A LU Decomposition Useful for Antenna Optimization

K. Jamil and E. H. Newman
Ohio State University ElectroScience Lab

Abstract - This paper describes a LU decomposition technique useful when solving a series of matrix equations in which only a small fraction of the original matrix changes from run to run. On the first run, the entire matrix must be computed and LU decomposed. However, on the second or subsequent runs, only those rows and columns of the matrix which have changed need be recomputed and re-LU decomposed. If only a small fraction of the matrix has changed, this results in a substantial saving in CPU time both in the computation of the original matrix and in its LU decomposition.

I. Description of Procedure

Many computational techniques in electromagnetics and other branches of engineering are based upon the solution of a matrix equation. For example, a Method of Moments (MM) solution with N unknowns requires setting up and solving an order N matrix equation of the form $[Z]I = V$. In this case, order N^2 CPU time is required to set up the $[Z]$ matrix, and order N^3 CPU time to solve the matrix equation by direct methods [1]. For a multidimensional optimization procedure, requiring thousands of runs, the total CPU time can become prohibitive. Assuming that the change in the geometry only impacts the lower numbered unknowns of the problem, this paper presents a LU decomposition procedure which saves CPU time since only the rows and columns of the $[Z]$ matrix which have changed need be re-LU decomposed. The technique has been implemented in the author's *Electromagnetic Surface Patch Code: Version V* (ESP5) for the MM analysis of the electromagnetic radiation and scattering from geometries which can be modeled as an interconnection of thin wires, polygonal plates, and polygonal dielectric volumes [2,3].

An ideal application of the method would be the design of an antenna on a large body of fixed geometry. Figure 1 shows a MM $[Z]$ matrix with N_A modes on the antenna and N_B modes on the fixed geometry body. On the first run, the entire order $N = N_A + N_B$ MM $[Z]$ matrix would need to be computed and LU decomposed. However, on the second and subsequent runs only the first N_A rows and columns of the $[Z]$ matrix would need to be recomputed and re-LU decomposed, resulting in a substantial saving in CPU time if $N_A << N_B$.

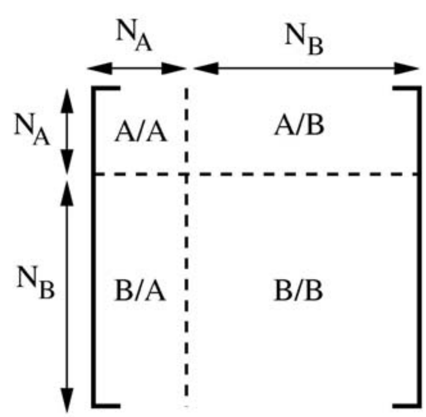


Fig. 1. The MM impedance matrix for an antenna on a large body.

II. Description of the Method

The appendix describes an LU technique that begins at the lower right hand corner of the matrix, and proceeds to the upper left corner. Figure 2 shows a snapshot of the $[Z]$ matrix part of the way through the LU process. At this point, the $N_2 \times N_2$ block in the lower right corner has been LU decomposed, but the first N_1 rows and columns have not. The important point is that as the method continues to LU decompose the first N_1 rows and columns, the elements in the $N_2 \times N_2$ block in the lower right corner do not change. Thus, if one is performing a series of MM computations in which the last N_2 expansion functions do not change, then the $N_2 \times N_2$ block in the lower right hand corner of both the $[Z]$ and LU of $[Z]$ matrices will not change. Only the first N_1 rows and columns of the $[Z]$ matrix need to be recomputed and re-LU decomposed. This reduces the number of elements that must be computed in the $[Z]$ matrix from N^2 to $2N_1N$. More importantly, only the first N_1 rows and columns need to be re-LU decomposed, thus reducing the solve time from $O(N^3)$ to $O(N_1N^2)$. If $N_1 << N$, this will result in a significant saving in CPU time. Note that the method works if N_1 changes, and thus one is free to change the number of expansion functions used to model the antenna.

III. The ESP5 Implementation

In the ESP5 implementation of the method, on the first run a LUD (LU to Disk) command causes the code to write two files to disk containing (1) the LU of the $[Z]$ matrix and (2) the detailed MM expansion function geometry. On the second or subsequent runs a DLU (Disk to LU) command causes the code to read the expansion function geometry from the disk file and to compare it to that for the present run in order to identify N_1 and N_2. Assuming $N_1 < N$, the code then reads the LU of the $[Z]$ matrix from the disk, and recomputes and re-LU decomposes only the first N_1 rows and columns.

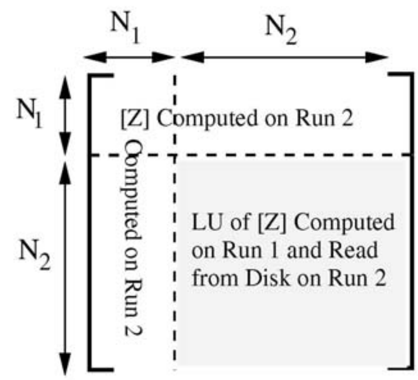


Fig. 2. The MM [Z] matrix part of the way through the LU procedure.

Table 1 shows CPU times for a problem involving $N = 4804$ wire expansion modes. On run 1, 6015 sec. were required to fill the $[Z]$ matrix, and 7289 sec. were required to do a full LU decomposition. Writing the LU of the $[Z]$ matrix to the disk required 166 sec., and a far zone pattern at 360 angles took 26 sec. On run 2, only the first mode changed, and thus $N_1 = 1$ and $N_2 = 4803$. In this case, 50 sec. were required to read the LU of the $[Z]$ from the disk, 2 sec. were required to re-compute and 13 sec. to re-LU decompose the 1st row and column of the $[Z]$ matrix. If $N_1 << N$ modes had changed, then these times would be approximately multiplied by N_1. For example, if the first $N_1 = 10$ modes had changed, then approximately 20 sec. would be required to recompute, and 130 sec. to re-LU decompose the $[Z]$ matrix.

Table 1. CPU times in sec. for runs 1 and 2 with N = 4804 wire modes, and with only the first mode changing on run 2.

Run	Comp. $[Z]$	LU $[Z]$	Write $[Z]$	Read $[Z]$	Comp. Pattern
1	6015	7289	166	N/A	26
2	2.0	13.0	N/A	50	26

Appendix

This appendix will describe a matrix LU procedure that begins at the lower right hand corner and proceeds to the upper left hand corner[1]. Referring to Figure 2, it has the advantage that if the $N_2 \times N_2$ block in the lower right hand corner of the original matrix does not change, then this block in the LU matrix also will not change, and one needs only re-LU decompose the first $N_1 = N - N_2$ rows and columns of the matrix. This reduces the total operation count and CPU time from order N^3 to $N_1 N^2$.

Assuming the order N $[Z]$ matrix is in the form of Figure 2, the following procedure will re-LU decompose into $[Z]=[L][U]$.

$for\ i = N_1 :1:-1$

$\quad for\ row = i:1:-1$

$$L_{row,i} = Z_{row,i} - \sum_{j=i+1}^{N} L_{row,j} U_{j,i}$$

$\quad end$

$\quad for\ col = i-1:1:-1$

$$U_{i,col} = (Z_{i,col} - \sum_{j=i+1}^{N} L_{i,j} U_{j,col}) / Z_{i,i}$$

$\quad end$

end

[1] The LU procedure described below may or may not be new. It was introduced to the author by Mr. Brian Lynch when he was a graduate student at The Ohio State University, Department of Electrical Engineering, ElectroScience Lab in 1989.

On run 1, set $N_1 = N$ to LU decompose the full $[Z]$ matrix.

The backward substitution to solve $[L]Y = V$ for Y proceeds as follows.

$for\ row = N:1:-1$

$$Y_{row} = (V_{row} - \sum_{j=row+1}^{N} L_{row,j} Y_j) / L_{row,row}$$

end

The forward substitution to solve $[U]I = Y$ for the solution vector I is

$for\ row = 1:N$

$$I_{row} = Y_{row} - \sum_{j=1}^{row-1} U_{row,j} I_j$$

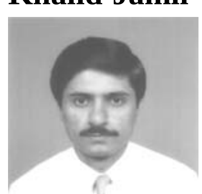
end

References

[1] G.H. Golub and C.F. Van Loan, *Matrix Computations*, 3rd Ed., Johns Hopkins Univ. Press, 1996.

[2] K. Jamil, "Enhancements to The Electromagnetic Surface Patch Code for Printed Antennas and Optimization," MSc. thesis, Ohio State Univ., Dept. of Elec. Engr. 2002.

[3] E.H. Newman, "A User's Manual for the Electromagnetic Surface Patch Code: Version V," Ohio State University, ElectroScience Lab, Unpublished Report, 2003.



Khalid Jamil was born in Lahore, Pakistan, on Feb 7, 1973. He received B.S.E.E. degree from University of Engineering and Technology, Lahore, Pakistan in 1996 and M.S. degree from The Ohio State University in 2002. Presently, he is pursuing his Ph.D. degree as a Graduate Research

Associate at the ElectroScience Lab, The Ohio State University. He worked as a Research Engineer at Advanced Engineering Research Organization, Pakistan, from 1996 to 2000. His research interests include computational electromagnetic techniques, scattering from random media and antennas.

Edward H. Newman was born in Cleveland Ohio on July 9, 1946. He received the BSEE, MS, and PhD degrees in electrical engineering from The Ohio State University in 1969, 1970, and 1974, respectively.

Since 1974 he has been a member of the Ohio State University, Department of Electrical Engineering, ElectroScience Laboratory, where he is currently a Professor. His primary research interest is in the development of method of moments techniques for the analysis of general antenna or scattering problems, and he is the primary author of the ``Electromagnetic Surface Patch Code" (ESP). Other research interests include electromagnetic shielding and antennas on automobiles, aircraft and similar platforms. He has published over 50 journal articles in these areas, and is a coauthor of the IEEE Press book "Computational Electromagnetics (Frequency Domain Method of Moments)."

Dr. Newman is a Fellow of the IEEE, and is a member of Commission B of URSI and the Electromagnetics Institute. He is a recipient of the 1986 and 1992 College of Engineering Research Award, and is a past chairman of the Columbus sections of the IEEE Antennas and Propagation and Microwave Theory and Techniques Societies.