An Algorithm for Solution of the Inverse Electromagnetic Liquid Metal Confinement Problem

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Abstract

The design of an electromagnetic levitation system for large amounts of liquid metal requires the solution of an inverse field problem. The objective is to calculate the electromagnetic pressure distribution which corresponds to a specified metallo-static head. This paper describes an algorithm for the design of coil systems which will produce the desired pressure distribution. The technique is illustrated by the design of a torroidal levitation system.

Keywords: electromagnetic confinement, liquid metals, pressure distribution, design methodology, algorithm.

1 Introduction

Defects found during the manufacture of equipment which require ultra-pure and/or highly reactive metals may be traced to contamination that occurred during their processing in the liquid state. Sources of the contaminants, in turn, can be often attributed to the vessels for the liquid materials. Thus, elimination of contact between the metal and its container would be highly advantageous and represents a goal that could be achieved by induction levitation.

In general, present-day electromagnetic levitation techniques of liquid metals are restricted to small amounts, typically in the order of few grams [1]. This can be attributed to the use of confinement systems which have inherent field singularities. That is, somewhere on the surface of the molten mass there is a zero, or negligible, amount of electromagnetic pressure to counter balance the metallo-static head. Consequently, at some location, which often coincides with the requirement of highest pressure, only the weak forces of surface tension are available for the support of molten material. This places serious limitation on the weight which can be levitated in a stable manner. As a result the technique is more or less confined to metallurgical research laboratories, and practical industrial applications which require the suspension of significant amounts of metal have not received sustained attention to date.

It has been demonstrated, both theoretically and experimentally, that multiple frequency excitation system can confine large amounts of liquid metal [2]. The physical realization of such a system requires the solution of a difficult inverse electromagnetic problem which can be stated as:

For a given geometry, what are the exact locations and magnitudes of excitation current sources which will generate a prescribed pressure distribution?

To the best knowledge of the author, a unique solution to this problem is not guaranteed since a number of alternate excitation systems can produce identical final results. Design of a practical system is beset by a confusing array of alternatives which are difficult both to realize and evaluate in a systematic manner. Consequently, the main goal of this paper is the development of a design technique which determines the locations of the individual turns within a coil.

A solution to the magnetic problem is an integral part of the algorithm, for which one of a number of alternate methods may be employed. One of these is the finite element method of Reference [3]. The purpose of this paper is to present a simple technique based on both easily identified physical parameters and clear cut design objectives. The approach takes into account limitations imposed by concerns such as the thermal cooling and physical protection of the coils. At the same time it specifies, a priori, a maximum value for the magnitude for the deviation for the confined shape from that specified. By relating the error to various spatial harmonics it is expressed in a physically meaningful manner The present technique combines an analytical approximation for the high frequency magnetic field with a linear optimization method which drives the solution towards the appropriate displacements of the coil windings. The resulting solution for the shape of a freely levitated body is expected to be robust since the effect of surface tension is considered to be negligible. In this case, both bulk confinement and surface stability will be realized through electromagnetic forces. Any additional effect due to surface tension will further aid confinement.

2 Design criteria for shape compliant electromagnetic pressure fields

Principles for stable electromagnetic levitation of large amounts of molten metal have been investigated and successfully implemented [2]. Its basic features can be summarized as:

a) the provision of a magnetic field which creates a metallo-static pressure distribution in conformity with the shape of the desired body,

b) the assurance of bulk stability for the liquid mass, such that it is prevented from falling apart due to the appearance of disturbances,

c) the automatic maintenance of surface stability which otherwise would lead to the eventual leakage of material and the destruction of confinement.

Implementation of these criteria lead to the design of multi-frequency confinement systems which both eliminate the presence of singularities in the pressure field and ensure surface stability. The removal of singularities is achieved by the superposition of pressures created by independent electromagnetic fields. These are produced by currents having different frequencies in a number of excitation coils. Surface stability is ensured by the mutual orthogonality of the fields [4].

3 Design considerations

Electromagnetic body forces produced by an interaction between the magnetic field and the current in a conducting medium are given by:

$$\bar{F} = \frac{1}{2} Re\ (\bar{J}^{*} \times \bar{B}) \qquad (1)$$

where: F is the body force (N/m^3), J* is the complex conjugate of the current density (A/m^2), and B is the flux density (Wb/m^2). When a conducting body is placed in an alternating magnetic field, confinement forces are produced by the tangential component of the flux density B_T, and the induced current $\bar{J}_i$. The normal component of the flux, B_n, and $\bar{J}_i$ produces a tangential force along the free surface of the body.

The principle objective of a design procedure is to match the magnitude of electromagnetic pressure with the prevailing metallo-static head everywhere on the surface. The tangential forces cause the liquid to recirculate within the body. When these flows are turbulent, waves are produced on the surface to reduce the overall stability of the confinement system [5]. Consequently, it is highly desirable to eliminate, or at least minimize, the magnitudes of B_n. This can be achieved by using high excitation frequencies which confine the magnetic flux within thin boundary layers immediately below the free surface.

Liquid metal processing is associated with high magnitudes of heat flux, and it is necessary to protect water cooled excitation coils against thermal damage. While the actual thickness of the thermal insulation depends upon the characteristics of the metal, practices of the induction heating industry can be used as guidelines for design. The accommodation of insulation introduces substantial gaps between the coils and the load which can seriously limit the magnitudes of magnetic flux. Hence, their thickness must be controlled to a bare minimum needed for coil integrity. With these considerations in mind, a general design approach for the shapes and locations of excitation coil is now investigated.

4 Solution of the inverse field problem

The required confinement field for a given geometry is determined by the variation of the metallo-static head. The final result of the analysis is known from the start, and the design task is reduced to determination of coil geometries and appropriate excitation current magnitudes. In this section, a solution to the inverse problem is described in the form of a series of design steps which are illustrated through the design of a levitation coil for an aluminium torus.

Step 1 *Preliminaries*

A confinement scheme for the desired shape and weight is first proposed. At this stage, it is necessary only to ensure qualitatively that all points on the free surface have tangential magnetic components. Should a single coil be inadequate to generate such a field, a multiple coil excitation system must be considered. The frequency of each coil current is different, hence the overall pressure distribution is the sum at the pressures contributed by each individual field.

Example: Consider the levitation of a torroidal aluminium ring of Figure 1, with the attributes specified in Table I.

PROPERTY	QUANTITY
Torus Diameter	200 mm
Ring Diameter	20 mm
Density	2700 kg/m^3
Torus Weight	0.533 kg

Table I Object geometry and weight

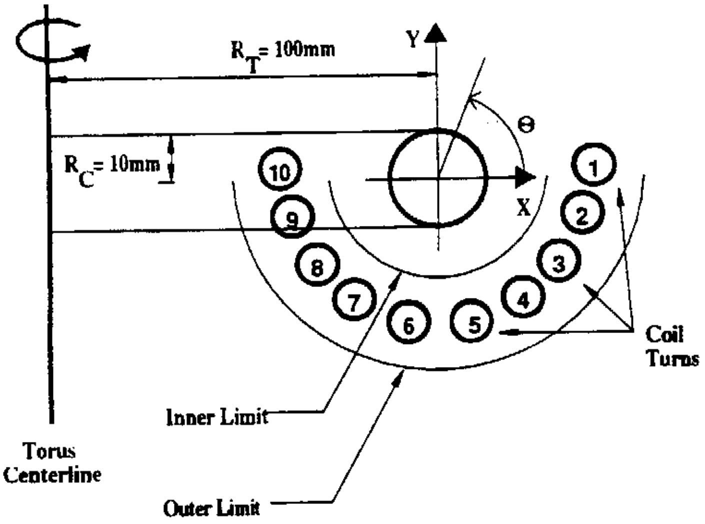

Figure 1 Relationships between load, coil geometry and limits

Since the torus has two planes of symmetry the design of the coil system is simplified, and the problem becomes two dimensional. When a high frequency current carrying single turn is wound parallel to the ring, a magnetic field is produced which has a tangential component everywhere on the surface. A coil, consisting of a set of turns placed at locations as yet to be defined, can be used to produce a pressure distribution that meets the requirements for levitation. Such a coil, shown in Figure 1, is initially assumed to have 20^0 spacing between its turns. It is now assumed that this estimate for coil design will result in a reasonable pressure field which can be improved upon by an iterative process. The purpose of this paper is to develop an algorithm that realizes the design objective for the generation of a compliant pressure field.

Step 2 *Inner limit for coil locations*

When the estimate for the numbers and shapes of the initial coils is completed, it is necessary to establish the minimum separation distances between these coils and the liquid metal. A first order analysis of the cooling requirement for the levitating coils will provide a guideline for minimum safe clearances. This establishes an inner limit for the location of the coils (See Figure 1).

Example: Since the temperature of a typical aluminium alloy melt is generally below 700 °C thermal insulation of the coils becomes an easy task. The magnitude of the heat flux at the coils can be estimated by the equality between the radiated heat flux from the ring and that conducted through the insulating refractory:

$$Q_r = \sigma\epsilon(T_s^4 - T_i^4) \qquad (2)$$

$$Q_c = k\frac{T_i - T_o}{\Delta X} \qquad (3)$$

where: Q_r=radiated heat flux (W/m^2), σ= Boltzman's constant, ϵ=emissivity of Al, T_s=Al temperature (K), T_i=insulator temperature facing the hot metal (K), T_o=coil temperature (K), k=thermal conductivity of the insulation (W/mK) , ΔX= thickness of the insulation (m). For a 10 mm thick insulation, the substitution of typical values given in Table II, results in approximate values for the inside temperature T_i=350 °C and a heat flux Q_c=6000 W/m^2. These values present no difficulty for water cooled coils.

PROPERTY	VALUE
Insulation thickness(ΔX)	10 mm
Thermal conductivity (k)	0.2 W/mK
Al temperature (T_s)	1000 K
Coil temperature (T_o)	300 K

Table II Approximate thermal parameters

A further allowance of 10 mm between the refractory and the ring is sufficient clearance for the disturbances caused by possible instabilities. Thus, the minimum inner limit is placed at 20 mm with respect to the surface of the ring.

__Step 3__ Outer limit for coil locations

The magnitude of magnetic flux decreases rapidly from a coil and in order to realize the required magnitude at the surface of the object, coil currents are in the order of several hundreds of amperes. This consideration leads to an outer limit for the location of the coils. In order not to exceed reasonable limits for power supply ratings, an outer limit for coil locations is in the vicinity of the inner (see Figure 1). The coils are placed between these two limits at arbitrary locations with assumed values for respective currents. As the solution of the inverse problem proceeds, the distance between the limits decreases to the overall width of the coils and defines their final positions.

Example: The individual turns of the coil, at the end of the design, will not be at uniform distances from the ring. Thus, the outer limit must be set far enough to allow sufficient space for their movements as the algorithm seeks the minimum error in pressure distribution around the ring. The initial arbitrary location for the outer limit is set at a distance of 60 mm from the surface. The turns of the coil are initially distributed uniformly along a 50 mm radius arc, centred on the torroidal ring of Figure 1.

__Step 4__ Target pressure distribution

The shape of the levitated object determines the target pressure distribution. The required pressure is determined by the density of the load and the depth of the metallo-static head at every point on the surface. This is the target distribution which ***must be satisfied*** by the final summation of all pressure distributions created by the magnetic fields.

Example: The metallo-static pressure distribution for the torroidal ring geometry is given by the simple relationship:

$$P = \frac{1}{2} P_o (1 - \sin \theta) \qquad (4)$$

where: $P_o=2\rho g R_C$ (N/m^2), R_C is the radius of the ring (m), θ (rad) is an angle measured counter clockwise and subtended from the centre of the ring. For the present case, P_o=529 N/m^2.

__Step 5__ Magnetic field produced by a single turn

The magnetic field produced by each turn within each coil is calculated either analytically or numerically depending on the configuration. Since high excitation frequencies are used, the depths of field penetrations are small in comparison with load dimensions. Consequently, a perfect conductor is a reasonable assumption for the load. This allows the solution of a DC magnetic field to represent the eddy current problem. The magnetic vector potential on the surface of the object becomes an arbitrary constant whose value set to zero. The normal component of the field does not penetrate, and only the tangential vector component exists on surface. The strength of the current sheet on the ring surface provides an estimate for the magnitude of the induced load current.

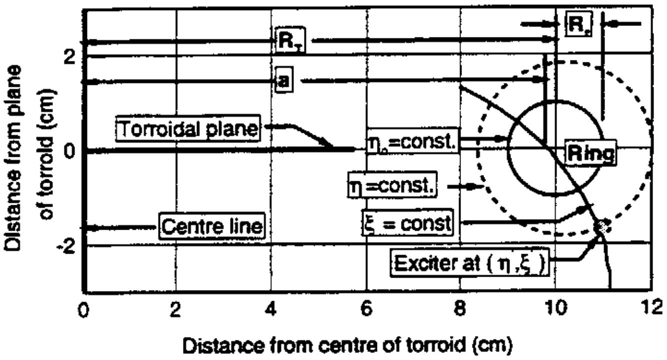

Figure 2 Definition of the torroidal coordinate system

Example: The tangential component of the magnetic field due to a current carrying loop located parallel to the ring is given by [6]:

$$B_{\zeta\, ij} = \frac{\mu_o\, I_j \sinh\eta' \,(\cosh\eta_o - \cos\zeta)^{3/2}}{\pi\, a \sinh\eta_o \,(\cosh\eta' - \cos\zeta')^{1/2}} \cdot \sum_{n=0}^{\infty} \frac{P^1_{n-1/2}(\cosh\eta')}{(1+\delta_{0n})\, P^1_{n-1/2}(\cosh\eta_o)} \qquad (5)$$

where: $B_{\zeta ij}$ is the magnetic flux density produced by the j-th turn of the i-th coil, I_j is the rms value of the current in the j-th turn, R_T is the radius of the torus, ζ' and η' are the torroidal coordinates of the loop, η_o and ζ are the torroidal coordinates for a point on the surface of the ring (see Figure 2), $P^1_{n-1/2}$ is the torroidal ring function [7]. Definition of the torroidal coordinates are shown in Figure 2, and the following relationships hold amongst the geometrical variables [8]:

$$a = \sqrt{R_T^2 - R_r^2} \qquad (6a)$$

$$\cosh\eta_o = R_T/R_r \qquad (6b)$$

Step 6 *The total magnetic pressure*

The magnetic field due to one coil is the sum produced by its individual turns. Since the coil frequencies are different, the fields become completely independent of each other. Under these conditions, the total magnetic pressure P_{mag} is given by:

$$P_{mag} = \sum_{i=1}^{N_c} \frac{1}{2\mu_o} \left(\sum_{j=1}^{n_i} B_{\zeta\, ij} \right)^2 \qquad (7)$$

where: N_c= number of coils, n_j= number of turns in the j-th coil.

Example: A ten turn coil has been selected for levitating the torroidal ring. For a given spatial distribution, between the inner and outer limits, their field contributions were determined by evaluating Eqn. 5 on the surface corresponding to the torroidal coordinate η_0 (see figure 2). The total pressure distribution is determined by Eqn. 7.

Step 7 *Error criterion*

The Fourier series representations of the target and electromagnetic pressure distributions allows the formulation of a physically meaningful error criterion. This approach identifies clearly the total lifting force as the DC term of the series, while higher order terms can be related to the shape of the object. The differences between the two pressure series indicate the required adjustments in:

a) the magnitudes of the excitation currents through the DC components,

b) the locations of the of the current loops through the spatial harmonic terms.

The criterion for the equality of total electromagnetic lifting force and the weight of the liquid metal mass can be satisfied through adjustments in the current magnitudes. The differences in spatial harmonics can be used as the driving term for the systematic re-positioning of the coil conductors for minimization of the harmonic rms error.

Example: The required pressure distribution for the torus, as given by Eqn. 4, is already in its Fourier form. The electromagnetic pressure given by Eqn. 7 is transformed into its Fourier components as:

$$P_{m\,DC} = \frac{1}{2\pi}\int_0^{2\pi} P_{mag}\, d\theta \qquad (8)$$

$$P_{m\,1s} = \frac{1}{2\pi}\int_0^{2\pi} P_{mag} \sin\theta\, d\theta \qquad (9)$$

$$P_{m\,1c} = \frac{1}{2\pi}\int_0^{2\pi} P_{mag} \cos\theta\, d\theta \qquad (10)$$

$$P_{m\,n} = \frac{1}{2\pi}\int_0^{2\pi} P_{mag}\, e^{jn\theta}\, d\theta \qquad (11)$$

where: $P_{m\,DC}$ is the average value of the magnetic pressure (N/m^2), $P_{m\,1s}$ and $P_{m\,1c}$ are its fundamental components (N/m^2), $P_{m\,n}$ is the magnitude of the n-th harmonic distortion term (N/m^2). Eqn. 8 and Eqn. 9 must be made identical to the two terms of Eqn. 6 respectively. Eqn. 10 is related to a shift in the location of the pressure centre in the X direction. Since this will result in a sideways displacement of the liquid metal, its magnitude must be reduced to the smallest possible minimum value. Eqn. 11 represents the deviation from the desired distribution. Its rms sum is a measure of the overall error in shape, and the purpose of the design method is to reduce this to as small a value as possible.

Step 8 *Error minimization*

Successive perturbation analyses in the location of each turn indicate the direction of its displacement needed for the minimization the pressure phase shift and overall spatial rms harmonic distortion. In this analysis, the location of any given turn is considered to be the centre of a small circle. The pressure field is recalculated for each displacement of a conductor along the circumference of the circle and the rms harmonic error for the total pressure is re-calculated with the positions of all other turns being unchanged. The magnitude of the error with each position is stored in memory along with the associated geometrical data. After the completion of one perturbation cycle, the conductor is positioned at the location corresponding to the minimum error. When the calculated new position is outside the assumed inner or outer limit, the conductor is placed on it and is prevented from crossing it. The analysis proceeds from turn to turn, and upon completion of one cycle for all coils, the total geometrical error is less than that at the start. The continuous decrease of error with iteration is caused by allowing physical displacements only if it results in the reduction of the local error. Next, the distance between the inner and outer limits is reduced, where possible, to narrow the band of coil locations. Thus, with each iteration an increasingly better estimate is obtained for the coil geometries.

Example: Application of the above minimization technique for the design of a 10 turn confinement coil shown in Figure 1 results in a reduction of the harmonic error from an initial value in the order of 40% to less than 2.5% after the completion of 16 iterations. The final positions of the turns are shown in Figure 3.

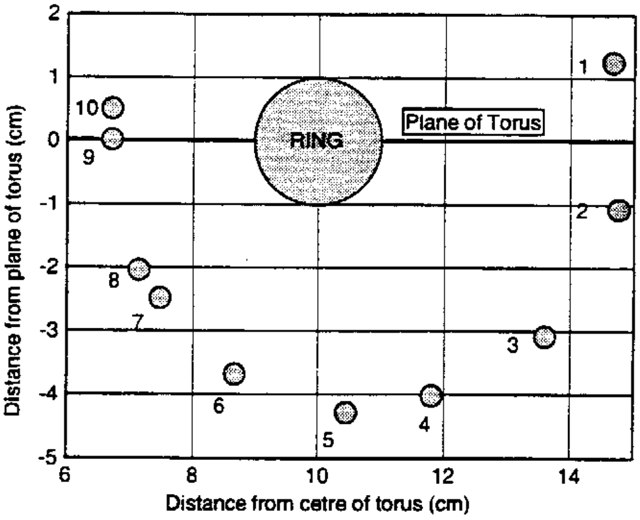

Figure 3 Final locations of turns 1 to 10 with respect to the torroidal plane and centre

The calculated value of the coil current is 335 A rms and an almost sinusoidal pressure distribution is obtained as seen in Figure 4. The results of a finite element analysis described in Section 5 below are also included to confirm the results. The small distortion in the waveform is due to the second harmonic pressure term which will cause a slight vertical elongation of the liquid torroidal ring.

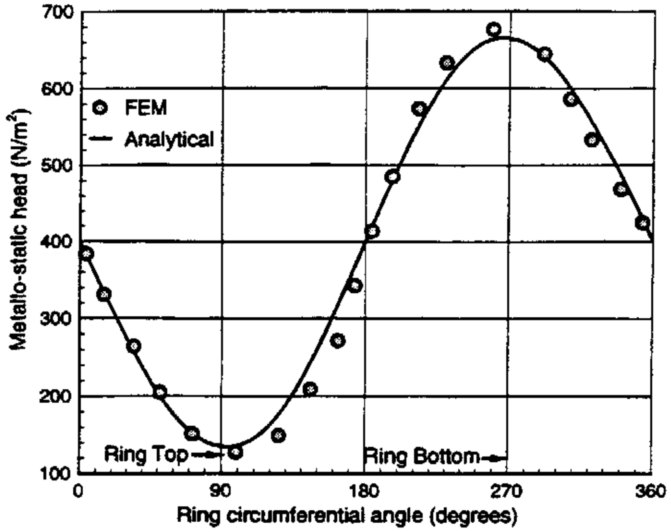

Figure 4 Electromagnetic pressure distribution

The variation of the vertical component of the force on the ring is calculated from the relationships given in Reference [7] and is shown in Figure 5. It is maximum at an angular position of 267^0 which almost coincides with the bottom of the ring. The magnetic field completely surrounds the ring to produce small negative forces on the top surface. The net force differential between the lower and upper parts of the ring creates the almost perfect sinusoidal form for net effective lifting force of 5.22 N.

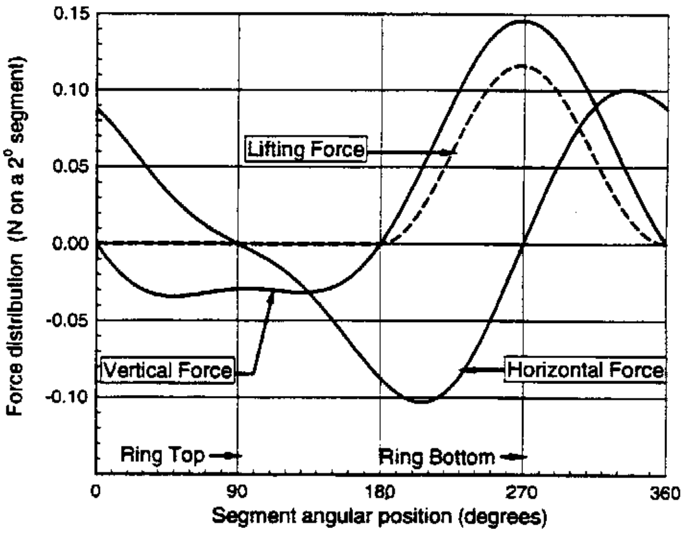

Figure 5 Force distributions on the ring

5 The design algorithm

The inverse method described above forms the basis of a design algorithm whose basic steps are described by the following pseudo-code:

Algorithm confine

{Function: solves for the pressure distribution required for the levitation of bulk amounts of liquid metal}

```
Start
 Procedure basic_parameters
 Procedure initial_geometry
 Procedure initial_limits
 do while spatial_error>target error
   do while coil_no ≤total_coil_no
     do while turn_no ≤ coil_turns
       do while total_lift ≤target_lift
         Procedure field_single_turn
         Procedure field_all_coils
         Procedure pressure
         Procedure Fourier
         calculate total_lift
         end while        ! end force calculation
       calculate spatial_error
       if spatial_error > target_error
         Procedure turn_position_purturbation
         end if
     end while            ! end calculations for each turn
   end while              ! end calculation for each coil
   Procedure limits
   end while              ! end calculation of error
Stop
```

The details of the procedures called for in the above algorithm change from problem to problem. These may be either solutions to analytical field problems or the results of numerical solutions. The choice regarding the method of solution does not, however, alter the overall algorithm.

6 Validation through finite element analysis

The shape of the magnetic field and the validity of the electromagnetic pressure profile can be confirmed by a number of methods. One of these is a coupled circuit approach which uses the self and mutual inductances for a large number of subdivisions of the load and the coils. This method produces good results and has been used successfully in a perturbation analysis of the shape of the liquid metal. Another is the calculation of the magnetic field using the finite element method. A commercial software package [9] (ANSOFT) was used for the solution for the high frequency eddy current problem. In order to minimize the impact of current penetration into the ring, the excitation frequency of the 335A coil was set to 500kHz. The shape of the magnetic field is shown in Figure 6. This result is consistent with the objectives of the design, and the calculated magnetic pressures are shown in Figure 4.

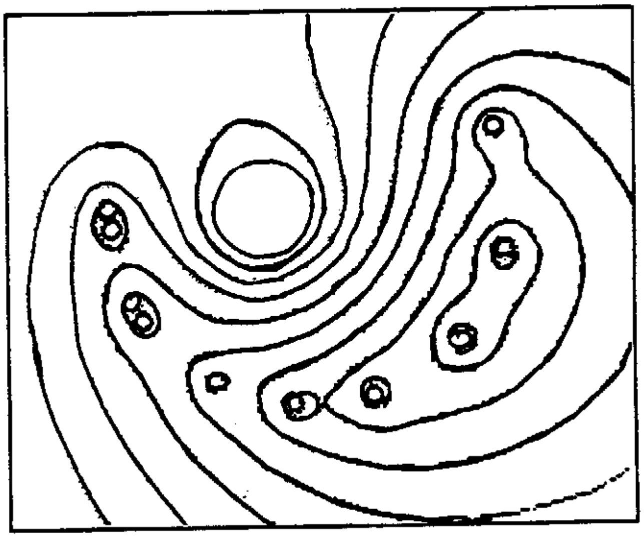

Figure 6 Spatial distribution of the electromagnetic field

7 Conclusions

The purpose of this paper was the development of an algorithm for the inverse electromagnetic field problem encountered in the design of levitation systems for large amounts of liquid metals. It is shown that an optimization procedure can be applied to determine the locations of lifting and confinement coils which define a specified metallo-static pressure distribution. The main thrust of the procedure relates to the adjustment of inner and outer limits which bound the coil locations. The relocation of each turn of a high frequency coil system is guided by the successive minimization of an error term. This is obtained from the differences between the space harmonics of the Fourier series for the desired shape and the calculated magnetic pressure field. The steps of the design procedure are illustrated by the development of a confinement system for a torroidal ring of liquid aluminium. A finite element analysis of the final design step shows that the described methodology produces excellent results for the pressure field. The various stabilities of the system have not been investigated in this paper, and are the subjects of a paper in preparation.

8 References

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