SOLUTION OF TEAM BENCHMARK PROBLEM #9
Handling Velocity Effects with Variable Conductivity

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Abstract

Users often raise the question of whether it is possible to analyze eddy current problems with velocity effects within codes that are not programmed to account for movement. This paper looks at a technique for applying a conventional boundary element technique to the analysis of a velocity induced eddy current by altering the conductivity of the conducting medium as a function of position. Results of the predicted B fields for v=0 m/s and v=10 m/s are compared to the analytical solution of a coil traveling axially down the center of a conducting tube. Good agreement is achieved; further refinement could be realized by iterating on conductivity if necessary.

The Boundary Element Approach

The problem to be analyzed is shown in Figure 1. The coil is excited at 50 Hz and is traveling down the pipe at velocity V. We analyze the problem with V=0 m/s and 10 m/s. The boundary element approach (BEM) employed asks what fictitious free surface currents K_f could be placed on the skin of this pipe to account for the magnetization of the iron and the eddy currents. Actually 2 sets of surface currents are employed. A skin of currents just inside the pipe shell perimeter is used to represent the fields everywhere in the pipe. Another set of currents just outside the shell models the field in the air. The surface currents on the air side at r just less than 14 mm, dictate the field in the air region 0<r<14 mm. The surface currents just outside the skin at r=20 mm, dictate the field for r>20 m. Once the surface currents are known, the magnetic field is found simply from Biot-Savart's law.

For the eddy current problem without movement, the pertinent equations for H and E are

$$\nabla\times\vec{H}=\sigma\vec{E}+\vec{J}_s \qquad (1)$$

$$\vec{E}=-j\omega\vec{A}-\nabla\Phi \qquad (2)$$

Writing (1) in terms of the vector potential A yields
With the specified gauge of (3), the curl curl equation can be replaced by

$$\nabla\times\nabla\times\vec{A}-k^2\vec{A}=\mu\vec{J}_s+\mu\sigma\nabla\Phi$$
$$\text{where } k^2=j\omega\mu\sigma,$$
$$\text{and } \nabla\cdot\vec{A}=\mu\sigma\Phi. \quad (3)$$

$$\nabla^2\vec{A}+k^2\vec{A}=-\mu\vec{J}_s. \quad (4)$$

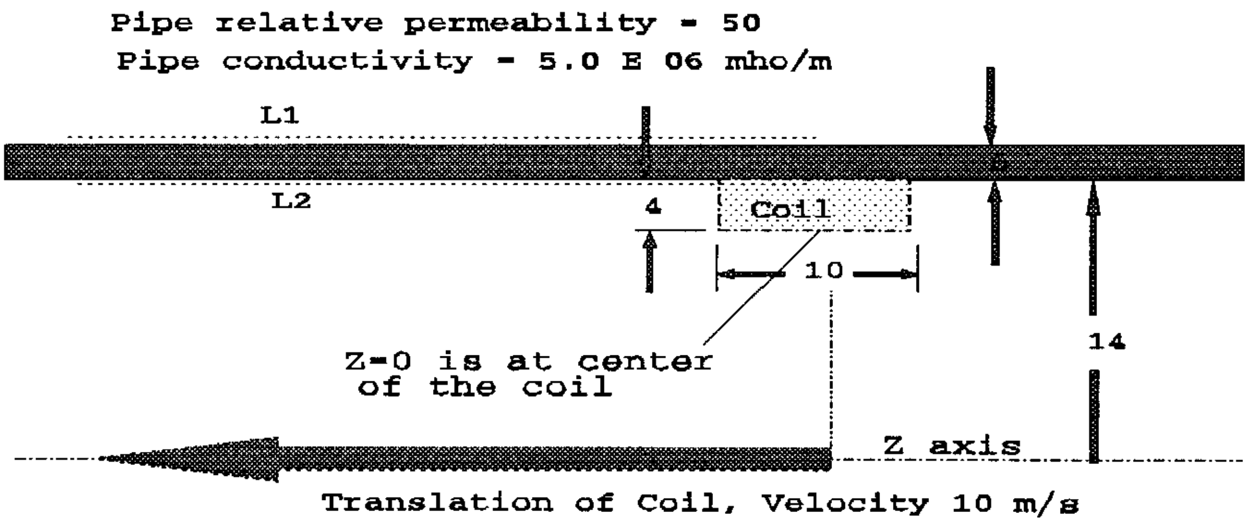

***Figure 1** Coil traveling axially down a conducting pipe with velocity V. All dimensions are in millimeters. The coil is excited at 50 Hz. L1 and L2 are displaced 3 mm outside and inside the pipe respectively.*

The integral solution for the vector potential due to a source current is [1,2]

$$A(r)=\mu\oint G(r,r')\,K_f(r')\,dS'.$$
where
$$G(r,r')=\frac{\mu}{2\pi}\int_0^{\pi}\frac{e^{jk|r-r'|}}{|r-r'|}dS' \quad (5)$$

Figure 2 helps to elucidate the approach. The fields in regions 1 and 2 are represented in terms of the surface currents and external impressed fields H_i and E_i as

$$\vec{H}^+=\vec{H}_i+\vec{H}(K_f^+) \quad (6)$$

$$\vec{H}^-=\vec{H}(K_f^-) \quad (7)$$

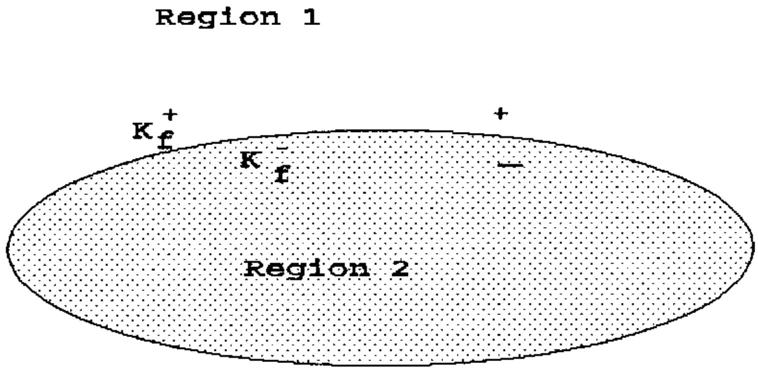

Figure 2 Two-region problem analyzed with BEM.

$$\vec{E}^{+}=\vec{E}_{i}+\vec{E}(K_{f}^{+}) \tag{8}$$

$$\vec{E}^{-}=\vec{E}(K_{f}^{-})=-j\omega A^{-} \tag{9}$$

It only remains to impose the boundary conditions on E and H which are

$$\hat{n}\times(\vec{E}_{2}^{+}-\vec{E}_{1}^{-})=-\hat{n}\times\vec{E}_{i} \tag{10}$$

$$\hat{n}\times(\vec{H}_{2}^{+}-\vec{H}_{1}^{-})=-\hat{n}\times\vec{H}_{i} \tag{11}$$

Here $\hat{n}$ is the outward normal to region 1. Note that the condition $\hat{n}\cdot\|\vec{B}\|=0$ is automatically insured by the use of the equivalent currents to directly compute B. Employing these boundary conditions yields the governing equations

$$j\omega\left[\mu_{2}\oint_{S'}G(k2,r,r')\,K_{f}^{+}(r')\,dS'-\mu_{1}\oint_{S'}G(k1,r,r')\,K_{f}^{-}(r')\,dS'\right]=-E_{t}^{i}=0 \tag{12}$$

Results

$$-\oint_{S'} K_f^{+}(r') \frac{\partial}{\partial n'} G(k2,r,r')\, dS' + \oint_{S'} K_f^{-}(r') \frac{\partial}{\partial n'} G(k1,r,r')\, dS' + 1/2\,(K_f^{+}(r) + K_f^{-}(r)) = -\hat{n}\times\vec{H}_i \qquad (13)$$

Equations (12) and (13) were applied to the problem both with the pipe having no relative permeability and with μ_r=50. Both in this case and those to follow, 179 linear boundary elements were used, resulting in 366 unknowns. The field was predicted along the lines L1 and L2 of Figure 1. The radial and axial fields for the nonmagnetic pipe with the coil traveling at zero velocity are shown in Tables I and II.

Table I Radial Magnetic Fields
nonmagnetic pipe, velocity = 0

z(mm)	Br on L1	Br on L1 analytic	Br on L2	Br on L2 analytic
0	1.05e-08		9.42e-08	
1.2	0.000184	0.000186	0.000824	0.000831
6	0.000688	0.000693	0.00345	0.00344
12	0.000627	0.000631	0.00119	0.0012
18	0.000396	0.000398	0.000532	0.000536
24	0.000237	0.000238	0.000264	0.000266
30	0.000143	0.000144	0.000142	0.000143
36	0.000089	0.00009	0.000081	0.000081
42	0.000057	0.000058	0.000049	0.000049
48	0.000038	0.000038	0.000031	0.000032
54	0.000026	0.000026	0.000021	0.000021
60	0.000018	0.000019	0.000014	0.000014
66	0.000013	0.000013	0.00001	0.00001
72	0.00001	0.00001	0.000007	0.000007

Table II Axial Magnetic Fields
nonmagnetic pipe, velocity = 0

z	Bz on L1	Bz on L1 Analytic	Bz on L2	Bz on L2 Analytic
0	0.000889		0.00241	

1.2	0.000869	0.000876	0.00237	0.0022
6	0.000478	0.000481	0.000166	0.000109
12	0.00001	0.000004	0.000676	0.000681
18	0.000143	0.000144	0.000477	0.00048
24	0.000149	0.00015	0.000315	0.000318
30	0.000124	0.000125	0.000211	0.000212
36	0.000097	0.000098	0.000144	0.000145
42	0.000075	0.000075	0.000101	0.000102
48	0.000058	0.000058	0.000073	0.000074
54	0.000045	0.000045	0.000055	0.000055
60	0.000035	0.000036	0.000041	0.000042
66.00001	0.000028	0.000028	0.000032	0.000033
72	0.000023	0.000023	0.000025	0.000026

As expected, the ferromagnetic pipe with μ_r=50 has a diminished axial field on L1 outside the pipe. The radial and axial magnetic fields are shown compared to the analytic solution in Tables III and IV.

Table III Radial Magnetic Fields
magnetic pipe, velocity = 0

z	Br on L1	Br on L1 analytic	Br on L2	Br on L2 analytic
0	7.94e-07		5.96e-07	
1.2	0.000015	0.000017	0.00131	0.00132
6	0.00006	0.000065	0.00539	0.0054
12	0.000062	0.000068	0.00154	0.00156
18	0.000048	0.000054	0.000515	0.000525
24	0.000037	0.000042	0.00018	0.000185
30	0.000029	0.000034	0.000064	0.000066
36	0.000024	0.000028	0.000023	0.000023
42	0.00002	0.000024	0.000008	0.000009
48	0.000017	0.000021	0.000003	0.000004
54	0.000015	0.000018	0.000001	0.000002

60	0.000013	0.000016	5.62e-07	6.65e-07
66.00001	0.000012	0.000015	3.61e-07	3.24e-07
72	0.00001	0.000013	2.87e-07	4.38e-07

Table IV Axial Magnetic Fields
magnetic pipe, velocity = 0

z	Bz on L1	Bz on L1 Analytic	Bz on L2	Bz on L2 Analytic
0	0.000087		0.000459	
1.2	0.000085	0.000093	0.000449	0.00064
6	0.000055	0.000061	0.000306	0.000335
12	0.000016	0.000018	0.000079	0.000088
18	0.000004	0.000001	0.000029	0.000033
24	0.000006	0.000005	0.000012	0.000016
30	0.000007	0.000006	0.000007	0.00001
36	0.000007	0.000006	0.000005	0.000008
42	0.000006	0.000006	0.000004	0.000007
48	0.000006	0.000006	0.000004	0.000006
54	0.000005	0.000005	0.000003	0.000005
60	0.000005	0.000005	0.000003	0.000005
66.00001	0.000005	0.000005	0.000003	0.000005
72	0.000004	0.000004	0.000003	0.000004

Velocity Effects

The remaining question is how to account for velocity effects. One alternative is to redefine the vector potential in terms of the axial velocity v of the pipe as [3]

$$\tilde{A}=Ae^{\left(-\frac{\mu\sigma vz}{2}\right)} \qquad (14)$$

The governing equation becomes

$$\nabla^2\tilde{A}-\alpha^2\tilde{A}=0$$
where
$$\alpha^2=\left(\frac{\mu\sigma v}{2}\right)^2+j\omega\mu\sigma. \tag{15}$$

Solution proceeds by solving for $\tilde{A}$.

The question in opening this paper seeks a solution without reformulating the program, i.e., using the same software as in the zero velocity case. We propose to trick the problem into thinking it is moving by altering the conductivity in front and to the rear of the coil. The defining vector potential equation with velocity is

$$\nabla^2 A-\mu\sigma\left(v\frac{\partial A}{\partial z}+\frac{\partial A}{\partial t}\right)=0. \tag{16}$$

In cylindrical coordinates, this becomes

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial A}{\partial\rho}\right)+\frac{\partial^2 A}{\partial z^2}-\mu\sigma v\frac{\partial A}{\partial z}-\mu\sigma\frac{\partial A}{\partial t}-\frac{A}{\rho^2}=0. \tag{17}$$

Terms 3 and 4 in (17) both share the common multiplier σ. One need merely to augment the conductivity to account for the effect of the velocity (term 3 in (17)). The steps for incorporating velocity are as follows:

1) Work the problem assuming v=0. Get A and $\frac{\partial A}{\partial z}$ along the tube (wherever eddy currents exist)

2) Examine the ratio $\left(\frac{v\frac{\partial A}{\partial z}+j\omega A}{j\omega A}\right)$.

3) Increase the conductivity by the ratio

$$\sigma_{new}=\sigma_{original}\left|\frac{v\frac{\partial A}{\partial z}+j\omega A}{j\omega A}\right|.$$

4). Repeat if necessary to refine the value of A and $\frac{\partial A}{\partial z}$.

Note that if the software used forces an entry of real conductivity as most do, the phase information of your final answer will not be correct. You are forced to use the absolute value of the ratio in step 3, but the magnitude should be correct.

Steps 1-3 were performed for problem 9 for the velocities v=1,10, and 100 m/s. The conductivity profiles along the tube for these three velocities are shown in Figure 3, Figure 4, and Figure 5 respectively. Note that as the velocity is increased, the conductivity becomes more symmetric, indicating the overwhelming

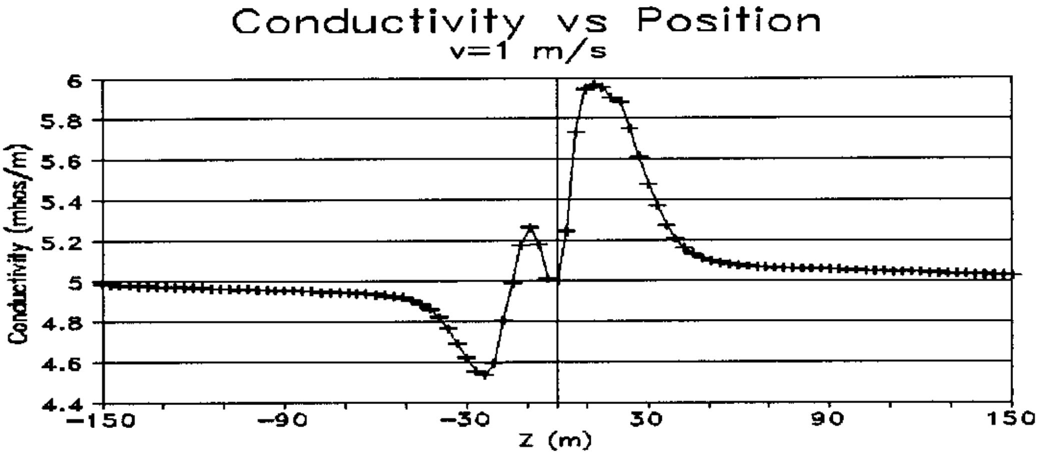

Figure 3 Conductivity in the tube for the v=1 m/s velocity case.

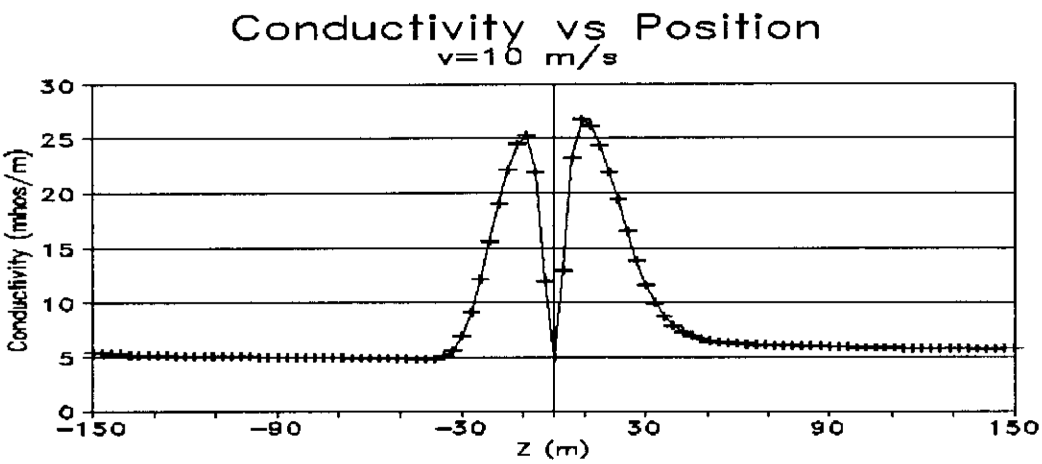

Figure 4 Conductivity in the pipe for the v=10 m/s velocity case.

influence of the $v\frac{\partial A}{\partial z}$ term compared to $j\omega A$.

Results for the v=10 m/s Velocity case

As seen in Figure 3, the effect of the velocity on the conductivity at v=1 m/s is slight. The analytic results differed generally only in the second decimal place from the analytic results for the v=0 m/s study. The v=10 m/s case was on the other

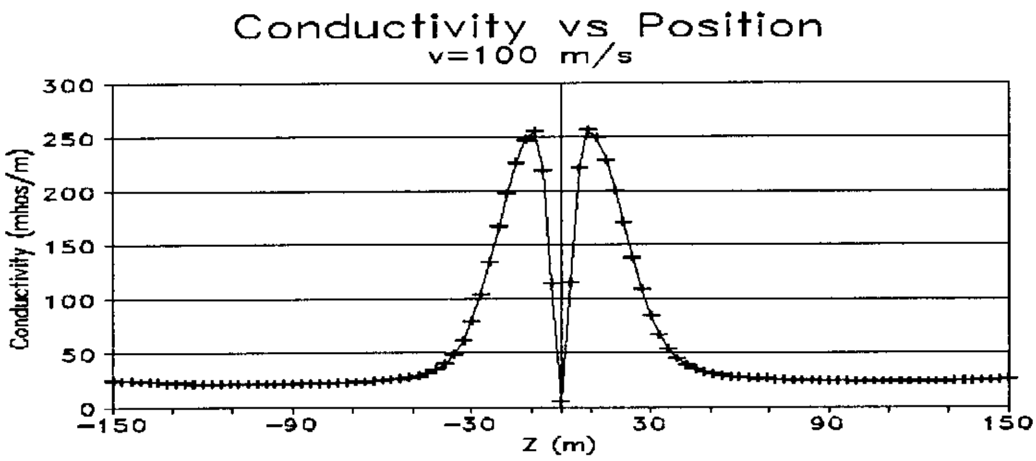

Figure 5 Conductivity in the tube for the v=100 m/s velocity case.

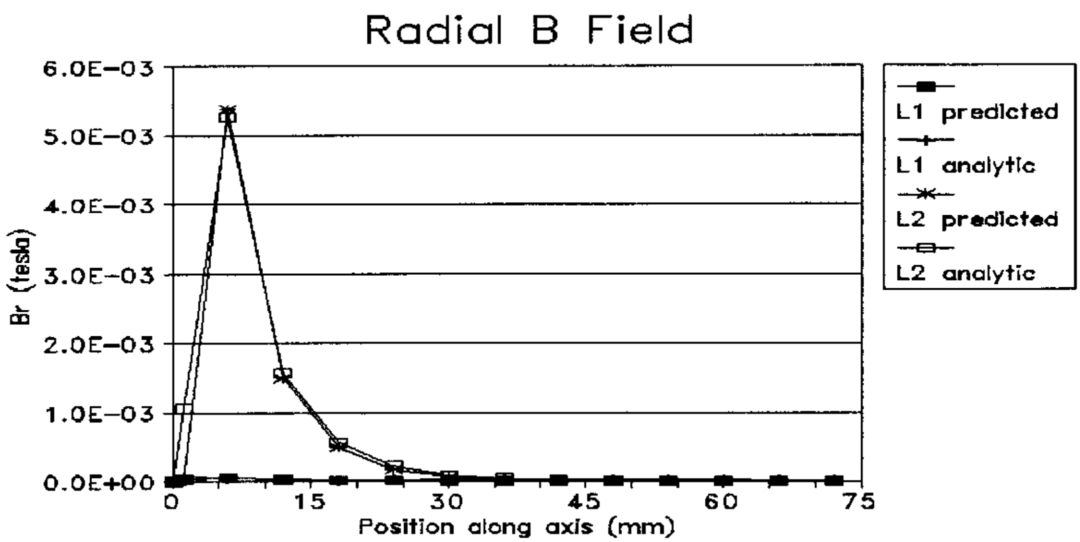

Figure 6 Radial field for the magnetic pipe, v=10 m/s.

hand quite dissimilar. It was thought that this would prove a good testing ground for the theory. Shown in Figure 6 is the radial field predicted along L1 and L2 with its analytic counterpart. The tabular comparison is shown in Table V.

Table V Radial Field Predictions
v=10 m/s, permeability = 50

z	Br on L1	Br on L1 analytic	Br on L2	Br on L2 analytic
0	0.000004	0	0.000007	0
1.2	0.000027	0.000059	0.000013	0.00106
6	0.000057	0.000055	0.005381	0.00525
12	0.000032	0.000028	0.001499	0.00157
18	0.00002	0.000005	0.000492	0.000561
24	0.000014	0.000023	0.00017	0.000217
30	0.000011	0.000031	0.000059	0.000089
36	0.000009	0.000032	0.000021	0.000039
42	0.000007	0.000031	0.000007	0.000019
48	0.000006	0.000028	0.000003	0.00001
54	0.000006	0.000026	8.27e-07	0.000006
60	0.000005	0.000023	2.42e-07	0.000003
66.00001	0.000004	0.000021	6.19e-08	0.000002
72	0.000004	0.000019	5.93e-08	0.000002

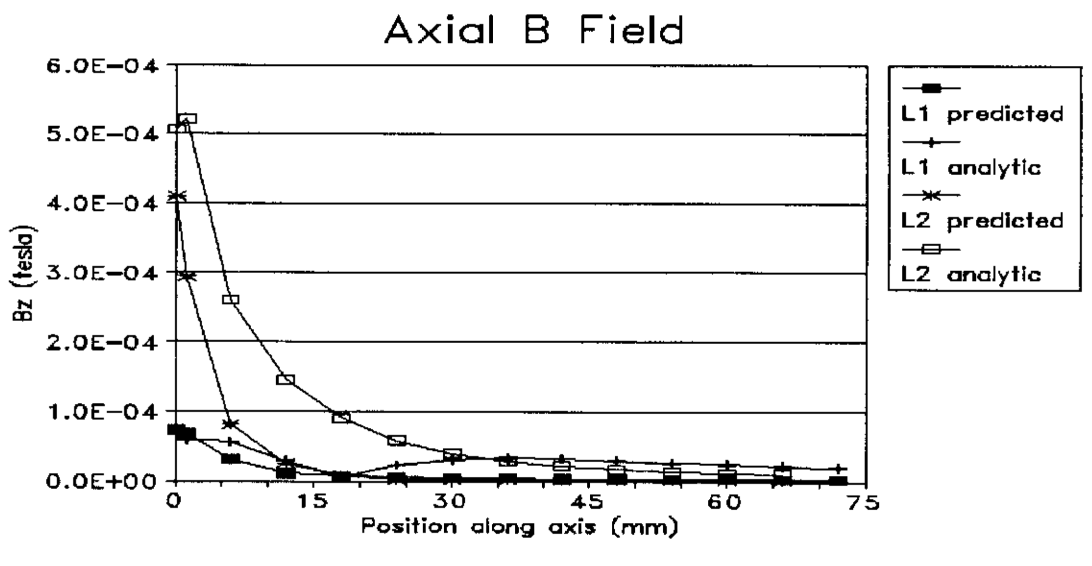

Figure 7 Axial magnetic field for the magnetizable pipe with coil traveling at v=10 m/s.

By comparison Figure 7 shows the axial B field predictions along with those obtained analytically. In both cases some error is seen in the smallest component of the field, but the resultant is very close. Table Vi displays this data along with the analytic predictions.

Table VI Axial Field Predictions
v=10 m/s, permeability = 50

z	Bz on L1	Bz on L1 analytic	Bz on L2	Bz on L2 analytic
0	0.000072		0.00042	
1.2	0.000068	0.000059	0.000409	0.000506
6	0.00003	0.000055	0.000293	0.000522
12	0.000011	0.000028	0.00008	0.000259
18	0.000007	0.000005	0.000025	0.000146
24	0.000006	0.000023	0.000007	0.000089
30	0.000004	0.000031	0.000002	0.000057
36	0.000004	0.000032	0.000001	0.000039
42	0.000003	0.000031	0.000001	0.000028
48	0.000002	0.000028	0.000001	0.000021
54	0.000002	0.000026	0.000001	0.000016
60	0.000002	0.000023	0.000001	0.000013
66.00001	0.000002	0.000021	0.000001	0.000011
72	0.000002	0.000019	9.20e-07	0.000009

The accuracy suggests that the method is quite effective.

Conclusions

Altering the conductivity to account for velocity effects is a relatively simple technique for accounting for velocity when the code does not implicitly have such capability. In this example, the conductivity was altered in the tube in regions to be piecewise continuous. Only 14 different conductivities were used to model Figure 4. Furthermore the ratio $\sigma_{new}=\sigma_{original}\left|\frac{v\frac{\partial A}{\partial z}+j\omega A}{j\omega A}\right|$. was computed in the center of the pipe at the radial line r=17 mm. In reality 3 further modifications would be necessary to get precise results.

1) Alter the conductivity to reflect radial changes in the ratio $\sigma_{new}=\sigma_{original}\left|\frac{v\frac{\partial A}{\partial z}+j\omega A}{j\omega A}\right|$..

2). Model a continuous change in conductivity as suggested by Figure 4.

3). Iterate on the solution to refine the conductivities with a closer estimate of $\sigma_{new}=\sigma_{original}\left|\frac{v\frac{\partial A}{\partial z}+j\omega A}{j\omega A}\right|$. after the first iteration.

The accuracy of the answers reflects the fact that the ratio does not change significantly as one varies the velocity. Also reasonable predictions of the fields are realized with a rather crude modeling of the conductivity.

If a complex conductivity is known, it can be inserted to correctly account for the $v\frac{\partial A}{\partial z}$ term. Since this is unknown a priori, one is forced to iteratively approached its corect value. The problem is worked first assuming it is zero, and then updating the value as suggested above. The accuracy of the results summarized below were obtained in a single iteration. They enable the user to obtain a close result without reformulating the Green's function integral. Many users do not have access to the code to make these alterations even if they could formulate the changes.

Acknowledgements

The BEM package used for these calculations was a program called Oersted from Integrated Engineering Software. The author wishes to thank Dalian Zheng for his help in checking the v=0 computations.

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