Design and Analysis of Multi-Frequency Unequal-Split Wilkinson Power Divider using Non-Uniform Transmission Lines

Derar Hawatmeh[1], Khair Al Shamaileh[2], and Nihad Dib[1]

[1] Department of Electrical Engineering, Jordan University of Science and Technology
P. O. Box 3030, Irbid 22110, Jordan
dfh_ee@hotmail.com, nihad@just.edu.jo

[2] Waseela for Integrated Telecommunication Solutions
P. O. Box 962487, Amman 11196, Jordan
khair.shamaileh@waseela-net.com

Abstract ─ In this paper, the design of miniaturized multi-frequency unequal-split Wilkinson power divider (WPD) using non-uniform transmission lines (NTLs) is presented. To achieve compactness, the uniform transmission lines of the conventional WPD are substituted by their equivalent NTLs. Moreover, two extra compact NTLs transformers are incorporated in each arm of the divider for output ports matching purposes. To prove the validity of the design procedure, two examples of single band and triple band NTL-based WPDs, with 2:1 split ratio, are presented. Both dividers are simulated using full-wave simulators. Furthermore, the proposed single band divider is fabricated and tested. Both simulation and measurements results are in good agreement. Besides the rejection of the odd harmonics over the band 1-5 GHz to a level lower than -15 dB, a total length reduction of 37%, and 16% in the single and triple band WPDs, respectively, is achieved.

Index Terms ─ Non-uniform transmission lines, power dividers, Wilkinson power divider.

I. INTRODUCTION

Microwave power dividers are essential components in modern microwave applications, such as antenna feed networks, phase shifters, and frequency mixers. Since its invention back in 1960 [1], the Wilkinson power divider (WPD) has been considered as one of the most important dividers in microwave circuits. Recently, WPDs have been notably addressed by researchers in many different aspects, such as reducing the size of their overall circuit area. The use of non-uniform transmission lines (NTLs) as one of the miniaturization techniques was presented in many papers [2-7]. In [2], an equal-split WPD was miniaturized using NTLs, and a size reduction of 52% was achieved. In [3], and as an extension to what was done in [2], a dual band WPD was proposed with 26% reduction in size (compared to the conventional dual-band one). In [4] and [5], NTL-based miniaturized Bagley power divider and branch line coupler, respectively, were presented. A general design procedure for NTL-based compact multi-band equal-split WPD was proposed in [6]. In [7], a reduced-size NTLs WPD, with modified topology, with high power split ratio was proposed in which the splitting ratio depends on the electrical lengths of its arms rather than the impedance values. Moreover, many miniaturization techniques were introduced in the literature to accomplish compactness, such as the use of stubs. In [8, 9], dual band compact WPDs were proposed in which stubs were incorporated to gain a significant size reduction of the circuit area. In [10], the WPD has been miniaturized using stubs, where artificial TLs have been used to accomplish the design. In [11], a stepped impedance interdigital coupling element has been incorporated to achieve the compactness for a single band WPD and to suppress the odd harmonics, as well.

Submitted On: Dec. 25, 2011
Accepted On: Feb. 23, 2012

In this paper, based on NTLs theory, a compact unequal-split WPD, with 2:1 split ratio, is presented. This is in contrast to [2, 3, 6], where equal-split NTLs WPDs were considered. To achieve compactness, the conventional uniform arms of the proposed dividers are replaced by their equivalent NTLs at specific design frequencies. The proposed divider is then simulated using two full-wave simulators to prove the validity of the design procedure. Furthermore, the designed single band unequal-split WPD is fabricated and measured, and both simulation and measurement results are in good agreement.

II. Design of compact NTLs

Figure 1 shows a schematic of the conventional unequal-split WPD, which can be designed using the following set of equations [12]:

$$Z_{02} = k^2 \times Z_{03} = Z_0 \times \sqrt{k \times (1 + k^2)}, \quad (1.\text{a})$$

$$Z_{03} = Z_0 \times \sqrt{\frac{1 + k^2}{k^3}}, \quad (1.\text{b})$$

$$R = Z_0 \times \left(k + \frac{1}{k}\right), \quad (1.\text{c})$$

where Z_{02} and Z_{03} are the characteristic impedances of the upper and lower arms, respectively; k^2 is the power splitting ratio between ports 3 and 2, which equals to $\frac{P_3}{P_2}$, and R is the isolation resistor between the two output ports.

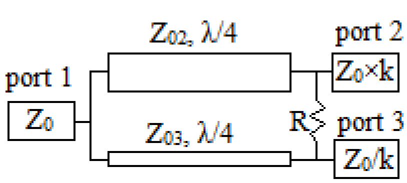


Fig. 1. A schematic of the conventional unequal-split WPD.

The key step in designing compact NTL-based WPD is to replace each arm of the conventional WPD with its equivalent NTL at a specific design frequency [2-7], keeping in mind that the length of the equivalent NTL should be less than the uniform transmission line's (UTL) length. Figure 2 shows a schematic of the proposed unequal-split NTL-based WPD.

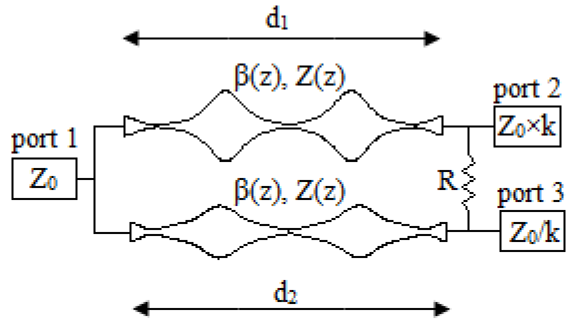


Fig. 2. A schematic of the unequal-split NTL-based WPD.

As shown in Figure 2, each equivalent NTL has varying characteristic impedance Z(z), and propagation constant β(z), compared to the conventional UTL, that has a constant characteristic impedance Z_0, and propagation constant β_0. The *ABCD* matrix of the UTL is given as follows [12]:

$$\begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \begin{bmatrix} \cos(\theta_0) & jZ_0 \sin(\theta_0) \\ \frac{j}{Z_0} \sin(\theta_0) & \cos(\theta_0) \end{bmatrix}, \quad (2)$$

where θ_0 is the electrical length of the UTL at the design frequency. In order to design the NTL section, it is firstly subdivided into a large number of uniform electrically short sections, and the overall *ABCD* matrix of the whole NTL can be obtained by multiplying the *ABCD* matrices of these uniform sections [2-7]. Then, the following truncated Fourier series expansion for the normalized characteristic impedance $\bar{Z}(z) = Z(z)/Z_0$ is considered:

$$\ln\left(\bar{Z}(z)\right) = \sum_{n=0}^{N} C_n \cos\left(\frac{2\pi nz}{d}\right). \quad (3)$$

An optimum designed compact NTL has to have its *ABCD* parameters as close as possible to the *ABCD* parameters of the UTL at a specific design frequency. For the single band NTL-based WPD, the optimum values of the Fourier coefficients C_n's can be obtained through minimizing the following error function [2, 5, 7, 13]:

$$Error = \sqrt{\frac{1}{4}\left(\left|A-A_0\right|^2+Z_0^{-2}\left|B-B_0\right|^2+Z_0^2\left|C-C_0\right|^2+\left|D-D_0\right|^2\right)}. \quad (4)$$

In the case of multi-band NTL-based WPD, the even-odd mode analysis carried out in [6] will be used. Figures 3 and 4 show the even and odd modes circuits of the NTL-based WPD, respectively. Using the even-mode equivalent circuits (Fig. 3), the NTLs are designed in such a way that the input reflection coefficient $|\Gamma_{in}|$ equals zero (or very small) at the design frequencies.

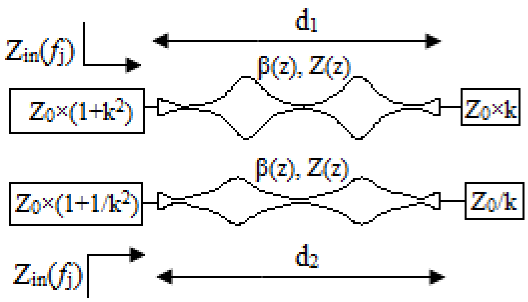


Fig. 3. Even-mode analysis of the unequal-split NTL-based WPD.

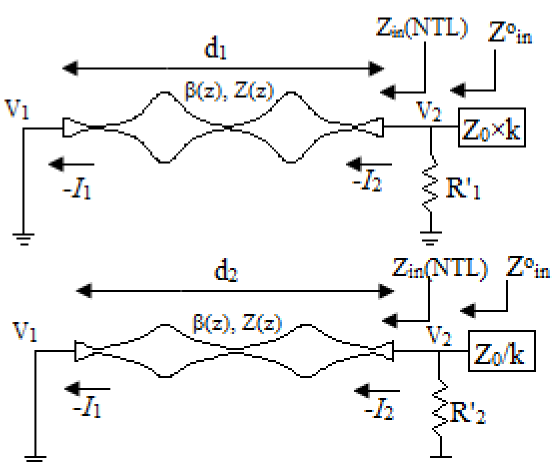


Fig. 4. Odd-mode analysis of the unequal-split NTL-based WPD.

Thus, the following error function, written in terms of the summation of the input reflection coefficients at M design frequencies ($f_j, j = 1, \ldots, M$), is used [6]:

$$Error_{input} = \sqrt{\sum_{j=1}^{M}\left|\Gamma_{in}(f_j)\right|^2}, \quad (5)$$

where,

$$\Gamma_{in}(f_j) = \frac{Z_{in}(f_j) - Z_s}{Z_{in}(f_j) + Z_s}, \quad (6)$$

where Z_S is the source impedance expressed by $Z_0(1+K^2)$ and $Z_0(1+1/K^2)$ for the upper and lower arms shown in Figure 3, respectively. It is worth mentioning here that both error functions (4) and (5) should be restricted by some constraints, such as reasonable fabrication and physical matching, as follows:

$$\bar{Z}_{min} \le \bar{Z}(z) \le \bar{Z}_{max}, \quad (7.a)$$

$$\bar{Z}(0) = \bar{Z}(d) = 1. \quad (7.b)$$

These optimization problems are solved using the Matlab "fmincon" routine.

The isolation resistor is found using the odd-mode equivalent circuits shown in Figure 4, where the *ABCD* matrix of each branch can be written as follows:

$$\begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}. \quad (8)$$

Setting V_1=0 leads to:

$$AV_2 - BI_2 = 0. \quad (9)$$

Solving for $\frac{V_2}{I_2}$,

$$\frac{V_2}{I_2} = \frac{B}{A} = Z_{in}^{NTL}. \quad (10)$$

Finally, the input impedances for both upper and lower branches are given as follows, respectively:

$$Z_{in}^{0} = \frac{R_1'.Z_{in}^{NTL}}{R_1' + Z_{in}^{NTL}}, \quad (11)$$

$$Z_{in}^{0} = \frac{R_2'.Z_{in}^{NTL}}{R_2' + Z_{in}^{NTL}}. \quad (12)$$

Perfect output port matching for the upper and lower branches, respectively, can be achieved by satisfying the following conditions:

$$\Gamma_{out}(f_j) = \frac{Z^o{}_{in}(f_j) - Z_0K}{Z^o{}_{in}(f_j) + Z_0K}, \quad (13.a)$$

$$\Gamma_{out}(f_j) = \frac{Z^o{}_{in}(f_j) - Z_0/K}{Z^o{}_{in}(f_j) + Z_0/K}. \qquad (13.b)$$

So, perfect output port matching at the design frequencies is achieved by keeping the output reflection coefficients as close as possible to zero by minimizing the following error function:

$$Error_{out} = \sqrt{\sum_{j=1}^{M} \left|\Gamma_{out}(f_j)\right|^2}, \qquad (14)$$

where R_1' and R_2' are the optimization variables that are determined using an optimization code. Clearly, the optimization must be run twice to find the value of the isolation resistor:

$$R = R_1' + R_2'. \qquad (15)$$

It is worth mentioning here that, compared to the conventional multi-band WPD [14], where a total number of M isolation resistors were used, the proposed multi-band NTL-based WPD has only one isolation resistor.

III. 2:1 NTL-based WPD

A. Single-band design

The conventional unequal-split WPD parameters can be calclulated using equations 1.a-1.c. Since the designed divider is of unequal-split type, output port 2 has an impedance of $R_2 = Z_0 \times k$, while output port 3 has an impedance of $R_3 = \frac{Z_0}{k}$ [12]. To obtain a 2:1 split ratio ($k^2 = 0.5$), the unequal-split WPD parameters are found to be: Z_{02}=51.5 Ω, Z_{03}=103 Ω, R=106.07 Ω, R_2=35.36 Ω and R_3=70.71 Ω (considering a reference impedance Z_0=50 Ω). Quarter-wavelength matching transformers are needed to match the output ports to 50 Ω. The characteristic impedances of these matching transformers are calculated as follows: for port 2: $\sqrt{35.36 \times 50} = 42.045\,\Omega$; and for port 3: $\sqrt{70.71 \times 50} = 59.46\,\Omega$. Now, each uniform microstrip line section is replaced by its equivalent compact NTL. Two output ports NTL matching transformers have been also designed. Considering an FR-4 susbtrate (with a thickness of 1.6 mm and dielectric constant of 4.6) and a design frequency of 1 GHz, the lengths (d) of the upper and lower WPD NTL arms are chosen to be 22.84 mm and 26.1 mm, respectively; while the lengths of the output ports matching transformers are chosen as 24.3 mm and 25.7 mm. It should be mentioned that the width of the upper (port 2) and lower (port 3) output ports at their ends are 3.92 mm and 2.18 mm, respectively; while the input port (50 Ω) width is 2.95 mm. Figure 5 shows the layout of the compact NTL-based 2:1 WPD compared to the conventional one. The proposed WPD is simulated using IE3D [15] and HFSS [16] full-wave simulators. The simulation results are shown in Figure 6, whereas Table 1 represents the values of the S-parameters at the design frequency.

Table 1: Values of the S-parameters of the single band NTL-based 2:1 WPD

		IE3D	HFSS	theoretical
At 1.0 GHz	S11 (dB)	-27	-25.55	$-\infty$
	S21 (dB)	-2.52	-2.4	-1.76
	S31 (dB)	-4.76	-4.77	-4.77
	S22 (dB)	-27.92	-25.1	$-\infty$
	S23 (dB)	-33.49	-23	$-\infty$
	S33 (dB)	-21.54	-24.94	$-\infty$

The slight differences between the theoretical and simulation results are thought to be due to dielectric losses, coupling, and discontinuities effects.

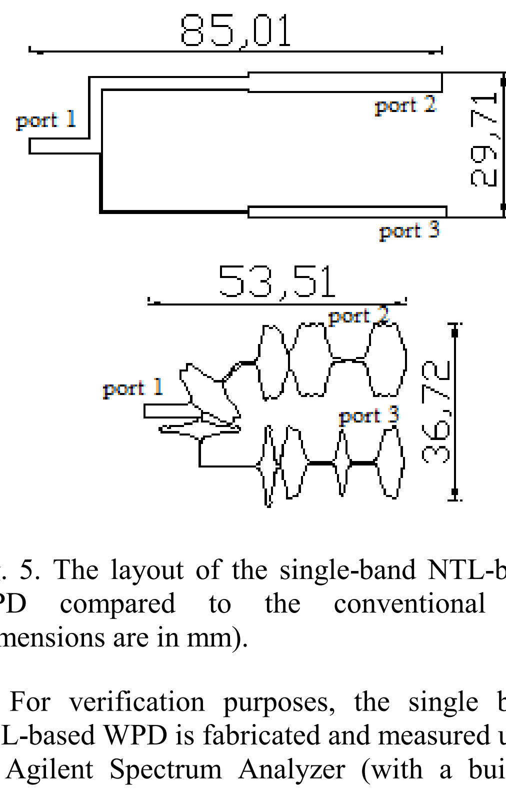
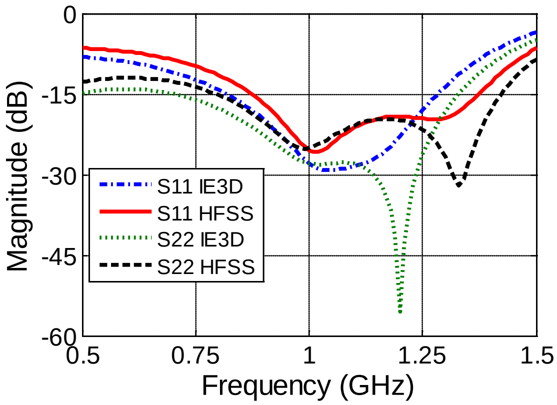


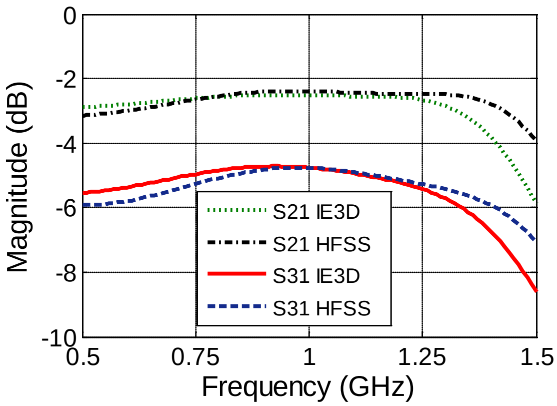
Fig. 5. The layout of the single-band NTL-based WPD compared to the conventional one (dimensions are in mm).

For verification purposes, the single band, NTL-based WPD is fabricated and measured using an Agilent Spectrum Analyzer (with a built in tracking generator extending from 0-1.5 GHz). Figure 7 shows the measured results, while Figure 8 shows a picture of the fabricated WPD.

Experimental results show an acceptable agreement between both simulated and measured results. The small discrepancies in the measured results could be due to conductor and dielectric losses, the use of the connectors and the errors in the measurements, keeping in mind that a spectrum analyzer (not a network analyzer) was used.



(a)



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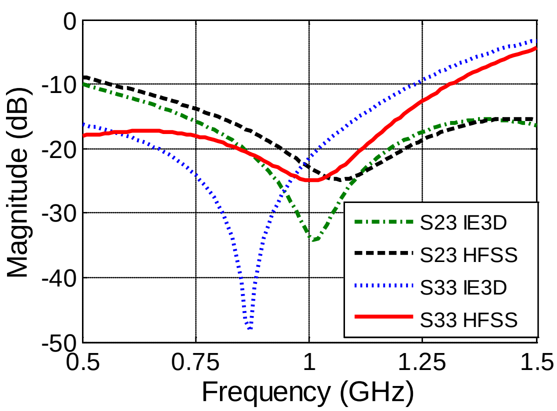

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Fig. 6. S-parameters of the designed NTL-based WPD using IE3D and HFSS.

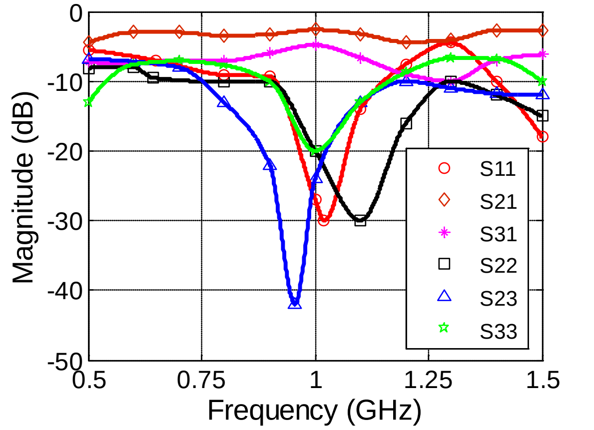


Fig. 7. Measured S-parameters of the fabricated NTL-based WPD.

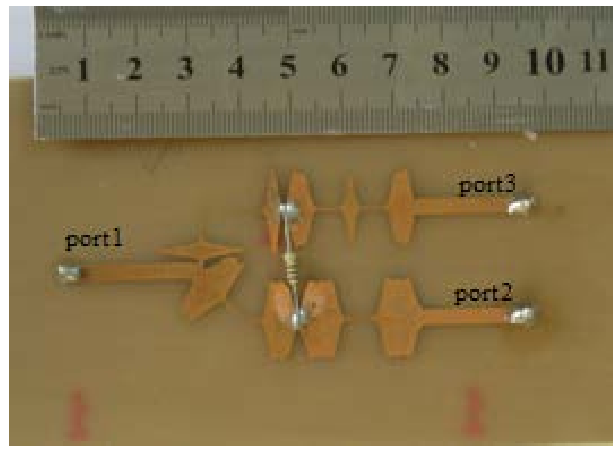


Fig. 8. Fabricated NTL-based 2:1 WPD.

B. Multi-band design

A triple frequency NTL-based 2:1 WPD is designed to operate at 0.5 GHz, 1.25 GHz, and 2 GHz, considering the same FR-4 substrate. The length of each WPD arm and output ports matching transformer is chosen to be $\frac{\lambda}{4}$ at the lowest design frequency. Using the design procedure described in the previous section, the layout of the designed triple band NTL-based WPD (compared to the conventional one) is shown in Figure 9. Three resistors (not shown in the figure) are needed in the conventional triple band WPD [14], while only a single one is needed in the NTL-based WPD.

The simulation results of the designed triple band NTL-based WPD (with an optimized isolation resistor of 105 Ω) are shown in Figure 10. A slight shift in the design frequencies is thought to be due to the coupling and discontinuity effects. The values of the S-parameters at the design frequencies are shown in Table 2.

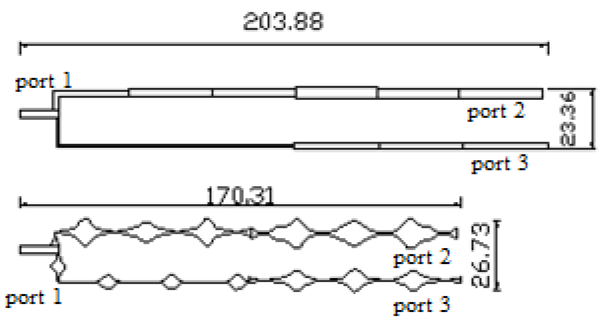
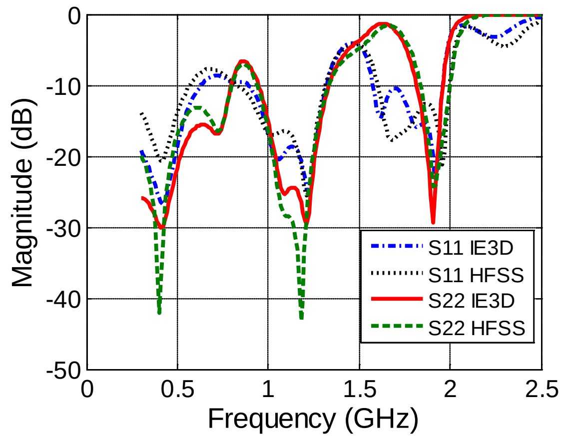
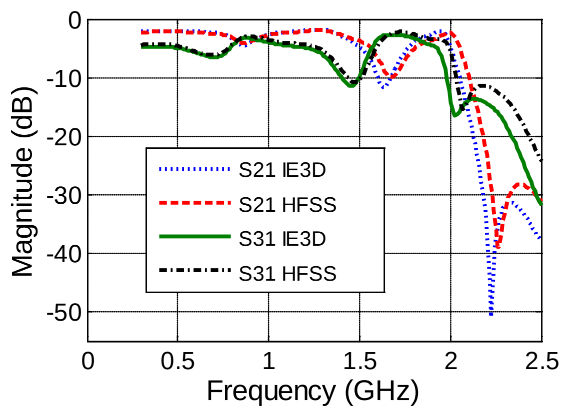


Fig. 9. The layout of the designed triple band NTL-based WPD compared to the conventional design (dimensions in mm).



(a)



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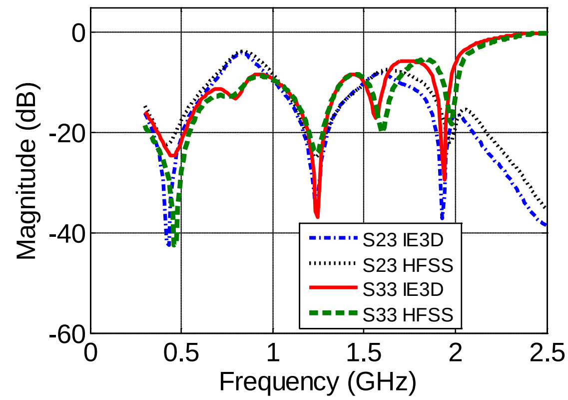

(c)

Fig. 10. S-parameters of the designed triple band NTL-based WPD.

Table 2: Values of the S-parameters of the triple band NTL-based 2:1 WPD

		IE3D	HFSS	theoretical
At 0.42 GHz	S11 (dB)	-26.7	-20.58	−∞
	S21 (dB)	-2.03	-2.12	-1.76
	S31 (dB)	-4.7	-4.3	-4.77
	S22 (dB)	-29.9	-31.8	−∞
	S23 (dB)	-41.9	-22.4	−∞
	S33 (dB)	-23.37	-27.72	−∞
At 1.22 GHz	S11 (dB)	-25.45	-25.4	−∞
	S21 (dB)	-1.896	-2	-1.76
	S31 (dB)	-5.07	-4.42	-4.77
	S22 (dB)	-28.01	-25.6	−∞
	S23 (dB)	-31.8	-24.24	−∞
	S33 (dB)	-28.3	-22.6	−∞
At 1.92 GHz	S11 (dB)	-23	-21.14	−∞
	S21 (dB)	-2.	-2.7	-1.76
	S31 (dB)	-4.8	-3.7	-4.77
	S22 (dB)	-26.5	-17.7	−∞
	S23 (dB)	-27	-20	−∞
	S33 (dB)	-17.4	-15	−∞

IV. COMPARISON BETWEEN CONVENTIONAL AND NTL-BASED WPDS

Using NTLs instead of UTLs, two main advantages are obtained: (1) the size reduction, and (2) the odd harmonics suppression. A total length reduction of almost 31 mm is achieved for the single band NTL-based WPD and about 33 mm for the triple band one. Since both structures have the same *ABCD* parameters at the design frequency only, the NTLs WPD behavior is completely different from the conventional one at other frequencies. Figure 11 shows the input port matching parameter (S_{11}) for both conventional and NTL-based single band WPD. It is clear that the third odd harmonic has been completely suppressed while the fifth odd harmonic is partially suppressed for the NTL-based WPD. Figure 12 shows S_{11} for both conventional triple band WPD and NTL-based triple band WPD. It is clear that the third and fifth odd harmonics are totally suppressed while the seventh odd harmonic is partially suppressed. Furthermore, a performance improvement is noticeable in the NTL-based WPD, since S_{11} is close to 0 dB at frequencies other than the design frequencies.

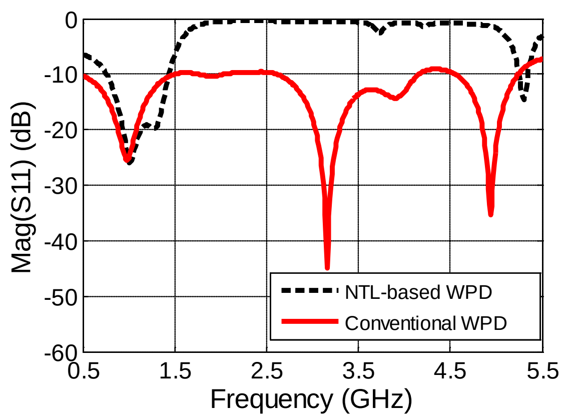

Fig. 11. S_{11} for both UTL-based and NTL-based single band WPDs.

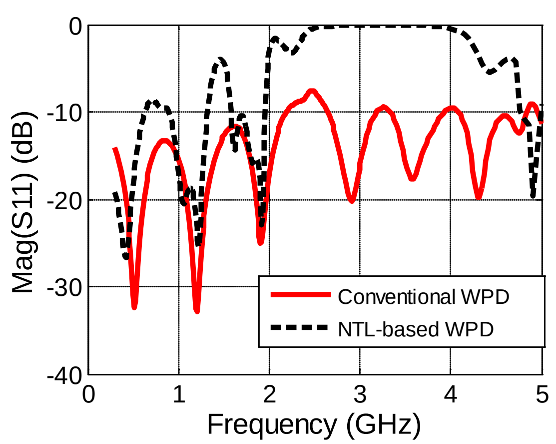

Fig. 12. S_{11} for both UTL-based and NTL-based triple band WPDs.

V. CONCLUSION

In this paper, the design and analysis of a compact 2:1 unequal-split NTL-based WPD was presented. In order to achieve compactness, each uniform transmission line was replaced by its equivalent NTL at the design frequency. Besides suppressing some of the odd harmonics of the design frequency, a length reduction about 30 mm was achieved compared to the conventional WPD. In addition to the length reduction and odd harmonics suppression, the number of isolation resistors in the multi-band design was reduced to one resistor regardless of the number of operation bands. This work will be extended to N-port, unequal-split multi-band WPD designs.

VI. ACKNOWLEDGMENT

This work was supported by the Deanship of Research at Jordan Univ. of Science and Technology (JUST).

REFERENCES

[1] R. Wilkinson, "An N-Way Hybrid Power Divider," *IRE Transactions on Microwave Theory and Techniques*, vol. MTT-8, no. 1, pp. 116-118, 1960.

[2] F. Hosseini, M. Khalaj-Amir Hosseini, and M. Yazdani, "A Miniaturized Wilkinson Power Divider Using Non-uniform Transmission Line," *Journal of Electromagnetic Waves and Applications*, vol. 23, pp. 917-924, 2009.

[3] K. Shamaileh and N. Dib, "Design of Compact Dual-Frequency Wilkinson Power Divider Using Non-uniform Transmission Line," *Progress In Electromagnetics Research C*, vol. 19, pp. 37-46, 2011.

[4] K. Shamaileh, A. Qaroot, and N. Dib, "Non-Uniform Transmission Line Transformers and Their Applications in the Design of Compact Multi-Band Bagley Power Dividers with Harmonics Suppression," *Progress In Electromagnetics Research*, vol. 113, pp. 269-284, 2011.

[5] F. Hosseini, M. Khalaj-Amir Hosseini, and M. Yazdany, "To Compact Ring Branch-Line Coupler Using Nonuniform Transmission Line," *Microwave and Optical Technology Letters*, vol. 51, no. 11, pp. 2679-2682, Nov. 2009.

[6] K. Shamaileh, A. Qaroot, N. Dib, and A. Sheta, "Design and Analysis of Multi-Frequency Wilkinson Power Dividers Using Non-uniform Transmission Lines," *International Journal of RF and Microwave Computer-Aided Engineering*, vol. 21, no. 5, pp. 526-533, September 2011.

[7] K. Shamaileh, A. Qaroot, N. Dib, and A. Sheta, "Design of Miniaturized Unequal Split Wilkinson Power Divider with Harmonics Suppression Using Non-Uniform Transmission Lines," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 26, no. 6, pp. 530-538, June 2011.

[8] L. Shao, H. Guo, X. Liu, W. Cai, and L. Mao "A Compact Dual-Frequency Wilkinson Power Divider with Open-ended Stubs," *Int. Symposium on Signals Systems and Electronics (ISSSE)*, vol. 1, pp. 1-4, 2010.

[9] Z. Wang, J. Jang, and C. Park, "Compact Dual-Band Wilkinson Power Divider Using Lumped Component Resonators and Open-Circuited Stubs," *Wireless and Microwave Technology Conference (WAMICON)*, pp. 1-4, June 2011.

[10] C.-H. Tseng and C.-H. Wu, "Compact Planar Wilkinson Power Divider Using Pi-Equivalent Shunt-Stub-Based Artificial Transmission Lines," *Electronics Letters*, vol. 46, pp. 1327-1328, 2010.

[11] P. Cheong, K. Lai, and K. Tam, "Compact Wilkinson Power Divider With Simultaneous Bandpass Response and Harmonic Suppression," *IEEE MTT-S International Microwave Symposium Digest*, pp. 1588-1591, 2010.

[12] D. Pozar, Microwave Engineering, New York: John Wiley, 3rd edition, 2005.

[13] M. Khalaj-Amirhosseini, "Nonuniform Transmission Lines as Compact Uniform Transmission Lines," *Progress In Electromagnetics Research C*, vol. 4, pp. 205-211, 2008.

[14] A. Qaroot, N. Dib, and A. Gheethan "Design Methodology of Multi-Frequency Unequal Split Wilkinson Power Dividers Using Transmission Line Transformers," *Progress in Electromagnetics Research B*, vol. 22, pp. 1-21, 2010.

[15] IE3D, V 14, 2007, www.zeland.com.

[16] HFSS: High Frequency Structure Simulation based on Finite Element Method, V. 10, 2005, Ansoft Corporation, www.ansoft.com.

Derar Fayez Hawatmeh received his B.Sc. in Communications and Electronics Engineering from Jordan University of Science and Technology (JUST), Irbid, Jordan in 2010. In 2010, he joined the Master program in the Electrical Engineering Department at JUST majoring in Wireless Communications. His research interests include the analysis and design of antennas, compact, planar, passive, and multifrequency and ultra-wideband microwave components for wireless applications.

Khair Ayman Al Shamaileh received his B.Sc. in Communications and Electronics Engineering from Jordan University of Science and Technology (JUST), Irbid, Jordan in 2009. In the same year, he joined the Master program in the Electrical Engineering Department at JUST majoring in Wireless Communications. He received his M. Sc. degree in 2011. His research interests include the analysis and design of compact, planar, passive, multi-frequency and ultra- wideband microwave components for wireless applications.

Nihad I. Dib obtained his B. Sc. and M.Sc. in Electrical Engineering from Kuwait University in 1985 and 1987, respectively. He obtained his Ph.D. in EE (major in Electromagnetics) in 1992 from University of Michigan, Ann Arbor. Then, he worked as an assistant research scientist in the radiation laboratory at the same school. In Sep. 1995, he joined the EE department at Jordan University of Science and Technology (JUST) as an assistant professor, and became a full professor in Aug. 2006. His research interests are in computational electromagnetics, antennas and modeling of planar microwave circuits.