Design and Optimization of Single, Dual, and Triple Band Transmission Line Matching Transformers for Frequency-Dependent Loads

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Abstract **–** This paper presents a design method for matching a frequency- varying load to a lossless transmission line at N frequency points using N transmission line sections. The formulation is based on the *ABCD* matrix of a cascaded N-section transmission line, terminated by a complex load. The unknown characteristic impedances and lengths of the sections are found by imposing the matching condition at N frequencies. Analytical solution for the nonlinear equations is known for the $N = 1$ case only; for $N > 1$, analytical solution is difficult to obtain, and therefore, the highly efficient particle swarm optimization method is used to find the design the parameters after casting the design into an optimization problem. Different numerical examples are presented and discussed that illustrate the validity of the proposed design method.

Keywords**:** Impedance matching, complex load, transmission line transformer, and particle swarm optimization.

I. INTRODUCTION

Multi-band impedance matching is desirable, especially with the emergence of multi-band operation in wireless communication systems [1-3]. Several papers have been published in which different techniques were proposed to design multiband matching transformers [4-14]. Recently, the particle swarm optimization (PSO) method is used to design a multi-band transmission line transformer (TLT) [15], and in [16], the PSO method is used to design a multi-band, two-way, equal split Wilkinson power divider. However, in all of these papers, the load impedance is assumed to be frequency independent, which might be considered as a limitation on the type of loads used. In [17, 18], a dual-band TLT to match complex impedances was presented and applied to wideband high-frequency amplifiers, but the formulation requires the solution of nonlinear equations through some zero-finding numerical routines. Finally, in [19] analysis method for matching complex loads using nonuniform microstrip transmission lines is presented. In this paper, we present an analysis and design method for matching a frequency- varying load to a lossless transmission line at N frequency points using N uniform TL sections. The analysis is based on the *ABCD* matrix of a cascaded N-section transmission line, terminated by a complex load. The unknown characteristic impedance and length of each section are found by imposing the matching condition at N frequencies. The resulting nonlinear equations are solved analytically for the $N = 1$ case only; for $N > 1$, analytical solution are difficult to obtain, and therefore, the highly efficient particle swarm optimization method is used after casting the design into an optimization problem.

The PSO algorithm is a multiple-agents optimization algorithm introduced by Kennedy and Eberhart [20-23] while studying the social behavior of groups of animals and insects such as flocks of birds, schools of fish, and swarms of bees. They quickly discovered they have stumbled on an easy and powerful optimization method. Since then, the method has been used in a wide range of optimization problems [24-27].

II. ANALYSIS

Referring to Fig. 1, the transmission (ABCD) matrix for the n[th] transmission line section is given by [28],

$$M_n = \begin{bmatrix} \cos(\beta l_n) & jZ_n \sin(\beta l_n) \\ jY_n \sin(\beta l_n) & \cos(\beta l_n) \end{bmatrix}, \quad n = 1, 2, \ldots, N \quad (1)$$

where $\beta = 2\pi/\lambda$, Z_n and l_n are, respectively, the characteristic impedance and length of transmission line section n, and $Y_n = 1/ Z_n$. The load impedance is in general complex and frequency dependent, i.e.,

$$Z_L(f) = R_L(f) + jX_L(f) \ . \quad (2)$$

The ABCD matrix for the load is defined by,

$$M_{N+1} = \begin{bmatrix} 1 & 0 \\ Y_L & 1 \end{bmatrix} \quad (3)$$

where $Y_L = 1/Z_L$. The overall transmission matrix is given by,

$$M = \prod_{n=1}^{N+1} M_n = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} . \quad (4)$$

The input impedance, looking into the input terminals of section 1, is given by,

$$Z_{in} = \frac{m_{11}}{m_{21}} . \quad (5)$$

The complex input reflection coefficient is calculated using,

$$S_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} . \quad (6)$$

The matching condition requires that,

$$Z_0 = Z_{in}(f_i),\ i = 1, 2, \ldots, N. \quad (7)$$

We should find Z_n and l_n, $n = 1, 2, \ldots, N$, such that the above equation is satisfied.

III. PSO ALGORITHM

As mentioned earlier, obtaining an analytical solution from the resulting nonlinear equations for the matching problem becomes extremely difficult, if not impossible, as the number of matching frequencies becomes ≥ 2, especially when the load is complex. Therefore, instead of going through lengthy analytical derivations, which will end up with the need for the use of optimization process too, we propose the use of the PSO method to design the multi-band multi-section TLT by solving the nonlinear equation (7) after casting it into an optimization problem. For a comprehensive study of the algorithm regarding its implementation, the parameters selection, applications, and other versions of the algorithm, the interested reader is referred to [20-27] and the references mentioned therein. The PSO algorithm, like other evolutionary algorithms, uses the concept of fitness or cost to guide the particles during their search for the optimum solution in the N-dimensional space.

For the current problem, the design is concerned with finding the characteristic impedances and lengths of N-section TLT that matches a lossless transmission line of characteristic impedance Z_0 to a complex load Z_L at N arbitrary frequencies (see Fig. 1). The PSO method is used to find the parameters of the matching transformer (i.e., the characteristic impedance and length of each section). The following cost function is used,

$$\textit{Cost Function} = \sum_{n=1}^{N} |S_{11}(f_n)| . \quad (8)$$

Although perfect matching at the design frequencies require the cost function to be exactly zero, however, we relaxed this condition and assumed that the matching is practically acceptable whenever the cost function reaches a value of 10^{-4} (-80 dB) or less. This value is used in terminating the PSO algorithm to indicate a solution is found.

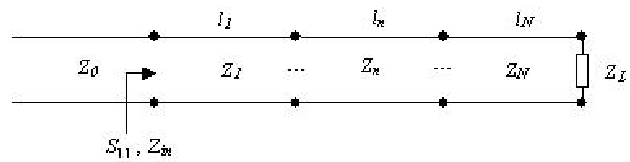

Fig. 1. An N-section, N-band matching transformer connected to a complex load.

IV. RESULTS

A. *Single-Section Transformer* (N = 1)

It is known that this case has analytical solution, and therefore there is no need to use the PSO method. The solution is given by,

$$Z_1 = \sqrt{\frac{Z_0(R_L^2 + X_L^2 - Z_0 R_L)}{R_L - Z_0}} \quad (9a)$$

$$l_1 = \frac{1}{2\pi} \tan^{-1}\left(\frac{Z_1(Z_0 - R_L)}{Z_0 X_L}\right) , \quad (9b)$$

provided that either of the following conditions is satisfied,

$$Z_0 < R_L \quad \text{or} \quad Z_0 > R_L + \frac{X_L^2}{R_L} .$$

In particular, there is always a solution if Z_L is real, which reduces to that of the quarter-wave transformer.

B. *Dual-Band Transformer* (N = 2)

For the dual and triple section transformer, it is extremely difficult, if not impossible, to find an analytical solution similar to the case of single section, and that is why the PSO algorithm is used to find the design parameters. Different two-section dual-band transformers are designed with various typical complex loads that could be encountered in practice. Some of the designed examples are implemented in microstrip lines and the resulting structure is simulated using the electromagnetic simulator software Ansoft Designer [29]. The substrate is FR4 with $\varepsilon_r = 4.5$ and thickness = 1.6 mm. The width and length of the microstrip sections are calculated from the characteristic impedance and electrical lengths using PCAAD [30].

1. The first transformer has a series RLC circuit as its load with R = 100 ohm, L = 10 nH, and C = 1 pF. The load impedance versus frequency is shown in Fig. 2. The design frequencies are f_1 = 1 GHz and f_2 = 2 GHz. The feeding line has Z_0 = 50 ohm. The obtained results for the two sections using the PSO were as follows: Z_1 = 107.78 ohm and Z_2 = 82.42 ohm, while $l_1/\lambda_1 = 0.2332$ and $l_2/\lambda_1 = 0.44$, where λ_1 is the wavelength at f_1. A typical convergence curve of the PSO algorithm is shown in Fig. 3(a), while the behavior of the reflection coefficient is shown in Fig. 3(b). Clearly, a perfect matching is achieved at the two design frequencies.

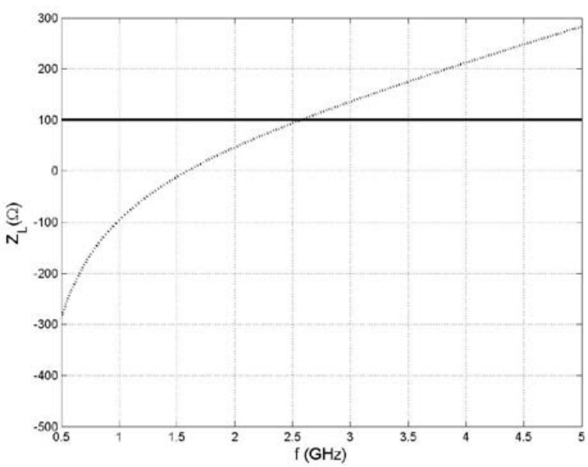

Fig. 2. Impedance of a series RLC load with R=100 ohm, L=10 nH, and C=1 pF.

2. The second example is the same except now the second design frequency is f_2 = 2.5 GHz. The obtained results were: Z_1 = 48.4612 ohm and Z_2 = 116.6992 ohm, while $l_1/\lambda_1 = 0.38242$ and $l_2/\lambda_1 = 0.1421$. Figure 4 displays the variation of $|S_{11}|$ with frequency where it is clear that a perfect matching is achieved at the two frequencies.
3. The third example is the same as the first example, except now f_2 = 1.1 GHz. The results for this case were: Z_1 = 29.6573 ohm and Z_2 = 103.1516 ohm, while $l_1/\lambda_1 = 0.4485$ and $l_2/\lambda_1 = 0.1323$. The reflection coefficient is shown in Fig. 5.
4. The fourth example is the same except now f_2 = 1.7 GHz, and C = ∞ (i.e., the capacitor in the load is replaced by a short circuit). The results for this case were: Z_1 = 90.4464 ohm and Z_2 = 146.7296 ohm, while $l_1/\lambda_1 = 0.3459$ and $l_2/\lambda_1 = 0.4890$. The corresponding reflection coefficient is shown in Fig. 6.

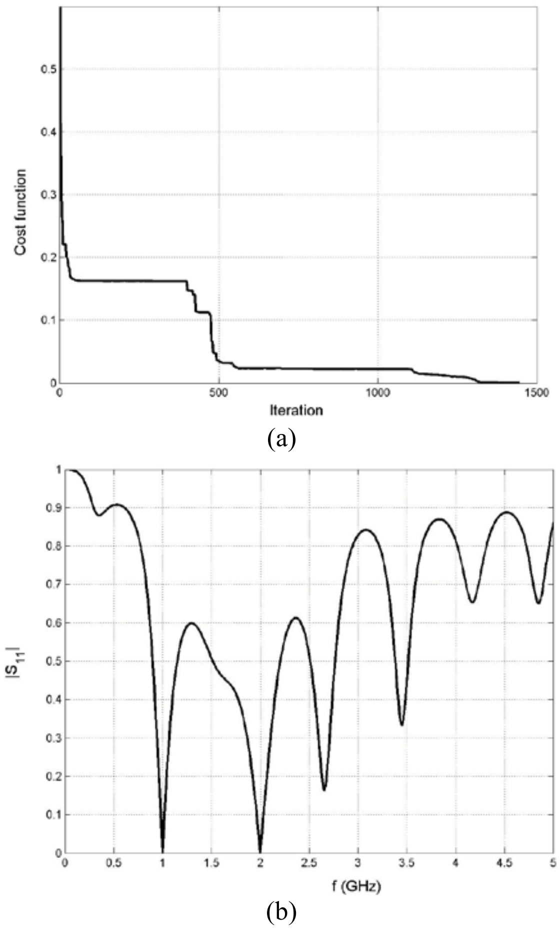

Fig. 3. (a) Convergence of the PSO algorithm. (b) Reflection coefficient versus frequency for example 1.

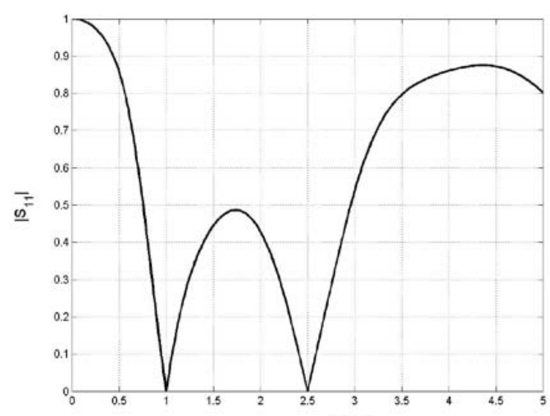

Fig. 4. Reflection coefficient versus frequency for example 2.

5. The fifth example is the same as the fourth except now the inductor is shorted out (i.e., L = 0), and the

capacitor is the same as in the first example. The obtained results were: Z_1 = 153.0174 ohm and Z_2 = 116.4692 ohm, while l_1/λ_1 = 0.2849 and l_2/λ_1 = 0.3918. The corresponding reflection coefficient is shown in Fig. 7.

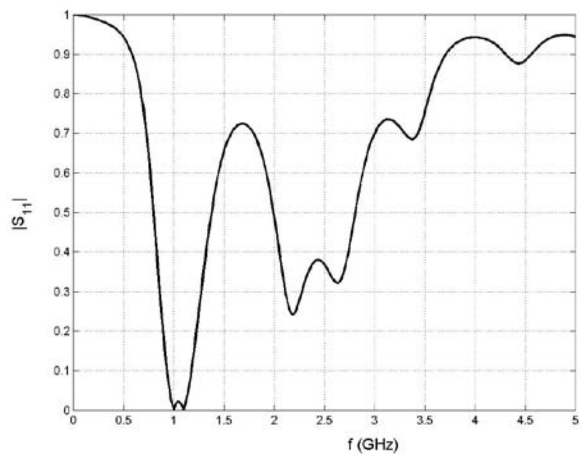

Fig. 5. Reflection coefficient versus frequency for example 3.

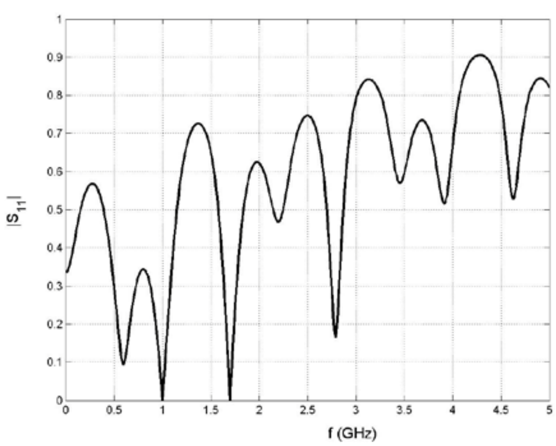

Fig. 6. Reflection coefficient versus frequency for example 4.

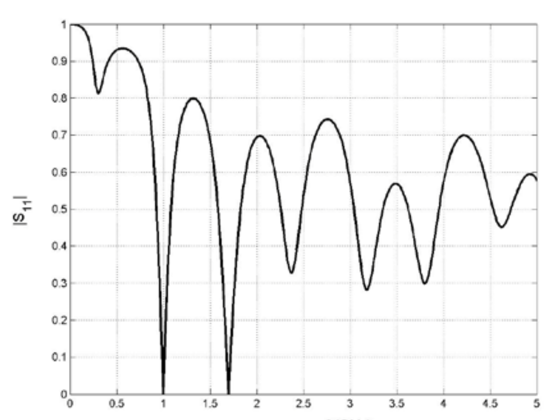

Fig. 7. Reflection coefficient versus frequency for example 5.

6. The sixth example has a parallel RLC as its load, with R = 100 ohm, L = 10 nH, and C = 1 pF whose impedance is shown in Fig. 8. The design frequencies are f_1 = 1 GHz and f_2 = 2 GHz. The obtained results were: Z_1 = 91.4413 ohm and Z_2 = 69.3849 ohm, while l_1/λ_1 = 0.2089 and l_2/λ_1 = 0.1473. The corresponding reflection coefficient is shown in Fig. 9.

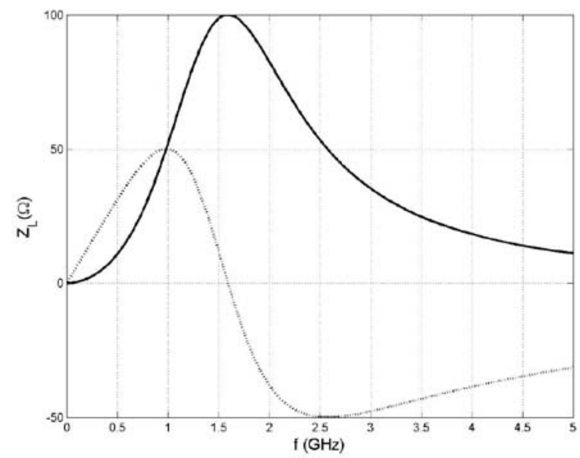

Fig. 8. Impedance of the parallel RLC load with R = 100 ohm, L = 10 nH, C = 1pF.

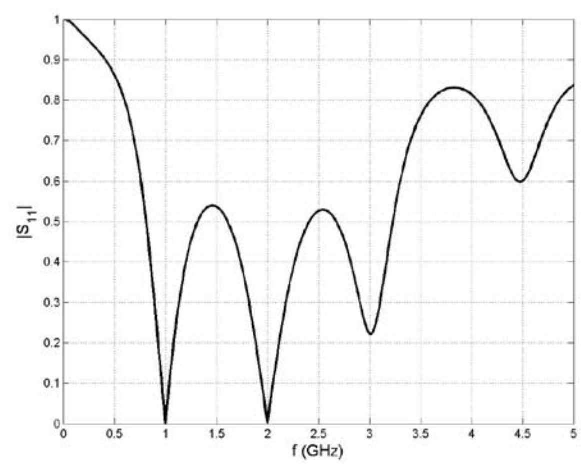

Fig. 9. Reflection coefficient versus frequency for example 6.

7. This example is the same as the sixth, except f_1 =0.9 GHz and f_2 = 2.4 GHz. The obtained results were: Z_1 = 118.2368 ohm and Z_2 = 92.0193 ohm, while l_1/λ_1 = 0.2457 and l_2/λ_1 = 0.2019. The reflection coefficient is shown in Fig. 10. The simulation results from Ansoft Designer are also shown, and an acceptable agreement is observed. The difference between the analytical results and simulation results in this example and the following ones are due to the difficulty in designing the microstrip lines that has the correct length and characteristic impedance as the theoretical values.

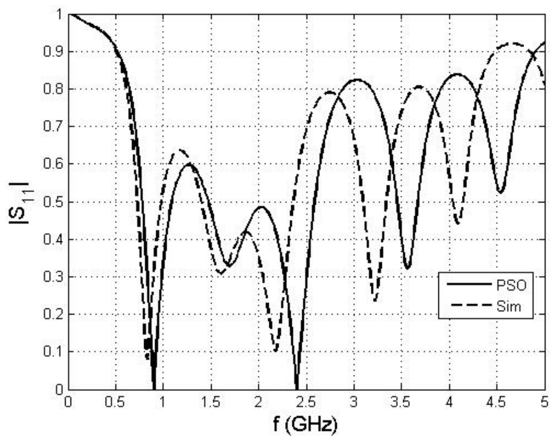

Fig. 10. Reflection coefficient versus frequency for example 7.

8. The load in this example is a dipole antenna, which has many applications in communications and microwave systems. The real and imaginary parts of the input impedance of a wire dipole are given by [31],

$$R_d = \frac{\eta}{2\pi}\{C + \ln(kl) - C_i(kl) + \frac{1}{2}\sin(kl)[S_i(2kl) - 2S_i(kl)] + \frac{1}{2}\cos(kl)[C + \ln(kl/2) + C_i(2kl) - 2C_i(kl)]\} \quad (10a)$$

$$X_d = \frac{\eta}{4\pi}\{2S_i(kl) + \cos(kl)[2S_i(2kl) - S_i(kl)] - \sin(kl)[2C_i(kl) - C_i(2kl) - C_i(\frac{2ka^2}{l})]\}, \quad (10b)$$

where C = 0.5772 (Euler's constant), $S_i(x)$ and $C_i(x)$ are the sine and cosine integrals, and a is the wire radius (which is assumed very small compared to the length l of the dipole). The calculated dipole impedance is shown in Fig. 11. The design frequencies were f_1 = 1 GHz and f_2 = 1.8 GHz, and the obtained design parameters were: Z_1 = 63.7305 ohm and Z_2 = 347.3256 ohm, while l_1/λ_1 = 0.1758 and l_2/λ_1 = 0.4729. The theoretical and simulation reflection coefficients are shown in Fig. 12, and good agreement is obtained. The difference between the results is due to the difficulty in simulating a dipole antenna that has input impedance as the theoretical one given by equation (10). The simulated dipole is a flat one, while the theoretical one is cylindrical.

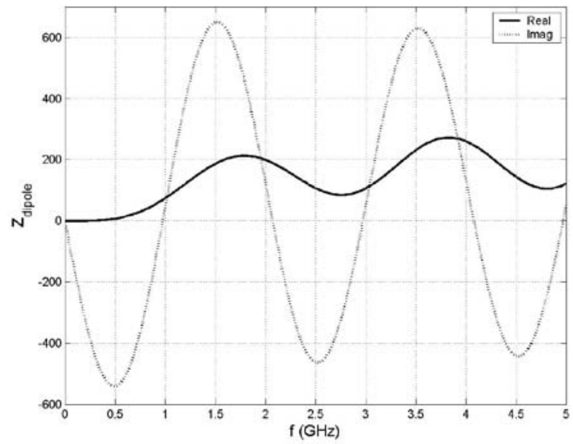

Fig. 11. Impedance of a wire dipole of radius a = $10^{-5}\lambda_0$, where λ_0 is the wavelength at 1 GHz.

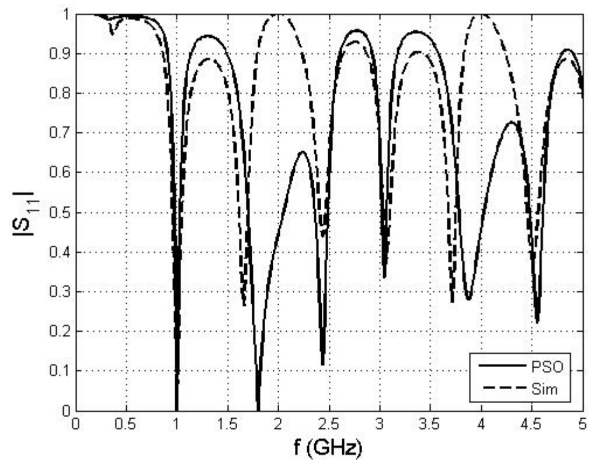

Fig. 12. Reflection coefficient versus frequency for example 8.

C. Triple-Band Transformer (*N* = 3)

Three design examples are given for this case:

9. The load is the series RLC circuit mentioned in example 1, and the design frequencies were: f_1 = 0.9 GHz, f_2 = 1.8 GHz, and f_3 = 2.4 GHz. The obtained design parameters were: Z_1 = 110.0161 ohm, Z_2 = 80.7006 ohm, and Z_3 = 102.0823 ohm, while l_1/λ_1 = 0.2758, l_2/λ_1 = 0.4187, and l_3/λ_1 = 0.0708. The corresponding reflection coefficient is shown in Fig. 13. A very good matching is obtained at the three design frequencies.
10. The load is the parallel RLC circuit mentioned in example 6, and the design frequencies were: f_1 = 0.9 GHz, f_2 = 1.8 GHz, and f_3 = 2.4 GHz. The obtained design parameters were: Z_1 = 63.3349 ohm, Z_2 = 105.6367 ohm, and Z_3 = 73.6901 ohm, while l_1/λ_1 = 0.4767, l_2/λ_1 = 0.2633, and l_3/λ_1 = 0.1773. The corresponding reflection coefficient is shown in Fig. 14. Again, a very good matching is obtained at the three design frequencies.

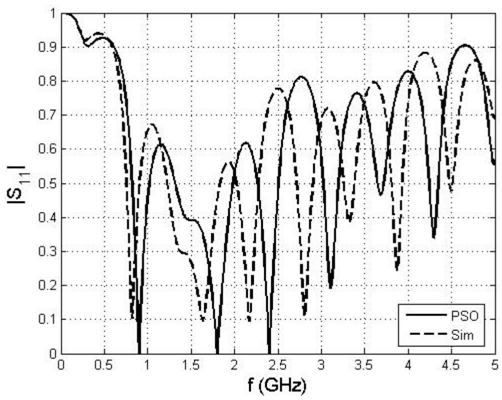

Fig. 13. Reflection coefficient versus frequency for example 9.

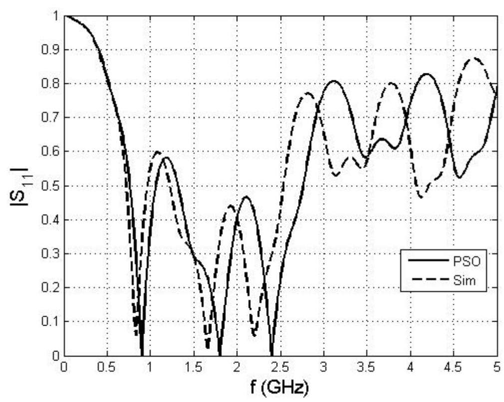

Fig. 14. Reflection coefficient versus frequency for example 10.

11. The load is the dipole antenna of example 8, and the matching frequencies are: f_1 = 0.9 GHz, f_2 = 1.8 GHz, and f_3 = 2.4 GHz. The obtained parameters were: Z_1 = 164.7378 ohm, Z_2 = 26.5777 ohm, and Z_3 = 56.5640 ohm, while l_1/λ_1 = 0.0940, l_2/λ_1 = 0.4748, and l_3/λ_1 = 0.1218. The resulting reflection coefficient from theory and simulation is shown in Fig. 15.

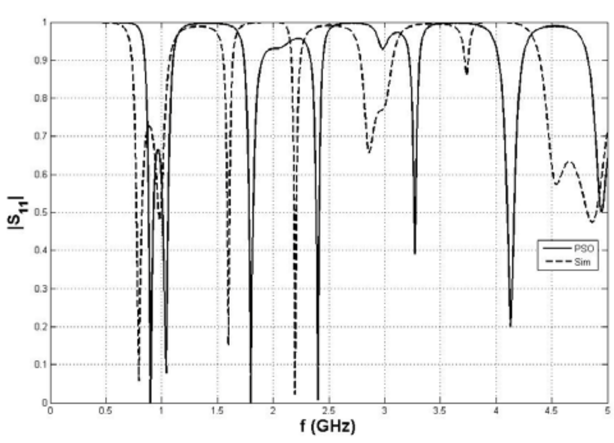

Fig. 15. Reflection coefficient versus frequency for example 11.

V. CONCLUSION

In principle, it is possible to impedance-match a complex load at N frequency points to a lossless line using ordinary N-section TLT. Analytical solution is known for the case N = 1 case. For N > 1, however, numerical solution is obtained through the use of the PSO algorithm. Different numerical examples were presented which illustrate the idea presented in this paper. It should be mentioned here that not all complex loads can be perfectly matched, and therefore, for some loads the PSO did not reach a solution. For single section, the conditions on the load are clear; for multiple sections, however, these conditions are not clear yet and remain an open area for research.

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