MODERN HIGH FREQUENCY TECHNIQUES FOR RCS COMPUTATION : A COMPARATIVE ANALYSIS

INVITED PAPER

by

F.A. MOLINET

Société MOTHESIM, La Boursidière, RN 186, 92357 Le Plessis-Robinson, France

Abstract

In this article, a comparative analysis is given of various asymptotic high frequency methods for the computation of the Radar Cross Section (RCS) of complex targets. After a brief revue of their principle, the limitations of the most popular methods and of their recent developments : Physical Optics (PO), Physical Theory of Diffraction (PTD), Geometrical Theory of Diffraction (GTD), Uniform Theory of Diffraction (UTD) and PTD extended to creeping waves, are analysed in relation with their theoretical foundations and the critical aspects of their application to the computation of RCS are discussed and illustrated by some numerical examples.

INTRODUCTION

Asymptotic high frequency techniques (GTD/UTD, PO/PTD) remain essential for the resolution of scattering problems involving large objects of arbitrary shape like airplanes, helicopters, tanks and ships, at radar frequencies. But at present, their implementation on a computer is rapidly changing. Especially, the shapes to which high frequency techniques are applied are of growing complexity and the need for general computer codes which manage automatically the geometrical modeling and ray searching is more and more pronounced. A question remains however which haunts the engineer who is in charge of the development of a general computer program for RCS calculations : what is the best choice GTD/UTD or PO/PTD?

In the past, the physical Optics (PO) approximation has generally been prefered to the Geometrical Theory of Diffraction (GTD) for its capability to calculate the field on caustics. However, at present, where the main objective is to reduce the RCS of military targets, the accuracy of PO even when one takes properly account of edge diffraction through the Physical Theory of Diffraction (PTD), is not always satisfactory especially away from the direction of specular reflections. More refinements are needed like multiple reflections or diffractions, creeping waves, whispering gallery modes etc... Now, GTD augmented by the Uniform Theory of Diffraction (UTD) is well adapted for treating such phenomena. Hence, the competition between the approaches PO/PTD and GTD/UTD is again open and one of the main criteria for the choice of a method, besides the suited accuracy, will be the easiness with which it can be applied to complex objects modeled by Computer Aided Design (CAD) techniques.

The main objective of this article is to analyse the arguments for and against each of the above mentioned methods by taking into account their most recent developments and especially the extension of PTD to creeping waves (generalized PTD).

In chapter 1, we first consider the methods based on the asymptotic expansion of surface currents. After having introduced some basic concepts which are important for our analysis we give a brief description of PO, PTD and generalized PTD.

In chapter 2 we consider the method based on the asymptotic expansion of the field at large distances from the object. Our analysis will be focused on the uniform GTD as a tool for RCS calculations of general complex targets.

The arguments for and against the uniform GTD and the generalized PTD are presented and analyzed in chapter 3, and illustrated by some numerical examples.

1 - ASYMPTOTIC DEVELOPMENT OF THE CURRENTS ON THE SURFACE OF A TARGET

1.1. Basic concepts

In a first step we assume that the exterior surface of the target is regular and that the incident time harmonic field, with the time dependence $e^{-i\omega t}$, is represented by the asymptotic expansion :

$$\left.\begin{aligned} \vec{E}^i(\vec{r}) &\sim e^{ikS^i(\vec{r})} \sum_{n=o}^{N} (ik)^{-n}\, \vec{e}^i_n(\vec{r}) + \mathcal{O}(k^{-N}) \\ \vec{H}^i(\vec{r}) &\sim e^{ikS^i(\vec{r})} \sum_{n=o}^{N} (ik)^{-n}\, \vec{h}^i(\vec{r}) + \mathcal{O}(k^{-N}) \end{aligned}\right\} \qquad (1)$$

where the phase $S^{i}(\vec{r})$ verifies the eikonal equation of GO and where the vector amplitudes $\vec{e}_n(\vec{r})$ and $\vec{h}_n(\vec{r})$ verify a system of coupled transport equations resulting from the application of the method of perturbations to the vector Helmholtz equation and the Gauss law, the small parameter being 1/kL where k is the wave number in vacuun ($k = 2\pi/\lambda$, λ = wavelength and where L is a characteristic length of the obstacle). The surfaces $S^{i}(\vec{r})$ = Const. are called wave fronts. They are orthogonal to the characteristics of the eikonal equation which is a first order partial differential equation. These characteristics or rays form a congruence in $\mathbb{R}^3$ and have therefore an envelope also called a caustic.

For a fixed value of the small parameter 1/kL, there is an optimum value of N in (1) for which the absolute value of the difference between the exact field and its asymptotic expansion is mimimum. Unfortunetely, no general rule exists which permits to predict the optimum value of N, and some experience is needed in fixing the number of terms of an asymptotic expansion. However since in most practical situations, it is not possible to construct more than the first two terms of an asymptotic expansion, this question never arises.

In an homogeneous medium, the rays are straight lines in the direction of $\overrightarrow{\nabla S}^{i}$. Some of them intercept the surface of the target and divide the space into an illuminated region and a shadow region separated by a surface Σ_o which is called the shadow boundary of the incident field (Fig. 1).

The shadow boundary is tangent to the surface S along the curve Γ separating the lit region S_e of S from the shadow region S_o. In the lit region, the incident field $(\vec{E}^{i},\vec{H}^{i})$ gives rise to an extended reflected field not strictly limited to the GO reflected field $(\vec{E}^{R},\vec{H}^{R})$, which can be represented away from the shadow boundary and possible caustics, by an asymptotic expansion similar to (1) :

$$\left.\begin{aligned} \vec{E}^{R}(\vec{r}) &\sim e^{ikS^{R}(\vec{r})} \sum_{n=0}^{N} (ik)^{-n}\, \vec{e}^{R}_{n}(\vec{r}) + \mathcal{O}(k^{-N}) \\ \vec{H}^{R}(\vec{r}) &\sim e^{ikS^{R}(\vec{r})} \sum_{n=0}^{N} (ik)^{-n}\, \vec{h}^{R}_{n}(\vec{r}) + \mathcal{O}(k^{-N}) \end{aligned}\right\} \qquad (2)$$

where the phase $S^R(\vec{r})$ and the amplitudes $(\vec{e}^R,\vec{h}^R)$ verify respectively the eikonal equation of GO and the system of coupled transport equations together with the Gauss law, resulting from the method of perturbations. As for the incident field,the characteristics or rays of the eikonal equation are orthogonal to the wave fronts $S^R(\vec{r})$ = Const. and form a congruence in $\mathbb{R}^3$. They have therefore an envelope or caustic which can be located outside (real caustic) or inside (virtual caustic) of the target. By analyzing the domain of validity of the solutions of the transport equations, it can be shown that the asymptotic expansions (1) and (2) are valid at every point in the illuminated region of $\mathbb{R}^3$ with the exception of those points located on or in the vicinity of the shadow boundaries and caustics. It is possible to use (1) and (2) at observation points located on the lit side S_e of S which are not too close to the curve Γ and to possible caustics and apply the boundary conditions at those points.

If $\hat{n}$ is the unit normal to S oriented to the outside of the volume delimited by S, we have :

- For a perfectly conducting body :

$$\hat{n} \times \vec{E}_t = 0 \ , \ \vec{r} \in S \tag{3}$$

- For an imperfectly conducting body characterized by an impedance Z :

$$\vec{E}_t - (\hat{n}.\vec{E}_t)\hat{n} = z \ \hat{n} \times \vec{H}_t \quad , \ \vec{r} \in S \tag{4}$$

where $(\vec{E}_t,\vec{H}_t)$ is the total field on the surface S.

The asymptotic expansions (1) and (2) do not give the total field on S but only an approximation of it. Other diffraction phenomena occur such as creeping waves or waves diffracted by edges if the surface S is not regular. But since the asymptotic expansions which are associated with this diffraction phenomena are defined with respect to an asymptotic sequence different from the power series $1/k^n$ where n is an integer, the boundary conditions (3) and (4) are separately verified by each species of waves. This property holds on for multiple reflected or diffracted rays of the same species, since in such a case the phase function of the asymptotic expansion is different which implies that the boundary conditions must be verified separately for each order of interaction.

Accordingly, we have for a perfectly conducting body :

$$\hat{n} \times \vec{E}_a = 0 \ , \ \vec{r} \in S_e \tag{5}$$

where $\vec{E}_a = \vec{E}^i + \vec{E}^R$, $\vec{H}_a = \vec{H}^i + \vec{H}^R$ with $\vec{E}^i$ and $\vec{E}^R$ given by (1) and (2). Similarly, for an imperfectly conducting body with a surface impedance Z we can write :

$$\vec{E}_a - (\hat{n} \cdot \vec{E}_a)\hat{n} = Z \ \hat{n} \times \vec{H}_a \quad , \ \vec{r} \in S_e \tag{6}$$

The conditions (5) or (6) involve the continuity of the phase at every point on S_e :

$$S^i(\vec{r}) = S^R(\vec{r}), \ \vec{r} \in S_e \tag{7}$$

It can be shown that (7) is equivalent to Fermat principle extended to the reflection of a wave. Consequently, the congruence of the rays associated with the generalized reflected field $\vec{E}^R$ can be related to the congruence of incident rays by applying the law of reflection on S_e.

As a matter of fact, the conditions (5) or (6) define completely the asymptotic expansion of $\vec{E}^R$ at any order n knowing the asymptotic expansion of $\vec{E}^i$. For n = 0, we find the reflected field of GO, which at a point on S_e, is independent on the curvature of the surface at that point an can therefore be calculated by replacing locally the surface and the incident wave front by their tangent planes. On the other hand, at n = 1, the vector amplitudes $(\vec{e}_n^{\,R}, \vec{h}_n^{\,R})$ on S_e depend on the curvature of the surface and of the incident wave front and for $n > 1$, they depend on higher order derivatives, superior to two, of the surface and the incident wave front.

The asymptotic expansion of the electric and magnetic currents on the surface can be directly deduced from (1) and (2). If we set :

$$\vec{J}_a(\vec{r}) = \hat{n}(\vec{r}) \times \vec{H}_a(\vec{r}) \ , \ \vec{r} \in S_e \tag{8}$$

$$\vec{M}_a(\vec{r}) = \vec{E}_a(\vec{r}) \times \hat{n}(\vec{r}) \ , \ \vec{r} \in S_e \tag{9}$$

and take into account (7), we get :

$$\vec{J}_a(\vec{r}) \sim e^{ikS^i(\vec{r})} \sum_{n=0}^{N} (ik)^{-n} \, \vec{j}^{\,a}_n(\vec{r}) \quad , \ \vec{r} \in S_e \tag{10}$$

$$\vec{M}_a(\vec{r}) \sim e^{ikS(\vec{r})} \sum_{n=0}^{N} (ik)^{-n} \vec{m}^a_n(\vec{r}) \quad , \vec{r} \in S_e \qquad (11)$$

where :

$$\vec{j}^a_n(\vec{r}) = \hat{n}(\vec{r}) \times \left[\vec{h}^R_n(\vec{r}) + \vec{h}^i_n(\vec{r})\right] \qquad (12)$$

$$\vec{m}^a_n(\vec{r}) = \left[\vec{e}^R_n(\vec{r}) + \vec{e}^i_n(\vec{r})\right] \times \hat{n}(\vec{r}) \qquad (13)$$

These expansions depend on the boundary conditions (5) or (6) through $\vec{h}^R_n(\vec{r})$ and $\vec{e}^R_n(\vec{r})$. For a perfectly conducting body, it follows from (4) that $\vec{M}_a(\vec{r})$ = 0 and $\vec{m}^a_n(\vec{r})$ = 0 so that it remains only an electric current given by (10).

As we have already mentioned, the currents $\vec{J}_a$ and $\vec{M}_a$ do not represent the total currents on S_e. Other waves of the same kind or not may reach a given point on S_e after a simple or a multiple interaction with the body. It is possible to associate an asymptotic development with the currents corresponding to each species of waves and the asymptotic expansion of the total currents is equal to the sum of the asymptotic expansions of the currents corresponding to each kind of waves.

With this basic concepts in mind, it is now possible to analyse the domain of validity of PO.

1.2. Physical Optics (PO) approximation

The PO approximation is usually stated in the following way :

(a) at every point $M(\vec{r})$ on the lit side S_e of a body, the Pysical Optics currents $\vec{J}_{OP}(\vec{r})$ and $\vec{M}_{OP}(\vec{r})$ are equal to the currents that would exist at the same point on a plane tangent to S_e at that point and submitted to the same boundary conditions.

(b) on the shadow side of the body the PO currents are zero.

This definition leads to the following expressions of the currents :

- For a perfectly conducting body :

$$\vec{J}_{OP}(\vec{r}) = \begin{cases} 2\hat{n}(\vec{r}) \times \vec{H}^i(\vec{r}) & , \vec{r} \in S_e \\ 0 & , \vec{r} \in S_e \end{cases} \qquad (14)$$

$$\vec{M}_{OP}(\vec{r}) = 0 \quad , \vec{r} \in S$$

- For an imperfectly conducting body :

$$\vec{J}_{OP}(\vec{r}) = \begin{cases} \hat{n}(\vec{r}) \times \left[\vec{H}^i(\vec{r}) + \vec{H}^R(\vec{r})\right] & , \vec{r} \in S_e \\ 0 & , \vec{r} \in S_o \end{cases} \tag{15}$$

$$\vec{M}_{OP}(\vec{r}) = \begin{cases} \left[\vec{E}^i(\vec{r}) + \vec{E}^R(\vec{r})\right] \times \hat{n}(\vec{r}) & , \vec{r} \in S_e \\ 0 & , \vec{r} \in S_o \end{cases}$$

where $(\vec{E}^i, \vec{H}^i)$ and $(\vec{E}^R, \vec{H}^R)$ are respectively the incident field on the surface and the reflected field on the tangent plane.

From this definition and without any further background it is difficult to find out general rules for the validity of the PO approximation. For this reason, it seems useful to relate this method to the asymptotic solutions introduced in § 1.1

It is easy to see from the asymptotic expansion (2) which is also known as the Luneburg-kline expansion of the reflected field, that the PO approximation corresponds to the leading term of the asymptotic expansions of the surface currents (11) and (12). Owing to the domain of validity of the method of perturbations described in § 1.1, we conclude therefore that PO is not valid at points on S_e located close to the shadow boundary Γ (See Fig. 1) or close to a real or virtual caustic of the incident or reflected field. For a convex object, bounded by a regular surface and illuminated by a divergent incident wave for instance, PO is not valid in the regions S_e and S_o close to the shadow boundary Γ shown on figure 2 in consequence of the proximity of shadow boundary Σ_o which is a surface tangent to S along Γ and to the virtual caustic of the reflected rays.

When the illuminated part of the surface has edges, PO is also not valid close to these edges. Moreover when the incident wave is convergent or when the surface S is concave, it may happen that some parts of S_e which are not close to Γ are in the vicinity of a caustic of the incident or reflected wave. PO is evidently not valid in such situations.

There exists a class of targets which, when illuminated by a plane wave, have no caustic of the reflected field at finite distance and hence all points of S_e are far from the caustic. It comprizes all targets built up with flat polygonal plates. For such geometrical shapes, the asymptotic expansions (11) and (12) are reduced to their first term n = 0, the other terms n ≠ 0, being identically zero.

For this class of targets, PO is very well suited for RCS calculations more especially as the other asymptotic methods founded on a direct calculation of the diffracted field like GO or GTD are not valid near the directions of specular reflections owing to the presence of a singular caustic (focal points) at infinity.

However, the practical implementation of PO for RCS computations of a complex target constituted by an assembly of plates or polyhedrals come up against two difficulties : the determination of the shadow boundary on the surface and the treatment of multiple interactions between plates.

The first difficulty can be surmounted by a refinement of the sampling of the surface of those plates which are partly illuminated or by the calculation of the intersection of the shadow boundary with the limits of a partly illuminated plate.

The second difficulty is generally solved by an association of GO and PO. However, this way to solve the problem is not completely satisfactory. It can be shownn for instance that the result does not verify reciprocity. We will come back to this problem in a more general framework in chapter 2.

The extension of PO to curved surfaces modeled by an assembly of flat plates is possible under some restrictive conditions. For an incident plane wave and an observation point at infinity, a general criteria valid for single diffractions has been proposed by Klement et al (1988). It states that the deviation between true and modeled surfaces should not exceed a value of $\frac{\lambda}{16}$. This criteria which has some similarity with the far field condition in antenna theory limits the maximum size of the panels which depend therefore on the radii of curvature of the true surface. There is another restriction more especially related to the asymptotic expansion of the reflected field which can no longer be limited to the leading term when $k\rho \lesssim 6$ where ρ is the smallest local radius of curvature of the true surface. Both rectrictions can be summerized in :

$$\Delta S \lesssim \frac{1}{2} \lambda \rho \;,\; \rho \gtrsim \lambda$$

where S is the surface of an elementary pannel.

For double diffractions, since the phase errors are additive, the deviation between the true and modeled surface should not exceed a value of $\lambda/32$.

Under the preceding conditions, PO gives accurate results for observation directions close to the direction of specular reflection. Away from these directions the accuracy of PO deteriorates because edge diffraction is incompletely taken into account.

1.3. The physical Theory of Diffraction (PTD)

PTD is a correction of PO which takes properly into account the edge diffraction in the illuminated region.

In order to introduce the method, we suppose that the illuminated part S_e of the surface of a target has a discontinuity of the tangent plane (edge) along the curve (C) and designate $\vec{J}(\vec{r})$ and $\vec{M}(\vec{r})$, with $\vec{r} \in S$, the exact currents on the surface S and $I(\vec{J},\vec{M},P)$ the radiation integral giving the field radiated by these currents at an observation point P faraway from S. The asymptotic expansion I_a of I comprizes the contribution I_S of the stationary points of S_e, corresponding to the GO field and an end point contribution I_b giving the field diffracted by the edge. We state :

$$\vec{J} = \vec{J}_{OP} + \vec{J}_F \quad , \vec{M} = \vec{M}_{OP} + \vec{M}_F \tag{16}$$

where $\vec{J}_F$ and $\vec{M}_F$ are corrections to the PO currents called fringe currents which for a non grazing incidence are essentially concentrated in the vicinity of (C) since, at high frequencies edge diffraction is found to be essentially a localized phenomenon. When the stationary points are not close to (C), $\vec{J}_F$ and $\vec{M}_F$ can be neglected in the stationary point contribution I_S to the radiation integral I, so that :

$$I_S(\vec{J},\vec{M};P) \simeq I_S(\vec{J}_{OP},\vec{M}_{OP};P) \tag{17}$$

since $I(\vec{J},\vec{M};P)$ is a linear function of the currents $\vec{J}$ and $\vec{M}$, we have from(16) :

$$I(\vec{J},\vec{M};P) = I(\vec{J}_{OP},\vec{M}_{OP};P) + I(\vec{J}_F,\vec{M}_F;P) \tag{18}$$

and

$$I_b(\vec{J},\vec{M};P) = I_b(\vec{J}_{OP},\vec{M}_{OP};P) + I_b(\vec{J}_F,\vec{M}_F;P) \tag{19}$$

Consequently :

$$I(\vec{J},\vec{M};P) \simeq I_a(\vec{J},\vec{M};P) = I_s(\vec{J},\vec{M};P)+I_b(\vec{J},\vec{M};P)$$

$$= I_s(\vec{J}_{OP},\vec{M}_{OP};P)+I_b(\vec{J}_{OP},\vec{M}_{OP};P)+I_b(\vec{J}_F,\vec{M}_F;P) \qquad (20)$$

$$\simeq I(\vec{J}_{OP},\vec{M}_{OP};P) + I_b(\vec{J}_F,\vec{M}_F;P)$$

The last equality shows that a correction to PO is given by the end point contribution of the asymptotic development of the radiation integral of the fringe currents. This method has been first proposed and applied to metallic strips by Ufimtsev (1962) who designated it "Pysical Theory of Diffraction". The term $I_b(\vec{J}_F,\vec{M}_F;P)$ is called the Ufimtsev correction of PO. It can be calculated without knowing explicitly the fringe currents $\vec{J}_F,\vec{M}_F$ by noting that $I_b(\vec{J},\vec{M};P)$ is the GTD solution for edge diffraction and according to (20) :

$$I_b(\vec{J}_F,\vec{M}_F;P) = I_b(\vec{J},\vec{M};P) - I_b(\vec{J}_{OP},\vec{M}_{OP};P) \qquad (21)$$

The leading term of the asymptotic expansion $I_b(\vec{J}_F,\vec{M}_F;P)$ has the same form as for the GTD solution, except for the diffraction coefficients which are given by the difference between the GTD diffraction coefficients and the corresponding PO diffraction coefficients. The latter hase been established by Unfimtsev for a half plane. They keep the same form for each face of a curved wedge, since alike the GTD solution, the tangent straight wedge approximation is applicable to the leading term of the asymptotic expansion of the PO radiation integral.

When the observation point is in the vicinity of a shadow boundary of the curved wedge, the stationary phase point approaches the edge and the asymptotic development :

$$I_a(\vec{J},\vec{M};P) = I_s(\vec{J},\vec{M};P) + I_b(\vec{J},\vec{M};P) \qquad (22)$$

is no longer valid since I_b tends to infinity.

A uniform asymptotic expansion of the radiation integral as given by the Uniform Asymptotic Theory or UAT (See Lee and Deschamps (1976)) is :

$$I_a(\vec{J},\vec{M};P) \simeq I_s(\vec{J}',\vec{M}';P)\ [\mathcal{F}(\xi) - \hat{\mathcal{F}}(\xi)] + I_b{}^e(\vec{J},\vec{M};P) \tag{23}$$

where $I_b{}^e$ is the partial non uniform edge contribution associated with the illuminated face and where I_S is the stationary phase contribution on the regular surface obtained by extending continuously the illuminated face of the wedge behind the edge, the continuity of the first and second derivatives being essential. The first term of the asymptotic expansion of the currents $(\vec{J}',\vec{M}')$ which appear in the expression of I_S are therefore $\vec{J}' = \vec{J}_{OP}$, $\vec{M}' = \vec{M}_{OP}$ while the correction terms $\vec{J}_F$ and $\vec{M}_F$ are zero since the surface is regular.

The functions $\mathcal{F}$ and $\hat{\mathcal{F}}$ in (23) are respectively the Fresnel function and the second term of its asymptotic expansion. They depend on a parameter which is completely defined by the position of the stationary phase point on the illuminated face and its virtual extension and on the position of the diffraction point on the edge.

By taking into account these remarks in (23), we get :

$$\begin{aligned} I_a(\vec{J},\vec{M};P) = I_s(\vec{J}_{OP},\vec{M}_{OP};P)\ [\mathcal{F}(\xi) - \hat{\mathcal{F}}(\xi)] \\ + I^e{}_b(\vec{J}_{OP},\vec{M}_{OP};P) + I^e{}_b(\vec{J}_F,\vec{M}_F;P) \end{aligned} \tag{24}$$

We recognize in the first two terms of the right hand side of (24), the uniform asymptotic expansion of $I(\vec{J}_{OP},\vec{M}_{OP};P)$. Hence, if we designate by $I_e(\vec{J},\vec{M};P)$ the radiation integral of the currents on the illuminated face, we have :

$$I_e(\vec{J},\vec{M};P) = I_a(\vec{J}_{OP},\vec{M}_{OP};P) + I_b{}^e(\vec{J}_F,\vec{M}_F;P) \tag{25}$$

where :

$$I_b{}^e(\vec{J}_F,\vec{M}_F;P) = I_b{}^e(\vec{J},\vec{M};P) - I_b{}^e(\vec{J}_{OP},\vec{M}_{OP};P) \tag{26}$$

Since the first term on the right hand side of (26) is the non uniform edge contribution associated with the illuminated face, its expression is given by GTD. The second term has the same expression as before when the observation point P was far from the shadow boundary. When P crosses the transition region, each of these two terms becomes infinite, however, their difference remains finite and continuous. This property is a consequence of the continuity of the solution (23) and of the continuity of the uniform asymptotic expansion of the radiation integral of the PO currents . When the second face is in the shadow region, the radiation integral $I_o(\vec{J},\vec{M};P)$ of the currents on that face, reduces to :

$$I_o(\vec{J},\vec{M};P) = I^o{}_b(\vec{J}_F,\vec{M}_F;P) = I^o{}_b(\vec{J},\vec{M};P) - I^o{}_b(\vec{J}_{OP},\vec{M}_{OP};P) \tag{27}$$

and since $I^o{}_b(\vec{J}_{OP},\vec{M}_{OP}) = 0$ on S_o, we have, for both faces :

$$I(\vec{J},\vec{M};P) = I_e(\vec{J},\vec{M};P) + I_o(\vec{J},\vec{M};P) \tag{28}$$

with :

$$\simeq I(\vec{J}_{OP},\vec{M}_{OP};P) + I_b(\vec{J}_F,\vec{M}_F;P)$$

$$I_b(\vec{J}_F,\vec{M}_F;P) = I_b(\vec{J},\vec{M};P) - I_b(\vec{J}_{OP},\vec{M}_{OP};P) \tag{29}$$

where $I_b(\vec{J},\vec{M};P)$ is the total edge diffracted field given by GTD.

By comparing (29) and (26) we see that Ufimtsev's correction has the same form whether the observation point lies inside or outside the transition regions at the shadow boundaries of the direct and reflected fields. The generalization of this result to the case when both faces are illuminated, is straightforward. Its extension to grazing incidence is also possible if the target is constituted by an assembly of flat plates.

Since $I_b(\vec{J}_F,\vec{M}_F;P)$ is equal to the difference between two non uniform edge contributions, it is locally a conical wave emanating from the edge. Hence, it is possible to define modified equivalent edge currents by identifying the first term of the asymptotic expansion of its radiation integral to $I_b(\vec{J}_F,\vec{M}_F;P)$. The modified equivalent edge currents have been first introduced by Knott and Senior. These currents can be used to extend PTD to observation points lying in the vicinity of caustics of the edge diffracted rays. Explicit expressions of Ufimtsev's correction and for the equivalent modified currents can be found in the articles by Knott and Senior (1973, 1974).

After this brief introduction on the foundations of PTD, we are now in a position to discuss the advantages and limitations of this method.

PTD has the following advantages :

- It gives a finite field on the caustics of the G.O. rays.
- It takes properly into account edge diffraction by a correction term to PO which remains valid within the transition regions at the shadow boundaries of the direct and reflected waves.
- It gives a finite field on the caustics of edge diffracted rays through the modified equivalent current method (MECM).
- It avoids the difficult problem of ray searching in the treatment of single interactions (reflection or edge diffraction)

The limitations of PTD are :

- It is not valid near grazing incidence on curved surfaces having edges or close to the shadow boundaries on a regular surface.
- It presents the same difficulties as PO for the treatment of multiple interactions.
- It is not adapted to targets without edges, of low RCS.

Twenty years ago, owing to the advantages mentioned before, PTD has been considered as the method which was best suitable for the computation of RCS and various computer codes founded on this technique have been developed in different laboratories around the world. Nowadays, it remains a popular method, but its domain of application is much more limited. Indeed the diffraction problems which are frequently encountered at present, concern targets which can roughly be separated into two classes :

(a) Complex targets the RCS of which is dominated by reflection and edge diffraction.

(b) Targets of low RCS at least in some directions of operational importance, resulting from weaker diffraction phenomena like creeping waves, edge diffracted creeping waves, tip diffraction, diffraction by a curvature discontinuity, etc...

Targets of class (a) generally involve multiple diffractions due to dihedral or trihedral angles which need the development of ray searching techniques. For targets of class (b) the extension of PTD to creeping waves is necessary.

In order to illustrate the importance of Ufimtsev's correction in PO we show on figure 3 the monostatic RCS at 9,5 GHz of a flat perfectly conducting rectangular plate with dimensions 1 m x 0,5 m when the observation direction moves in the symetry plane perpendicular to the smallest side from $\theta = 0$ (normal incidence) to $\theta = 90°$ (grazing incidence), the direction of the incident electric field being perpendicular to this plane. The dashed line corresponds to the PO result whereas the solid line gives the PTD result. We see that for $\theta > 60°$, the difference between the maxima given by PTD and PO is larger than 10 dB and for grazing incidence the difference is about 30 dB. The contribution of the correction term in the same angular domain is shown on figure 4.

A shape for which PTD is well suited is shown on figure 5. It is a tank modeled by about 100 flat plates with a total of 350 edges. On figure 5b only the plates illuminated by the incident field are represented.

1.4. The generalized Physical Theory of Diffraction

The generalized PTD consists in completing the PO current on the lit side and on the shadow side of a target by adding other contibutions of the same order or of higher order and to generalize Ufimtsev's procedure to correct the radiation of these currents at edges.

Multiple reflections give rise to additive currents of the same order as the PO currents since the leading term of the corresponding asymptotic expansion is given by GO. Extension of Ufimtsev's correction to the radiation of these currents at edges is straightforward. It consists simply in substituting to the direct incident field, the field reaching the surface and the edges after multiple reflections. However, in order to get an expression of the total radiated field verifying reciprocity, it is necessary to add to these currents those corresponding to incident waves which undergo multiple reflections and one single edge diffraction before reaching the point on the surface where additive terms of the PO currents are calculated.

A more difficult problem is the generalization of PTD to creeping wave currents. In order to show how this problem can be solved, we consider first a two dimensional geometry having an ogival cross section as shown on figure 6. We assume that the incident electric (or magnetic) field is parallel to the generatrixes and that its direction of propagation lies in the symetry plane of this geometry and is perpendicular to the generatrixes.

For monostatic diffraction, the ordinary PTD gives correctly the diffraction by the edge A but gives not the contribution of the edge B which is located in the shadow region where the PO currents are zero. This deficiency of PTD still exists for bistatic diffraction where it is even more pronounced for large bistatic angles. It can be overcome by replacing the PO currents by those currents that would exist on the same surface if the edges A and B were absent. More precisely, the currents on the upper and lower surfaces are calculated by assuming that these surfaces are extended continuously behind the points A and B. The field on a regular, perfectly conducting curved surface and hence the currents, are well known and can be expressed by a Fock integral which reduces to the GO field and to the PO currents on the illuminated side of the surface away from the shadow boundaries Q_1 and Q_2 and to the creeping wave field and creeping wave currents in the deep shadow. We have now to extend Ufimtsev's correction to these currents. Since edge A is in the illuminated region far away from the shadow boundaries, the classical Ufimtsev's correction apply as long as the observation direction AP shown on figure 7 does not cross the surface and is not situated inside the transition region close to the tangents AT_1 and AT_2 to wedge A, where edge diffraction and surface diffraction can no longer be separated. Elsewhere Ufimtsev's correction remains formally the same and is given by (21) or (29), but now the total edge diffracted GTD field $I_b(\vec{J},\vec{M};P)$ must be calculated by taking into account surface diffraction. General formulas for edge diffraction, valid in the transition regions of edge diffracted rays, have been recently published by Michaeli (1989 b, c) for a perflectly conducting curved wedge. Improved solutions which tend uniformly to the GTD solution outside the transition regions of edge diffracted rays have also been established by Liang (1990), Liang, Chuang and Pathak (1990). By using these solutions, a generalized Ufimtsev's correction for the edge A can be performed.

The field close to a regular, perfectly conducting curved surface has been given by Pathak (1979) and put in the form of a spectrum of inhomogeneous local plane waves by Michaeli (1989 b). Following this procedure which has been extended to coated surfaces by Molinet (1990), we can apply Ufimtsev's correction at edge B to each component of the spectrum and obtain a correction term in the form of a spectral integral. This procedure belongs to the technique of the Spectral Theory of Diffraction (STD) developed by Rhamat-Samii and Mittra (1977).

When the observation point cannot be reached by direct space waves emanating from the wedge, but only by a creeping wave it is necessary to modify Ufimtsev's correction by multiplying it by a factor taking into account surface diffraction. This factor is the same as the one which arises in the expression of the diffracted field, between an edge diffracted space wave and an edge diffracted creeping wave.

It is also possible to extend the generalized PTD to observation points lying in the vicinity of caustics of edge diffracted space or creeping rays, by introducing modified equivalent currents.

One of the main advantages of the generalized PTD is its capability to give the correct RCS on regular or degenerated caustics of the reflected field and to recover the GTD results away from these caustics. Other advantages occur in overlapping transition regions where the generalized PTD gives always a finite result whereas uniform GTD may have numerical difficulties, especially for bistatic diffraction. A typical example is shown on figures 8a to 8c. It corresponds to a plane wave illuminating the front edge of a two-dimensional wing represented on figure 8a. The echo width has been computed at 40 GHz by the uniform GTD (Fig. 8b) and the generalized PTD (Fig. 8c) at bistatic observation angles varying from 90° to 180°. We see that if we disregard the fine structure of the lobes which is not present in the GTD diagram due to undersampling, the results obtained by both methods are close except for a bistatic angle approaching 180° where the GTD result blows up, in spite of the fact that the GTD formulas are uniform. It is a numerical problem which could be avoided by extracting analytically the limit or by augmenting the precision of the computer. However, analytical extraction of the limits is not always possible a priori, especially on a complex target.

The extension of the formulas of the generalized PTD to three-dimensional bodies is straightforward. But the practical implementation of the method is much more cumbersome since it is necessary to determine the geodesics followed by the creeping rays to reach an arbitrary point on the shadowy side of the surface and on its edges. This is the main difficulty in the application of the generalized PTD to complex targets. But as will be seen in chapter 2 techniques for searching geodesics on a complex target, the geometry of which being described by a computer, are now developed for the application of GTD/UTD.

2 - ASYMPTOTIC EXPANSION OF THE FIELD AT LARGE DISTANCES

2.1. Geometrical Theory of Diffraction (GTD)

Geometrical Theory of Diffraction has been developed by Keller in the early 1950. We give only a brief synthesis on the method since more detailed descriptions of GTD are given in several articles and textbooks the references of which may be found in a feature article on GTD written by Molinet (1987).

The field scattered by a target at large distance from it compared to the wavelength can be thought of as being the sum of different asymptotic expansions corresponding to different asymptotic sequences. As we have shown in § 1.1, the incident and the reflected fields can be represented by an asymptotic expansion with respect to the sequence k^{-n} (n = 0, 1,...) of powers of the small parameter k^{-1}. Similarly, it can be shown that the fieLd diffracted by an edge may be represented by an asymptotic expansion with respect to the sequence $k^{-n-\frac{1}{2}}$ and more generally, the appropriate sequence for the diffraction by a line discontinuity in the m^{th} derivative of the surface is found to be $k^{-n-\frac{1}{2}-(m-1)}$.

In order that these asymptotic expansions be solutions of Maxwell equations, they must verify the eikonal and transport equations obtained by substituting them into the vector Helmholtz equation. In addition they must verify the Gauss law $\nabla . \vec{E} = 0$.

It follows as a consequence of these equations, that the field propagates along rays which are orthogonal to the surfaces of constant phase $S(\vec{r})$ = const. called wavefronts and that once its value is known on an initial surface $S_o(\vec{r})$ = Const., it can be calculated in space along all rays crossing this surface except on their envelope called caustic, where the transport equation is singular. Moreover, by imposing the continuity of the phase $S(\vec{r})$ on the surface of the object which scatters the incident field, it is possible to relate the congruence of diffracted rays to the incident rays and to establish the law of reflection and diffraction (Keller's cone). Now, owing to these properties and without further specifications on the nature of the diffraction phenomenon and on the boundary conditions on the surface of the scatterer, we can formally relate the field along a reflected or diffracted ray to the field of the corresponding incident ray at its point of interaction with the scatterer by an operator which can be put in the form of a diffraction dyadic and which is independent of the position of the observer along the reflected or diffracted ray. Generally the diffraction dyadic has only be constructed for the leading term of the asymptotic expansion of the diffracted field. It depends on the diffraction phenomenon and on the boundary conditions which occur through polarization dependent diffraction coefficients. Explicit expressions of the diffraction dyadic and of the associated diffraction cofficients may be found for the diffraction by the edge of a curved wedge or by a smooth surface in the original work of Keller and his co-workers (Keller 1958), Levy and Keller (1959). At present most of the diffraction coefficients which are needed for the computation of the diffracted field are available for a perfectly conducting body and with some restrictions for a coated body. However, the asymptotic expansions introduced so far are not uniform. They break down near caustics and in the transition regions adjacent to shadow and reflection boundaries of incoming rays.

2.2. Uniform asymptotic solutions

The shortcomings of GTD near shadow boundaries and caustics have been recognized in the early state of development of the theory. They have been progressively surmounted by the construction of uniform expansions with the aid of boundary layer theory or by direct generalization of uniform asymptotic expansions of exact solutions of canonical problems. This last procedure is generally much simpler since it does not involve stretched boundary layer coordinates, but its complexity grows rapidly if a large number of caustic regions and transition regions are to be covered. In addition, since the asymptotic expansion depends on the asymptotic sequence used, different uniform expansions may be obtained for the same canonical problem.

This happened for the wedge problem for which two different asymptotic solutions have been developed : The Uniform Theory of Diffraction (UTD) due to Kouyoumjian and Pathak (1974) and the UNiform Asymptotic Theory (UAT) proposed by Lee and Deschamps (1976). When UTD is augmented by the "Slope Diffraction Coefficient", both UAT and UTD provide very similar numerical results, the observed differences being of no practical importance (Boersma and Rhamat-Samii (1980)).

The uniform UTD and UAT mentioned so far are only valid in the regions 1,2 and 3 of figure 9. In the transition regions of the diffracted rays (regions 4 and 5), and in the deep shadow of those rays, surface diffraction cannot be neglected. In these regions, the field may be described by hybrid diffraction coefficients expressed in terms of products of edge diffraction coefficients and launching coefficients of creeping waves as proposed by Albertsen and Christiansen (1978). A more rigorous justification of this procedure has been given by Michaeli (1989 a) who developed in addition new solutions valid in overlapping transition regions when the incident field grazes one of the faces of a curved wedge.

Uniform solutions have also been established in caustic regions. Ludwig (1966) first succeeded in the construction of a uniform expansion at a smooth convex caustic and a cusped caustic. The universal founction appearing in his expansion for a smooth caustic is the Airy function and its derivative. Ludwig's solution reduces to the GO solution away from the caustic in the illuminated region and remains finite at the caustic. This solution has been recently extended to the dark side of a regular caustic by Pathak and Liang (1990) using complex ray theory developed by Felsen and his co-workers (Ikuno and Felsen (1988)). However since complex ray theory is difficult to implement on a three-dimensional object, an approximate method has been tested by the author for an observation point crossing a branch of a cautic at infinity. It consists in computing the field at a point M in the shadow region of the caustic by modifying the uniform solution valid in the illuminated ragion as follows : the point M is replaced by its symetric with respect to the caustic and the sign of the argument of the Airy function and of its derivative is changed into its opposite.

In RCS calculations it happens frequently that the caustic is degenerated into a line or a focal point. For a flat plate for instance the caustic is degenerated into a focal point at infinity. In this case, where the field is strongly affected by the presence of the caustic, a modified ray solution is not adequate. Other methods have been developed. One of them consists basically in finding an integral form of the solution, having the same asymptotic expansion as the solution given by GO or GTD away from the caustic. At observation points located on the caustic and its neighbourhood, the field is then determined by a numerical evaluation of that integral. However, such a method is not applicable to RCS computation of a complex target since it needs, an a priori knowledge of the asymptotic solution away from the caustic. A more efficient method on a computer consists in applying the generalized PTD or at least PO in the angular domain influenced by the presence of the caustic.

2.3. Conception of a general computer program founded on GTD/UTD

The basic ideas underlying the construction og a general computer program founded on GTD/UTD have been presented together with some illustrations in an Agard Lecture Series by Molinet (1989). First, it seems necessary to separate all the algorithms depending on the geometrical modeling of the target from those depending on the GTD/UTD solutions since they concern two different types of specialists. Secondly, the computer Aided Design of the geometry being performed by existing codes like CATIA, EUCLID etc, which do not exactly compute the geometrical parameters needed for RCS computation, a special set of geoemtrical data must be extracted from the CAD code and arranged in order to optimize the computer time needed for ray searching.

These concepts led to the construction by MOTHESIM of a general computer library for GTD/UTD applications called PROMETHEE. The library was first coupled to ray searching algorithms on targets modeled by simple shapes, developed by THOMSON-CSF and integrated in an interactive code called SARGASSES which simulates the radiation of antennas on structures such as an aircraft or a spacecraft. At present, the targets are modeled by a biparametric polynomial representation with restrictions (Bezier polynomials). Figure 10 shows a theoretical target consisting of a rotational symetric body having a concave convex shape to which two wings of variable thickness are attached. This shape is modeled by 32 Bézier squares (24 for the body and 8 for the wings). They can be subdivided in smaller domains if necessary. Ray searching techniques are also developed for such representations. There is no difficulty in finding singly reflected or edge diffracted rays. However the search for multiplying reflected rays becomes rapidly cumbersome and carefully tested elimination procedures must be developed in order to reduce the complexity of computer calculation.

The determination of the geodesic curves followed by the creeping rays on a surface described by Bézier polynomials presents also some difficulties, especially when the geodesic crosses a limit of a Bézier square. But this problem has already been solved and efficient algorithms for creeping rays searching are now developed by MOTHESIM in collaboration with another French firm CORETECH specialized in CAD.

3 - COMPARISON BETWEEN GTD/UTD AND GENERALIZED PTD FOR RCS COMPUTATION

In the preceeding chapters we have seen that both methods need ray searching techniques and asymptotic solutions up to a given order which we will now analyse in more detail.

For GTD/UTD, following classes of rays are generally sufficient for most practical applications :

a) multiple reflections limited to 3 interactions,

b) edge diffraction limited to 2 interactions,

c) creeping rays on two separate regular surfaces,

d) edge excited creeping rays limited to 2 interactions,

e) combinations of an edge excited creeping wave and a creeping wave excited at a regular shadow boundary,

f) single tip and corner diffractions,

g) all combinations of rays of classes b, c, d , e , f with three or less reflections.

These rays correspond to an asymptotic solution limited at the order k^{-1}. When edge diffraction is limited to one interaction and tip and corner diffractions are suppressed, we get a solution limited to the order $k^{-\frac{1}{2}}$. In many applications such a model in which (c) is also limited to one interaction, is sufficient. However for balistic forms or other geometries where no edges are present, an asymptotic solution of order $k^{-\frac{3}{2}}$ is needed which implies that single diffraction by a curvature discontinuity must be taken into account.

For the same level of accuracy the generalized PTD differ from GTD/UTD by the fact that only two reflections are needed, all other classes of rays remaining identical. An economy of one interaction in edge diffraction can also be obtained by using equivalent edge currents in GTD/UTD or modified equivalent edge currents in the generalized PTD, but this is not always practical, especially when the radiation of the equivalent currents is intercepted by another part of the target.

In a ($k^{-\frac{1}{2}}$)- GTD/UTD model, limited to two reflections, the gain in the complexity of the ray searching algorithms when one passes from two reflections to one reflection, is appreciable and constitutes a serious advantage for the generalized PTD. However, this advantage is not so important when three reflections are needed in the GTD/UTD model since for more than one reflection, new concepts must be found for ray searching which avoid the exponential behaviour of combinatory techniques. Generally these concepts use framing procedures which can be easily extended to more than two reflections.

Another important parameter is the number of observation points where the field associated with each ray species has to be computed. In the generalized PTD, the field must be determined at every point on the surface of the target.

We must therefore choose a repartition of points adapted to its geometry and use interpolation algorithms for calculating the field at other points on the surface. When the target is described by Bézier polynomials, we can choose for instance the four corners of each elementary Bézier square. Anyway, the number of points where the rays have to be found is very large and constitutes a disavantage of the generalized PTD compared to GTD/UTD for monostatic diffraction. However this assertion does not hold for bistatic diffraction since in the generalized PTD the rays reaching a regular point of the surface depend only on the incident field.

A main difficulty in the implementation of a GTD/UTD program is the correction of the field near caustics or rays which encounter single or multiple reflections. The solution of this problem comprizes two steps : the detection of the caustic and the calculation of the field. When the caustic is regular, it can easily been detected by a test on the curvature of the reflected wave since in the illuminated region of a regular caustic ray searching algorithms converge even if the observation point is very close to the caustic. When the caustic is degenerated other tests must be used involving the curvature of the Bezier square and the curvature of the wave front reaching that square after having encountered one or more reflections or diffractions. Proper delimitation of caustic regions especially when they are degenerated is a complicated problem which is still under investigation, for multiply reflected rays. However the probability of the occurrence of caustics of multiply reflected rays is generally weak, so they can be ignored in a first step and corrected after inspection of the scattering diagram.

Once the caustic regions are detected GTD/UTD must be replaced by other techniques in order to get the correct value of the field in these regions. For a regular caustic, Ludwig's method extended in the region on the shadow side of the caustic by the method described in § 2.3 is adequate. For a degenerated caustic it is necessary to use PO or generalized PTD. At that state of the discussion, the following question arises : why do not use generalized PTD troughout the observation space ? In order to answer this question, we consider first the different operations which are needed to apply PO to a target described by Bezier polynomials. The first step consists in dividing the initial Bezier squares into smaller squares the dimensions of which are approximatively $\frac{\lambda}{\sqrt{2}} \times \frac{\lambda}{\sqrt{2}}$ as mentioned in § 1.2.

Then after having approximated the elementary Bezier squares by two flat triangles having the same extremities as the Bezier squares, as shown on figures (11) and (12), a test is made on each triangle in order to know if it is in the shadow or not of the incident rays reaching it. Owing to the large number of triangles which is needed to describe a complex target (about 10^6 for an airplane at 10 GHz) these tests are very time consuming on a computer.

Hence, since generalized PTD rather than PO must be used for an accurate computation of the RCS of a target, this method is not practicable with a reasonable computer time on very large targets compared to the wavelength (several hundreds of wavelengths in both dimensions). For these targets, the best suited method is GTD/UTD completed by the generalized PTD in caustic regions. Since ray searching algorithms for edge diffraction are identical in GTD/UTD and in the generalized PTD, it is possible to use this last method in caustic regions instead of PO, without augmenting drastically the computer time, the advantage being a better continuity of the diagram at the periphery of the caustic region.

Another advantage of GTD/UTD is the possibility to reduce the number of points necessary to construct the RCS diagram. Indeed, since outside caustics and shadow boundaries, the GTD field is a superposition of local plane waves, interpolation algorithms can be used for the phase and amplitude of the field associated with each individual ray before summing all the contributions.

This property is particularly important for very large targets for which it leads to a computer time saving by a factor of 10 to 100.

For targets of moderate size (several tens of wavelengths in both dimensions) like missiles, the generalized PTD seems to be the best suited method since besides its advantages in caustic regions where no tests are needed, it has some other interesting properties like its capability to give better results than GTD limited to real rays on concave convex surfaces, on surfaces with small radii of curvature ($k\rho \simeq 1$) on surfaces with sharp or blunt corners etc...

Acknowledgement :

The author wishes to thank his colleagues at MOTHESIM, Th. George and C. Louet who provide the numerical results. Thanks are also addressed to the French military administration DRET who supported some of the developments mentioned in the text.

ALBERTSEN N.C. and P.L. CHRISTIANSEN, (1978), Hybrid diffraction coefficients for first and second order discontinuities of two-dimensional scatterers, SIAM J. Appl. Math., 34, pp. 398-414.

BOERSMA J. and Y. RHAMAT-SAMII, (1980), Comparison of two leading uniform theories of edge diffraction with the exact uniform asymptotic solution, Radio Sci., Vol. 15, N° 6, pp. 1179-1194.

IKUNO H. and L.B. FELSEN, (1988), (a) Complex ray interpretation of reflection from concave-convex surfaces, (b) Complex rays in transient scattering from smooth targets with inflection points, IEEE Trans. Ant. Prop., Vol. 36, pp. 1260-1271 and 1272-1280..

KELLER J.B., (1958), A Geometrical Theory of Diffraction, Proceedings of Symposia in Applied Math., Vol. VII : Calculus of Variation and its Application, pp. 27-52, MC. Graw-Hill, Book Co., New-York.

KLEMENT D., J. PREISSNER, V. STEIN, (1988), Special problems in applying the Physical Optics method for backscatterer computations of complicated objects, IEEE Trans. Ant. Prop., Vol. 36, n° 2, pp. 228-237.

KNOTT E.F. and T.B. SENIOR, (1973), A program incorporating diffraction for the computation of radar cross sections, Radiation Lab., Univ. Michigan, Ann Harbor, Rep. 011758-2-T.

——————————— (1974), Comparison of three high frequency diffraction techniques, Proc. IEEE, Vol. 62, pp. 1468-1474.

KOUYOUMJIAN R.G. and P.H. PATHAK, (1974), A geometrical Theory of Diffraction for an edge in a perfectly conducting surface, Proc. IEEE, Vol. 62, pp. 1446-1461.

LEE S.W. and G.A. DESCHAMPS, (1976), A Uniform Asymptotic Theory of electromagnetic diffraction by a curved wedge, IEEE Trans. Ant. Prop., Vol. 24, pp. 25-34.

LEVY B.R. and J.B. KELLER, (1959), Diffraction by a smooth object, Comm. Pure and Appl. Math., Vol. 12, pp. 159-209.

LIANG M.C., (1990), Ph. D. Thesis, The Ohio State University.

LIANG M.C., C.W. CHUANG and P.H. PATHAK, (1990), A geometrical uniform GTD ray solution for the diffraction of a wedge with convex faces (Preprint, submitted to IEEE Ant. Prop.).

LUDWIG D., (1966), Uniform asymptotic expansion at a caustic, Comm. Pure and

Appl. Math., Vol. 19,pp. 215-250.

MICHAELI A., (1989), Transient functions for high-frequency diffraction by a perfectly conducting wedge, IEEE Trans. Ant. Prop., Vol. 37, (a) Part I : Canonical solution for a curved sheet (pp. 1073-1079), (b) Part II : A partially uniform solution for a general wedge angle (pp. 1080-1085), (c) Part III : Extension to overlapping transition regions (pp. 1086-1092).

MOLINET F.A., (1987), Geometrical Theory of Diffraction (GTD), Part I : Foundations of the theory; Part II : Extensions and future trends of the theory, Feature articles in IEEE Ant. Prop. Newsletter, August pp 6-17 and October pp. 5-16

MOLINET F.A., (1989), GTD-UTD : Brief history and successive development of theory and recent advances - Applications to antennas on ships and aircraft, AGARD Lecture Series N° 165, pp. 8.01-8.22.

MOLINET F.A., (1990), Quelques applications des méthodes asymptotiques à des corps métalliques recouverts de matériaux, Ecoles CEA-EDF-INRIA, June 18-20, 1990, INRIA Rocquencourt (FRANCE), Conference book pp. 361-368.

PATHAK P.H. (1979), An asymptotic analysis of the scattering of plane waves by a smooth convex cylinder, Radio Sci., Vol. 14, pp. 419-425.

PATHAK P.H. and M.C. LIANG, (1990), On a Uniform Asymptotic Solution valid across smooth caustics of rays reflected by smoothly indented boundaries, IEEE Trans. Ant. Prop. Vol. 38, pp. 1192-1203.

RHAMAT-SAMII Y. and R. MITTRA, (1977), A spectral domain interpretation of high-frequency diffraction phenomena, IEEE Trans. Ant. Prop., Vol. 25, pp. 676-687.

UFIMTSEV P. Ya., (1962), Method of edge waves in the Physical Theory of Diffraction, Air Force System Command, Foreign Tech. Div., Document ID N° FTD-HC-23-259-71, 1971, (Translation from the Russian version published by Soviet Publication House, Moscow, 1962).

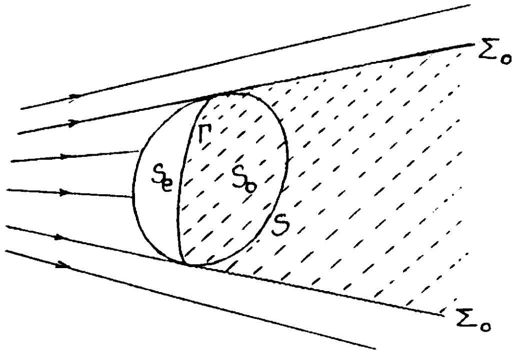


Fig. 1

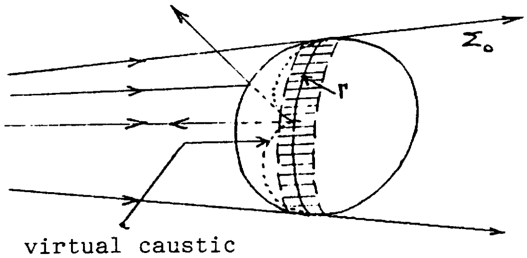


Fig. 2

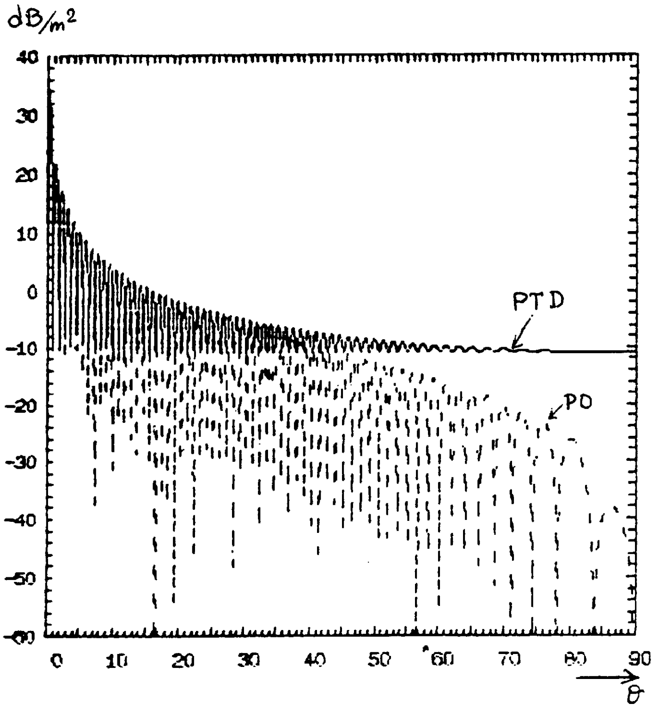


Fig. 3 : RCS of a rectangular plate

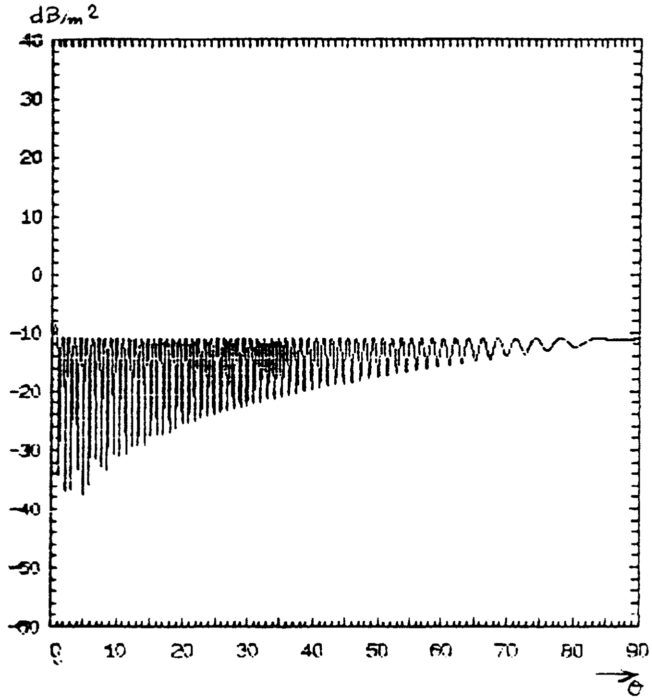
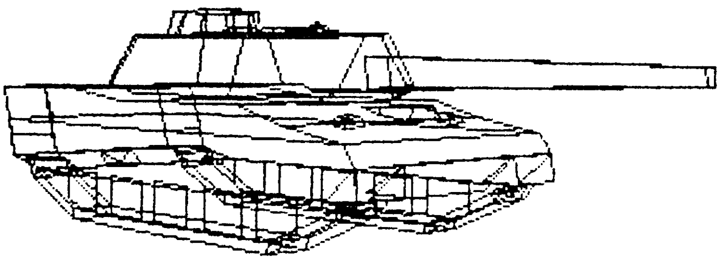
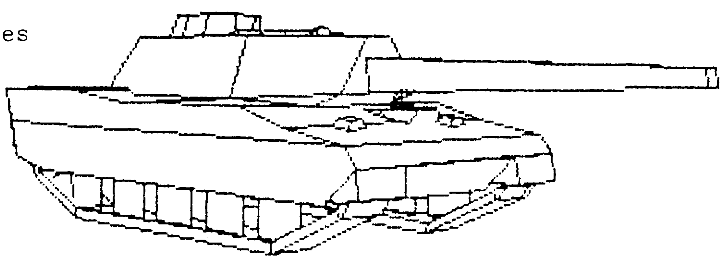


Fig. 4 : Ufimtsev's correction



(a)

Fig. 5 : A tank modeled by flat plates



(b)

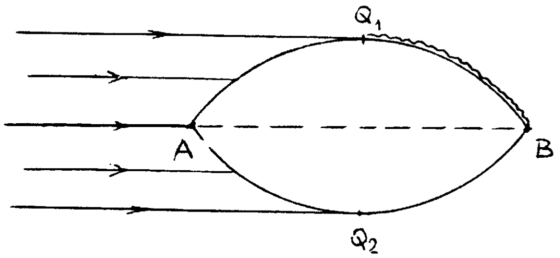


Figure 6

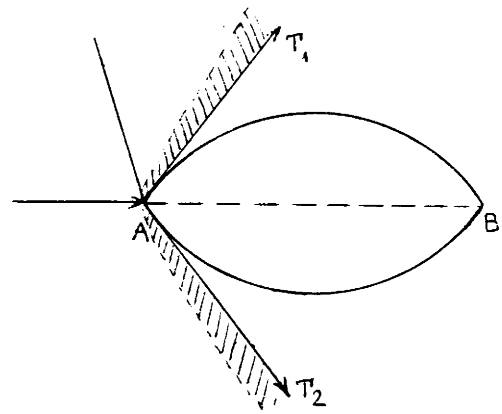
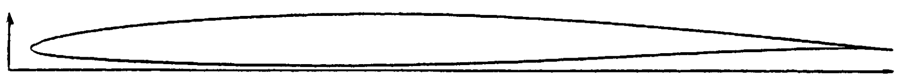
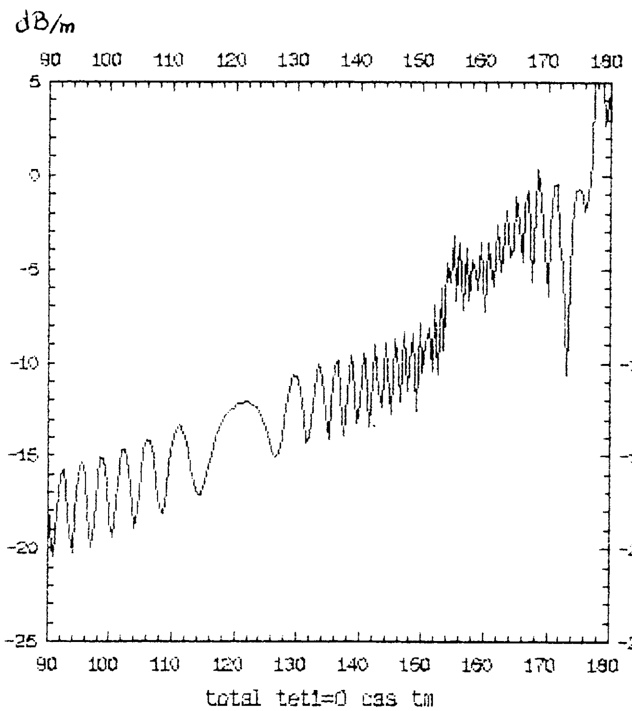


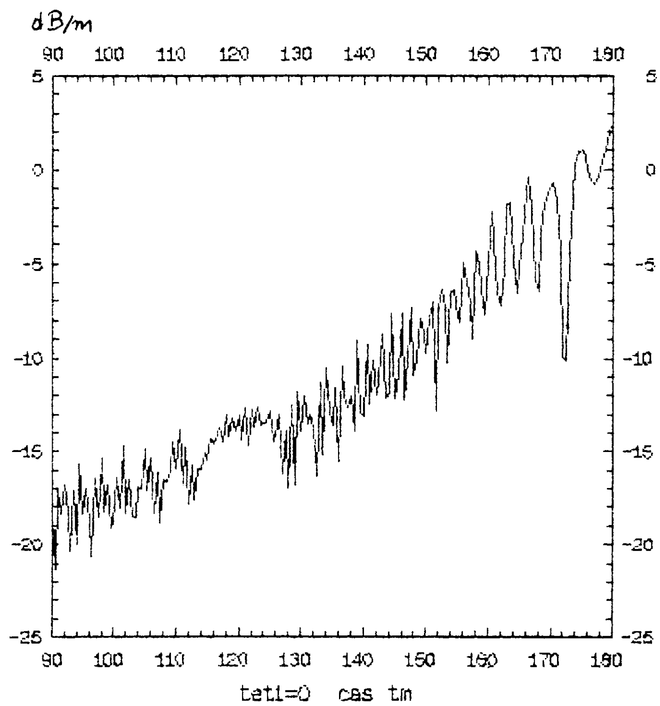
Figure 7



8(a): Geometry of the cross section of the wing,
length = 1 meter



8(b) : GTD/UTD



8(c) : Generalized PTD

Fig. 8 : Echowidth of a two-dimensional wing at 40 GHz, TM polarization

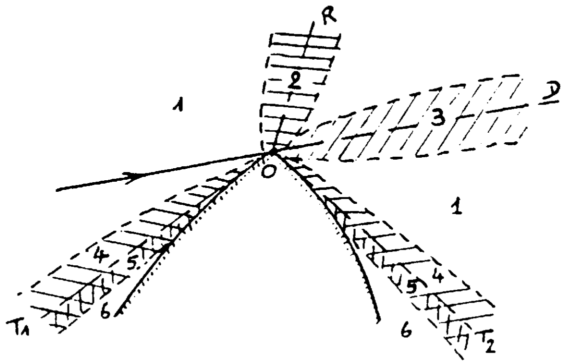


Fig. 9 : Domains of validity of various uniform edge solutions

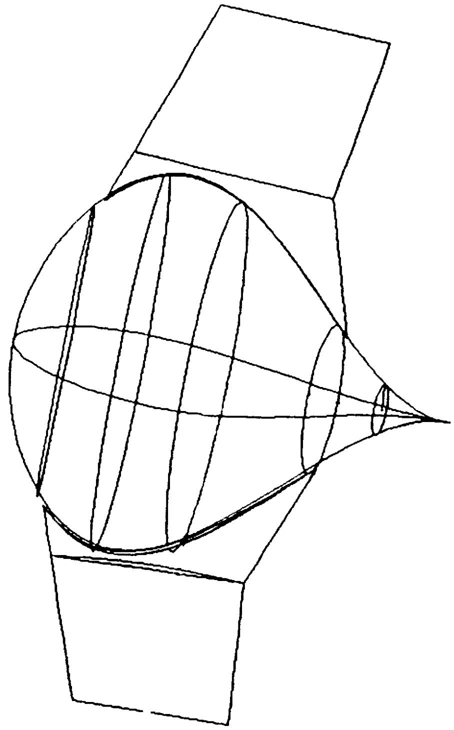


Fig. 10 : Typical shape modeled with Bezier polynomials

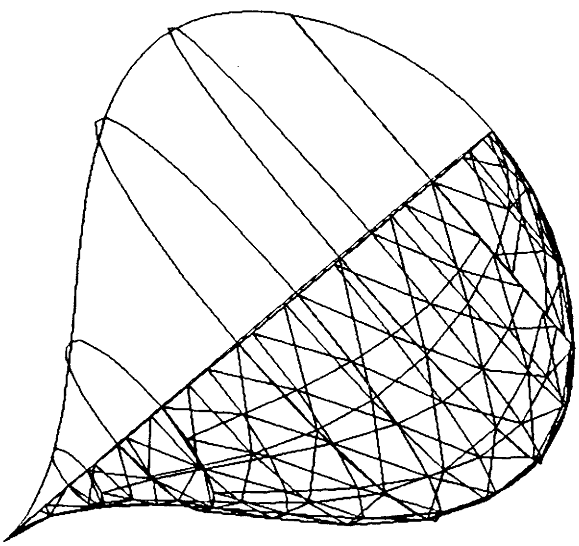


Fig. 11 : Subdivision of Bezier squares in flat triangles.Direction of incidence perpendicular to the symetry axis. Only those triangles which are illuminated are represented

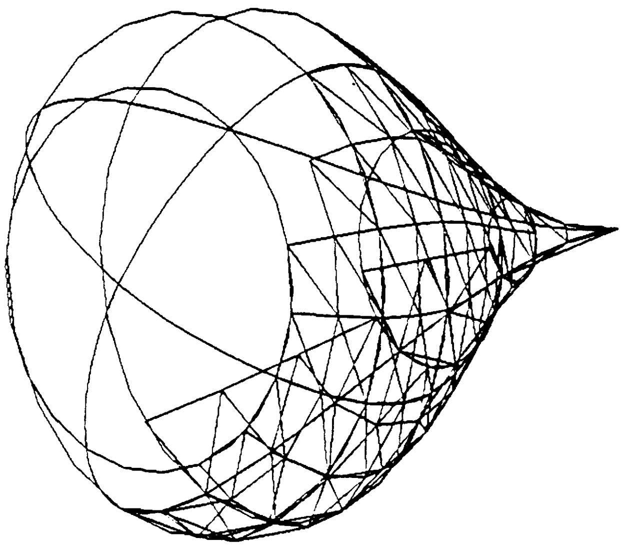


Fig. 12 : Same legend as in Fig. 11 with a direction of incidence making an angle of 30° with the symetry axis.