A User-friendly Computer Code for Radiated Emission and Susceptibility Analysis of Printed Circuit Boards

Jan Carlsson[1] and Per-Simon Kildal[2]
[1] Swedish National Testing and Research Institute, Borås, Sweden
[2] Chalmers University of Technology (CTH), Gothenburg, Sweden

Abstract - A user-friendly computer code, PCB-MoM, that is intended to be used in EMC applications for predicting radiated emission and susceptibility of printed circuit boards (PCB) is presented. The formulation is based on an electric field integral equation (EFIE) expressed in the frequency domain. The EFIE is solved by the method of moments using two-dimensional pulse basis functions and one-dimensional pulse test functions. In order to incorporate dielectric material in the substrate a spectral domain formulation is used. The code has been validated by comparison with previously published results and results obtained by other methods and codes.

I. Introduction

Knowledge of emission and susceptibility of printed circuit boards is important in order to control the electromagnetic compatibility of an electronic device. The main advantage of computation compared with measurements, is that the former can be done already during the design phase of the device. Thereby, a costly redesign due to a failure in passing an EMC test can be avoided. Still, it is important to realise that computation is not a substitute to EMC tests, but rather a complement. As a complement the computation can give us insight into different coupling phenomena etc. on the circuit board, which can be very difficult to understand through measurements. It is also very easy by computation to test different methods for reducing the radiated emission or increasing the susceptibility level. As an example, the re-routing of the clock signals on a printed circuit board in order to reduce the radiated emission, can hardly be done by experiments.

The purpose of this paper is to present a user-friendly computer program called PCB-MoM that can be used for analysing planar conducting structures, such as PCB, regarding radiated emission and susceptibility as well as crosstalk. The formulation used by the program is based on an electric field integral equation (EFIE) expressed in the frequency domain. In order to solve the EFIE the method of moments [1] is used. The formulation and choice of basis and test functions is based on that in [2]. The dielectric material in-between the structure and the ground plane is taken into account by a spectral domain technique similar to the one presented in [3]. The spectral domain approach is speeded up by making use of asymptotic extraction, as explained in [4]. The circuit board is treated as a grounded single layer structure. The approach is readily extended to a multilayer structure of arbitrary number of layers by means of the G1DMULT algorithm presented in [5].

II. Theory for conducting plane surfaces in homogenous region

Referring to Fig. 1, we know that the incident field, $\mathbf{E}^{inc}$, will induce a surface current on the conducting structure and that the surface current in turn will produce a scattered field, $\mathbf{E}^{scat}$. If the structure is assumed to be a perfect conductor, we know that the tangential component of the electric field at the surface of the structure must vanish, i.e. $\left(\mathbf{E}^{inc}+\mathbf{E}^{scat}\right)_{\tan}=0$. If we want to consider a finite conductivity, the above formula becomes:

$$\left(\mathbf{E}^{inc}+\mathbf{E}^{scat}\right)_{\tan}=Z_s\mathbf{J} \tag{1}$$

where Z_s is the surface impedance (in Ohms) and $\mathbf{J}$ is the induced surface current density (in A/m) on the structure.

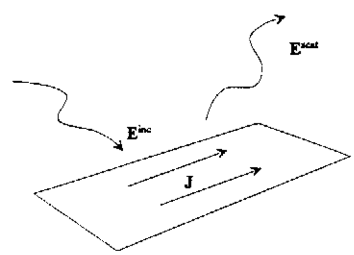

Fig. 1. Planar conducting structure subject to an incident electromagnetic field.

For the case of thin planar conducting structures, we can assume the conducting sheet to be infinitesimally thin and that the current only can flow in two orthogonal directions. Without loss of generality, we can assume the conducting sheet to be placed in the xy-plane and the two orthogonal current directions are in the x- and y-directions. From any standard textbook on electromagnetics, e.g. [6], we can find expressions for the scattered field from a surface current density expressed in terms of a vector and a scalar potential (2).

$$E_x^{scat} = -j\omega A_x - \frac{\partial\Phi}{\partial x},\ E_y^{scat} = -j\omega A_y - \frac{\partial\Phi}{\partial y} \quad (2)$$

where

$$\begin{cases} A_{x,y} = \dfrac{\mu}{4\pi}\displaystyle\iint_S J_{x,y}\frac{e^{-jkr}}{r}ds \\ \Phi = \dfrac{1}{4\pi\varepsilon}\displaystyle\iint_S \sigma\frac{e^{-jkr}}{r}ds,\ \sigma = \dfrac{j}{\omega}\left(\dfrac{\partial J_x}{\partial x} + \dfrac{\partial J_y}{\partial y}\right) \end{cases} \quad (3)$$

A is the vector potential, Φ is the scalar potential and σ is the charge density which is related to the current density through the continuity equation. r is the distance between the source point (the current) and the field point. For field points on the surface of the structure, r is given as: $r = \sqrt{(x-x')^2 + (y-y')^2}$, where (x',y') is the source point and (x,y) the field point. In order to solve the coupled integral equations for J_x and J_y obtained by enforcing the boundary conditions in (1), we use the method of moments [1] and the same type of basis and test functions as introduced by Glisson & Wilton [2]. The first step in using the method of moments is to expand the unknowns in series of known basis functions with unknown coefficients. The choice here is to use so called pulse sub-domain basis functions. These functions are unity over a rectangular area and zero elsewhere. Thus, we express the current densities and charge density in the following way:

$$J_x = \sum_{n=1}^{N} J_{xn}\Pi,\ J_y = \sum_{m=1}^{M} J_{ym}\Pi,\ \sigma = \sum_{i=1}^{I}\sigma_i\Pi \quad (4)$$

where Π represents the two-dimensional pulse function. J_{xn}, J_{ym} and σ_i are the coefficients for the current and charge densities, respectively. The locations of the pulse functions for the current and charge elements are shown in Fig. 2.

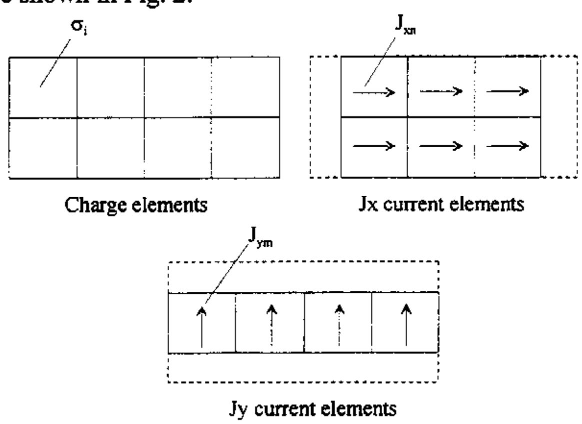

Fig. 2. Definition of current and charge elements.

Insertion of the expansions (4) in the integral equations gives:

$$E_x^{inc}(x,y) = \frac{j\omega\mu}{4\pi}\sum_{n=1}^{N} J_{xn}F(S_{xn},r) + \frac{\partial}{\partial x}\left[\frac{1}{4\pi\varepsilon}\sum_{i=1}^{I}\sigma_i F(S_{ci},r)\right] + Z_s(x,y)J_x(x,y) \quad (5)$$

$$E_y^{inc}(x,y) = \frac{j\omega\mu}{4\pi}\sum_{m=1}^{M} J_{ym}F(S_{ym},r) + \frac{\partial}{\partial y}\left[\frac{1}{4\pi\varepsilon}\sum_{i=1}^{I}\sigma_i F(S_{ci},r)\right] + Z_s(x,y)J_y(x,y) \quad (6)$$

where $F(S,r) = \iint_S \frac{e^{-jkr}}{r}ds$, S_{xn} represents the area of Jx current element number n, S_{ym} the area of Jy current element number m and S_{ci} the area of charge element number i. The next step is to define two sets of testing functions, one for equation (5) and one for equation (6). The choice here is to use functions that are constants along a line in x- and y-directions, respectively, Fig. 3.

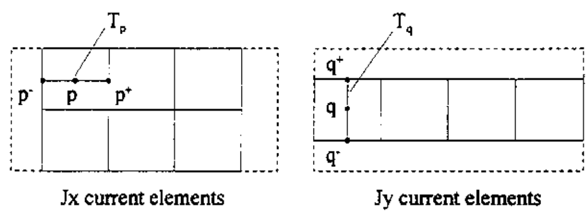

Fig. 3. Definition of test functions.

$$T_p = \begin{cases} 1, & \bar{p} \le x \le \overset{+}{p},\ y = y_p \\ 0, & \text{elsewhere} \end{cases},\ p = 1\ldots N \quad (7)$$

$$T_q = \begin{cases} 1, & x = x_q,\ \bar{q} \le y \le \overset{+}{q} \\ 0, & \text{elsewhere} \end{cases},\ q = 1\ldots M \quad (8)$$

Multiplying equation (5) by the test functions (7) and multiplying equation (6) by the test functions (8) and integrating over each test function gives a matrix equation of the form:

$$\begin{bmatrix} [E_x^{inc}] \\ [E_y^{inc}] \end{bmatrix} = \begin{bmatrix} [Z_{xx}] & [Z_{xy}] \\ [Z_{yx}] & [Z_{yy}] \end{bmatrix}\begin{bmatrix} [J_x] \\ [J_y] \end{bmatrix} \quad (9)$$

where the sub-matrices are defined by:

$[E_x^{inc}]$ is a column vector of dimension N and the elements: $(E_x^{inc})_p = \Delta p E_x^{inc}(p)$

$\left[J_x\right]$ is a column vector of dimension N and the elements: $\left(J_x\right)_n = J_{xn}$

$\left[Z_{xx}\right]$ is a matrix of dimension N by N and the elements:

$$\left(Z_{xx}\right)_{pn} = \frac{j\omega\mu}{4\pi}\Delta p F\left(S_{xn}, r_p\right) + \Delta_{yp} Z_{sp} +$$

$$+\frac{j}{4\pi\omega\varepsilon}\left\{\frac{1}{\Delta_{xcn}^{-}}\left[F\left(S_{cn}^{-}, r_p^{+}\right) - F\left(S_{cn}^{-}, r_p^{-}\right)\right] - \right.$$

$$\left. -\frac{1}{\Delta_{xcn}^{+}}\left[F\left(S_{cn}^{+}, r_p^{+}\right) - F\left(S_{cn}^{+}, r_p^{-}\right)\right]\right\}$$

$\left[Z_{xy}\right]$ is a matrix of dimension N by M and the elements:

$$\left(Z_{xy}\right)_{pm} = \frac{j}{4\pi\omega\varepsilon}\left\{\frac{1}{\Delta_{ycm}^{-}}\left[F\left(S_{cm}^{-}, r_p^{+}\right) - F\left(S_{cm}^{-}, r_p^{-}\right)\right] - \right.$$

$$\left. -\frac{1}{\Delta_{ycm}^{+}}\left[F\left(S_{cm}^{+}, r_p^{+}\right) - F\left(S_{cm}^{+}, r_p^{-}\right)\right]\right\}$$

and similar for the other sub-matrices. In deriving the above expressions we have used a finite-difference approximation for the derivatives of the currents in the expression for the charge density. Note that in the above expressions Z_{sp} and Z_{sq} are non-zero only for $n = p$ and $m = q$, respectively. Thus, the surface impedance effects only the diagonal elements in the matrices. They are also allowed to be complex. This means that we can model series impedances consisting of either a resistance and an inductance or a resistance and a capacitance (simulating components on a PCB). The excitation vector, $\left[E_x^{inc}\right]$, can represent either an incident field, for a radiated susceptibility analysis, or voltage sources, for a radiated emission analysis. For incident field excitation all elements of the excitation vector are non-zero and for voltage source excitation only one or a limited number of the elements are non-zero.

Equation (9) can easily be solved for the unknown current distribution by matrix inversion. However, special care must be taken when computing the matrix elements since the integrand involved in the integration of Green's function becomes singular for the self-terms (when the observation point is located within the source rectangle). Fortunately, the singularity is integrable and can be treated by changing to polar coordinates.

III. Including the ground plane

Equation (9) together with the matrix elements given above is valid for a conducting sheet (representing the conducting traces on a PCB) in the xy-plane and situated in free-space. In order to also include the case when the conducting sheet is placed over an infinitely large and perfectly conducting plane (z=constant) we use image theory. Since the image currents will be the opposite of the currents on the sheet we do not have to increase the size of the matrix equation, it is sufficient to modify the matrix elements according to.

$$\begin{bmatrix}\left[E_x^{inc} + E_x^{refl}\right] \\ \left[E_y^{inc} + E_y^{refl}\right]\end{bmatrix} = \begin{bmatrix}\left[Z_{xx} - Z_{xx}^{im}\right] & \left[Z_{xy} - Z_{xy}^{im}\right] \\ \left[Z_{yx} - Z_{yx}^{im}\right] & \left[Z_{yy} - Z_{yy}^{im}\right]\end{bmatrix}\begin{bmatrix}\left[J_x\right] \\ \left[J_y\right]\end{bmatrix} \quad (10)$$

where the matrices without superscript are the same as before and the matrices with the superscript "*im*" are computed in the same way as the corresponding matrix without the superscript but with the distance r changed from $\sqrt{(x-x')^2 + (y-y')^2}$ to $\sqrt{(x-x')^2 + (y-y')^2 + (2h)^2}$, when evaluating the integrals over the Green's function. h is the distance between the conducting sheet and the ground plane (i.e. the ground plane is assumed to be defined by the plane $z=-h$). The excitation vectors will be given by: $E_{x,y}^{inc} + E_{x,y}^{refl} = E_{x,y}^{inc}\left(1 - e^{-j2kh\cos\theta}\right)$ for the case of incident field excitation and will remain the same as for the case without a ground plane for the case of voltage source excitation. Thus, the matrix filling time will be approximately doubled when we have a ground plane, compared to the case without a ground plane, but the matrix inversion time will remain the same.

Another case of interest is when we have one or several connections from the conducting sheet to the ground plane. In order to solve this case we have to introduce currents in the z-direction as well. Following the same procedure that was used for the case when we only had x- and y-directed currents, similar expressions for the currents can be derived. Details can be found in [7].

IV. Including the substrate

In order to extend the above theory so that also planar structures on a grounded dielectric substrate could be analysed, the natural way would be to exchange the free space Green functions to that for a grounded dielectric substrate. However, this approach requires that slowly converging integrals of Sommerfelds type have to be evaluated, see e.g. [8]-[10]. Another approach is to perform a Fourier transform of the structure along the two uniform directions, i.e. to perform the computation in the spectral domain. This will give a spectrum of 1D field problems instead of one 3D field problem [5]. This approach will also result in slowly converging integrals, see e.g. [3], [11]. However, the convergence can be speeded up by extracting the asymptotic part of these integrals and treat this separately [12]. It can also be shown that the asymptotic part just as well can be computed in the spatial domain and in fact is equal to the impedance elements as given in Section III but computed with a permittivity equal to $\varepsilon_{eff} = (1+\varepsilon_r)/2$, [4]. Thus, in order to take the grounded dielectric substrate into account we only have to add a correction factor to the impedance elements already computed (using the permittivity $\varepsilon_{eff} = (1+\varepsilon_r)/2$). The correction factors for the impedance elements in (9) are:

$$\left(Z_{ij}\right)_{kl}^{diel-as} = \frac{-1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\tilde{E}_{ij}^{diel} - \tilde{E}_{ij}^{diel,as}\right)\tilde{J}_{jl}\tilde{T}_{jk}^{*}dk_x dk_y \tag{11}$$

where $i = x$ or y, $j = x$ or y, $k = p$ or q, $l = m$ or n. $\mathbf{E}^{diel,as}$ are the asymptotic field expressions and $\mathbf{E}^{diel}$ are the exact field expressions.

$$\tilde{f}\left(k_x, k_y\right) = \iint_S f(x,y)e^{jk_x x}e^{jk_y y}dxdy$$

represent quantities in the spectral domain and * denotes complex conjugate. The asymptotic field expressions in the spectral domain correspond, in the spatial domain, to the field from a single current element over a ground plane in a homogenous region. Expressions for the asymptotic part can be found in [4] and [12]. Exact field expressions can be found in e.g. [3]. The integrals (11) converge fast since the asymptotic field expressions approach the exact field expressions for large $k_x^2 + k_y^2$.

V. The user interface

The above theory has been implemented in a user-friendly computer program, PCB-MoM, that can be used on an ordinary PC running Windows 95 or NT 4.0. In using the PCB-MoM program, analysing a printed circuit board (PCB) is a three-step procedure: defining the geometry, performing a simulation and finally visualising the computed results. Each of these steps is devoted an own page in the program. Switching between these pages is done simply by clicking on the corresponding page tab, Fig. 4. In order to define the geometry the conducting segments on the PCB are simply drawn on the screen using the built-in CAD-like interface, Fig. 4.

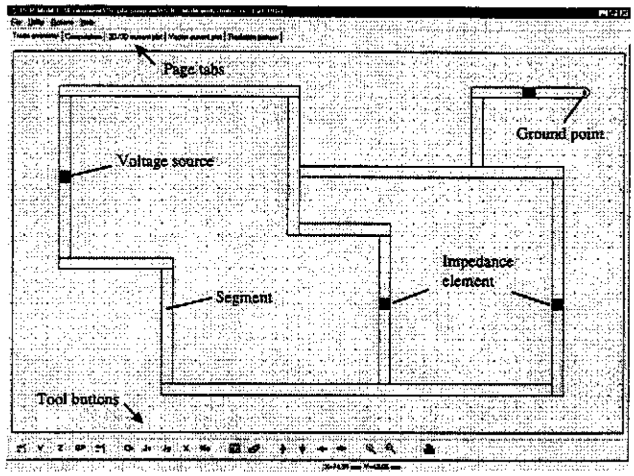

Fig. 4. The geometry definition page showing a simple structure.

The subdivision of the segments into current and charge elements is done automatically, but can also be set manually by the user. Lumped, discrete voltage sources and impedance elements can easily be defined by pointing out the locations in the layout. In the same way, ground points (i.e. metal connections to the underlying ground plane) can be defined. When the geometry is defined a simulation can be performed. All relevant parameters for the simulation is set on an own page, Fig. 5.

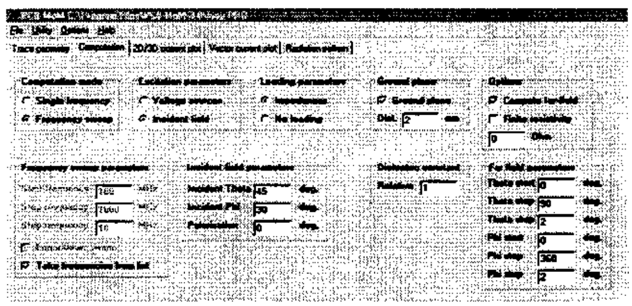

Fig. 5. Part of the computation page where parameters for the simulation are set.

On this page the excitation is either defined as voltage sources or as an incident plane wave. The existents of a ground plane and the dielectric constant for the region between the structure and the ground plane are also defined on the computation page. When a simulation is started the program will, if requested by the user, give a time estimate for how long the simulation will take. The time estimate is based upon constants saved in a file. Since the constants are dependent on the used computer they are created by the program through a calibration procedure. Before a simulation is started, the program analyses the input data and, if possible, a symmetrical matrix filling procedure will be used in order to reduce the simulation time. Fig. 6 shows the required time for filling both a full matrix and a symmetrical matrix for a structure in free space.

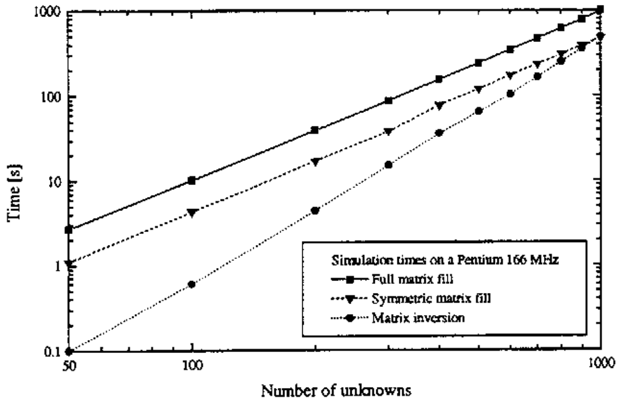

Fig. 6. Simulation times for a structure in free space on a Pentium 166 MHz.

The results from a simulation are primarily the current densities on the conducting traces. In addition the near-field in selected points and the far-field in selected angular ranges can be obtained. The computed current densities can be viewed in different ways, as a 3D plot, as a 2D plot or as a vector plot. Figures 7 and 8 show two different ways of visualisation of the current density.

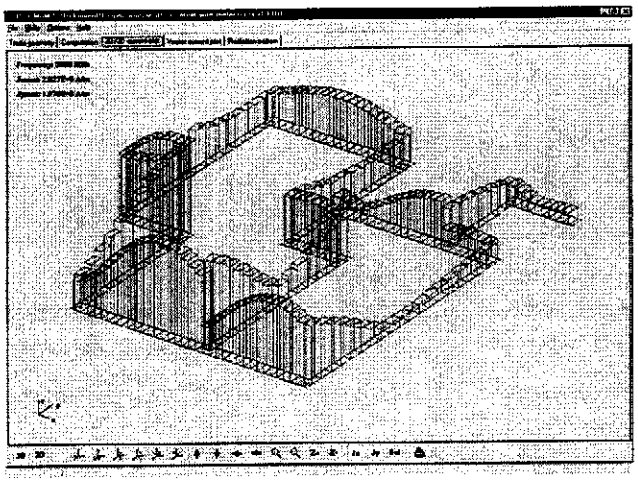

Fig. 7. Current density at 2 GHz for the structure in Fig. 4 shown as a 3D-plot.

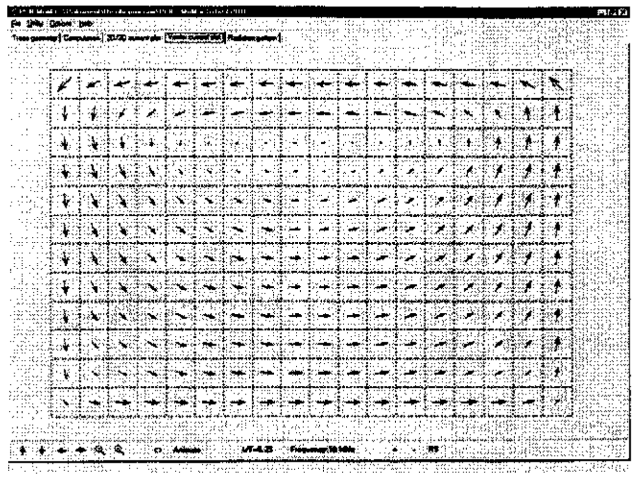

Fig. 8. Current density on a plate shown as a vector plot. Excitation is done by a current carrying straight conductor in the vicinity of the plate (not shown in the figure).

The computed radiation pattern in the far-field can be viewed as a polar plot. All computed quantities can also be listed in the program and are saved in ASCII-files. This makes it easy to perform further analysis or plotting using another program.

VI. Test cases and examples

The PCB-MoM program has been extensively tested against previously published results, against results obtained with other codes and methods, and against measurements. At low frequencies the code has been tested against results obtained with ordinary circuit theory. Results for intermediate frequencies have been tested against ordinary transmission line theory and multiconductor transmission line theory. High frequency results and problems involving dielectric material have been compared with published results obtained by other methods and also with computations done with other programs. As an example the computed and measured radiated emission from a simple PCB with a 10 MHz clock oscillator is shown in Fig. 9. For the computation it is necessary to know the output voltage from the clock oscillator at the desired frequencies. The voltage was measured with a spectrum analyzer. Since this measurement is not trivial, the uncertainty is quite high and it probably explains the disagreement between measurement and simulation. However, it is interesting to note that the overall behaviour of the radiated field is predicted well.

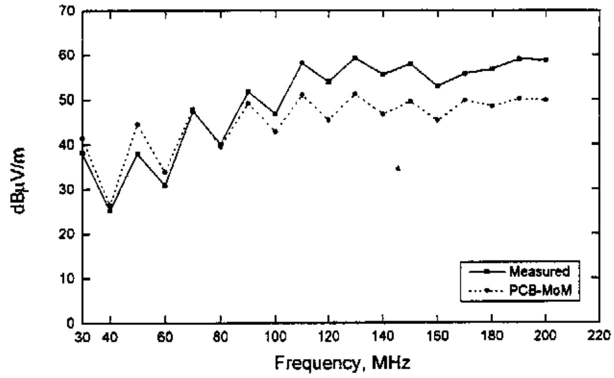

Fig. 9. Measured and computed radiated emission from a simple PCB with a 10 MHz clock oscillator.

The configuration in Fig. 10 was considered in order to verify the results against multiconductor transmission line theory . One of the lines in Fig. 10 was excited with a 1V voltage source and the other ends of both lines were terminated with 1 kΩ resistors. The computed quantity was the current through the R_f resistor in line 2. Using the multiconductor transmission line theory the per unit length parameters, inductance matrix L and capacitance matrix C, were first computed with LC-Calc [13] which is a finite difference program. The matrices were found to be:

$$\mathbf{L}=\begin{bmatrix}0.383 & 0.067\\ 0.067 & 0.383\end{bmatrix}[\mu\text{H/m}]$$

$$\mathbf{C}=\begin{bmatrix}30.0 & -5.25\\ -5.25 & 30.0\end{bmatrix}[\text{pF/m}]$$

The per unit length parameters were then used in BMTL [14] which is a finite difference time domain program that solves the multiconductor transmission line equations. In the BMTL program the conductors were divided into 50 elements and 8192 time steps were used for the computation. This gives a frequency resolution of approximately 11.4 MHz. In the PCB-MoM program the conductors were divided into 20 current elements along the length and 4 in the transverse direction. As can be seen from the results in Fig. 10 the agreement is good.

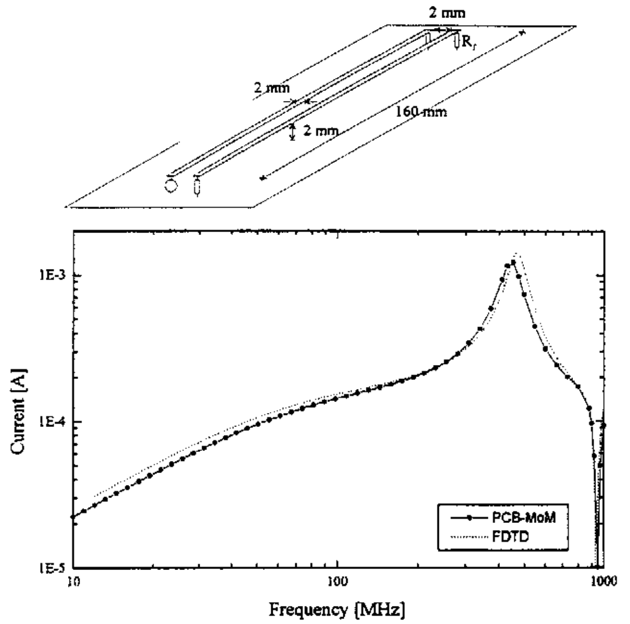

Fig. 10. Computation of crosstalk between two parallel lines that are terminated to a common ground plane. All resistors are 1 kΩ. The environment is free space. Computed quantity is current through the R_f resistor. Computation done with PCB-MoM and with multi-conductor transmission line theory implemented in a FDTD program.

As a test of the capability of computing the near field, the square plate in Fig. 11 was considered. The plate was excited with a normal incident plane wave with the E-field polarised along the x-axis. The computed quantities were the total field components E_x and E_z close to the surface of the plate. Since PCB-MoM gives the scattered field when the excitation is an incident field, the total field was computed as: $\mathbf{E}^{tot}=\mathbf{E}^{scat}+\mathbf{E}^{inc}$, where the incident field is given by: $\mathbf{E}^{inc}=\hat{x}e^{jkz}$. For the computation the plate was divided into 25 by 25 charge elements. The results in Fig. 11 agree very well with the results reported in [15] and the results obtained with the FDTD program XFDTD. In [15] two different method of moments programs and a UTD program were used. Also the E_x component, although not shown here, agrees very well with results obtained with XFDTD and results reported in [15].

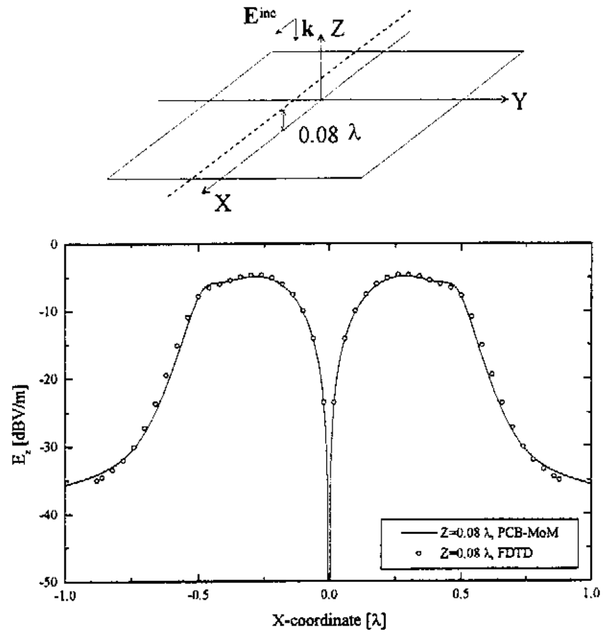

Fig. 11. Computed E_z along the line: $-\lambda < x < \lambda$, $y = 0, z = 0.08\lambda$ over a square plate at $z = 0$ excited with an incident plane wave. Plate dimension $\lambda \times \lambda$. Solid line represents PCB-MoM, circles the XFDTD program.

For testing the capability of treating conducting structures on a dielectric substrate the loaded rectangular loop shown in Fig. 12 was analysed. The loop was fed by a voltage generator with an amplitude of 1 Volt and an internal resistance of 50 Ω. The computed quantity was the current at the fed point and the result was compared with result obtained by the commercial FDTD program XFDTD. As can be seen in Fig. 12 the agreement between the two approaches is good. However, it can be noted that the current amplitude predicted by XFDTD around 700 MHz is slightly too large. It should not be higher than 20 mA due to the internal resistance of the source.

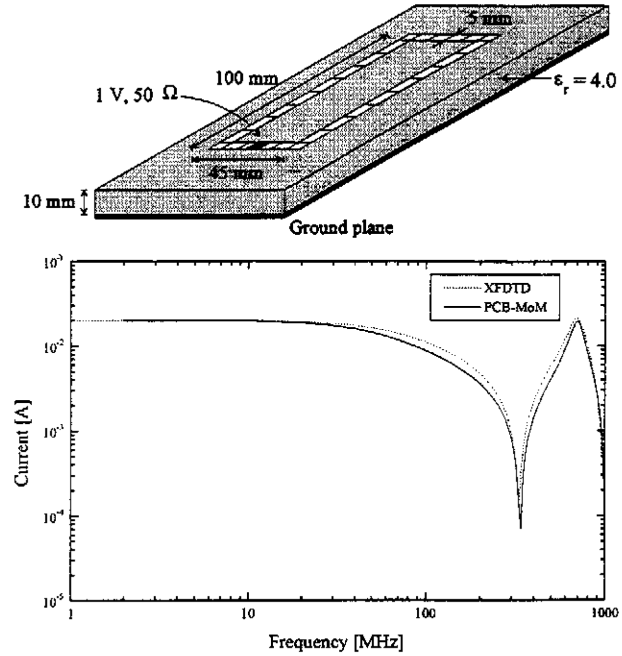

Fig. 12. Computed current amplitude at the feed point of a loaded rectangular loop on a grounded dielectric substrate with permittivity 4.0. Solid line represents PCB-MoM, dotted line the XFDTD program.

As another test of the capability of treating dielectric material the centre fed microstrip dipole in Fig. 13 was considered. The length of the dipole was varied and the input impedance at the feed point was calculated. The impedance was computed as $Z = U/I$, where U is the excitation voltage and I is the current in the current element where the voltage source is placed. Fig. 13 shows the impedance, real and imaginary parts, computed by the PCB-MoM program using 19 current elements. The agreement with results reported in [9] is found to be good. It can also be noted that the half and full wavelength resonances are predicted accurately.

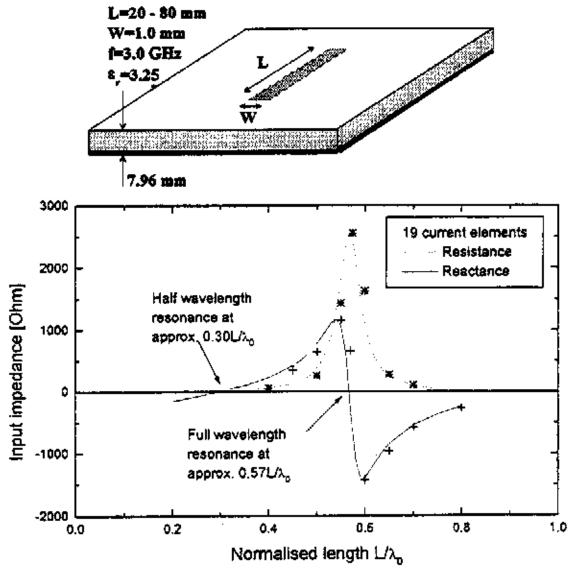

Fig. 13. Computed input impedance for a centre fed microstrip dipole with varying length. Stars and crosses represent data extracted from [9].

VII. Development history

From the beginning the program was aimed to be used for planar structures in free space and was later modified to treat also structures on a dielectric substrate. The free space routines and the user interface were all written in Delphi by Borland. Since the Pascal used in Delphi does not have support for complex numbers several routines had to be written for treating them. This also includes matrix inversion routines. The routines for treating dielectric material were written in C++, also by Borland, and added to the original program as a DLL (Dynamic Link Library).

VIII. Conclusions

We have presented a user-friendly method of moments program designed to be used in EMC applications where the interest is in determining the radiated emission and susceptibility of printed circuit boards. The intention has been to design a tool that can be of help to EMC engineers. The program has been extensively tested and the agreement with results obtained by other methods and codes has been found to be good. The code can be obtained from the first author (jan.carlsson@sp.se).

IX. References

[1] R. F. Harrington, *Field Computation by Moment Methods*. New York: Macmillan, 1968.

[2] A. W. Glisson and D. R. Wilton, "Simple and efficient numerical methods for problems of electromagnetic radiation and scattering from surfaces," *IEEE Trans. Antennas Propagat.*, vol. AP-28, pp. 593-603, Sep. 1980.

[3] D. M. Pozar, "Input impedance and mutual coupling of rectangular microstrip antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 1191-1196, Nov. 1982.

[4] J. Carlsson and P.-S. Kildal, "A physical model for the asymptotic field expression for a conducting patch on a dielectric substrate," Submitted to *IEEE Trans. Antennas Propagat.*

[5] Z. Sipus, P.-S. Kildal, R. Leijon, and M. Johansson, "An algorithm for calculating Green's functions for planar, circular cylindrical and spherical multilayer substrates," Accepted for publication in *Applied Computational Electromagnetics Society Journal.*

[6] C. A. Balanis, *Advanced Engineering Electromagnetics*: John Wiley & Sons inc., 1989.

[7] J. Carlsson, "A method of moments program for radiated emission and susceptibility analysis of printed circuit boards," SP Swedish National Testing and Research Institute report 1998:03.

[8] J. R. Mosig, "Arbitrarily shaped microstrip structures and their analysis with a mixed potential integral equation," *IEEE Trans. Microwave Theory and Tech.*, vol. 36, pp. 314-323, Feb. 1988.

[9] M. Marin, S. Barkeshli, and P. H. Pathak, "Efficient analysis of planar microstrip geometries using a closed-form asymptotic representation of the grounded dielectric slab Green's function," *IEEE Trans. Microwave Theory and Tech.*, vol. 37, pp. 669-679, 1989.

[10] M. I. Aksun and R. Mittra, "Spurious radiation from microstrip interconnects," *IEEE Trans. Electromagn. Compat.*, vol. 35, pp. 148-158, 1993.

[11] E. H. Newman and D. Forrai, "Scattering from a microstrip patch," *IEEE Trans. Antennas Propagat.*, vol. AP-35, pp. 245-251, March 1987.

[12] D. M. Pozar, "Improved computational efficiency for the moment method solution of printed dipoles and patches," *Electromagnetics*, vol. 3, pp. 299-309, Jul-Dec 1983.

[13] J. Carlsson, "Computer code LC-Calc," *(also in J. Carlsson, Crosstalk on printed circuit boards, SP Swedish National Testing and Research Institute report 1994:14).*

[14] J. Carlsson, "A FDTD program for computing responses on branched multi-conductor transmission lines," SP Swedish National Testing and Research Institute report 1998:16.

[15] R. Paknys and L. R. Raschkowan, "Moment method surface patch and wire grid accuracy in the computation of near fields," *Applied Computational Electromagnetics Society Journal*, vol. 12, pp. 16-25, Nov 1997.