Generalized Scattering Matrix (GSM) Approach for Multilayer Finite Array Analysis

(*Invited Paper*)

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Abstract **—** The paper presents the Generalized Scattering Matrix (GSM) approach for analyzing multilayer printed array structures. The analysis involves computation of the overall GSM of an infinite array structure, Floquet modal based analysis of mutual coupling between array elements followed by estimation of finite array characteristics. A slot-fed patch array of 225 elements is used as an example for numerical results. It is found that the input match of the edge elements significantly differs from that of the elements in the center region of the array. The advantages and disadvantages of the approach are discussed at the end.

Index Terms**—**Finite Array, GSM Approach, Multilayer Array, Floquet Analysis, Mutual Coupling.

I. INTRODUCTION

The purpose of this paper is to provide a general overview of the GSM approach for analyzing multilayer finite printed array structures. Multilayer printed patch structures are used for enhancing the bandwidth performance [1,2] of a printed array. They are also used as a multiband frequency selective surface [3], wide band screen polarizers [4] and for realizing photonic band-gap materials [5]. The GSM approach is very convenient for analyzing such structures. The GSM approach essentially is a modular approach, where each layer of a multilayer structure is analyzed independently and then characterized in terms of a matrix. The matrix is called the GSM of the layer, because the reflection and transmission characteristics of the layer with respect to several incident modes are embedded within the matrix. The complete characterization of a multilayer structure is obtained by cascading the individual GSMs of the layers.

The GSM of an array essentially characterizes the periodic array that is extended to infinity in the transverse directions. In addition, the GSM is associated with an ideal Floquet excitation, defined by uniform amplitude and linear phase progression. For a finite array or a tapered excitation, the analysis involves few additional steps. In this paper, we outline the steps and illustrate their mathematical foundations. We demonstrate that the results of an infinite array can be utilized to predict the performances of a finite array with an arbitrary excitation. The predicted result would be *exact* if we define a "finite array" as *a physically infinite array with a finite number of excited elements*. The remaining elements are non-excited, though they *must be physically present*. Such a finite array is impractical. A real finite array, however, has only a finite number of physical elements. In many situations radiation characteristics of a real finite array can be approximated as that of a finite array defined above, because the non-excited elements generally do not contribute significantly to the radiated fields, particularly in the main lobe region.

Section II briefly outlines the GSM approach for an infinite multilayer array. Section III formulates the mutual coupling that plays the most important role in the performance of a finite array. Section IV presents the analytical procedure of a finite array employing the mutual coupling data. Numerical results of a multilayer slot-fed finite patch array antenna are shown in Section V and the important conclusions are made in Section VI.

II. THE GSM APPROACH

The GSM approach of a multilayer finite array involves the following steps:

- Computation of GSM of each layer,
- Combining GSMs of the individual layers to obtain overall GSM of the structure,
- Mutual coupling computation between the array elements using Floquet modal theory,
- Active element pattern computation,
- Computations of finite array pattern and return loss of the elements.

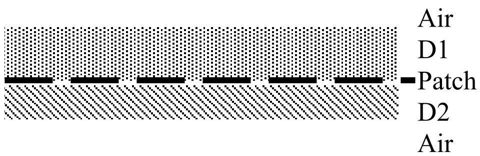
A typical multilayer array consists of four types of basic building blocks: printed elements layer, dielectric layer, dielectric interface and aperture (slot aperture, for instance) layer. The GSM of a printed element layer and slot aperture layer are usually obtained using Galerkin's MoM analysis [6]. The GSM of a dielectric layer and the interface are determined using Floquet modal analysis [7]. The GSMs of the individual layers are then combined to obtain the overall GSM of the multilayer structure. The

GSM analysis and the cascading formulas are outlined in the following Section.

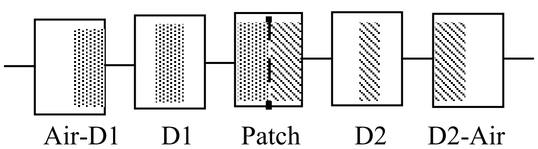
A. The GSM

The GSM essentially represents the input-output characteristics of an infinite array structure with respect to a set of Floquet modes. For a multilayer array structure, the GSMs of individual layers are determined and then combined together to obtain the overall GSM of the structure, as typically done in a mode-matching analysis of waveguide horns or filters.

To illustrate the GSM approach pictorially, consider a three-layer periodic array structure (patch-dielectric-patch) as shown in Fig. 1(a). The three-layer-structure is equivalent to five modules connected in cascade as shown in Fig. 1(b).



(a)



(b)

Fig. 1. (a) A three-layer array structure, (b) Modular representation of the array (D1 and D2 represent dielectric layers).

Identical cell-sizes and cell-orientations for the periodic arrays are assumed. Also, the structure is assumed to be infinite extent along *x* and *y* directions and is under Floquet excitation.

The GSM of a module is defined through the relation between incident and reflected voltages as below

$$\begin{bmatrix} a_1^- \\ a_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1^+ \\ a_2^+ \end{bmatrix} \quad . \tag{1}$$

In the above $[a_1^+]$ and $[a_2^+]$ are the incident voltage vectors with respect to the Floquet modes at the two sides (or ports) of the module and $[a_1^-]$ and $[a_2^-]$ are the corresponding reflected voltage vectors. The [S] matrix at the right-hand side of (1) is called the GSM of the layer. It consists of four sub-matrices, namely $[S_{11}]$, $[S_{12}]$, $[S_{21}]$, and $[S_{22}]$, respectively.

The overall GSM of the multilayer structure is obtained by combining the individual GSMs of the layers or modules. The cascading formula for two modules A and B is given by [7, p. 190]

$$[S_{11}^{AB}] = [S_{11}^{A}] + [S_{12}^{A}][I - S_{11}^{B} S_{22}^{A}]^{-1}[S_{11}^{B}][S_{21}^{A}], \tag{2a}$$

$$[S_{12}^{AB}] = [S_{12}^{A}][I - S_{11}^{B} S_{22}^{A}]^{-1}[S_{12}^{B}], \tag{2b}$$

$$[S_{21}^{AB}] = [S_{21}^{B}][I - S_{22}^{A} S_{11}^{B}]^{-1}[S_{21}^{A}], \tag{2c}$$

$$[S_{22}^{AB}] = [S_{22}^{B}] + [S_{21}^{B}][I - S_{22}^{A} S_{11}^{B}]^{-1}[S_{22}^{A}][S_{12}^{B}]. \tag{2d}$$

This formula can be applied repeatedly to obtain the overall GSM of a multilayer array. The GSM cascading formulas is applicable only if the layers have identical periodicities and have identical cell orientations. This insures that a Floquet modal vector function has an identical expression for all the layers. If the layers have different periodicities, then the process is more involved as detailed in [7, 8].

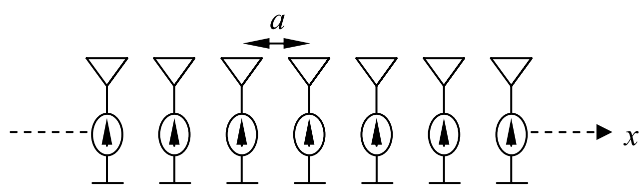


Fig. 2. Infinite linear array.

III. MUTUAL COUPLING FORMULATION

An accurate analysis of a finite array with an arbitrary excitation necessitates an estimation of the mutual coupling between the array-elements. The mutual coupling between the elements can be estimated by invoking the Floquet modal analysis of an infinite array [9].

The mutual coupling between the elements is generally quantified in terms of the following three measurable quantities:

(a) Mutual impedance,
(b) Mutual admittance,
(c) Scattering parameters.

The above three measurable quantities are related to each other by simple algebraic relations. In this Section we will first derive the mutual impedance from Floquet impedance of an infinite array [7,9]. We first consider a one-dimensional array. The result can be extended for a two-dimensional array.

A. Mutual Impedance

Consider the infinite array shown in Fig. 2. The elements are arranged along the x-direction with element spacing a. Suppose the elements are excited uniformly with linear phase progression, known as Floquet excitation. Suppose ψ is the phase difference between two adjacent elements. Then following the definition of mutual impedance, the input voltage for the 0-th element can be obtained as

$$V_0(\psi) = \sum_{n=-\infty}^{\infty} I_n Z_{0n} \,. \quad (3)$$

In the above V_0 is the input voltage for the element located at $x = 0$, I_n is the input current of the n-th element and Z_{0n} is the mutual impedance between the two elements that are located at $x = 0$ and at $x = na$, respectively. For Floquet excitations, the input currents can be expressed as

$$I_n = I_0 \exp(-jn\psi) \quad (4)$$

where I_0 is the input current for the element at $x = 0$. The input impedance seen by the $n=0$ element is

$$Z_0(\psi) = \frac{V_0(\psi)}{I_0} \,. \quad (5)$$

Substituting (3) and (4) in (5) we obtain

$$Z_0(\psi) = \sum_{n=-\infty}^{\infty} Z_{0n} \exp(-jn\psi) \,. \quad (6)$$

For a Floquet excitation, the above input impedance must be equal to the Floquet impedance $Z^{FL}(\psi)$. Therefore we obtain

$$Z^{FL}(\psi) = \sum_{n=-\infty}^{\infty} Z_{0n} \exp(-jn\psi) \,. \quad (7)$$

The right hand side of (7) is the Fourier series expansion of the Floquet impedance where the Fourier coefficients are equal to the mutual impedances. Thus, the mutual impedance Z_{0n} is readily obtained in terms of the Fourier integral as follows,

$$Z_{0n} = \frac{1}{2\pi} \int_{-\pi}^{\pi} Z^{FL}(\psi) \exp(jn\psi) d\psi \,. \quad (8)$$

If the two elements are located at $x = ma$ and $x = na$, respectively, then the mutual impedance between these two elements can be expressed as

$$Z_{mn} = \frac{1}{2\pi} \int_{-\pi}^{\pi} Z^{FL}(\psi) \exp\{-j(m-n)\psi\} d\psi \,. \quad (9)$$

Equation (9) establishes the relation between the Floquet impedance and the mutual impedance between the elements. It is important to observe that equation (9) yields the mutual impedance in the array environment. Also observe that Z_{mn} and Z_{nm} are identical because $Z^{FL}(\psi) = Z^{FL}(-\psi)$ [7, p. 130]. The symmetry property of $Z^{FL}(\psi)$ can be utilized to express Z_{mn} in the following convenient form from computational point of view:

$$Z_{mn} = \frac{1}{\pi} \int_{0}^{\pi} Z^{FL}(\psi) \cos\{(m-n)\psi\} d\psi \,. \quad (10)$$

The mutual impedance deduced in (10) includes the effects of scattering from the intermediate and surrounding elements that are open-circuited. The element-by-element approach [10] typically ignores the scattering effects; therefore the present formulation for mutual coupling is generally more accurate than the element-by-element approach.

It is worth pointing out that for some arrays the Floquet impedance Z^{FL} may have a finite number of singularities due to resonances of selective Floquet modes with the guided wave modes supported by the array structures. Under such a situation, a singularity extraction technique [11] must be employed to compute the integral near a singular point.

The mutual admittance between the two elements in array environment can be obtained as

$$Y_{mn} = \frac{1}{\pi} \int_{0}^{\pi} Y^{FL}(\psi) \cos\{(n-m)\psi\} d\psi \quad (11)$$

where $Y^{FL}(\psi)$ is the Floquet admittance, reciprocal to the Floquet impedance $Z^{FL}(\psi)$ and Y_{mn} is the mutual admittance between the m-th and the n-th elements. The distance between the two elements is $(m-n)a$. The scattering parameters between the elements also follow the similar relation. If S_{mn} represents the scattering parameter defined as the voltage received by the m-th element when the n-th element is excited with all other elements including the m-th element are matched terminated, then

$$S_{mn} = \frac{1}{\pi} \int_{0}^{\pi} \Gamma^{FL}(\psi) \cos\{(m-n)\psi\} d\psi \quad (12)$$

where $\Gamma^{FL}(\psi)$ is the reflection coefficient of an array element under Floquet excitation. Since $\Gamma^{FL}(\psi) = \Gamma^{FL}(-\psi)$, one can see that $S_{mn} = S_{nm}$. It should be noted that the integral for S_{mn} does not have any singularity because the magnitude of $\Gamma^{FL}(\psi)$ does not exceed beyond unity. Thus, from computational point of view, scattering matrix formulation is advantageous as compared with impedance/admittance formulation for a finite array analysis.

The mutual coupling formulation can be extended for a two dimensional planar array. For a rectangular lattice structure, the mutual impedance involves a two-dimensional integral with variables ψ_x and ψ_y, where ψ_x and ψ_y represent the phase difference between the adjacent elements along x and y directions, respectively. For a triangular lattice, the formulation is slightly different (see [7, p.141]).

IV. Finite Array: Active Impedance and Radiation Patterns

The mutual coupling information between the elements is utilized to determine the active impedance or return loss of an element with respect to given amplitude and phase distributions. The active impedance of an element depends on the amplitude and phase distributions and the load-conditions of the non-excited elements [7, p.146]. In the present study we will assume that the non-excited elements of a "finite array" are match terminated. This assumption is somewhat justified because a matched element have a small scattered field, thus closely resembles the absence of an element.

Under such a situation the scattering matrix relation will yield the exact active input impedance solution. The relation in this situation is

$$[V^-] = [S][V^+] \tag{13}$$

where $[V^+]$ and $[V^-]$ are the incident and reflected voltage vectors. Elements of $[S]$ are obtained using (12). Equation (13) can be utilized to obtain the complex reflection coefficients of the elements with respect to a given amplitude distribution of a finite array.

The radiation pattern of the finite array for this particular situation can be obtained directly from superposition. The result becomes [7, p.150]

$$E^{array} = E^a[P][V^+] \tag{14}$$

where E^a is the active element pattern, which is defined as the radiation pattern of an element in array environment while other elements are match terminated. The vector form of the active element pattern can be obtained from Floquet analysis and the final expression becomes [7, p.109]

$$\vec{E}_a(\theta,\phi) = \hat{\theta}\{jV_{00}^{TM}\sqrt{ab/\lambda_0^{\,2}}\} - \hat{\phi}\{jV_{00}^{TE}\sqrt{ab/\lambda_0^{\,2}}\cos\theta\} \tag{15}$$

In the above V_{00}^{TM} and V_{00}^{TE} are the modal voltages at the array aperture for the TM_{00} and TE_{00} Floquet modes, respectively. The modal voltages are functions of scan direction (θ,ϕ). The gain can be determined by normalizing element and array structures.

V. Results

To illustrate the GSM approach, we consider a multilayer finite array of slot-fed patch elements. Figure 3 shows the element and array structure. The element numbering scheme is also shown pictorially. We computed the Floquet return loss (return loss under Floquet excitation) versus scan angle and plotted in Fig. 4. The return loss is generally good near the bore-sight scan, however a sharp resonant spike is observed near 39 degree scan angle along the E-plane. The resonant spike causes a complete mismatch and the array is ceased to radiate at this angle. This phenomenon is known as scan blindness. The TM_0 surface wave mode, supported by the grounded dielectric structure, is responsible for this blindness. At that scan angle, the surface wave mode has a perfect phase-match with the element phase causing a resonance[1]. The surface wave resonance for the D-plane scan is not present because the resonant condition is not satisfied for the square grid structure under consideration. For the H-plane scan, the resonance does not occur because the surface wave is not excited at the first place due to polarization mismatch between the patch mode and the surface wave mode.

Figure 5 shows the active element pattern cuts for the array. The patterns are normalized with respect to the incident power. The active element gain is about 6.39 dBi, which is 0.23 dB lower than that of a 100% aperture-efficient element. This gain loss is due to back side radiation of the feed slot. The E-plane pattern has a null (blind angle) near 39 degree which is consistent with the return loss behavior. For the E and H-plane patterns, the cross-polarization components do not exist because of symmetrical geometry. The cross-polarization level is substantial at the D-plane scan, particularly near 60-degree off-boresight.

Figure 6 shows the radiation pattern of a finite array of 15×15 elements. Two scan angles were considered in this case. The radiation patterns of the finite array were computed using (14). The amplitude taper (Gaussian) was 10 dB for both cases. For the bore-sight beam, the peak gain is about 28.95 dBi, which is about 1 dB lower than that of a uniform excitation. The scanned beam has a peak gain of 28.06 dBi. The side lobes are 25 dB below the beam peaks in both cases.

Equation (12) was utilized to compute the mutual coupling in terms of the array scattering parameters.

[1] As stated at the introduction, the present analysis assumes infinite array with a finite number of excited elements. For a real finite array, the effects of edge diffraction may be included approximately through complex reflection coefficient of the surface wave mode due to the ground plane and dielectric truncation [12].

Figure 7 shows the coupling level of the array elements with respect to the center elements. The E-plane elements are tightly coupled than the H-plane elements.

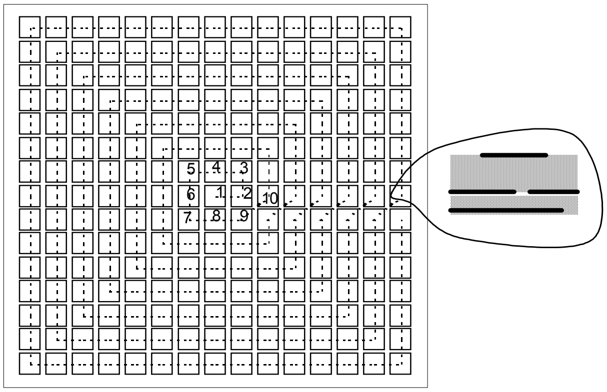


Fig. 3. A 15×15 element array of slot-fed patch elements. The element numbering scheme is also shown.

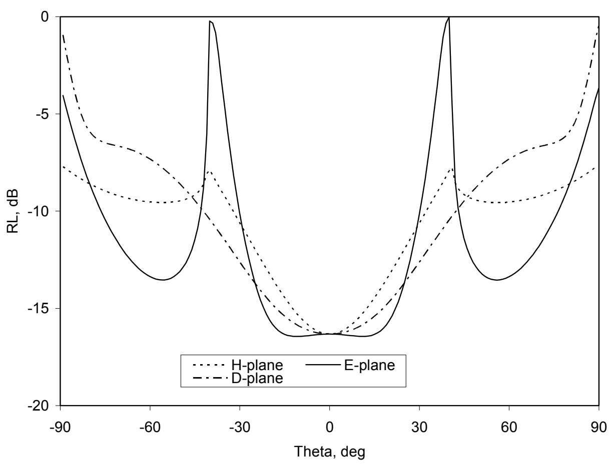


Fig. 4. Floquet return loss of a slot-fed patch array versus scan angle. Cell size =0.6 × 0.6, patch size=0.24 × 0.47, slot size =0.017 × 0.25, patch side ε_r =2.53, thickness=0.058, feed side ε_r =9.8, thickness=0.026, 50 Ohms feed line. All dimensions are in wavelength in free space.

Figure 8 shows the active return loss of the elements in the 15×15 elements patch array with uniform and tapered distributions, respectively. For the tapered array, Gaussian amplitude distributions with 10 dB taper for both planes were considered. For the plots, elements were numbered according to the numbering scheme depicted in Fig. 2. Four cases were considered as indicated at the inset of Fig. 8. The elements were designed to have about -16 dB bore-sight match under Floquet excitation. It is found that the active return loss varies from element to element. In particular, elements near the edge have noticeably different return losses than the rest. For the

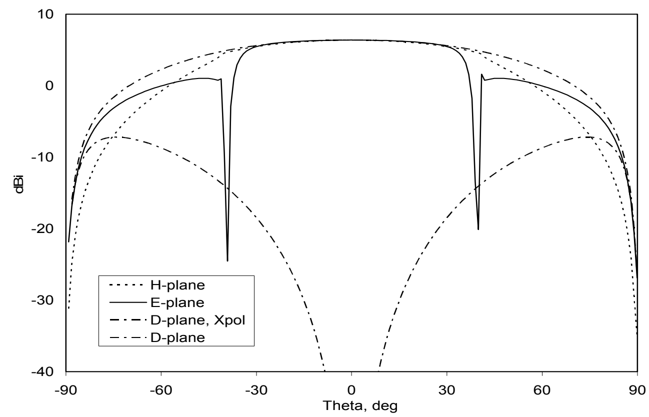


Fig. 5. Active element pattern cuts of the array. Element dimensions are sane as in Fig. 4.

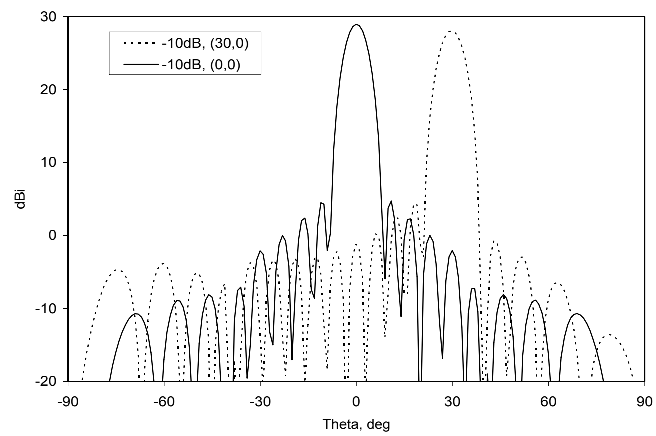


Fig. 6. Radiation pattern of 15 x 15 element patch array.

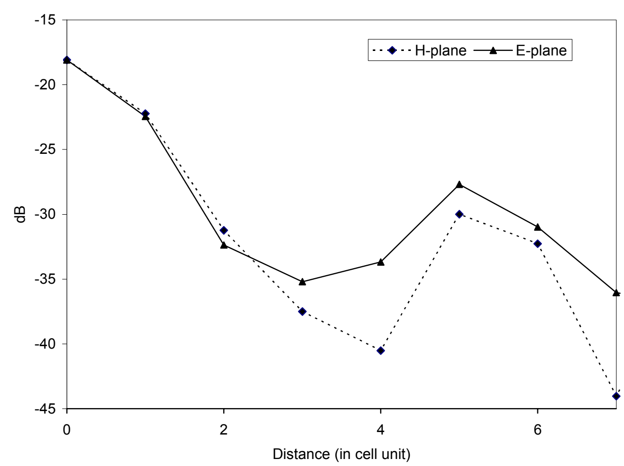


Fig. 7. Mutual coupling between patch elements in array environment. Patch dimensions are in Fig. 4.

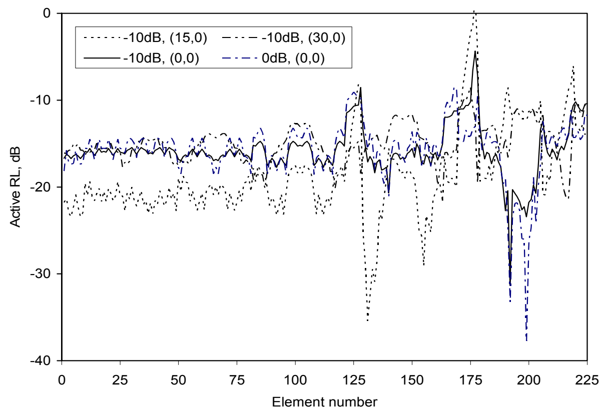


Fig. 8. Active return loss of the 15 x 15 element patch array in Fig. 3. The patch dimensions are in Fig. 4. The element numbering scheme is shown in Fig. 3.

bore-sight scan, the return loss for most of the elements lies below –15 dB. The return loss deteriorates for the off-bore-sight scans.

VI. Conclusions

In this paper we demonstrated the GSM approach for analyzing multilayer finite array. We considered a finite slot-fed patch array as an example. The mutual coupling between the elements, active element pattern, active return loss and array patterns were computed and results are shown. It is found that the mutual coupling is stronger between the E-plane elements than H-plane elements. The active return loss is substantially different for the elements near the edge, than the elements at the center region of the array.

The GSM approach is a modular approach as compared to an integrated approach. Computationally, the GSM approach for finite array analysis is much more efficient than FEM and FDTD approaches, because the problem size of a GSM is limited to a cell only. Furthermore, the matrix size of the MoM based GSM approach is much smaller as compared to a grid-based approach. However, a grid-based approach is more versatile because it can be applied to non-periodic geometries also without additional complexity.

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