LIMITATIONS OF THE CONVENTIONAL METHODS OF FORCE AND TORQUE PREDICTION

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Abstract: The calculation of forces and torques developed in electromechanical devices, and their variation with changes in position or excitation, is often what the designer is, ultimately, interested in. This paper addresses some of the problems associated with the force and torque calculations based on numerical field solutions for 2-D magnetostatic problems. The problem of calculating the cogging torque characteristic of a neodymium-iron-boron permanent-magnet motor is considered and the technique of torque measurement is described briefly.

1. INTRODUCTION

Forces developed in the air-gap of an electric machine can be resolved into two components: normal and tangential. The resultant of the tangential components provides the useful electromagnetic torque, and the resultant of the normal components has to be accommodated in the bearings. One of the sources of difficulty is that the size of the useful tangential force component is usually small compared with the radial force. Errors are more likely to occur when the force of interest is calculated in the presence of a much larger force field [1-2].

Forces can be obtained from numerical field solutions by evaluating the Maxwell stress tensor along a given integration path, by the virtual work concept, or by integration of $I(d\vec{l} \times \vec{B})$. The latter is only applicable in systems containing current carrying conductors. In general, all these methods tend to give rise to errors in the forces that are greater than the errors in the field solution.

When the problem solution is obtained in terms of the magnetic vector potential, flux densities are obtained from the curl operation

$$\vec{B} = \nabla \times \vec{A} \tag{1.1}$$

This involves numerical differentiation and, therefore, any errors in the potential solution lead to larger errors in flux density values. This affect all subsequent post-processing operations.

The Virtual Work Method

The force acting on a movable part of a device may be evaluated by determining the variation of the magnetic stored energy of the entire device when a small displacement takes place. For the simple case where only one-dimensional movements are considered, the force is given by

$$F = \frac{(W_1 - W_2)}{d_{12}} \tag{1.2}$$

where W_1 and W_2 represent the stored energies at the two distinct positions; d_{12} is the positional displacement and F is the estimate to the force at the intermediate position $\{(d_1 + d_2)/2\}$.

The Method of Maxwell Stress Tensor

The basis of the method is the calculation of the force and torque directly from the field distribution. The force and torque are evaluated by integrating the force density over a contour surrounding the part of interest. For a known two-dimensional flux density distribution (**B**) and a contour C enclosing a body (or the movable part of a device), the total force and torque acting on the body are given by:

$$\vec{F} = \int_C \left[\frac{1}{\mu_o} \vec{B}(\vec{B}.\vec{n}) - \frac{1}{2\mu_o} B^2 \vec{n} \right] dC \tag{1.3}$$

and

$$\vec{T} = \vec{r} \times \vec{F} \tag{1.4}$$

where $\vec{r}$ is a vector function with its origin taken at the action point for the torque and $\vec{n}$ is the unit vector normal to the contour.

2. ACCURACY OF FORCE CALCULATIONS

The finite element method produces a solution to the problem in terms of scalar or vector potentials. The potential distribution is only a numerical approximation to the true potential distribution, i.e. there is an inherent error in the potentials, commonly referred to as error in the approximation or shape function [3-4].

In conventional formulations, the forces and torques are directly related to magnetic flux densities, not to the vector or scalar potentials. Flux densities and magnetic field strengths are obtained from potential solutions by means of numerical differentiation. All the familiar sources of errors are present in numerical differentiation but errors in the approximation function are the most critical, even when small. This is because they are magnified by differentiation algorithms [5]. Therefore, errors in field distributions are, generally, greater than those of the corresponding potential distributions. This helps to explain why some formulations for predicting force and torque are more prone to inaccuracy problems.

In the following, an attempt is made to enumerate and analyse the factors that affect the accuracy of force and torque calculations. To simplify the analysis, it is assumed that the discretization is appropriate to the problem (i.e. the finite elements are properly shaped to model the non-uniformity of the field). The factors that affect the accuracy of force and torque calculations may be summarized as follows:

(i) For some algorithms, the quality of the force and torque prediction is dictated by the accuracy that can be achieved in determining the flux distribution. This sensitivity is evident in the Maxwell stress equations (1.3) and (1.4), but not so obviously from the virtual work equation (1.2). Here, it is worth remembering that the differences in stored energies are due to the different field configurations associated with the system displacement;

(ii) Regions where a considerable amount of field energy is stored, like geometries with pole tips, are the most critical for the force and torque calculations. In the method of Maxwell stress they contribute the main component of the line integral in equation (1.3). Similarly, in some formulations of the virtual work method these regions also contribute the main component of the area integral used to calculate the magnetic stored energy. In other words, the main contribution to the net force and torque is due to stresses (or stored energy) in regions where pronounced concentration and non-uniformity of the field occur; these areas being where an accurate field solution is most difficult to obtain;

(iii) Problems involving the computation of tangential force are often more difficult. This is because the tangential component of the force can be of a much smaller magnitude than the component in the normal direction [6]. This can be illustrated by considering a hypothetical system where the tangential force is small but non-zero. The diagram in figure 1(a) represents the "true" field distribution **B** at some point, while the approximate field distribution **B'** is shown in figure 1(b). Although the error in the magnitude of **B'** is small, the incorrect field direction will result in wrong prediction of the tangential force. Therefore, slightly incorrect flux direction is a point of concern because this causes errors in force and torque values calculated by whatever method;

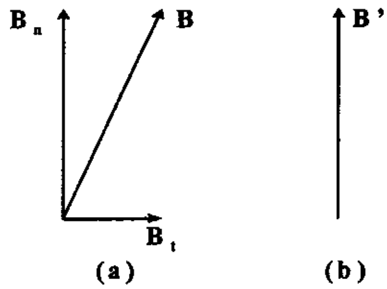

Fig. 1: two flux distributions of similar magnitude.

(iv) incorrect flux directions result from the fact the finite-element methods use some form of energy minimization to drive convergence. The homogeneous Neumann boundary conditions, for example, are not exactly satisfied and this affects, even locally, the flux direction [3]. Different flux directions imply different values of stored energies. Special algorithms are used to perform the numerical differentiation of the shape function and this feature is code-dependent. For example, some packages use the first of the two methods discussed by Binns *et all* [7] in which the continuity of the normal flux density is not imposed at the air-iron interface.

3. PROBLEMS OF IMPLEMENTATION

The above discussion identifies the factors which may affect the accuracy of force and torque obtained by whatever method. The following discussion is concerned with the aspects of each method that could result in difficulties in implementation and substantial numerical errors.

The Virtual Work Method

In contrast to the easy realization of its formulation, the implementation of the method requires a judicious choice of the positional displacement. This choice is problem-dependent and has to take into consideration errors of conflicting nature. If, in an attempt to improve the accuracy of the approximate derivative expressed in equation (1.2), a small displacement is used, the energy values will be of similar magnitude and the subtraction (W_1-W_2) will be more sensitive to round-off error. On the other hand, a larger displacement may not be adequate to model the true non-linear characteristic that represents the variation of the system's energy with respect to position [8].

The method requires careful planning of the model and this must be followed by a critical examination of the results. In order to reduce discretization errors, one single finite element mesh must be used in all the solutions representing the sequence of the disturbed movable part [9]. In some cases the results might show that the variation of energy with respect to position is not consistent with the physical realization of the actual device, and a model re-definition may have to be done. From the observations made above it is evident that the method is computationally expensive.

The Method of Maxwell Stress tensor

Once the field distribution **B** in equation (1.3) is an approximation to the true one, i.e. there is an inherent error in the numerical field distribution, the independence of the results relative to the choice of the integration contour disappears; the definition of integration contours thus assumes a great importance. This aspect has to be considered very early, during the planning of the finite-element model, and adds complexity in the construction of the mesh.

The energy minimization used by the finite-element method produces a numerical solution to the problem that is optimal for a given discretization. This scheme has no concern for variations in local energy accuracy. Consequently, the resulting fields are globally optimal, even though may possess considerable local error [10]. The forces and torques in equations (1.3) and (1.4) are related only to flux densities of the elements crossed by a given contour (*C*). This makes evident why the method is so sensitive to mesh artifact and to the location of the integration contour [1].

4. FINITE ELEMENT MODEL AND TORQUE MEASUREMENT

In order to investigate the numerical problems associated to the conventional methods, the problem of calculating cogging torque in a small permanent-magnet motor has been chosen. Cogging torque values in small permanent-magnet motors are typically in the range of millinewton-metres and, therefore, they are very difficult to compute and measure accurately.

Cogging torque is defined as the non-uniform torque that arises when only the excitation field is present (i.e. the armature current is absent). Cogging torque is a saliency effect that arises from the interaction between a salient pole on one member of the machine (rotor or stator) and the teeth on the other member. The interaction implies a magnetic field distribution which depends on the rotor position. In dc permanent-magnet motors, the interaction between the edges of the magnets and the teeth, situated on the opposite side of the air-gap, causes alternate cycles of restoring and anti-restoring torques as the rotor moves.

The test machine uses radially oriented neodymium-iron-boron magnets to provide a four-pole rotor excitation. A cross-sectional drawing of the motor is shown in figure 2. The magnet arc spans 90 mechanical degrees and the stator has 24 evenly spaced slots. The cogging torque characteristic is therefore periodic, with a period of 15 mechanical degrees.

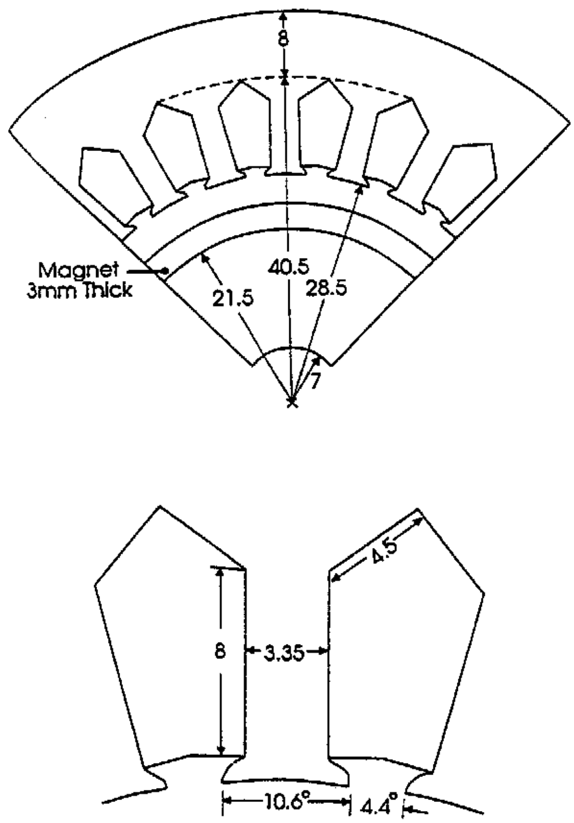

Fig. 2: Top: view of one-quarter of the test motor; Bottom: enlarged drawing of one tooth. Dimensions in millimetre.

The cogging torque is zero at any rest position where the edges of the magnets (interpolar regions) are radially aligned with the centerline of stator teeth. Such positions are stable equilibrium points for the rotor. Alignment of the interpolar regions with the centerline of the stator slots also implies symmetry, hence zero torque; but these positions are unstable equilibrium points for the rotor. The cogging torque is expected to be a smooth function of displacement in the absence of magnetic saturation.

The cogging torque characteristics of the motor were determined experimentally by displacing the rotor shaft and measuring the torque induced. The rotor shaft was rotated, via a flexible coupling and a torque sensor, by a rotary table mounted so that its axis was collinear with the rotor shaft. The rotary table was driven by a stepping motor via a worm-wheel gearbox. The resolution of the stepping motor was 200 steps per revolution and the gearbox had a speed ratio of 90:1 giving an angular resolution for rotor displacement of 0.02 degree. The estimated backlash in the gearbox was less than 0.1 degree. The torque was measured using a Lord six-component sensor. A data acquisition system capable of recording and processing up to 1000 measurements per second was used. The resolution of this system was 1.4 millinewton-metre. The effect of friction was eliminated from the measured torque-position curves by moving the shaft in one direction a total of 30 degrees (two slot pitches) and than reversing direction. Torque readings were taken in both directions and the results were averaged.

In order to investigate the accuracy obtainable by the conventional methods, a single pole pitch of the motor was modeled, subject to periodicity conditions. A commercially available two-dimensional magnetic field analysis package (MagNet Release 4) was used to solve the field problems. Initial investigations showed that magnetic saturation was not present anywhere in the machine. Therefore, in subsequent runs, all materials in the motor were considered to be magnetically linear, and a linear solver was utilized.

Values of cogging torque were computed by both the virtual work and the Maxwell stress tensor method. Each torque characteristic is associated with a series of problems representing eleven rotor positions separated by 1.5 mechanical degree (10% of the period). In order to guard against mesh artifact in the results, a single finite-element mesh is used in all eleven runs belonging to one curve. Rotor movement is simulated by redefining material properties.

To gain some idea how sensitive the methods are to mesh artifact and fineness of discretization, solutions are obtained utilizing two different meshes. Results were firstly obtained on a reasonably fine mesh containing 952 nodes with 1834 first-order elements. A second, coarser mesh was created by deletion of nodes in the clusters of elements close to the corners of stator teeth in figure 2. The number of elements was thereby reduced to 1546, the number of nodes to 808. The two meshes only differ in the air-gap region, because this is a critical region regarding energy transfer and it is where most significant field variations occur.

5. NUMERICAL RESULTS

Figure 3 shows zoomed views of the air-gap zone illustrating different node densities and corresponding flux plots.

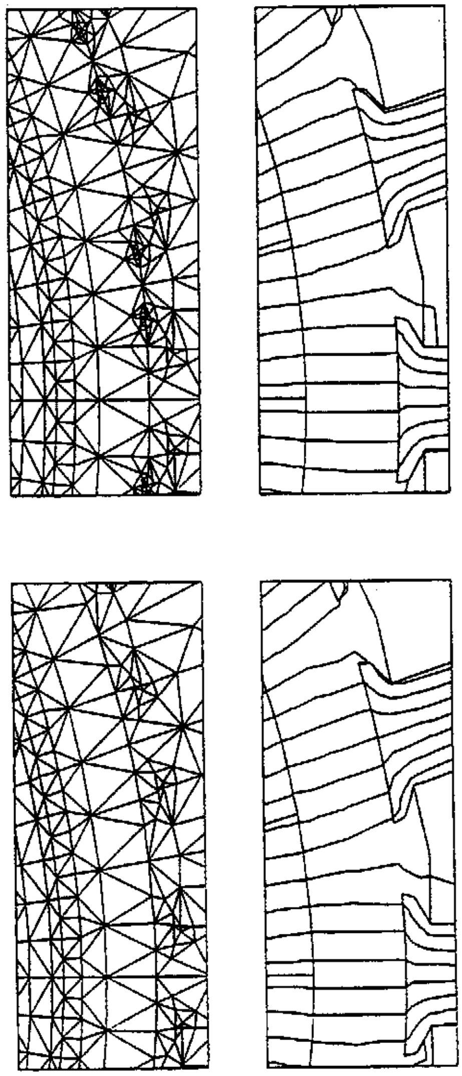

Fig. 3: Mesh detail in air-gap zone
Top: finer mesh;
Bottom: coarse mesh.

The second mesh is a great deal coarser near the tooth tips, and flux density values in the air gap and near tooth tips are expected to loose accuracy accordingly. While most of the flux lines trace roughly similar courses on the two meshes, tooth-tip flux density distributions differ. In the regions situated slightly to the right of the corners, flux lines do not impinge the laminations at right angles, despite the contrast between the permeability values, taken as 1:10000. This helps to explain why errors in the finite-element approximations are more accentuated in these regions. The low order polynomials used in the finite-element solutions are not adequate in approximating the sharp variations in potential values that occur in these regions.

The virtual work method was used in its classical form, evaluating the stored total energy for successive rotor positions, then subtracting to give energy differences. The computed cogging torque characteristics are presented in figure 4, along with measurements.

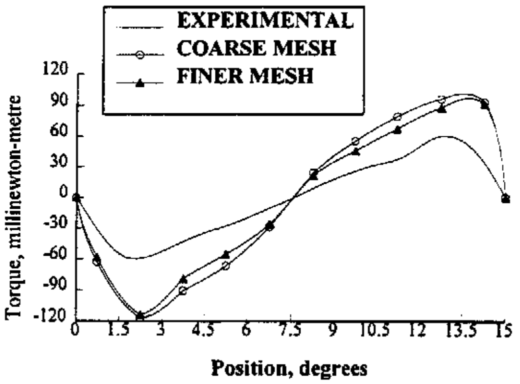

Fig. 4: Cogging torque characteristics, virtual work method.

The error in torque prediction is quantified in terms of the torque magnitude error and peak torque position error. Table I summarizes the errors for the two sets of data related to the virtual work method.

Mesh	Magnitude error, %	Position error, %
Coarse	99	5
Finer	92	5

Table I: Errors, virtual work method.

The energy difference δW between successive rotor positions are in the range 0.5-2.9 mJ in a total stored energy W of about 5.4 Joules. Inspection of the results has shown that data associated with this method have produced curves with the right shape, correctly exhibiting zero average torque over the 15° period. Also, the positions associated with peak torque are close to those obtained in the measurements with errors of the order of 5%. Errors in peak-to-peak torque, however, are very high, exceeding 90%. Visibly, the small differences in stored energies affect the accuracy of the computed torque sufficiently to render the straightforward virtual work approach questionable for this class of problem. The sensitivity of the method in its classical form to numerical error is seen here to be very high.

Torque computations based on the Maxwell stress method were performed using different integration contours, consisting of single arcs spanning one pole pitch. Stress integration over an arc of radius 27.0 mm resulted in more accurate values. In both meshes, this contour crosses the centre of the second layer of air-gap elements. Values for the cogging torques using this contour are presented in figure 5, along with the curve that represents measured values. Table II summarizes the errors for the two sets of data related to the Maxwell stress method.

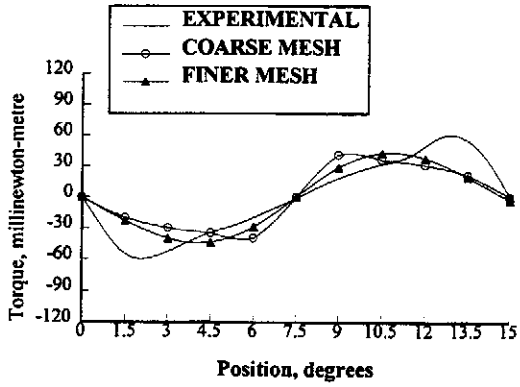

Fig. 5: Cogging torque characteristics, Maxwell stress method.

Mesh	Magnitude error, %	Position error, %
Coarse	33	23
Finer	27	13

Table II: Errors, Maxwell stress method.

None of the two curves related to the Maxwell stress method follow the curve of experimental results very closely. The errors in torque magnitude are significantly smaller than those obtained by the classical virtual work approach. Prediction of peak torque position, however, is even worse for the computations based on the Maxwell stress method. These results are not ideal, but this is to be expected when dealing with a geometry that contains sharp corners. The geometry contains 12 corners per pole pitch and this affects the level of accuracy of computed torques.

Further refinements of the mesh containing 952 nodes with 1834 elements (finer mesh) have not led to any significant improvement in results. Torque computations based on the virtual work method remain excessively high. The Maxwell stress method continues to predict peak torque at the wrong positions.

6. CONCLUSIONS

Calculation of forces and torques from numerical field solutions is a very difficult task. Usually, the solution is obtained in terms of potential distributions and, therefore, force is not the primary quantity in the computational analysis.

Among the various methods for the evaluation of force and torque, the methods of virtual work and Maxwell stress tensor have been chosen for a detailed numerical investigation. These methods have been used to solve a notoriously difficult problem: prediction of cogging torque in a small permanent-magnet motor.

Torque computations based on both methods have been compared to measured values. Numerical results have shown that, for both methods, the refinement of the finite-element mesh at an earlier stage has led to improvements in the results. For this particular problem, computations not reported in the paper and based on very coarse meshes with less than 700 nodes have produced oscillatory torque characteristics not consistent with the physical understanding of the problem. Mesh refinement at this level of discretization has, in fact, produced improvements in the results. Disappointingly, at the level of discretization of the finer mesh (952 nodes with 1834 elements), the accuracy of computed torques has not increased significantly as a result of an increase of mesh fineness.

The virtual work method in its classical form fails to predict cogging torque accurately. This is mainly due to the energy values corresponding to the two adjacent positions being of very similar magnitude.

The main problems of Maxwell stress method are related to its sensitivity to mesh artifact and to the location of the integration contour.

The key to accurate torque computation is to avoid numerical differentiation entirely. In many problems involving numerical differentiation and integration, the order of differentiation and integration can be so rearranged that all integrations are done numerically, all differentiations analytically. Torque computations based on the technique of mean and difference potentials use the magnetic vector potential directly and can produce results which agree with measured values to within a few percent, within the limits of measurement accuracy and the approximations inherent in two-dimensional analysis.

7. REFERENCES

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