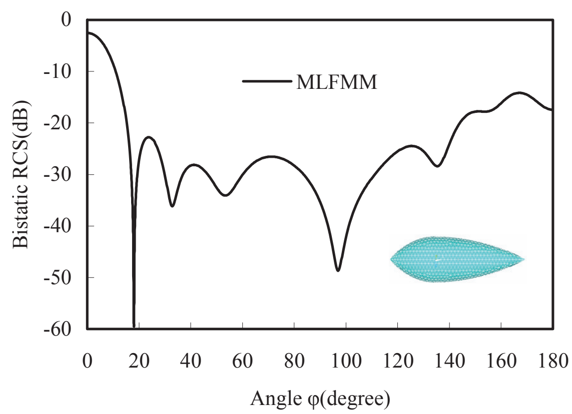

Fig. 3. Bistatic RCS for horizontal polarization at 5GHz for Double-Ogive.

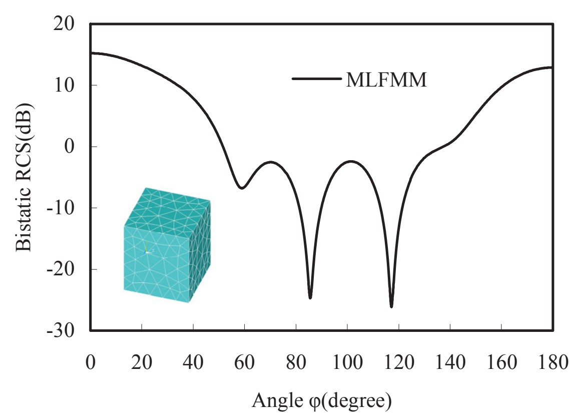

Fig. 4. Bistatic RCS for horizontal polarization at 350MHz for PEC Cube.

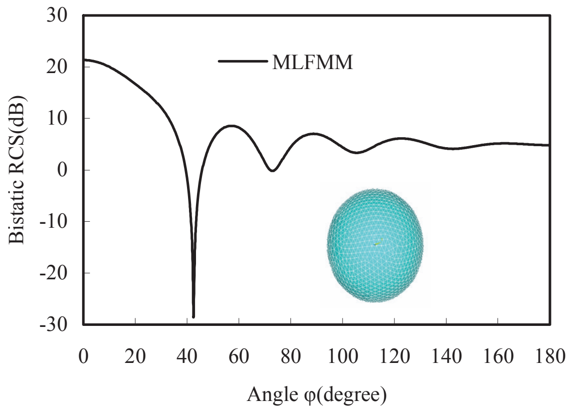

Fig. 5. Bistatic RCS for horizontal polarization at 200MHz for PEC Sphere.

Figures 6 to 9 show the convergence history of GMRES(m) algorithms with different preconditioners for all examples. It can be observed that the adaptive preconditioned GMRES has almost the same convergence history as that of the optimal preconditioner.

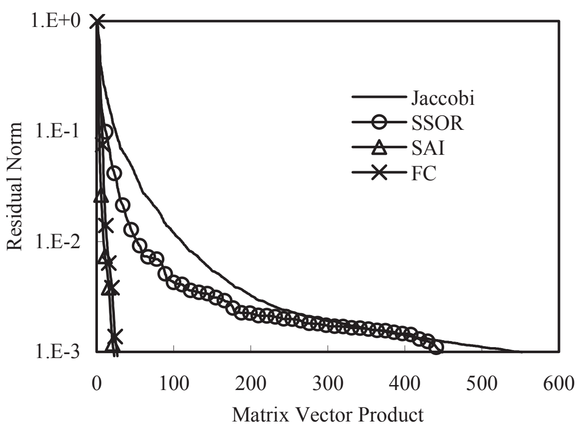

Fig. 6. Convergence history of GMRES algorithms with different preconditioners on the almond example.

Since a good preconditioner depends not only on its effect on convergence but also on its construction and implementation time. Tables 1-4 list the construction time and total solution time of GMRES algorithms with different preconditioners on all examples. According to these results, we can easily find that the proposed adaptive preconditioning method using FC requires more construction time than other preconditioners. As a control method for the choice of preconditioners, the adaptive preconditioner has to prepare all of the preconditioners for choice. Therefore, large time costs during the process of construction of all the preconditioners. However, the new method shows its efficiency on convergence in these examples. Furthermore, the initial time of adaptive preconditioner is negligible when compared with the total CPU time cost in monostatic RCS computation. Therefore, this proposed method is suitable for analysis of monostatic scattering.

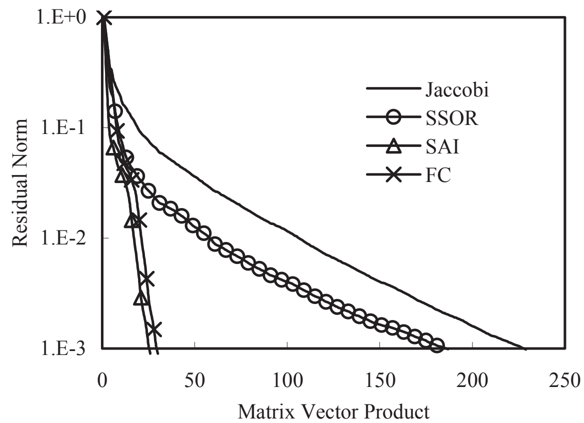

Fig. 7. Convergence history of GMRES algorithms with different preconditioners on the double-ogive example.

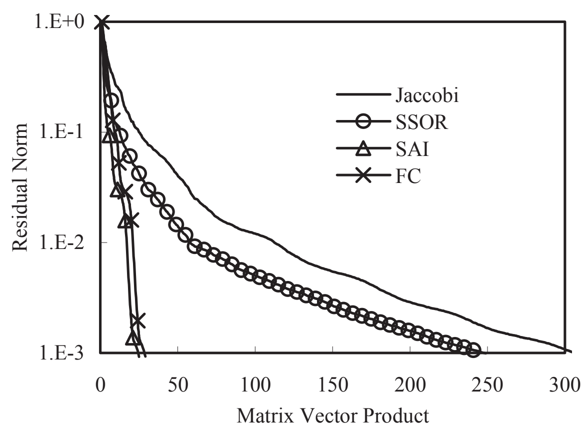

Fig. 8. Convergence history of GMRES algorithms with different preconditioners on the cube example.

V. CONCLUSIONS AND COMMENTS

In this paper, fuzzy controller is presented and used for building robust adaptive preconditioning method for efficiently solving large dense linear systems that arise in EFIE formulation of electromagnetic scattering problems. The main idea is to make a choice of preconditioners which performs the highest convergence rate. Numerical experiments on several examples are preformed and comparison with general preconditioners are made, which shows the new method is more efficient

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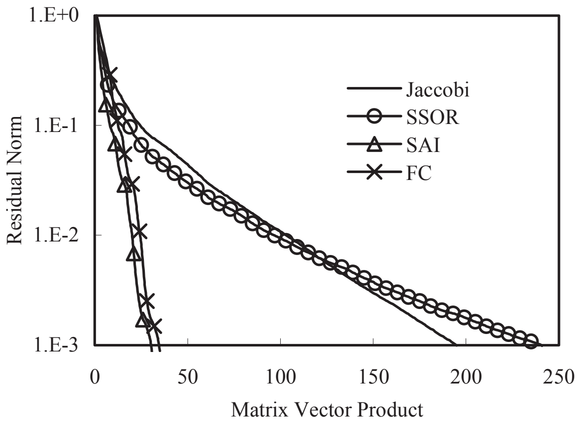

Fig. 9. Convergence history of GMRES algorithms with different preconditioners on the sphere example.

Table 1: Comparison of the cost and performance of different preconditioners on the almond example (Time: Second)

Almond	Construc-time	Number of Iterations	Sol-time	Total -time
Jacobi	/	552	31.73	31.73
SSOR	/	449	28.25	28.25
SAI	18.42	23	1.61	20.03
FC-AP	25.45	27	1.77	27.32

Table 2: Comparison of the cost and performance of different preconditioners on the double-ogive example (Time: Second)

Double ogive	Construc-time	Number of Iterations	Sol-time	Total-time
Jacobi	/	229	22.94	22.94
SSOR	/	187	20.56	20.56
SAI	11.61	26	2.86	14.47
FC-AP	16.77	30	3.19	19.96

Table 3: Comparison of the cost and performance of different preconditioners on the cube example (Time: Second)

Cube	Construc-time	Number of Iterations	Sol-time	Total -time
Jacobi	/	308	33.38	33.38
SSOR	/	249	29.91	29.91
SAI	23.02	31	3.17	26.19
FC-AP	33.45	35	3.48	36.93

Table 4: Comparison of the cost and performance of different preconditioners on the sphere example (Time: Second)

Sphere	Construc-time	Number of Iterations	Sol-time	Total-time
Jacobi	/	195	31.44	31.44
SSOR	/	241	42.02	42.02
SAI	17.33	31	5.48	22.81
FC-AP	24.72	35	5.98	30.70

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