On The Optimum Directivity of Dipole Arrays Considering Mutual Coupling

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Abstract – Linear half-wave dipole array is optimized on the basis of the directivity considering mutual coupling between the array elements, in horizontal, vertical and three dimensional field patterns. Simulated results for the value of directivity without and with mutual coupling are presented in both broad-side and end-fire configurations, for different number of elements in the array. It is observed that mutual coupling can enhance as well as degrade the overall directivity of the array.

Keywords: Half wave dipole, directivity, broad side array, end fire array, and mutual coupling.

I. INTRODUCTION

Mutual coupling is a phenomenon that depends upon the adjacent array elements and greatly affects the characteristics of an antenna array. Mutual coupling, expressions for mutual impedance and procedure for obtaining currents after mutual coupling in a linear dipole array are discussed in [1-4]. An advanced method of measuring mutual impedance easily is given in [5]. Modeling and estimation of mutual coupling in uniform linear array of dipoles is presented in [6]. Method of computing mutual impedance conducive to digital computer application is given in [7]. Mutual impedance calculated considering scattering from the individual elements is given in [8] and mutual impedance calculated using Finite Difference Method (FDM) is discussed in [9]. Fundamentals of linear arrays can be found in [2, 3, 10]. The optimum directivity of an antenna or its array is given in [11, 12], and approximate directivity expressions for a linear array of uniformly spaced elements are discussed in [13, 14] when the amplitudes for all the array elements are real.

In this paper the directivity of the linear dipole array is improved taking mutual coupling into account rather than trying to compensate the effect. Linear array design by compensating mutual coupling phenomenon is discussed in [15, 16]. Simple expressions to calculate the directivity are presented for a linear array of any large number of identical elements with constant inter element spacing, where elements can have different complex amplitude coefficients. Optimum directivity values and corresponding graphs with respect to the separation between the elements are given, with mutual coupling compared to the case of without mutual coupling for the three amplitude configurations uniform, Dolph-Tchebyscheff and binomial, obtained by simulating in Matlab. The results show that the factors such as amplitudes, phase difference between adjacent elements and total number of elements in the array together greatly influence the effect of mutual coupling on the array directivity. Directivity values for the uniform three dimensional field pattern without mutual coupling for the case of half-wave dipole arrays in both broad-side and end-fire directions are validated by [11, 17], whereas mutual coupled currents are verified with some of the results in [15, 18]. Effect of mutual coupling on the maximum directivity of the array by changing the dipole length [2] from 0.5 λ to 1.5 λ is also given by observing the change in near and far field regions, resulting from the change in the dipole length.

Consider an array of N identical elements with any amplitude and phase excitation, placed along the x axis, symmetrically with respect to origin, as shown in Fig. 1 with constant separation between adjacent elements as d.

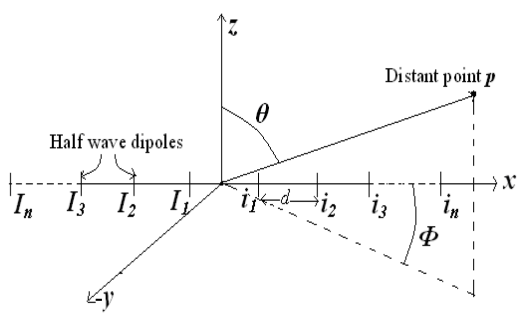


Fig. 1. Linear array of N half wave dipoles with uniform spacing d along the x axis.

The field pattern of this array if the total number of elements in the array is even can be given as,

$$E = E_0[I_n e^{-[N-1]j\psi} \ldots + I_2 e^{-3j\psi} + I_1 e^{-j\psi} + i_1 e^{+j\psi} + i_2 e^{+3j\psi} \ldots + i_n e^{+[N-1]j\psi}] \quad (1)$$

where Ψ = ½ kd sinθ cosφ. And for odd number of elements it is,

$$E = E_0[I_n e^{-\frac{[N-1]}{2} j\psi} .... + I_2 e^{-2j\psi} + I_1 e^{-j\psi} + I_0 + i_1 e^{+j\psi} + i_2 e^{+2j\psi} .... + i_n e^{+\frac{[N-1]}{2} j\psi}] \quad (2)$$

where $\Psi = kd \sin\theta \cos\varphi$. I_n is the current excitation coefficient in the left half of the array and i_n is the excitation coefficient in the right half of the array of dipoles with respect to origin. In the case of without mutual coupling $I_1 = i_1$, $I_2 = i_2, ...I_n = i_n$. However, they are not equal if we consider mutual coupling between the elements. E_0 is the normalized electric field of a single element and in the case of a half-wave dipole [3]. If we represent the amplitudes of the above array,

$I = [I_n, ...I_2, I_1, i_1, i_2, ...i_n]$ as

$I = [I_1, I_2, I_3 .....I_i ....I_N]$

where $I(i)$ represents the current in the i^{th} element of the array and N represents the total number of elements in the array, then $E \times E^*$ (E^* is the complex conjugate of E) becomes,

$$\begin{aligned} E \times E^* = \Big\{ & \sum_{i=1}^{N} [I(i) * I(i)^*] + [\sum_{i=1}^{N-1} I(i) I(i+1)^*] e^{-2j\psi} \\ & + [\sum_{i=1}^{N-1} I(i)^* I(i+1)] e^{2j\psi} + [\sum_{i=1}^{N-2} I(i) I(i+2)^*] e^{-4j\psi} \\ & + [\sum_{i=1}^{N-2} I(i)^* I(i+2)] e^{4j\psi} + ... \\ & + [\sum_{i=1}^{2} I(i) I(i+N-2)^*] e^{-2(N-2)j\psi} \\ & + [\sum_{i=1}^{2} I(i)^* I(i+N-2)] e^{2(N-2)j\psi} + \\ & I(1) I(N)^* e^{-2(N-1)j\psi} + I(1)^* I(N) e^{2(N-1)j\psi} \Big\} \left| E_0{}^2 \right| \end{aligned} \quad (3)$$

note that $E \times E^*$ is not equal to $|E|^2$ if the amplitudes of array elements are not real which will be the case for the array with mutual coupling. Taking the real part of the above expression we get,

$$\begin{aligned} \mathrm{Real}(E \times E^*) = \Big\{ & \sum_{i=1}^{N} \mathrm{Real}[I(i) * I(i)^*] \\ & + 2 * \{ \sum_{i=1}^{N-1} \mathrm{Real}[I(i) I(i+1)^*] \cos(2j\psi) \} \\ & + 2 * \{ \sum_{i=1}^{N-2} \mathrm{Real}[I(i) I(i+2)^*] \cos(4j\psi) \} + ... \\ & + 2 * \{ \sum_{i=1}^{2} \mathrm{Real}[I(i) I(i+N-2)^*] \cos(2(N-2)j\psi) \} \\ & + 2 * \{ \mathrm{Real}[I(1) I(N^*)] \cos(2(N-1)j\psi) \} \Big\} \left| E_0{}^2 \right| \end{aligned} \quad (4)$$

with same Ψ = ½ $kd \sin\theta \cos\varphi$, for both even and odd number of elements in the array. If we define $U = \mathrm{Re}\{ E \times E^* \}$, as the radiation intensity of the total array, and then the directivity of the array become, [3]

$$D = \frac{4\pi U_{\max}}{P_{rad}} \quad (5)$$

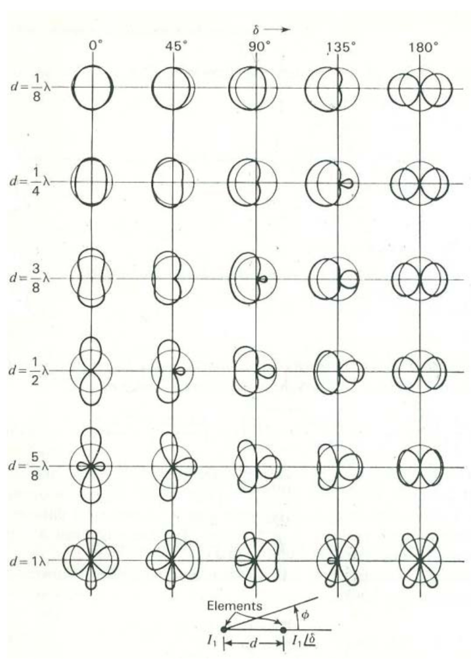
where P_{rad} is total radiated power given by,

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} U \sin\theta \, d\theta \, d\varphi . \quad (6)$$

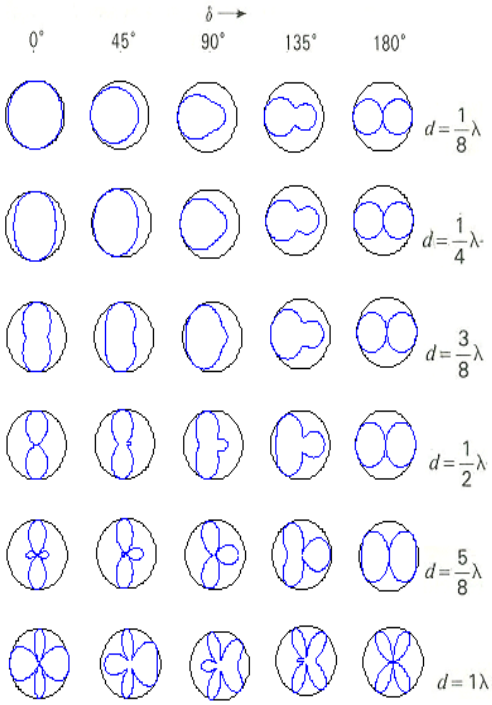
II. EFFECT OF MUTUAL COUPLING ON HORIZONTAL FIELD PATTERN

The radiation pattern produced by a half wave dipole in the horizontal (or azimuthal) direction is uniform as an isotropic source and it is a figure '8' pattern in the vertical (or polar) direction. However the radiation pattern or array factor of a two element dipole array in the horizontal direction varies according to separation d and phase difference δ between the elements as well as mutual coupling between them.

Polar plots of horizontal field pattern of a vertical half wave dipole array oriented along the z axis, as a function of phase difference δ and spacing d between two elements, excited with same magnitude of current is given in Fig. 2, with and without mutual coupling. The circles indicate the field pattern of a single reference half wave dipole antenna which is uniform like an isotropic source.



Without mutual coupling. [2]



With mutual coupling.

Fig. 2. Field pattern of a two element half wave dipole array vs spacing and phase difference in horizontal plane with and without mutual coupling.

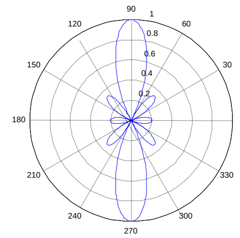
We can see a noticable change in pattern as d & δ vary, due to mutual coupling. In particular there is noticeable enhancement in the size of the back lobe in some cases, which tends to minimize the directivity.

III. BROAD-SIDE CONFIGURATION

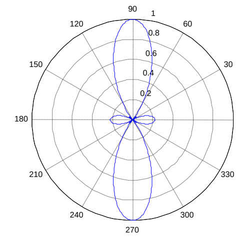
In any broad-side array where the relative phase difference between the currents is zero, we analyze mainly three amplitude patterns which are uniform, Dolph-Tchebyscheff (for 20dB side lobe level) and binomial arrays.

The polar field patterns drawn in linear scale, of a three element half wave dipole array along x axis, for each of the above three amplitude configurations with d equal to 0.72 λ, 0.67 λ and 0.67 λ, respectively, which give maximum directivity without mutual coupling, are given in Fig. 3.

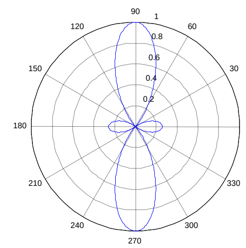
The field patterns after considering mutual coupling between the elements are shown in Fig. 4 with same separation d between the elements. D decreases for the uniform amplitude case while increases for the other two amplitude patterns. Notice that the major change in the field patterns with mutual coupling is the change in the side lobe level.



d = 0.72 λ, D = 5.8579
Uniform array

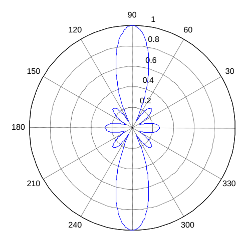


d = 0.72 λ, D = 5.8579
Dolph-Tchebyscheff array

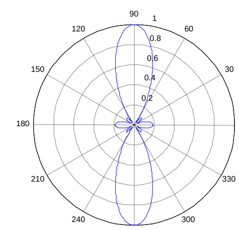


d = 0.67 λ, D = 5.118
Binomial array

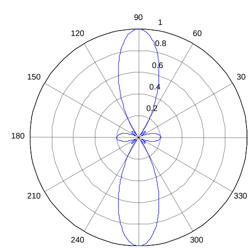
Fig. 3. Field patterns of 3 element half wave dipole broad-side uniform, Dolph-Tchebyscheff and binomial arrays with optimum separation 'd', and maximum directivity 'D' without mutual coupling.



d = 0.72 λ, D = 5.79
Uniform array



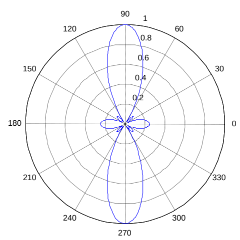
d = 0.67 λ, D = 5.5923
Dolph-Tchebyscheff array



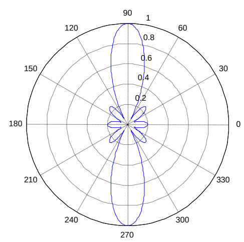
d = 0.67 λ, D = 5.5149
Binomial array

Fig. 4. Field patterns after mutual coupling with same amplitude coefficients and spacing.

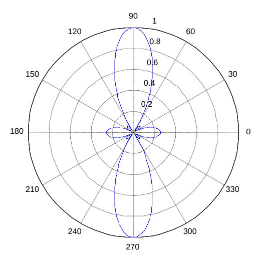
The above field patterns are improved for maximum directivity by slightly adjusting the separation between the dipoles. Fig. 5 shows the optimized field patterns after considering mutual coupling, with their corresponding maximum directivity for each of the three amplitude configurations, obtained for d = 0.7 λ, 0.69 λ, and 0.69 λ, respectively. We can notice the increase in the maximum directivity values than those shown in Fig. 4.



d = 0.7 λ, D = 5.8307
Uniform array



d = 0.69 λ, D = 5.6439
Dolph-Tchebyscheff array



d = 0.69 λ, D = 5.5587
Binomial array

Fig. 5. Optimized field patterns of the three amplitude arrays with mutual coupling.

The variation of directivity for a uniform array of half wave dipoles ranging from 3 to 10, with distance between the elements varying from 0.1 λ to 1 λ is given in Fig. 6. From the graph we can see that there is an increase in the directivity of the array due to mutual coupling for d around 0.5 λ to 0.9 λ at which most broad-side arrays operate and it is less at remaining distances. However maximum directivity of the array is less with mutual coupling than the directivity without coupling.

The optimum separation to maximize directivity for a uniform broad-side array with different number of elements is given in Table 1 with and without mutual coupling. Note that the directivity of the array decreases due to mutual coupling for small N. However, there is little enhancement due to mutual coupling in the optimum directivity for large number of elements in the array. Also the optimum spacing between the elements decreases when mutual coupling is taken into account in order to optimize the directivity for a uniform array.

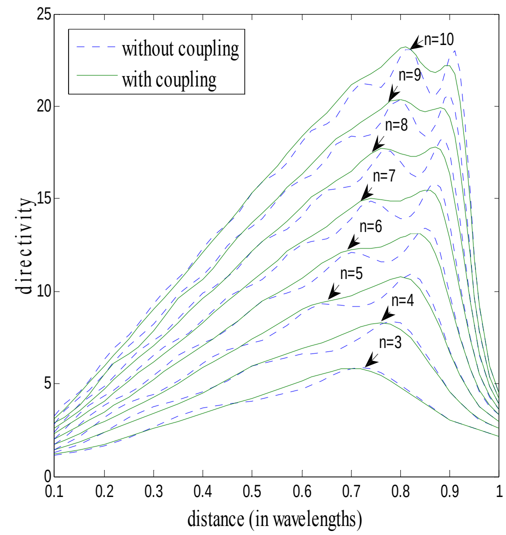


Fig. 6. Variation of directivity with separation d, in uniform n element broad-side array with and without mutual coupling.

Table 1. Uniform broad-side array.

	d	*D*	*d**	*D**
N=3	0.72 λ	5.8579	0.7 λ	5.8307
N=4	0.78 λ	8.387	0.76 λ	8.3221
N=5	0.82 λ	10.901	0.8 λ	10.7664
N=6	0.85 λ	13.3941	0.83 λ	13.1562
N=7	0.87 λ	15.8495	0.85 λ	15.4865
N=8	0.89 λ	18.2178	0.87 λ	17.7859
N=9	0.9 λ	20.6493	0.8 λ	20.3893
N=10	0.81 λ	23.0761	0.81 λ	23.1784
N=20	0.86 λ	49.601	0.85 λ	49.7916
N=30	0.91 λ	76.9691	0.87 λ	77.1831
N=50	0.92 λ	132.51	0.9 λ	132.87

d = Optimum separation without coupling.
D = Corresponding maximum directivity.
d* = Optimum separation with coupling.
D* = Maximum directivity with coupling.
N = Total number of elements.

For Dolph-Tchebyscheff broad-side array the optimum distance between the elements and the corresponding maximum directivity are given in Table 2, with and without mutual coupling.

Table 2. Broad-side Dolph-Tchebyscheff array.

	d	*D*	*d**	*D**
N=3	0.67 λ	5.4019	0.69 λ	5.6439
N=4	0.73 λ	7.7082	0.76 λ	8.0623
N=5	0.77 λ	10.2386	0.8 λ	10.6029
N=6	0.8 λ	12.899	0.83 λ	13.2309
N=7	0.83 λ	15.6228	0.85 λ	15.9254
N=8	0.85 λ	18.3781	0.87 λ	18.6261
N=9	0.86λ	21.1917	0.8 λ	21.312
N=10	0.88 λ	23.908	0.81 λ	24.094

Maximum directivity increases due to mutual coupling for this array with a slight adjustment in the spacing between elements.

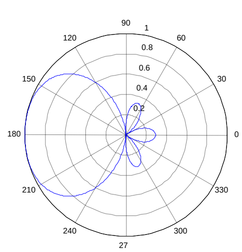
For the binomial broad side array the optimum separation between the elements and the corresponding directivity are shown in Table 3. Although the directivity of a binomial array is lower relative to uniform and Dolph-Tchebyscheff arrays without coupling, mutual coupling increases the overall directivity of the array because of the larger current amplitudes in the middle of the array. The spacing between the elements needs to be increased for this array to get better directivity.

Table 3. Binomial broad-side array.

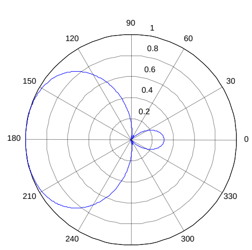
	d	*D*	*d**	*D**
N=3	0.67 λ	5.118	0.69 λ	5.5587
N=4	0.70 λ	6.6013	0.74 λ	7.6064
N=5	0.73 λ	7.9028	0.77 λ	9.4228
N=6	0.75 λ	9.0766	0.80 λ	11.0186
N=7	0.76 λ	10.1601	0.81 λ	12.4034
N=8	0.77 λ	11.1627	0.83 λ	13.6332
N=9	0.78λ	12.1052	0.84 λ	14.6996
N=10	0.79 λ	12.9993	0.85 λ	15.6872

IV. END-FIRE CONFIGURATION

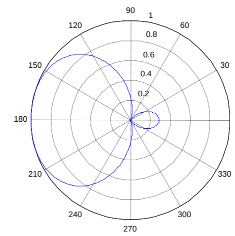
In end-fire arrays, if the relative phase difference between the elements is $\delta = kd,$ the effect of mutual coupling on the field pattern is more pronounced than in broad-side arrays. The polar field patterns drawn in linear scale, of uniform, Dolph-Tchebyscheff (for 20dB side lobe level) and binomial arrays in end-fire configuration, without mutual coupling between them are given below for a three element array. The optimum distance *d* between the elements to maximize the directivity is 0.37 λ, 0.35 λ, and 0.34 λ, respectively, for the uniform, Dolph-Tchebyscheff and binomial arrays while the directivities for these field patterns are shown in Fig. 7.



d = 0.37 λ, D = 3.0822
Uniform array



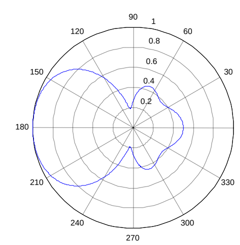
d = 0.35 λ, D = 2.8941
Dolph-Tchebyscheff array



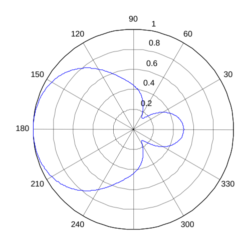
d = 0.34 λ, D = 2.8143
Binomial array

Fig. 7. End fire field patterns of 3 element half wave dipole uniform, Dolph-Tchebyscheff and binomial arrays at optimum separation and with maximum directivity without coupling.

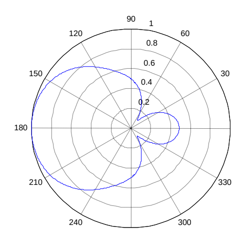
The above field patterns after mutual coupling for same distance *d* between the elements are given in Fig. 8. Observe that there is significant distortion in the shape of the field patterns due to mutual coupling in end-fire arrays, compared to broad side arrays as evidenced from the increase in the back lobe level. As a result the maximum directivity of the array also decreases as shown. When the separation between the elements is changed to improve directivity for each of the three amplitude configurations, the field patterns in Fig. 9 result for *d* = 0.35 λ, 0.29 λ, and 0.28 λ, respectively.



d = 0.37 λ, D = 2.6501
Uniform array

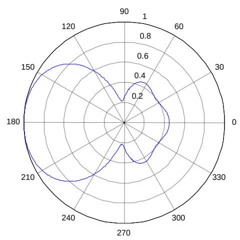


d = 0.35 λ, D = 2.5589
Dolph-Tchebyscheff array

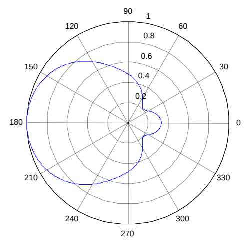


d = 0.34 λ, D = 2.5115
Binomial array

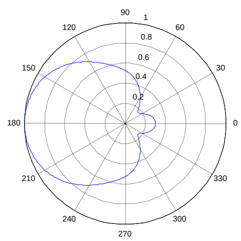
Fig. 8. Field patterns with coupling.



d = 0.35 λ, D = 2.6577
Uniform array



d = 0.28 λ, D = 2.6358
Dolph-Tchebyscheff array

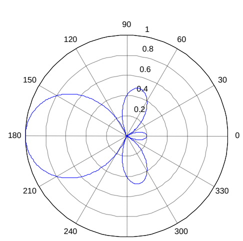


d = 0.29 λ, D = 2.6058
Binomial array

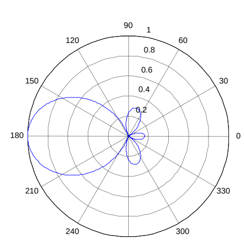
Fig. 9. Optimum patterns with coupling.

Directivity can be increased further by changing the phase excitation between the elements from $\delta = kd$ to $\delta = pkd$ where p is a constant, which is called as Hansen-Woodyard end-fire array. The optimum field patterns obtained this way for the three amplitude excitations neglecting mutual coupling between the dipoles are given in Fig. 10.

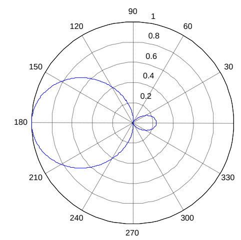
However, if we consider mutual coupling between the dipoles, directivity decreases for the three arrays and becomes 3.0039, 1.8317, and 1.419, respectively. We can notice that the directivity for the Dolph-Tchebyscheff and binomial amplitude patterns is very lower compared to before. The new optimum values for p to maximize directivity in the presence of mutual coupling are found to be 1.4, 1.8, and 1.9, for d = 0.27 λ, 0.21 λ, and 0.19 λ, respectively which produces more directivity as shown in Fig. 11. So the effect of mutual coupling is dominant in this type of array and should be considered before designing the array.



p =1.6, d = 0.27
D = 3.9399
Uniform array

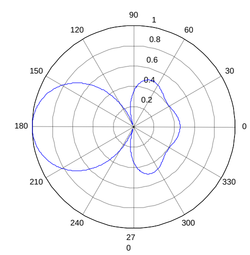


p = 3.9; d = 0.12 λ
D = 4.8319
Dolph-Tchebyscheff array

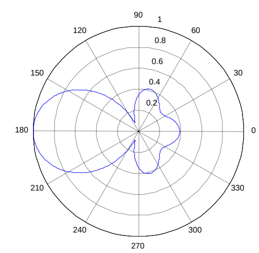


p = 5.2; d = 0.09 λ, D = 4.301
Binomial array

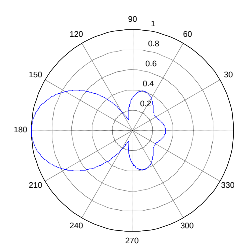
Fig. 10. Field patterns of a 3 element half wave dipole array with the three amplitude patterns and with optimum phase and separation, without mutual coupling.



p =1.4; d = 0.27 λ
D = 3.2262
Uniform array



p = 1.8; d = 0.21 λ
D = 3.8761
Dolph-Tchebyscheff array



p = 1.9; d = 0.19 λ, D = 3.9694
Binomial array

Fig. 11. Optimized field patterns with change in phase, with mutual coupling.

Variation of directivity with separation between the elements for a two element end-fire array is given in Fig.12. In the end fire operation where elements generally will be placed below half wave length distance (from 0.2λ to 0.5λ approximately), directivity severely decreases due to mutual coupling although there is some increase after half wave length separation. We can observe that mutual coupling has very little effect on the directivity at multiples of half wave length separation. Directivity variation with separation between the elements d and total number of elements N for end fire arrays is shown in Fig.13 which shows the degradation in the directivity due to mutual coupling.

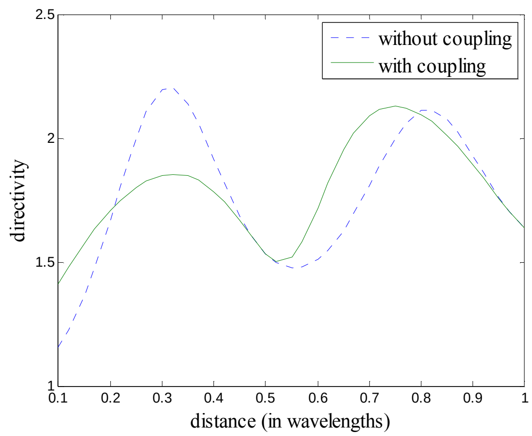


Fig. 12. Variation of directivity with separation, for a 2 element end-fire array with and without mutual coupling.

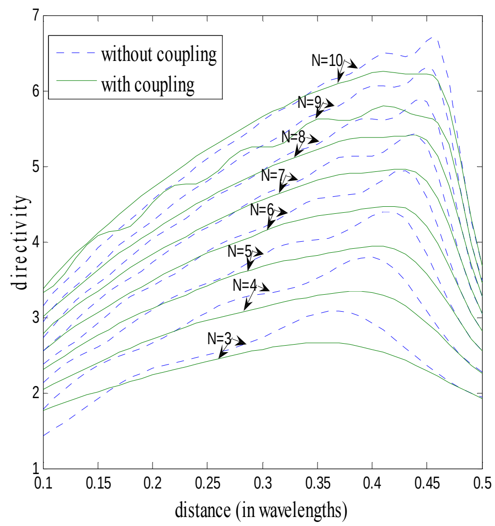


Fig. 13. Variation of directivity with distance in a uniform end-fire array with and without coupling.

Maximum directivity and corresponding optimum separation of a uniform end-fire array is shown in Table 4.

Table 4. Uniform end-fire array.

	d	*D*	*d**	*D**
N=2	0.32 λ	2.2044	0.32 λ	1.8549
N=3	0.37 λ	3.0822	0.35 λ	2.6577
N=4	0.40 λ	3.7922	0.38 λ	3.3426
N=5	0.41 λ	4.396	0.39 λ	3.9395
N=6	0.43 λ	4.9466	0.42 λ	4.4734
N=7	0.44 λ	5.4348	0.43 λ	4.9596
N=8	0.44 λ	5.8683	0.44 λ	5.405
N=9	0.45 λ	6.3121	0.41 λ	5.8047
N=10	0.45 λ	6.7074	0.40 λ	6.2601
N=50	0.48 λ	16.0109	0.47 λ	15.9856
N=70	0.48 λ	19.0824	0.47 λ	19.1346
N=100	0.49 λ	22.9193	0.48 λ	23.1129

with and without mutual coupling, where *N* represents total number of elements in the array. Directivity of the array decreases due to mutual coupling between the elements in uniform end-fire configuration. However, there is little improvement in the directivity if there is large number of elements in the array.

For the Dolph-Tchebyscheff array the maximum directivity is given in Table 5 with corresponding optimum spacing. Maximum directivity decreases for this array due to coupling while the optimum distance with mutual coupling may be below or above the optimum distance obtained without mutual coupling.

Table 5. Dolph-Tchebyscheff end-fire array.

	d	*D*	*d**	*D**
N=3	0.35 λ	2.8941	0.29 λ	2.6358
N=4	0.37 λ	3.505	0.33 λ	3.3119
N=5	0.39 λ	4.0713	0.36 λ	3.8871
N=6	0.41 λ	4.592	0.42 λ	4.4097
N=7	0.42 λ	5.0761	0.43 λ	4.8945
N=8	0.43 λ	5.5216	0.44 λ	5.3425
N=9	0.44 λ	5.9338	0.41 λ	5.8013
N=10	0.44 λ	6.3312	0.43 λ	6.1517

For the binomial end-fire array the optimum distance between the elements and the corresponding maximum directivity is given in Table 6, which shows that maximum directivity decreases when mutual coupling is considered.

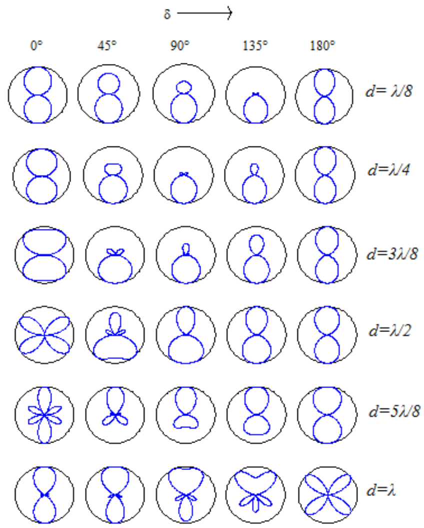
Table 6. Binomial end-fire array.

	d	*D*	*d**	*D**
N=3	0.34 λ	2.8143	0.28 λ	2.6058
N=4	0.36 λ	3.2384	0.31 λ	3.0803
N=5	0.37 λ	3.5671	0.34 λ	3.3006
N=6	0.38 λ	3.8408	0.35 λ	3.4236
N=7	0.39 λ	4.0769	0.37 λ	3.5135
N=8	0.4 λ	4.2815	0.38 λ	3.5883
N=9	0.4 λ	4.4682	0.4 λ	3.6629
N=10	0.41 λ	4.6333	0.39 λ	3.7167

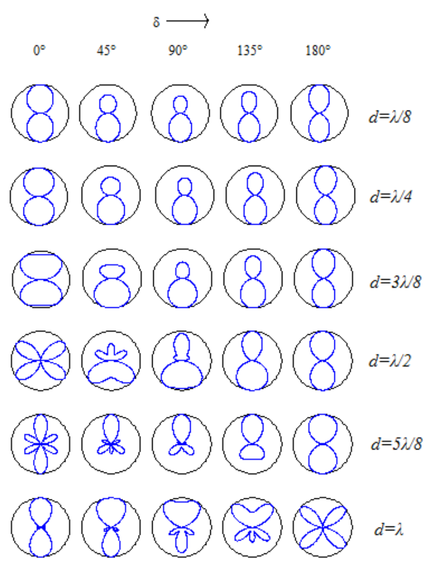
V. EFFECT OF MUTUAL COUPLING ON VERTICAL FIELD PATTERN

For the half wave dipole array along x axis centered at the origin, mutual coupling affects the directivity of the vertical plane field pattern of constant azimuthal angle. Also note that $\Psi = \frac{1}{2}\, kd \sin\theta \cos\varphi$ in the array factor now changes to $\Psi = \frac{1}{2}\, kd \sin\theta$ for $\varphi = 0$ in vertical plane.

Applying the principle of multiplication of field patterns, polar plots of vertical plane field pattern of a 2 element half wave dipole array as a function of phase difference δ and spacing d between the elements fed with the same magnitude of current are shown in Fig. 14, with and without mutual coupling. The patterns show considerable change in some cases due to mutual coupling especially if there is relative phase difference δ between the elements.



Without mutual coupling



With mutual coupling

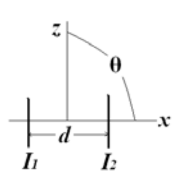
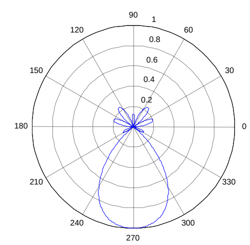
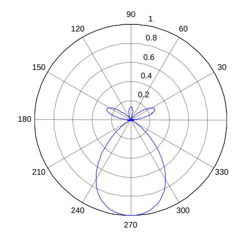


Fig. 14. Field pattern of a 2 element half wave dipole array in vertical direction with variation of spacing and phase difference between the dipoles, before and after considering mutual coupling.

Polar field patterns of a 3 element end-fire half wave dipole array with optimum distance *d* between the elements as 0.85 *λ* and 0.81 *λ* and where maximum directivity can be achieved for uniform and binomial amplitude coefficients with phase difference between the elements as δ = *kd* without mutual coupling are shown in Fig. 15 with their corresponding directivities.



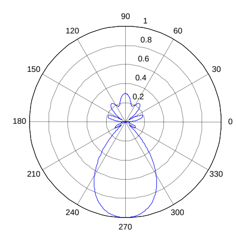
d = 0.85 λ, D = 4.0704
Uniform array



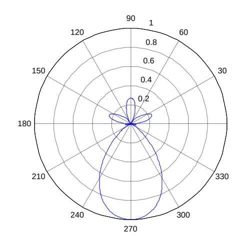
d = 0.81 λ, D = 3.8959
Binomial array

Fig. 15. Polar field pattern of 3 element uniform and binomial amplitude arrays in end-fire direction at optimum spacing.

When mutual coupling is considered between the elements with same amplitudes and separation between the elements, the patterns change to those shown in Fig. 16. As in the case of horizontal plane field pattern, the side lobe level is increased by mutual coupling, however the directivity is increased for the case of binomial array.



d = 0.85 λ, D = 4.056
Uniform array



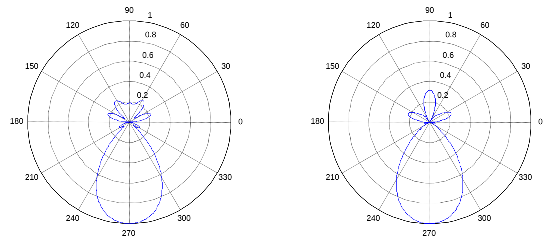
d = 0.81 λ, D = 4.1394
Binomial array

Fig. 16. Above field patterns after considering mutual coupling.

The optimum distance between the elements changes with mutual coupling to improve the directivity. The corresponding field patterns are shown in Fig. 17 along with their directivities.

The new value of *d* that gives maximum directivity is found to be 0.82 *λ* for both uniform and binomial amplitude arrays.

Optimum value of *d* between the elements and corresponding directivity for both uniform and binomial amplitude coefficients in end-fire configuration are given in Tables 7 and 8, before and after mutual coupling consideration for different *N*.



d = 0.82 λ, D = 4.1332
Uniform array

d = 0.82 λ, D = 4.2939
Binomial array

Fig. 17. Optimum field patterns with mutual coupling for maximum directivity.

Table 7. Uniform end-fire array in vertical direction.

	d	*D*	*d**	*D**
N=2	0.79 λ	3.414	0.74 λ	3.4838
N=3	0.85 λ	4.0704	0.82 λ	4.1332
N=10	0.91 λ	6.7963	0.9 λ	7.0594
N=50	0.97 λ	14.8917	0.94 λ	15.3628

Unlike the case of azimuthal patterns, the maximum directivity is slightly improved due to mutual coupling in the vertical plane even for the lower values of *N*.

The directivity in the binomial amplitude end-fire also improves due to mutual coupling after slightly adjusting the spacing between elements as shown in Table 8.

Table 8. Binomial end fire array in vertical direction.

	d	*D*	*d**	*D**
N=3	0.81 λ	3.8959	0.82 λ	4.2939
N=10	0.91 λ	6.7963	0.90 λ	7.0594

VI. EFFECT OF MUTUAL COUPLING ON 3 -D FIELD PATTERN

The maximum directivity and corresponding optimum spacing between the elements for the uniform array placed along the *x* axis in broad-side and end-fire configuration are given in Tables 9 and 10, calculated by considering both polar and azimuthal angles into account. We can clearly see that mutual coupling is degrading the overall directivity of the array in both configurations especially more in the end-fire array. Also the optimum spacing between the elements does not remain the same after considering mutual coupling.

Table 9. Uniform broad-side array.

	d	*D*	*d**	*D**
N=2	0.67 λ	5.0217	0.67 λ	5.0217
N=3	0.76 λ	8.6101	0.74 λ	8.5405
N=10	0.92 λ	34.4619	0.84 λ	33.9866
N=50	0.6 λ	82.6351	0.62 λ	83.8385

Table 10. Uniform end-fire array.

	d	*D*	*d**	*D**
N=2	0.33 λ	3.8437	0.36	3.2705
N=3	0.36 λ	4.2401	0.30	2.8679
N=10	0.46 λ	19.1332	0.43	17.0622
N=50	0.48 λ	94.6537	0.48	88.9445

VII. EFFECT OF CHANGING LENGTH

We observed that maximum directivity is degraded in half wave dipole array due to mutual coupling in both broad-side and end-fire configurations. The change in the maximum directivity of the overall array by adjusting the length [2] of the each dipole in the array can be observed here, by noting the change in near and far field regions, resulting from the change in the dipole length. Variation in the directivity with length of a 2 element uniform broad-side array placed along the *x* axis, is given in Table 11. Here *d* represents the optimum separation where maximum directivity (calculated for the three dimensional field pattern of the array) occurs.

Table 11. Variation of maximum directivity with dipole length, for a 2 element broad-side array.

l	*d*	*D*
0.5 λ	0.67 λ	5.0217
0.6 λ	0.66 λ	5.2952
0.7 λ	0.66 λ	5.6695
0.8 λ	0.65 λ	6.1819
0.9 λ	0.65 λ	6.8924
1 λ	0.64 λ	7.8691
1.1 λ	0.63 λ	9.1178
1.2 λ	0.63 λ	10.2192
1.3 λ	0.63 λ	9.5801

We can see that maximum directivity increases as the length of the dipole increases up to some extent and the optimum length is around 1.2 λ. If the length increases further, the directivity decreases and more side lobes appear. Also we can see that the optimum separation between the dipoles which gives maximum directivity decreases as the length of both dipoles increases.

The directivity variation in the case of 2 element end-fire dipole array is given in Table.12 which shows that directivity decreases after considering mutual coupling for dipole lengths less than 1 λ. However, if we increase the length of the dipole above 1 λ, more directivity can be obtained by adjusting the separation between the dipoles. From the table we can see that maximum directivity occurs for dipole length around 1.3 λ.

Maximum directivities that can be obtained by changing the dipole length of all elements are given in Tables 13 and 14 for both broad side and end fire configuration. We can see that there is improvement in the total directivity of the array by changing the length and this optimum length is found to be around 1.23 λ for a broad-side array, and in end fire arrays this optimum length is even more. There is much increase in the directivity of the end-fire array by mutual coupling than compared to broad-side array by changing the length of the dipole from 0.5 λ which we can observe. Also the optimum dipole length along with separation between the dipoles is not remained same after considering mutual coupling.

Table 12. Variation of directivity with dipole length for a 2 element end-fire array.

l	*d*	*D*	*d**	*D**
0.5 λ	0.33 λ	3.8437	0.36 λ	3.2705
0.6 λ	0.33 λ	4.003	0.37 λ	3.6289
0.7 λ	0.33 λ	4.2184	0.36 λ	3.8663
0.8 λ	0.33 λ	4.5106	0.36 λ	4.1023
0.9 λ	0.33 λ	4.9107	0.37 λ	4.3463
1 λ	0.32 λ	5.4686	0.35 λ	4.5489
1.1 λ	0.32 λ	6.2226	0.18 λ	8.0466
1.2 λ	0.32 λ	7.1033	0.26 λ	8.1592
1.3 λ	0.33 λ	7.5118	0.28 λ	8.3672
1.4 λ	0.35 λ	6.0750	0.29 λ	7.1492

Table 13. Optimization of directivity by changing length in uniform broad-side array.

	l	*d*	*D*	*l**	*d**	*D**
N=2	1.23	0.63	10.3129	1.23	0.63	10.3129
N=3	1.24	0.74	18.0823	1.23	0.73	18.2829
N=10	1.24	0.91	69.848	1.22	0.9	75.2991
N=50	1.23	0.59	174.2973	1.23	0.57	174.2695

l = optimum length (in wave lengths) without mutual coupling.
*l** = optimum length (in wave lengths) with coupling.

Table 14. Optimization of directivity by changing length in uniform end-fire array.

	l	*d*	*D*	*l**	*d**	*D**
N=2	1.29	0.33	7.5295	1.27	0.28	8.3925
N=3	1.31	0.35	8.2584	1.31	0.32	8.5948
N=10	1.44	0.46	32.1662	1.44	0.41	55.098
N=50	1.54	0.48	120.5469	1.42	0.42	267.2029

VIII. CONCLUSION

The effect of mutual coupling is very less in uniform dipole arrays for multiples of half wave length separation between the elements. The maximum directivity of a λ /2 uniform dipole array decreases due to mutual coupling although there is little increase in broad-side arrays for large number of elements in the array. Therefore we need to minimize the mutual coupling effect if there are few number of elements in the array and if there are large number of elements we need to use the effect for maximum directivity. The degradation with mutual coupling is more severe in uniform end-fire arrays than broad-side arrays. However, there is an increase in the maximum directivity due to mutual coupling with little adjustment in separation between the elements for Dolph-Tchebyscheff and binomial broad-side arrays even with little number of elements in the array. Directivity of the uniform dipole array with mutual coupling can be increased with even less number of elements in the array by adjusting the dipole length to be above 1 λ for broad-side arrays and above 1.25 λ end-fire arrays, being aware of the fact that there will be change in the far and near field regions of the array with the change in the dipole length.

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I want to dedicate this to my parents for their love and support. Also the authors express sincere thanks to the faculty of Electrical department, University of South Alabama for their valuable assistance.

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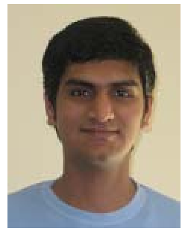

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