Characterization of Biaxial Materials Using a Partially-Filled Rectangular Waveguide

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***Abstract*―** A technique is proposed to measure the permittivity and permeability parameters of a sample of biaxial material placed into a rectangular waveguide. By constructing the material as a cube, only a single sample is required to find all six material parameters. The sample is inserted into the waveguide in multiple orientations, and the transmission and reflection coefficients of the sample region are measured using a vector network analyzer. The material parameters are then found by equating the measured S-parameters to those determined theoretically using a mode-matching technique. The theoretical details are outlined and the extraction process is described. A stacked dielectric cube is characterized experimentally to demonstrate the feasibility of the approach, and results are compared to those obtained using a reduced-aperture waveguide technique.

Index Terms – Anisotropic, biaxial, material measurement, permeability, permittivity, and waveguide.

I. INTRODUCTION

Engineered materials, formed from composites of various constituents with both dielectric and magnetic properties, are gaining interest for use in antenna apertures due to their useful electromagnetic properties [1, 2]. These materials are often anisotropic, and their constitutive parameters are hard to predict theoretically. Thus it is important to develop methods to accurately characterize the behavior of anisotropic materials experimentally, so that the constitutive parameters may be used in the analysis and design of antenna systems.

Rectangular waveguide systems are often used to measure the electromagnetic properties of materials due to high signal strength, ease of sample preparation, and the ability to analyze the sample interaction analytically [3]. The authors have recently developed a method for characterizing the properties of biaxially anisotropic materials using a reduced-aperture waveguide system [4]. By using a sample holder of cubical shape, a single sample of biaxial material may be measured in three different orientations, providing the required number of reflection and transmission measurements to determine the six unique constitutive parameters. The fields in the sample region are computed analytically, and the mode-matching approach is used to determine the theoretical S-parameters of the cascaded system consisting of the sample holder and the empty waveguide transitions. This technique has the drawbacks that the sample must fit tightly within the conducting sample holder (to preclude air gaps), the restricted aperture of the sample holder reduces the energy transmitted through the sample, and a special sample holder must be constructed.

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This paper introduces an alternative technique that doesn't require a special sample holder. In this technique a cubical sample is inserted directly into a full-aperture waveguide, leaving spaces on each side of the sample. The sample is centered within the guide cross section, and mode-matching techniques are again used to find the S-parameters. This approach eliminates the presence of gaps along the sidewalls (although not along the top and bottom walls), reduces reflections from the conducting restriction, and does not require a special sample holder. Drawbacks include accurately centering the sample in the guide, and dealing with a more complicated field structure in the sample region, including finding the modal propagation constants by solving a transcendental equation.

II. THEORETICAL S-PARAMETERS FOR A CUBICAL BIAXIAL MATERIAL SAMPLE IN A RECTANGULAR WAVEGUIDE

Dependable extraction of the biaxial properties of a material sample depends on having an accurate model for the theoretical S-parameters of the measurement system. The system considered here is designed in such a way that simple mode-matching techniques can be used to find the S-parameters with a computational accuracy that is easily quantified [5].

Consider the system shown in Fig. 1. A cubical sample of material is centered within the cross-section of a rectangular waveguide such that the cross-sectional view is shown in Fig. 2. The material is assumed to be biaxial along the orthogonal axes A, B, and C, such that the tensor permittivity and permeability are given by,

$$\varepsilon = \varepsilon_0 \begin{bmatrix} \varepsilon_A & 0 & 0 \\ 0 & \varepsilon_B & 0 \\ 0 & 0 & \varepsilon_C \end{bmatrix} \qquad (1)$$

and

$$\mu = \mu_0 \begin{bmatrix} \mu_A & 0 & 0 \\ 0 & \mu_B & 0 \\ 0 & 0 & \mu_C \end{bmatrix}, \qquad (2)$$

respectively, where ε_A, μ_A, etc., are relative parameters. A TE_{10} rectangular waveguide mode is assumed to be incident upon the sample from the region $z < 0$, as shown in Fig. 3. Due to the material discontinuities at the sample interfaces, an infinite spectrum of empty waveguide modes is reflected back into this region, an infinite spectrum of modes is created in the sample region, $0 \le z \le d$, and an infinite spectrum of modes is transmitted into the region $z > d$. The empty waveguide sections on the sending and receiving ends are assumed to be of sufficient length that only the dominant empty waveguide TE_{10} mode propagates to the ends of the sections. Thus, dominant-mode reflection and transmission coefficients can be measured at these ports using a vector network analyzer, and the S-parameters of the sample determined by shifting these measurements to the sample planes $z = 0$ and $z = d$. To determine the biaxial material properties, the theoretical S-parameters are needed at these planes.

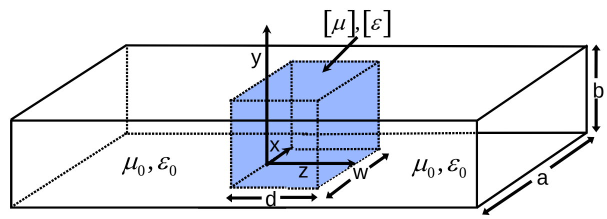

Fig. 1. Cubical sample of biaxial material centered inside a rectangular waveguide.

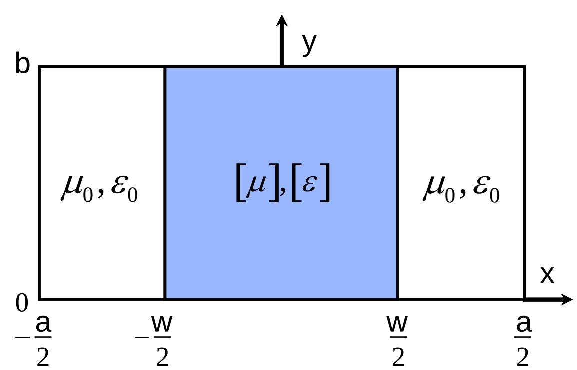

Fig. 2. Cross-sectional view of the sample inside the waveguide.

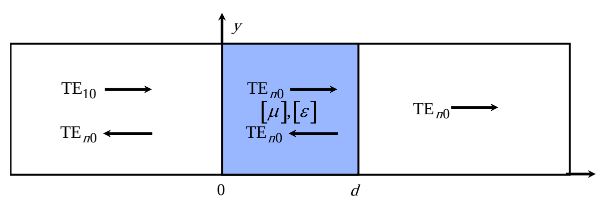

Fig. 3. Side view of the sample inside the waveguide showing the presence of higher-order modes in each region.

Since the electric field of the dominant TE_{10} mode of the empty guide is even about x, the incident field will only couple to modes with a similar symmetry. The field structure of the empty waveguide modes is well known [6], and it is easily seen that only modes of the type TE_{n0} will be excited in the empty guides. Thus, the transverse fields in the region $z < 0$ may be expanded as,

$$E_y(x,z) = a_1^i \bar{E}_1(x) e^{-j\bar{\beta}_1 z} + \sum_{n=1}^{N} a_n^r \bar{E}_n(x) e^{+j\bar{\beta}_n z} \quad (3)$$

$$H_x(x,z) = -a_1^i \bar{H}_1(x) e^{-j\bar{\beta}_1 z} + \sum_{n=1}^{N} a_n^r \bar{H}_n(x) e^{+j\bar{\beta}_n z}, \quad (4)$$

while the field in the region $z > d$ may be written as,

$$E_y(x,z) = \sum_{n=1}^{N} a_n^t \bar{E}_n(x)\, e^{-j\bar{\beta}_n(z-d)} \quad (5)$$

$$H_x(x,z) = -\sum_{n=1}^{N} a_n^t \bar{H}_n(x) e^{-j\bar{\beta}_n(z-d)}. \quad (6)$$

Here a_1^i is the known amplitude of the incident TE_{10} mode, while a_n^r and a_n^t are modal amplitudes to be determined by applying appropriate boundary conditions at the interfaces between the samples and the empty waveguide sections. Once these are found, the theoretical S-parameters are given by,

$$S_{11}^T = \frac{a_1^r}{a_1^i} \quad (7)$$

$$S_{21}^T = \frac{a_1^t}{a_1^i}. \quad (8)$$

In equations (3) to (6), $\bar{\beta}_n$ is the real phase constant of the TE_{n0} mode given by,

$$\bar{\beta}_n = \sqrt{k_0^2 - \bar{k}_{c,n}^2} \quad (9)$$

with $k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$ is the free-space wave number, and $\bar{k}_{c,n}$ the cutoff wavenumber,

$$\bar{k}_{c,n} = \frac{n\pi}{a},\ n = 1,2,3, ... \quad (10)$$

Also in equations (3) to (6), the field structure of the empty waveguide modes is given by,

$$\bar{E}_n(x) = -\frac{j\omega\mu_0}{\bar{k}_{c,n}} \sin\bar{k}_{c,n}\left(x - \frac{a}{2}\right) \quad (11)$$

$$\bar{H}_x(x) = \frac{\bar{E}_n}{\bar{Z}_n}, \quad (12)$$

where $\bar{Z}_n$ is the TE-wave impedance

$$\bar{Z}_n = \frac{\omega\mu_0}{\bar{\beta}_n}. \quad (13)$$

The field structure of the waveguide modes in the sample region is somewhat more complicated. Assuming that the biaxial cube is aligned so that the axes A, B, and C lie along some choice of the directions x, y, and z, the fields within the region $0 \le z \le d$ may be expanded as,

$$E_y(x,z) = \sum_{n=1}^{N}\left[a_n^+ e^{-j\beta_n z} + a_n^- e^{+j\beta_n z}\right] E_n(x) \quad (14)$$

$$H_x(x,z) = \sum_{n=1}^{N}\left[-a_n^+ e^{-j\beta_n z} + a_n^- e^{+j\beta_n z}\right] H_n(x)' \quad (15)$$

where a_n^+ and a_n^- are amplitudes to be determined by application of the boundary conditions at $z = 0$ and $z = d$. Here the modal fields are given for $x > 0$ by,

$$E_n(x) = \begin{cases} E_n^I(x), & 0 \le x \le \frac{w}{2} \\ E_n^{II}(x), & \frac{w}{2} < x \le \frac{a}{2} \end{cases} \quad (16)$$

$$H_n(x) = \begin{cases} H_n^I(x), & 0 \le x \le \frac{w}{2} \\ H_n^{II}(x), & \frac{w}{2} < x \le \frac{a}{2}' \end{cases} \quad (17)$$

where

$$E_n^I(x) = \frac{j\omega\mu_z}{k_{c,n}^I} \cos\left(k_{c,n}^I x\right), \quad (18)$$

$$E_n^{II}(x) = -\frac{j\omega\mu_0}{k_{c,n}^{II}} \sin\left(k_{c,n}^I \frac{d}{2}\right) \frac{\sin k_{c,n}^{II}\left(x - \frac{a}{2}\right)}{\cos k_{c,n}^{II}\left(\frac{d-a}{2}\right)}, \quad (19)$$

$$H_n^I(x) = \frac{E_n^I(x)}{Z_n^I},\ H_n^{II}(x) = \frac{E_n^{II}(x)}{Z_n^{II}}, \quad (20)$$

with

$$Z_n^I = \frac{\omega\mu_x}{\beta_n},\ Z_n^{II} = \frac{\omega\mu_0}{\beta_n}. \quad (21)$$

These fields are found by solving the wave equation for a biaxial medium [7],

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\mu_z}{\mu_x}\left(\omega^2\mu_x\varepsilon_y - \beta^2\right)\right] H_z(x,z) = 0 \quad (22)$$

and applying the boundary conditions at the interfaces $x = a/2$ and $x = w/2$. Note that because of symmetry, the boundary conditions at $x = -a/2$ and $x = -w/2$ are satisfied automatically.

The complex propagation constants β_n are found by solving the transcendental equation,

$$\mu_0 k_{c,n}^I \sin\left(k_{c,n}^I \frac{w}{2}\right) \sin\left(k_{c,n}^{II} \frac{a-w}{2}\right) = \mu_z k_{c,n}^{II} \cos\left(k_{c,n}^I \frac{w}{2}\right) \cos\left(k_{c,n}^{II} \frac{a-w}{2}\right) \quad (23)$$

where the cutoff wavenumbers are related to the complex propagation constants by,

$$\left(k_{c,n}^{I}\right)^2 = \frac{\mu_z}{\mu_x}\left(\omega^2\mu_x\varepsilon_y - \beta_n^2\right) \qquad (24)$$
$$\left(k_{c,n}^{II}\right)^2 = k_0^2 - \beta_n^2 \,.$$

It is not as straightforward to number the modes in the partially-filled guide as it is to number the modes in the empty waveguide extensions. One approach is to start with the solution for β_n under the condition $w = 0$, which is given by equations (9) and (10) since the sample region is empty, and then slowly increase the width of the sample, solving for β_n at each step using the previous solution as an initial guess. Then the modes can be numbered according to the starting empty waveguide mode. This is the approach suggested in [8] for thin samples. However, as the sample width increases, modes may switch between propagating and evanescent, and solving equation (23) using previous results as initial guesses becomes problematic. Instead, when the material loss is not too high, the following approach can be used. First the imaginary parts of the material constants are set to zero (implying a lossless material). Then the real values of β are found by looking for zero crossings of equation (23) between $\beta = 0$ and $\beta = \omega\sqrt{\mu_x\varepsilon_y}$. These are the propagating modes, and are assigned the index n according to the order of the zero crossing. Next β is replaced by $-j\delta$ in equation (23), and real values of δ are found by looking for zero crossings starting from $\delta = 0$ and continuing until a prescribed number of modes have been found. These are the evanescent modes, and they are numbered continuing after the highest order propagating mode. Finally, the imaginary parts of the material parameters are restored, and the zero crossing values found earlier for β and δ are used as initial guesses in a Newton's method root search (possibly with a small imaginary perturbation) to find the complex propagation constants for both the propagating and the evanescent modes.

The unknown modal amplitudes are found by enforcing continuity of E_y and H_x at the interfaces $z = 0$ and $z = d$,

$$a_1^i \bar{E}_1(x) + \sum_{n=1}^{N} a_n^r \bar{E}_n(x) = \sum_{n=1}^{N}[a_n^+ + a_n^-]E_n(x) \qquad (25)$$

$$-a_1^i \bar{H}_1(x) + \sum_{n=1}^{N} a_n^r \bar{H}_n(x) = \sum_{n=1}^{N}[-a_n^+ + a_n^-]H_n(x), \qquad (26)$$

$$\sum_{n=1}^{N}\left[a_n^+ e^{-j\beta_n d} + a_n^- e^{+j\beta_n d}\right]E_n(x) = \sum_{n=1}^{N} a_n^t \bar{E}_n(x), \qquad (27)$$

$$\sum_{n=1}^{N}\left[-a_n^+ e^{-j\beta_n d} + a_n^- e^{+j\beta_n d}\right]H_n(x) = -\sum_{n=1}^{N} a_n^t \bar{H}_n(x). \qquad (28)$$

Note that, for convenience, the same number of modes, N, is used in the empty waveguide and sample holder regions.

Equations (25) to (28) are a system of functional equations. They may be transformed into a system of linear equations by applying appropriate testing operators as follows. First, equation (25) is multiplied by $\bar{E}_m(x)$ and integrated between 0and $a/2$. Next, equation (26) is multiplied by $\bar{H}_m(x)$ and integrated between 0and $a/2$. Then equation (27) is multiplied by $\bar{E}_m(x)$ and integrated between 0and $a/2$. Finally, equation (28) is multiplied by $\bar{H}_m(x)$ and integrated between 0 and $a/2$. Note that all integrals can be computed in closed form. The result is a linear system of $4N \times 4N$ equations of the form,

$$\begin{bmatrix} -C_{mn} & D_{mn} & D_{mn} & 0 \\ E_{mn} & F_{mn} & -F_{mn} & 0 \\ 0 & D_{mn}^{'} & D_{mn}^{''} & -C_{mn} \\ 0 & -F_{mn}^{'} & F_{mn}^{''} & E_{mn} \end{bmatrix} \begin{bmatrix} a_n^r \\ a_n^+ \\ a_n^- \\ a_n^t \end{bmatrix} = a_1^i \begin{bmatrix} C_{m1} \\ E_{m1} \\ 0 \\ 0 \end{bmatrix} \qquad (29)$$

where each of the quantities C_{mn}, D_{mn}, etc., are $N \times N$ submatrices, and a_n^r, a_n^+, etc., are the unknown modal coefficients. Once the matrix of equation (29) has been solved, equations (7) and (8) can be used to compute the desired S-parameters.

III. EXTRACTION PROCEDURE

Since there are six independent complex quantities to determine, $(\varepsilon_A, \varepsilon_B, \varepsilon_C, \mu_A, \mu_B, \mu_C)$, the extraction process requires a minimum of six complex measurements. These may be obtained by measuring S_{11} and S_{21} with the material axes A, B, and C aligned along three properly chosen directions. Consider the orientations,

$$(A, B, C) \rightarrow (x, y, z) \qquad (30)$$

$$(A, B, C) \rightarrow (z, x, y), \tag{31}$$

$$(A, B, C) \rightarrow (y, z, x). \tag{32}$$

Measurement under these orientations gives the S-parameters $(S_{11,1}^{\text{meas}}, S_{21,1}^{\text{meas}})$, $(S_{11,2}^{\text{meas}}, S_{21,2}^{\text{meas}})$, $(S_{11,3}^{\text{meas}}, S_{21,3}^{\text{meas}})$, respectively. The material parameters can be found by solving the system of six nonlinear equations in six complex unknowns,

$$S_{11,n}^{\text{thy}}(\varepsilon_A, \varepsilon_B, \varepsilon_C, \mu_A, \mu_B, \mu_C) - S_{11,n}^{\text{meas}} = 0 \quad n = 1,2,3 \tag{33}$$

$$S_{21,n}^{\text{thy}}(\varepsilon_A, \varepsilon_B, \varepsilon_C, \mu_A, \mu_B, \mu_C) - S_{21,n}^{\text{meas}} = 0 \quad n = 1,2,3. \tag{34}$$

It may be difficult to solve this set of equations using standard methods such as Newton's method, since extremely accurate initial guesses may be required. Alternatively, a subset of the material parameters may be found using fewer equations and then these parameters may be used as known quantities to solve for the remaining parameters. This approach is possible since only the three parameters ε_y, μ_x, and μ_z appear at any given orientation.

Although many possible measurement combinations are possible, the results shown here were obtained using a three-step process. First, measurements are made with the orientations,

$$(A, B, C) \rightarrow (x, y, z) \tag{35}$$

$$(A, B, C) \rightarrow (-z, y, x), \tag{36}$$

giving the S-parameters $(S_{11,1}^{\text{meas}}, S_{21,1}^{\text{meas}})$ and $(S_{11,2}^{\text{meas}}, S_{21,2}^{\text{meas}})$, respectively. The first of these orientations is labeled 1 in Fig. 4. The second orientation corresponds to a rotation of the cube by 90° and is labeled 2 in Fig. 4. The measured S-parameters only implicate ε_B, μ_A, and μ_C, which can be found by solving the set of simultaneous equations,

$$S_{11,n}^{\text{thy}}(\varepsilon_B, \mu_A, \mu_C) - S_{11,n}^{\text{meas}} = 0, n = 1 \tag{37}$$

$$S_{21,n}^{\text{thy}}(\varepsilon_B, \mu_A, \mu_C) - S_{21,n}^{\text{meas}} = 0, n = 1,2, \tag{38}$$

using Newton's method. Next, a measurement is made using orientation 3 in Fig. 4,

$$(A, B, C) \rightarrow (y, -x, z) \tag{39}$$

giving $(S_{11,3}^{\text{meas}}, S_{21,3}^{\text{meas}})$. This measurement implicates ε_A, μ_B, and μ_C. However, μ_C is now known, so ε_A and μ_B may be found by solving,

$$S_{11,3}^{\text{thy}}(\varepsilon_A, \mu_B) - S_{11,3}^{\text{meas}} = 0 \tag{40}$$

$$S_{21,3}^{\text{thy}}(\varepsilon_A, \mu_B) - S_{21,3}^{\text{meas}} = 0, \tag{41}$$

using Newton's method. Finally, a measurement is made under orientation 4 of Fig. 4,

$$(A, B, C) \rightarrow (z, x, y) \tag{42}$$

implicating ε_C, μ_A, and μ_B. Since both μ_A, and μ_B are now known, ε_C can be found by solving,

$$S_{21,4}^{\text{thy}}(\varepsilon_C) - S_{21,4}^{\text{meas}} = 0 \tag{43}$$

using Newton's method. Note than when both reflection and transmission data is available, but only one S-parameter is required, a choice is made to use transmission data since experience shows that it is more robust.

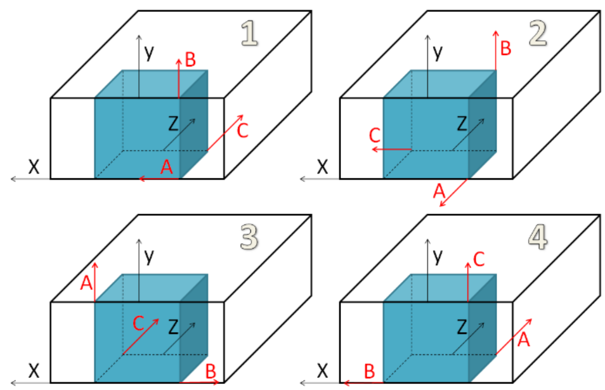

Fig. 4. Four cube orientations used in the three-step process.

IV. MEASURED RESULTS

A. Experimental setup

Demonstration of the partially-filled waveguide technique is undertaken by performing appropriate measurements at S-band and employing the extraction algorithm developed in the previous section. The waveguide system consists of two 5 inch (12.7 cm) long sections of WR-284 rectangular guide with coaxial transitions connected at the ends. These are attached through test port cables to an Agilent E5071C vector network analyzer (VNA); the assembled system is shown in Fig. 5. The VNA is calibrated at the open ends of the waveguide sections using a through-reflect-line (TRL) method. A cubical material sample is inserted in the end of one of the waveguide sections so that its surface is flush to the open-ended waveguide surface and centered between the walls, as shown in Fig. 6. The two sections of guide are then assembled and the reflection coefficient S_{11}, and transmission coefficient, S_{21}, are measured. Assembly is done using precision alignment pins to ensure high repeatability between measurements. The measurement process is repeated with the sample inserted into the appropriate orientations described in section III. Note that the S-parameters must be phase corrected to account for the fact that the rear surface of the cube is not located at the calibration plane. All measurements were made with a -5dBm source power, 64 averages, and a 70 kHz IF bandwidth.

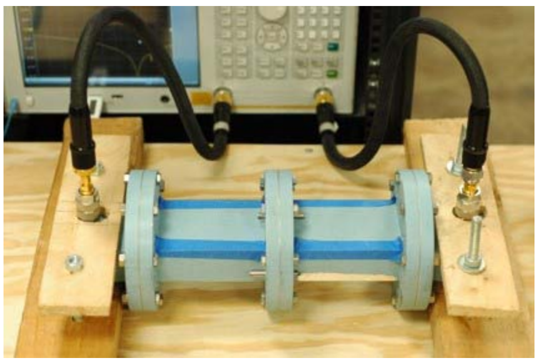

Fig. 5. Assembled S-band waveguide system.

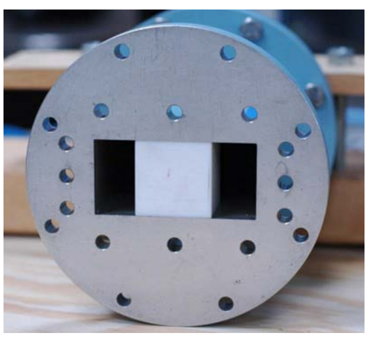

Fig. 6. Cubical sample inserted into waveguide section.

B. Experimental results

To test the characterization procedure using an anisotropic material, a cube was constructed by gluing together Rogers RO3010 circuit board substrate and Rogers RT/duroid 5870 substrate. The 3010 board has a thickness of t_1 = 1.27 mm, a dielectric constant of $\varepsilon'_{r1} = 10.2$ and a loss tangent of $\tan\delta_1 = 0.0022$. The 5870 board has a thickness of $t_2 = 3.4$ mm, a dielectric constant of $\varepsilon'_{r2} = 2.33$, and a loss tangent of $\tan\delta_2 = 0.0012$. The resulting cube, shown in Fig. 7, has uniaxial dielectric properties and isotropic magnetic properties. If the B direction is chosen to be aligned perpendicular to the layer interfaces, then it is expected that ε_A and ε_C should be identical, but different from ε_B. The sample was constructed approximately 0.01 mm larger than the inner dimensions of the sample holder so that when inserted it would compress slightly and eliminate air gaps between the MUT and the sample holder walls.

The simple geometry of the uniaxial cube allows ε_A, ε_B, and ε_C to be estimated using closed-form expressions. At the highest frequency considered in the measurements, the free-space electrical length of the stack period is $k_0(t_1 + t_2) = 0.387$. Since $k_0\sqrt{\varepsilon'_{r1}}(t_1 + t_2) \ll 2\pi$, the following approximate formulas may be used to determine the biaxial material constants [9],

$$\varepsilon_B = \left[\frac{1}{\varepsilon_{r2}} - \frac{\varepsilon_{r1}-\varepsilon_{r2}}{\varepsilon_{r1}\varepsilon_{r2}}\frac{t_1}{t_1+t_2}\right]^{-1} \tag{44}$$

$$\varepsilon_A = \varepsilon_C = \varepsilon_{r2} + (\varepsilon_{r1} - \varepsilon_{r2})\frac{t_1}{t_1+t_2} \tag{45}$$

where $\varepsilon_{r1} = \varepsilon'_{r1}(1 - j\tan\delta_1)$ and $\varepsilon_{r2} = \varepsilon'_{r2}(1 - j\tan\delta_2)$. Using the board parameters gives $\varepsilon_B = 2.95 - j0.0038$ and $\varepsilon_A = \varepsilon_C = 4.47 - j0.0081$. Because of internal reflections, the results of [10] suggest deviations of up to 10% between these approximations and the values measured using the partially-filled waveguide. Also, the slight anisotropy of the boards themselves suggests that ε_A should differ slightly from ε_C [11].

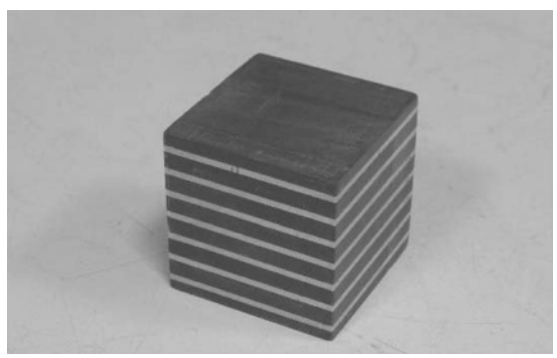

Fig. 7. Layered cube constructed from alternating layers of Rogers substrates.

The S-parameters of the uniaxial cube were measured 10 separate times, recalibrating between the measurements, and the material parameters were extracted using the 3-step procedure outlined in section III. The average values are shown in Figs. 8 and 9.

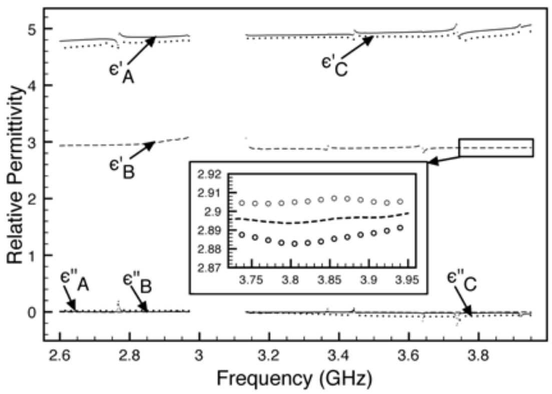

Fig. 8. Relative permittivities (mean values) of the uniaxial cube extracted using 10 measurement sets. Inset shows 2-σ confidence interval.

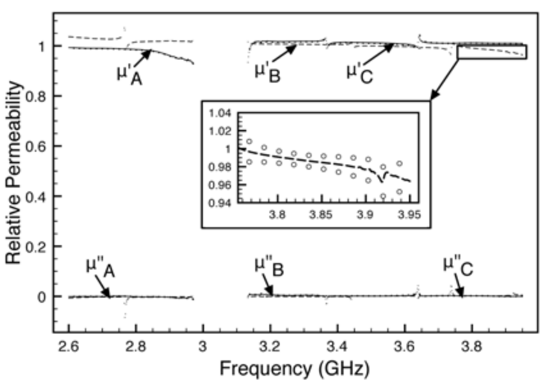

Fig. 9. Relative permeabilities (mean values) of the uniaxial cube extracted using 10 measurement sets. Inset shows 2-σ confidence interval.

The results for ε_A and ε_C are slightly higher than predicted by the closed-form expression (36), while ε_B is very close to the predicted value. The extracted values of μ_r are all close to unity, as expected, since the cube is non-magnetic. Note that the variance between measurement sets is quite small, such that showing the error bars in the figures would be distracting. Instead, the smaller insets in the figures show 95% (2-σ) confidence intervals for portions of the data sets; these intervals are typical across the entire frequency range.

The narrow confidence intervals suggest that noticeable variations in the extracted parameters are due to systematic errors, such as imperfect machining and alignment of the sample layers, or the presence of glue between sample layers, or air gaps between the sample and the waveguide walls, accentuated in certain frequency ranges by an ill-conditioning of the extraction process. This can be observed as gaps in the data in the frequency range 3 GHz - 3.15 GHz. There is amplified propagation of experimental uncertainties near frequencies where the sample is a half-wavelength long, a problem inherent to all guided-wave techniques in which both permittivity and permeability are determined (including the Nicolson-Ross-Wier closed-form method for isotropic materials [12-13]). Typically, the propagated error becomes so large that extraction is completely unreliable. This is a drawback of using a cubical sample holder, since the thickness of the material cannot be reduced below a half-wavelength. Experience has shown that a frequency range within approximately ±5% of the half-wavelength frequency should be avoided, and data within that range is not displayed in the figures, producing the observed gaps. It is possible, however, to interpolate the values of the parameters in the gaps. Figures 10 and 11 show the extracted parameters obtained by fitting a fifth-order polynomial to the data.

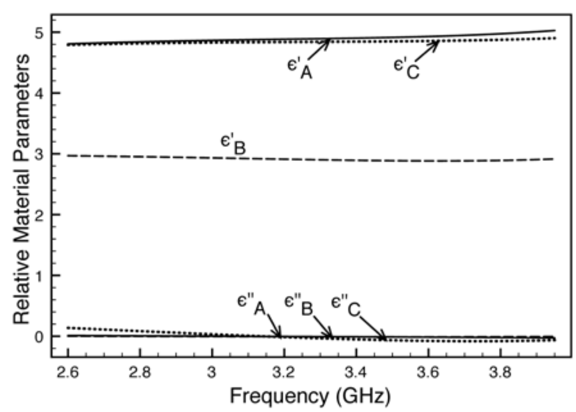

Fig. 10. Extracted relative permittivities of the uniaxial cube fitted to a fifth-order polynomial.

The extracted values of permittivity and permeability are quite similar to those obtained using the reduced-aperture waveguide described in [4]. To provide a direct comparison, the material parameters ε_A and ε_B extracted using both methods are directly compared in Figs. 12 and 13, while a comparison for the parameter μ_A is shown in Fig. 14. Results for the parameters ε_C, μ_B, and μ_C are quite similar. Note that the reduced-aperture waveguide technique also has difficulties near half wavelength frequencies, but because the propagation constants of the modes are different than those for the partially-filled guide, the gaps appear in the range 3.55 GHz - 3.75 GHz, and do not coincide with those of Figs. 8 and 9. This suggests that combining data from both techniques may ameliorate the half-wavelength issue.

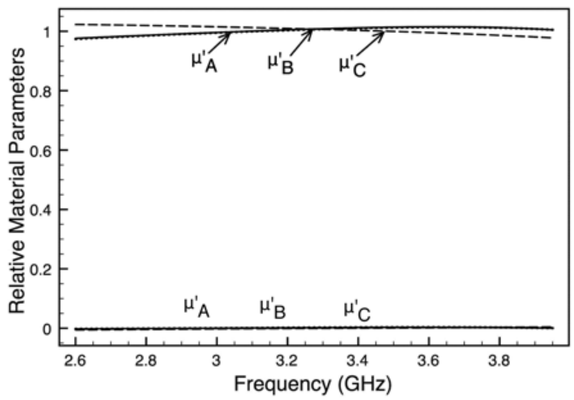

Fig. 11. Extracted relative permeabilities of the uniaxial cube fitted to a fifth-order polynomial.

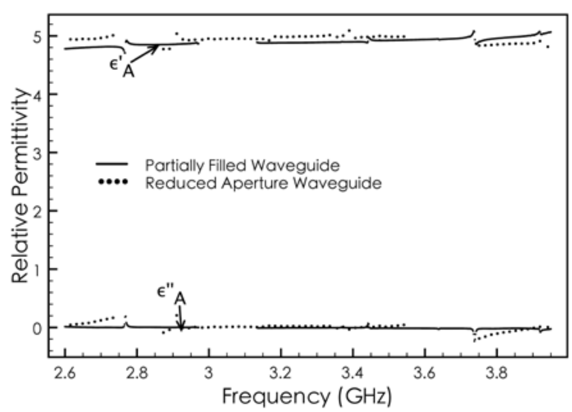

Fig. 12. Comparison of ε_A for the uniaxial cube extracted using the partially-filled waveguide technique and the reduced-aperture waveguide technique [4].

V. CONCLUSION

This paper introduces a method for measuring the electromagnetic properties of a biaxial material using a partially-filled rectangular waveguide. The proposed technique is validated using experimental data, and its accuracy is found to be commensurate with that of the reduced-aperture waveguide technique, without the need for a special sample holder, and with less worry about air gaps between the sample and the waveguide walls. The drawback to the method is the need for a more complicated theoretical analysis. A combination of both techniques may provide a means for overcoming the difficulties with accurately extracting the parameters near frequencies where the sample is a half wavelength in thickness.

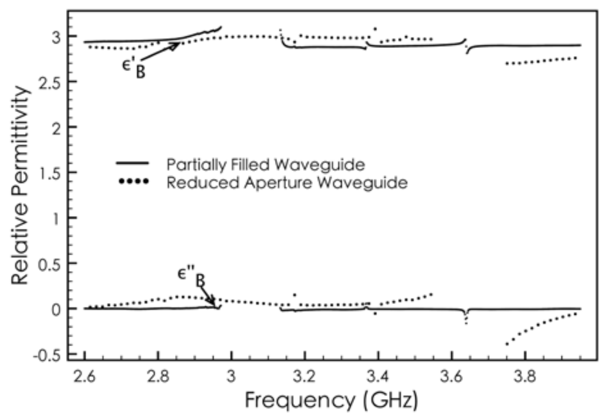

Fig. 13. Comparison of ε_B for the uniaxial cube extracted using the partially-filled waveguide technique and the reduced-aperture waveguide technique [4].

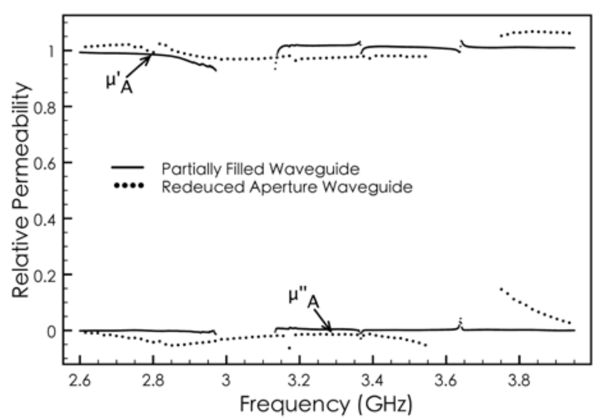

Fig. 14. Comparison of μ_A for the uniaxial cube extracted using the partially-filled waveguide technique and the reduced-aperture waveguide technique [4].

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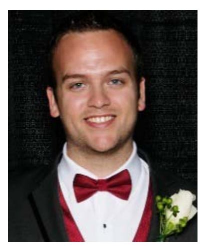

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