CALCULATION OF ELECTRIC FIELDS IN THE HUMAN KNEE PRODUCED BY A KNEE-THIGH ELECTRODE PAIR

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ABSTRACT. Calculations of the electric fields produced in the cartilage of the human knee by a knee-thigh electrode pair excited by a low-frequency voltage waveform are presented and discussed. The simulations were performed using a three-dimensional finite-difference frequency-domain technique based upon an equivalent circuit model for solving Maxwell's equations. The leg model used in these calculations was obtained from the anatomically segmented human-body model of Gandhi. The calculated average electric fields within the medial and lateral cartilage were found to be 2.92 V/m and 2.51 V/m respectively for a voltage excitation of one volt at a frequency of 1 kHz applied between the electrodes. A special truncation technique, which incorporates information from a physically larger coarse model in the excitation of a finer subsection of the model, was utilized to obtain higher resolution results in the region of interest.

1 INTRODUCTION

Osteoarthritis, the most common form of arthritis, affects approximately 21 million Americans [Lawrence et al., 1998]. Symptoms can range from stiffness and mild pain that comes and goes to severe joint pain and even disability. To help alleviate these symptoms various therapeutic techniques are being examined. For example, pulsed electric- and magnetic-field applicators are being investigated for their potential to help relieve the inflammation and pain associated with osteoarthritis [Zizic et al., 1995, Trock et al., 1994]. A recent survey of 20 double-blind, randomized and placebo-controlled studies involving pulsed electromagnetic fields found pain alleviation was confirmed in most of these trials [Quittan, et al., 2000].

In an effort to understand possible mechanisms contributing to these observed effects, it is important to calculate the characteristics of the electric fields and current densities produced by various applicator configurations in the knee region. In this paper, we describe calculations of the electric fields (E) and current densities (J) produced in knee cartilage from a pair of electrodes placed on the surface of the knee and thigh with a sinusoidal excitation of one volt at 1 kHz applied between them.

2 METHODS OF CALCULATION

2.1 Solution Method

The simulations in this paper were performed using a three-dimensional finite-difference frequency-domain technique for solving Maxwell's equations [Buechler, 1997, Buechler et al. 1997]. The computational space is subdivided into mathematical cells called Yee cells [Yee, 1966]. These Yee cells, which are characterized by the interleaving of electric and magnetic fields in space, define the electrical properties (conductivity σ, permittivity ε, and permeability μ) of the different media in the model. By applying Maxwell's curl equations in integral form to a nonuniform Yee cell and using a finite-difference approximation [Buechler, 1997] an RLC equivalent circuit can be obtained for each Yee cell face (Fig. 1), where the edge voltages are related to electric fields by $V = -\int E \cdot dl$ and where the face loop currents are related to magnetic fields passing through the cell face by $I = \oint H \cdot dl$. The resulting equivalent-circuit equations can be written in the form **Ax** = **b**, where **x** is the unknown loop current vector (related to the magnetic fields passing through the Yee cell face), **b** is the Yee cell edge voltage vector (related to the Yee cell edge electric fields), and **A** is the RLC impedance matrix, which is complex, symmetric, and very sparse. The quasi-minimal residual (QMR) method [Freund et al., 1994] is used to solve this nonHermitian system because of its constant work and storage requirements per iteration and good convergence properties. The resulting calculational method, which extends Gandhi's impedance method [Gandhi et al., 1984, Orcutt et al., 1988] to higher frequencies by including inductance effects, is valid at any frequency for which the finite-difference approximation to Maxwell's equations is valid and therefore applies to both quasi-static and higher frequency problems. In addition to applied electric- and magnetic-field sources, the method allows voltage and current sources, including source impedances, between any two points in the model. This method complements other recent approaches to analyzing internal electric fields produced in an anatomically realistic model of the human body by either external field or electrodes [Furse et al., 1998, Dawson et al., 1998, Dawson et al., 2000, Dimbylow et al., 1998, Dimbylow et al., 2000].

2.2 Leg Model

For this application problem, we were interested in the **E** produced within the cartilage of the human knee. We obtained a 151 x 148 x 310 mathematical cell model of the various tissues types in the human leg from Gandhi [Gandhi, 1995]. Each mathematical cell had uniform x,y,z dimensions (1.974 mm x 1.974 mm x 3.00 mm) and was assigned properties of permittivity and conductivity according to its tissue type.

2.3 Media properties

The leg model included muscle, fat, cancellous bone, compact bone, cartilage, skin, blood, ligament, and air. Impedance measurements presented in the USAF Report by Gabriel and Gabriel [1996] were used to assign permittivity and conductivity values to these different tissue types over the frequency range of interest. Since the leg model did not distinguish between parallel and transverse muscle, we assigned muscle an average of the two measured values. Extrapolated data were used to characterize cancellous bone, compact bone, cartilage, and blood. Tendon properties were substituted for those of ligaments due to the lack of measured data for ligament. Tables 1a and 1b summarize the media properties over the frequency range from 500 Hz to 100 kHz.

2.4 Electrode Model

2.4.1 Physical description

The knee-thigh electrode model consisted of two metal electrodes, as shown in Fig. 2: one centered over the knee cap and one placed over the mid-thigh anteriorly, with a sinusoidal excitation voltage applied between them. The physical dimensions of the knee electrode were 4.5" x 6.25", with the long dimension circumferentially wrapped around the knee. The dimensions of the thigh electrode were 4" x 6.75", again with the long dimension circumferentially wrapped around the thigh. To model these electrodes, we placed metal, two cells thick, on the surface of the leg assuming direct electrode contact with the leg surface to account for the presence of electrode gel. The extent of the metal electrode circumferentially was calculated slice-by-slice using the definition of a half perimeter of an ellipse. In a few locations, additional metal cells were added to the two-cell layer to eliminate gaps in the electrodes where the vertical position between two adjacent slices of the knee was changing rapidly.

2.4.2 Truncation Technique

For the calculation of the **E**-fields within the cartilage of the knee, a fine model is needed in the region of interest. This requirement is met by the original model consisting of cells with dimensions of 1.974 mm x 1.974 mm x 3.00 mm. However, the memory limitation of our computer (688 MB) required that the total number of cells in this double-precision complex calculation be restricted to about one million. Therefore use of a truncated model that did not include the full region under the thigh electrode was necessary.

We started with a coarser model that encompassed the entire electrode region, as shown in Fig. 2, but with cells that were twice as long in the z-direction as in the original model (1.974 mm x 1.974 mm x 6.00 mm). The resultant 94 x 96 x 82 cell model contained 739,968 cells total. Then using the results from this model, we made a second simulation with the finer model, which was a subsection of the coarser model. This 94 x 96 x 110 cell model with cells 1.974 mm x 1.974 mm x 3.00 mm contained 992,640 cells; its location with respect to the knee and electrodes is shown in Fig. 3. The upper boundary of the finer model occurs at the center of the thigh electrode, leaving the original source connection between the two electrodes intact. In addition, the voltage distribution in the plane of truncation from the coarse model calculation was used as a source to include the effect of the truncated portion of the model.

2.4.3 Excitation

In the equivalent-circuit formulation, voltage and current sources as well as their source impedances can be easily created between any two points in the model. This allowed excitation between knee and thigh electrodes having different diameters. Pulsed excitations are realized by taking the Fourier transform of the desired waveform and by making independent runs for the dominant frequency components. The results can then be recombined using superposition.

In this study, we are interested in the **E**-fields produced within the knee by a low-frequency voltage waveform applied between the electrodes. For simplicity in later calculations, we used an excitation voltage of $V = 1.0\sin(2\pi ft)$ volts, where $f = 1$ kHz, in combination with zero source impedance. Although we chose 1 kHz as the excitation frequency, the results should be approximately valid for other frequencies in the range of about 500 Hz to 100 kHz. An analysis of the tissue properties over the frequency range from 500 Hz to 100 kHz reveals that the resistive properties dominate over the reactive properties. This is demonstrated by the ratio of media conductivities to the product of angular frequency and permittivities (i.e., the ratio $\sigma/\omega\varepsilon$) for all media types in this model, as given in Table 2. With the exception of dry skin, all media possess $\sigma/\omega\varepsilon$ ratios of greater than five, with most being considerably greater. Since the media are essentially resistive over the frequency range from 500 Hz to 100 kHz and there is little change in the conductivities over this range (except for dry skin), the magnitude of **E** will be essentially independent of frequency in this range. Because perfectly dry skin is unlikely to be encountered in actual practice, where electrode gel is used to couple the electrodes to the body, and since the skin layer is thin compared to other tissue volumes, we believe the presence of the skin will not alter the frequency dependence of the results significantly.

2.5 Approximations

A number of approximations have been made in these calculations to obtain the most accurate solutions possible within the region of interest (the cartilage of the knee) given the constraints of the computer and tissue models. These approximations include perfectly conducting metal electrodes, exclusion of air cells, sinusoidal excitation, and model truncation. All except the common perfectly conducting metal approximation are discussed below.

A relatively large number of air cells were included in the leg model obtained from Gandhi. These air cells, which add to the overall problem size and have the potential of causing convergence problems, can be removed by setting the currents within them equal to zero (i.e. replacing them by a magnetic wall).

Since the media types in our model are essentially resistive for low frequencies (500 Hz to 100kHz) and media conductivities are fairly constant over this range, we have calculated the **E** in the knee for a sinusoidal excitation at a single frequency (1 kHz). This is a good approximation for the fields over the frequency range from 500 Hz to 100 kHz for all media types in this model except for dry skin. Below 500 Hz, the rise in permittivity values, which result in a drop in the $\sigma/\omega\varepsilon$ ratios below 5.0 for muscle (2.80 at 100 Hz) and ligament (4.95 at 100 Hz), combined with the reduction in conductivities for muscle (0.293 S/m at 100 Hz) and ligament (0.303 S/m at 100 Hz), reduces the validity of this approximation for these two media types below 500 Hz. For higher accuracy results for frequency components below 500 Hz for models containing muscle and ligament, additional calculational runs should be done.

The final assumption was the use of source information from a coarser 94 x 96 x 82 cell model to allow the solution of a finer 94 x 96 x 110 cell subsection of the model in the region of interest. As stated above in section 2.4.2, we used a coarser model that encompassed the entire electrode region, but with cells that were twice as long in the z-direction as in the original model (1.974 mm x 1.974 mm x 6.00 mm). After the completion of the runs for the larger 94 x 96 x 82 cell coarser model, we did an additional run to validate this coarse-fine truncation technique. We set up the problem as if we were going to calculate the fields in the truncated 94 x 96 x 110 fine-cell model using the original source and the voltage distribution in the plane of truncation, but instead used a 94 x 96 x 55 coarse-cell model to represent this same subsection to allow direct comparison of the results from the previous coarse-cell calculation within the subsection. The differences between the E-field values for the two sets of data over this subsection were small (less than 1%).

3 RESULTS

3.1 Region of interest

In these calculations, the regions of primary interest are the lateral and medial knee cartilage. These regions are contained within four successive z planes of 3-mm thickness each. Therefore all the following results will be limited to these four planes. The media types for the z planes containing the lateral (z = 67) and medial (z = 68) cartilage are shown in Fig. 4 and Fig. 5 respectively.

3.2 Results within cartilage

The electric fields produced by the knee-thigh electrodes were found at the peak of the 1V sinusoidal excitation. Cell-centered field components E_x, E_y, and E_z were obtained by averaging the four corresponding field components at the edges of the cell. A cell-centered electric-field magnitude was then calculated from these cell-centered field components. The cell-centered magnitudes of the electric field in the cross-sectional plane (z = 68) containing the medial cartilage (outlined region) are shown in Fig. 6. The maximum electric field magnitude anywhere in the slice is 48.4 V/m, and the average electric field within the cartilage is 2.92 V/m. Similarly, the cell-centered E-field magnitudes in the cross-sectional plane (z = 67) containing the lateral cartilage are shown in Fig. 7, with an average electric field of 2.51 V/m within the cartilage and a slice maximum of 56.5 V/m. The average cartilage electric field magnitudes within the other two planes (z = 65 and z = 66) were 2.76 V/m and 2.70 V/m respectively. The maximum electric field magnitude in the entire model was 146.6 V/m, located directly underneath the thigh electrode at the edge closest to the knee electrode.

From the normalized electric-field data, the edge current-density components were calculated using the property values of the surrounding media types. By averaging the four corresponding edge current densities, the cell-centered current-density components J_x, J_y, and J_z can be obtained and used to calculate the cell-centered current-density magnitude. The cell-centered magnitudes of the current density for the cross-sectional plane (z = 68) containing medial cartilage (outlined region) are shown in Fig. 8. The average current density within the cartilage is 0.376 A/m^2 and the maximum current density anywhere in this plane is 0.728 A/m^2. The average current density within the lateral cartilage plane (not shown) was 0.317 A/m^2, with a maximum value of 1.28 A/m^2 located in the plane. The maximum current density magnitude anywhere in the entire model was 9.83 A/m^2, located in tissue underneath the knee electrode near the edge closest to the thigh electrode.

4 DISCUSSION AND CONCLUSIONS

The resultant electric-field pattern in the human knee due to this knee-thigh electrode excitation is very nonuniform, as seen in Fig. 6 and 7. The regions of the highest electric field and current density magnitudes are located underneath the edges of the electrodes closest to the other electrode. This can be explained by current crowding into the edges and corner of the electrodes and by the current density boundary conditions, which require the continuity of the normal components across interfaces between low conductivity media (such as fat) and higher conductivity media (such as muscle). As a result, the highest electric field magnitude anywhere in the entire region modeled is about 50 times higher than the average **E** in the medial cartilage.

Since the media within the knee are essentially resistive for frequencies between 500 Hz and 100 kHz, the E-field waveform for an arbitrary low-frequency voltage excitation will have approximately the same shape as the excitation waveform provided that the majority of its frequency components fall within that range. Thus our results using a normalized sinusoidal excitation can be scaled to obtain the responses for different waveforms having frequency components primarily in this resistive range.

A recent survey of various double-blind, randomized and placebo-controlled clinical studies with pulsed electromagnetic fields with both electric-field and magnetic-field applicators confirmed bone healing and pain alleviation in most of the trials (results for enhanced healing of ulcers and the reduction of spasticity were contradictory); however, the optimal dosimetry for therapy with electromagnetic fields is yet not established [Quittan et al., 2000]. Electric-field and magnetic-field applicators have different characteristics. Electric-field applicators, as shown here, have high fields near the surface of the electrodes but achieve moderate field levels in the cartilage. Magnetic-field (coil) applicators produce more uniform, but much lower intensity, field patterns [Buechler et al., 2001]. Correlation of clinical outcomes and applicator field distributions could be used to help understand possible mechanisms contributing to the observed clinical effects. Once more is known about the dose-response mechanism for the treatment of osteoarthritis by electric or magnetic fields, this calculational method may be useful in the design of optimal treatment applicators.

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Table 1a. Media Conductivities (σ in S/m)

Media	(500 Hz)	(1 kHz)	(100 kHz)
avg. muscle	0.410	0.433	0.477
fat	0.0241	0.0246	0.0262
cancellous bone	0.080	0.080	0.080
cartilage	0.180	0.190	0.190
dry skin	1.77×10^{-4}	1.8×10^{-4}	8.9×10^{-4}
blood	0.700	0.700	0.700
compact bone	0.020	0.020	0.021
ligament	0.379	0.391	0.398

Table 1b. Media Relative Permittivities (ε_r)

Media	(500 Hz)	(1 kHz)	(100 kHz)
avg. muscle	2.76×10^6	9.15×10^5	5.67×10^3
fat	5.44×10^4	2.08×10^4	9.54×10^1
cancellous bone	2.20×10^4	1.30×10^4	5.00×10^2
cartilage	6.50×10^4	3.00×10^4	2.60×10^3
dry skin	1.23×10^3	1.17×10^3	9.34×10^2
blood	5.30×10^3	5.30×10^3	5.20×10^3
compact bone	4.00×10^3	2.70×10^3	2.50×10^2
ligament	1.51×10^6	4.52×10^5	4.87×10^2

Table 2. Ratio of Conductivity (σ) to Product of Angular Frequency and Permittivity ($\omega\varepsilon$)

Media	$\sigma/\omega\varepsilon$ (500 Hz)	$\sigma/\omega\varepsilon$ (1 kHz)	$\sigma/\omega\varepsilon$ (100 kHz)
avg. muscle	5.33	8.51	15.1
fat	15.9	21.2	49.4
cancellous bone	131	111	28.8
cartilage	99.6	114	13.1
dry skin	5.18	2.76	0.172
blood	4750	2370	24.2
compact bone	180	133	15.1
ligament	9.01	15.6	147

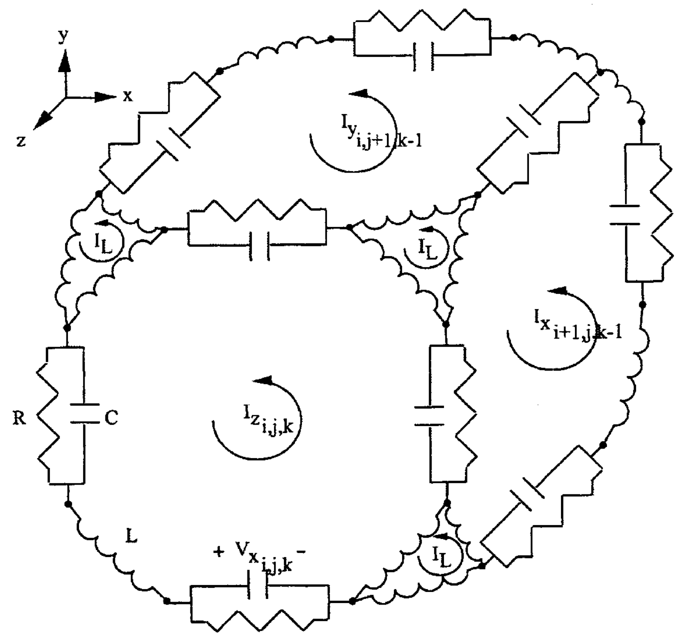

Fig. 1. RLC equivalent circuit on each face of a typical Yee cell.

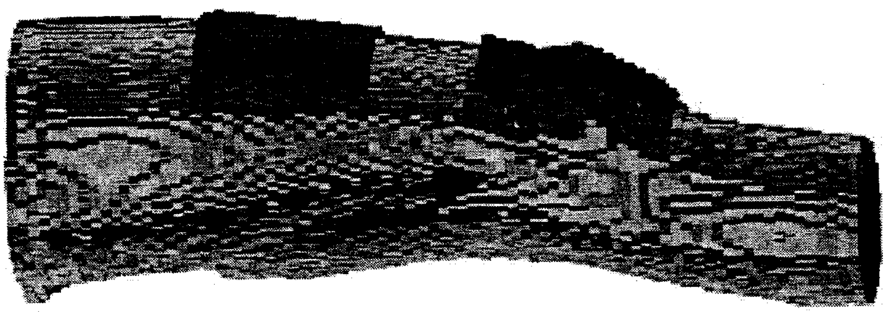

Fig. 2. Coarse knee-thigh electrode model consisting of 739,968 calculational cells each of size 1.974 mm x 1.974 mm x 6.000 mm. The lighter shading shows the surface of the skin. The darker shading indicates the positions of the thigh and knee electrodes.

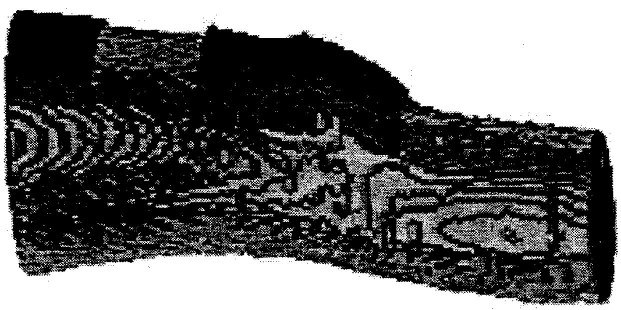

Fig. 3 Truncated fine knee-thigh electrode model consisting of 992,640 calculational cells of size 1.974 mm x 1.974 mm x 3.000 mm. The truncation occurs at the center of the thigh electrode.

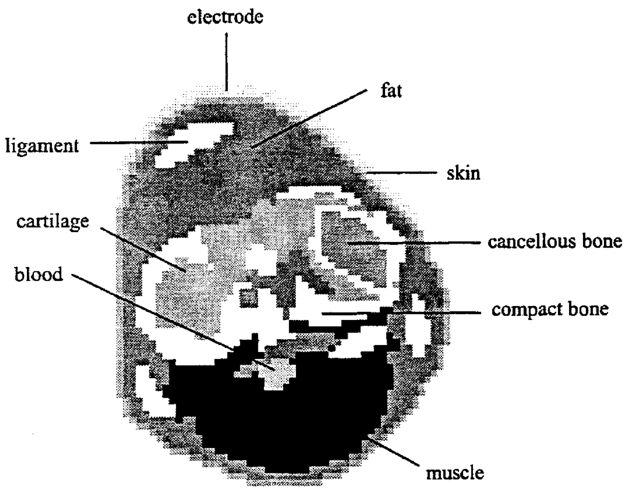

Fig. 4 Media types within cross-sectional plane (z = 67) containing lateral cartilage.

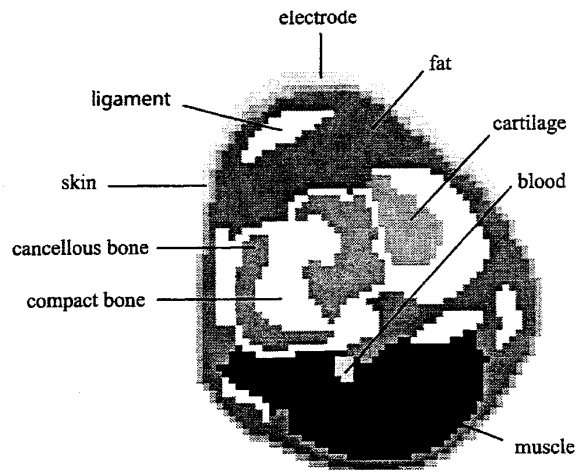

Fig. 5. Media types within cross-sectional plane (z = 68) containing medial cartilage.

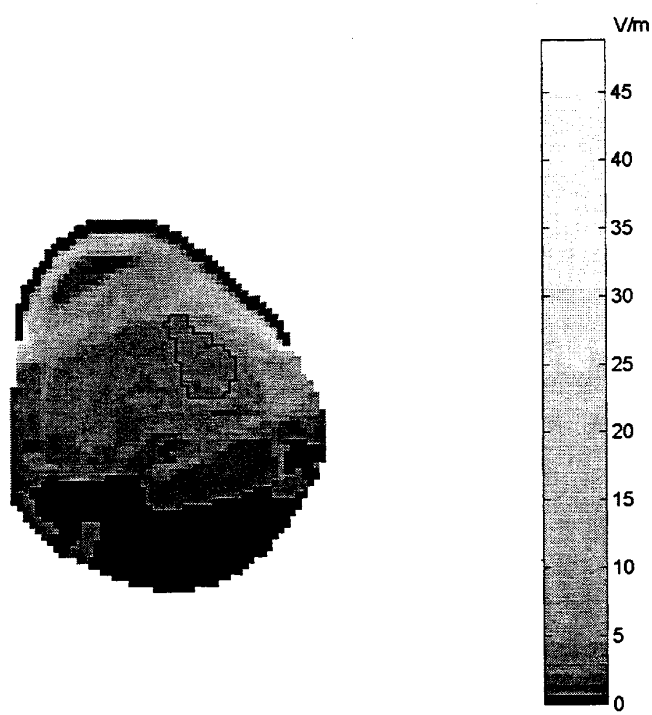

Fig. 6. Cell-centered magnitudes of **E** in cross-sectional plane z = 68 for a voltage excitation of 1 V at 1 kHz applied between the two electrodes. The maximum slice **E** is 48.4 V/m. The average **E** in the medial cartilage of this slice is 2.92 V/m. The cartilage region and knee electrode are outlined in black.

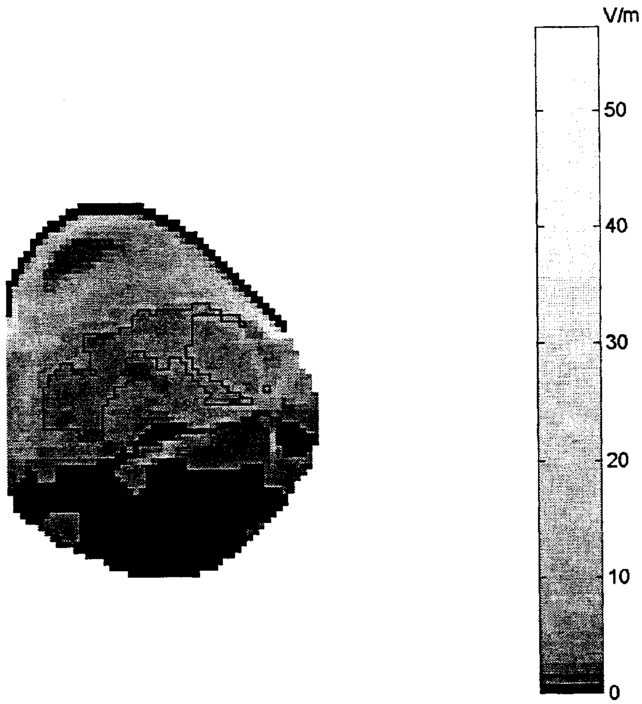

Fig. 7. Cell-centered magnitudes of **E** in cross-sectional plane z = 67 for a voltage excitation of 1 V at 1 kHz applied between the two electrodes. The maximum slice **E** is 56.5 V/m. The average **E** in the lateral cartilage of this slice is 2.51 V/m. The cartilage region and knee electrode are outlined in black.

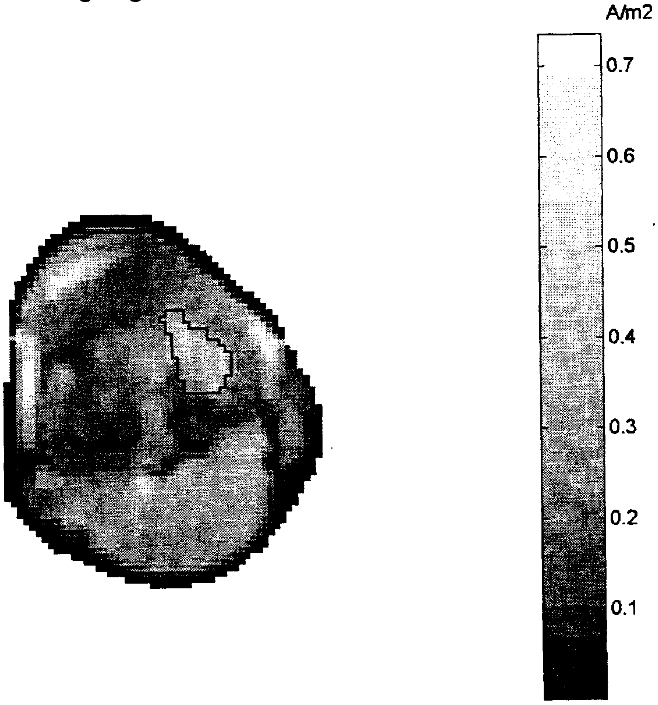

Fig. 8. Cell-centered magnitudes of **J** in cross-sectional plane z = 68 for a voltage excitation of 1 V at 1 kHz applied between the two electrodes. The maximum slice **J** is 0.728 A/m^2. The average **J** in the medial cartilage of this slice is 0.376 A/m^2. The cartilage region and knee electrode are outlined in black.