Direction of Arrival Estimation in a Multipath Environment: an Overview and a New Contribution

(*Invited Paper*)

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Abstract **—** This paper proposes a new computationally efficient algorithm for direction-of-arrival (DOA) estimation in a multipath environment using a uniform linear array (ULA) of equispaced sensors. The paper starts by presenting a comprehensive overview of the classical MUSIC algorithm used for DOA estimation of uncorrelated signals. The effect of different factors related to the signal environment as well as the sensor array is investigated. The concept of spatial smoothing required in the case of correlated signals encountered in multipath propagation environments is then discussed. This then leads to the development of a new computationally efficient DOA estimation algorithm that is proposed for a multipath environment with unknown correlated signals. The algorithm comprises two stages: a first stage for discriminating uncorrelated signals, and a second stage for resolving the directions of arrival of correlated signals using covariance differencing and iterative spatial smoothing. Simulation results show that the proposed algorithm operates at a much lower computational cost compared to standard methods. The proposed algorithm also offers a hardware saving by reducing the number of sensors required to detect a given number of signals.

I. INTRODUCTION

The area of signal processing using sensor arrays to estimate the directions of radio signals has drawn considerable interest in recent years. This is due to the opportunities that this area offers in satisfying the increasing demand of wireless communication networks for higher capacity, larger coverage areas, and lower interference effects. Direction-of-Arrival (DOA) estimation methods based on eigen value evaluation of the signal covariance matrix are known to have high-resolution capabilities and yield accurate estimates [1]. The MUltiple SIgnal Classification (MUSIC) algorithm, proposed by Schmidt [2, 3], is an example of these methods that gained most of the research interest since it uses an accurate data model with a sensor array of arbitrary form. Section II of this paper highlights the concept of adaptive antenna arrays, or what is known as smart antennas, and the benefits that they offer for wireless communication systems. Section III demonstrates the sensor array geometry and the signal model used to develop the DOA estimation algorithm. Detection of radio signals incident on a uniform linear array (ULA) of equi-spaced sensors using the MUSIC algorithm is illustrated in Section IV. This section also provides a performance evaluation of MUSIC by studying the effect of changing parameters related to the signal environment, as well as the sensor array. Section V explains the concept of using spatial smoothing for the detection of correlated signals encountered in practical multipath propagation environments. Performance evaluation of classical spatial smoothing methods is also presented. Finally, Section VI proposes a new technique for DOA estimation in a multipath environment based on covariance differencing and iterative spatial smoothing. It is shown that the proposed technique offers noticeable advantages including lower computational time and reduced array size.

II. ADAPTIVE ANTENNA ARRAYS: CONCEPT AND BENEFITS

A block diagram of a typical adaptive (or smart) antenna array system is illustrated in Fig. 1. The system consists of an array of fixed set of elements (or sensors) that are connected to a signal processing unit. This unit contains Direction Finding (DF), or what is known as Direction of Arrival (DOA) estimation, algorithms to estimate the directions of the signals coming from the mobile users. The signal processing unit then adjusts the weights of a beamforming network in order to maximize the array output towards intended users and minimize it towards interferers [4]. This paper focuses on the DOA part of the system.

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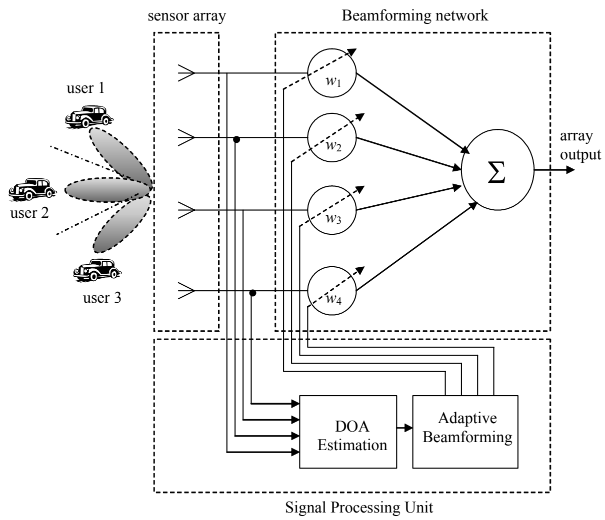


Fig. 1. Block diagram of an adaptive antenna array system.

The use of adaptive antenna arrays in mobile communication networks brings a number of benefits as summarized in what follows.

A. Extended Coverage

The *range extension factor* (REF) that an N-element adaptive antenna array offers is given by [5]:

$$\text{REF} = \frac{r_{array}}{r_{conv}} = N^{\frac{1}{\alpha}}\,, \tag{1}$$

where α is an exponent modeling the path loss, and r_{conv} and r_{array} are the ranges covered by the conventional antenna (with single element) and the antenna array, respectively. The *extended area coverage factor* (ECF) that an adaptive antenna array provides is then calculated as [5]:

$$\text{ECF} = \left(\frac{r_{array}}{r_{conv}}\right)^2 . \tag{2}$$

The inverse of the ECF represents the reduction factor in the number of base stations required to cover the same area that is covered by a single antenna. Fig. 2 demonstrates the improvement gained in coverage area when deploying adaptive antenna arrays in the base stations for different path loss values. It can be seen from Fig. 2 that the coverage area can be almost doubled, compared with conventional single antenna base stations, if an antenna array of six elements is used with α=5, or an antenna array of three elements is used with α=3. More examples that demonstrate the improvement in the coverage range of wireless systems can be found in [6–8].

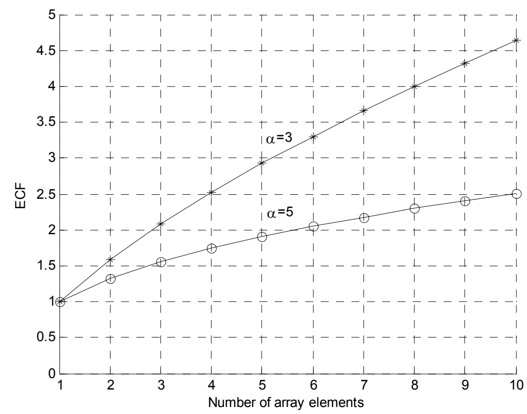


Fig. 2. Improvement of coverage range using adaptive antenna arrays.

B. Reduced Transmission Power

The *array gain G* that is achieved by an N-element adaptive antenna array is expressed as [5]:

$$G = 10\log_{10} N\,. \tag{3}$$

This gain leads to a reduction in the transmission power of the base station. If the required base station reception sensitivity is kept the same, then the power requirement of the base station with N-element array is reduced to N^{-1} and, correspondingly, the required output power of the power amplifier of the base station can be reduced to N^{-2} [5]. Fig. 3 illustrates the reduction in the transmission power of the base station as the number of elements of the antenna array is increased.

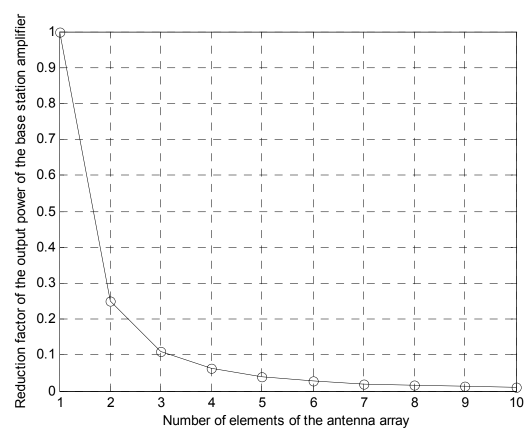


Fig. 3. Reduction in transmission power at the base station using adaptive antenna arrays.

C. Improved Signal Quality

The additional gain offered by adaptive antenna arrays leads to better output *SINR* (signal to interference ratio).

For an antenna array of N elements with a number of interferers smaller than N–1, the output *SINR* in a single propagation environment (i.e., without multipath fading) can be evaluated as [9]:

$$SINR_{out}\,(\text{dB}) = 10\log_{10} N + SNR_{in}\,(\text{dB})\ , \tag{4}$$

where SNR_{in} represents the input *SNR* in dB. Fig. 4 shows the improvement in the output *SINR* value for different values of SNR_{in} using antenna arrays of different sizes. It is clearly seen from Fig. 4 that higher $SINR_{out}$ is achieved as more elements are used in the sensor array.

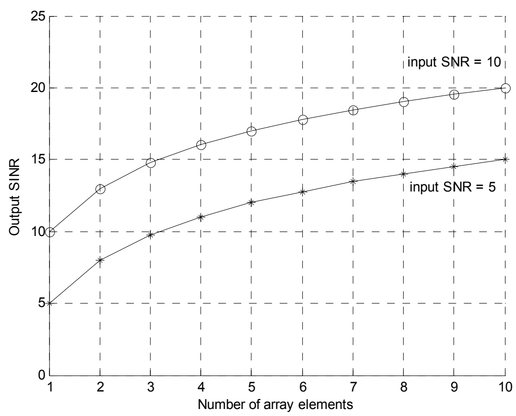


Fig. 4. Improvement of output *SINR* using adaptive antenna arrays.

D. Improved System Capacity

Mitigating the effect of interference is another major capability of adaptive antenna arrays. With conventional antennas, a small portion of the transmitted power is actually received by the intended users while most of the transmitted power is considered to be a source of interference for other users. The radiation pattern of an antenna array is determined by positions of the array elements as well as the amplitude and phase of their feeding currents [10]. By adjusting these parameters, the radiation pattern of the antenna array can be optimized such that the main beams are formulated at the directions of the intended users and null-patterns are placed at the directions of the interferers. Reducing the effect of interference in GSM networks, as an example, allows the reduction of the frequency reuse patterns and this, consequently, leads to an increase in the system capacity. Examples that demonstrate the capacity improvement gained by using adaptive antenna arrays can be found in [11–13].

E. Introduction of New Services

Due to the high signal quality and increased system capacity offered by adaptive antenna arrays, a wide range of applications can be introduced to 3G and future 4G wireless networks including location-based services. When using adaptive antenna arrays, the network will have access to spatial information of users. This information can be used to estimate the positions of users much more accurately compared to the methods used in existing networks. Positioning can be used in services such as emergency calls and location specific billing [14].

III. PROBLEM FORMULATION

A. Sensor Array Geometry

An antenna array consists of a set of elements (or sensors) that are spatially distributed at known locations with reference to a common fixed point [15]. The array sensors can be fashioned into different geometries such as linear, circular, semi-circular and planer arrays. In this paper, the case of a linear sensor array is considered where the centres of the sensors are aligned along the same axis. If the linear array sensors are spaced at equal distances, then the array is known as a uniform (or equispaced) linear array (ULA), which is the type considered in this paper.

B. Array Signal Model

Fig. 5 demonstrates an equispaced linear array of N omni-directional sensors, with spacing d, used to receive M narrowband signals $s_m(t)$ incident with azimuth angles of arrival θ_m, $1 \le m \le M$. The N-dimensional received data vector $\mathbf{u}$ at time t is given by:

$$\mathbf{u}(t) = \sum_{m=1}^{M} \mathbf{a}(\theta_m) s_m(t) + \mathbf{n}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t)\ , \tag{5}$$

where $\mathbf{n}$ is a noise vector modeled as temporally white and zero-mean complex Gaussian process, and $\mathbf{a}(\theta_m)$ is the array response (or steering) vector, corresponding to the DOA of the m^{th} signal, and is defined as:

$$\mathbf{a}(\theta_m) = \begin{bmatrix} 1 & e^{-j\varphi_m} & e^{-j2\varphi_m} & e^{-j(N-1)\varphi_m} \end{bmatrix}^T , \tag{6}$$

where ^{T} is the transpose operator, and φ represents the electrical phase shift from element to element along the sensor array [16], and is expressed as:

$$\varphi_m = 2\pi \left(\frac{d}{\lambda}\right) \sin(\theta_m)\ , \tag{7}$$

where λ is the wavelength of the incident signals. The combination of all possible steering vectors forms the array manifold matrix $\mathbf{A}$.

As seen from equation (5), the intersection of the source waveforms (i.e., signals) with one or more of the steering vectors of matrix $\mathbf{A}$ builds up the signal model. The array manifold can be a multi-dimensional matrix if other signal characteristics such as the elevation angle of incidence and the signals distance from the sensor array are considered. However, for simplicity, this

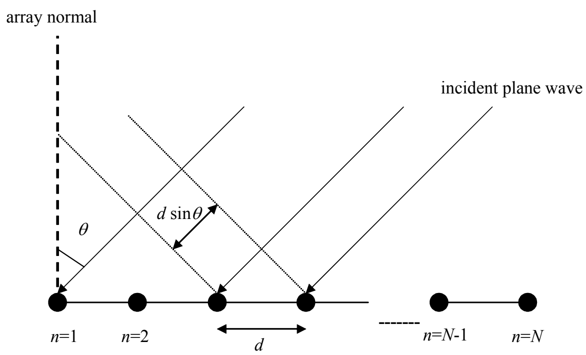


Fig. 5. Equispaced linear array of *N* sensors.

paper considers the case of one-dimensional array manifold (i.e., a function of the azimuth angle of incidence only).

Physical array calibration is very important to adjust the array response to the incident signals. Inaccuracies in the array response (or steering) vectors lead to a decrease in the accuracy of the DOA estimation process [17]. A basic form of array calibration is carried out such that the array manifold is measured by rotating the antenna array relative to a fixed signal source under controlled multipath conditions.

IV. DETECTION OF UNCORRELATED SIGNALS: THE MUSIC ALGORITHM

A. Theory

The MUltiple SIgnal Classification (MUSIC) algorithm was first introduced by Schmidt [2, 3]. The algorithm starts by applying temporal averaging over *K* snapshots (or samples) taken from the signals incident on the sensor array. This averaging process leads to forming a spatial correlation (or covariance) matrix **R** defined as:

$$\mathbf{R} = \frac{1}{K}\sum_{t=1}^{K} \mathbf{u}(t)\,\mathbf{u}(t)^{H} \,, \tag{8}$$

where ^{H} denotes the Hermitian operator. Substituting $\mathbf{u}(t)$ from (5) into (8) results in:

$$\mathbf{R} = \frac{1}{K}\sum_{t=1}^{K} \mathbf{A}(\theta)\mathbf{s}(t)\,\mathbf{s}(t)^{H}\,\mathbf{A}(\theta)^{H} + \mathbf{n}(t)\,\mathbf{n}(t)^{H} \,, \tag{9}$$

$$\mathbf{R} = \mathbf{A}\,\mathbf{R}_{ss}\,\mathbf{A}^{H} + \sigma_n^2\mathbf{I} \,, \tag{10}$$

where $\mathbf{R}_{ss}$ is the signal covariance matrix, σ_n^2 is the noise variance and **I** is an identity matrix of size *N*×*N*. Since matrix **A** contains the linearly-independent steering vectors, and the signal covariance matrix $\mathbf{R}_{ss}$ is non-singular as long as the incident signals are uncorrelated, then this implies that *N–M* of the eigen values of the covariance matrix **R** are equal to σ_n^2. The eigen values of matrix **R** are the values $\{\gamma_1\ \gamma_2\ ...\ \gamma_N\}$ such that:

$$\left|\,\mathbf{R} - \gamma_i\mathbf{I}\,\right| = 0 \,. \tag{11}$$

The basic idea behind the MUSIC algorithm is that the eigen vectors corresponding to the smallest *N–M* eigen values form the noise subspace, and are also orthogonal to the steering vectors that make up matrix **A**. The eigen vector associated with a particular eigen value γ_i is the vector $\mathbf{q}_i$ that satisfies the following equation:

$$(\,\mathbf{R} - \gamma_i\mathbf{I}\,)\,\mathbf{q}_i = 0 \,. \tag{12}$$

Therefore, by exploiting the orthogonality between the steering vectors making matrix **A** and the noise subspace, the MUSIC angular or spatial spectrum is defined as:

$$P(\theta) = \frac{1}{\mathbf{a}(\theta)^{H}\,\mathbf{V}_{\mathrm{n}}\mathbf{V}_{\mathrm{n}}^{H}\,\mathbf{a}(\theta)} \,, \tag{13}$$

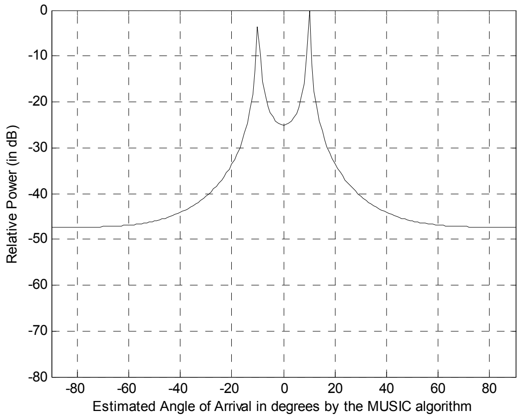
where $\mathbf{V}_{\mathrm{n}}$ is the matrix of eigen vectors corresponding to the noise subspace of matrix **R**. Orthogonality between **a** and $\mathbf{V}_{\mathrm{n}}$ will minimize the denominator and hence will give rise to peaks in the MUSIC spectrum. Those peaks correspond to the directions of arrival of the signals impinging on the sensor array. The standard MUSIC algorithm has a high computational load and storage requirements due to the exhaustive search through all possible steering vectors to estimate the direction of arrivals. Thus, other versions of MUSIC as well as other DOA estimation algorithms have been introduced in [20–25] to reduce the computational load and storage requirements.

B. Results and Discussion

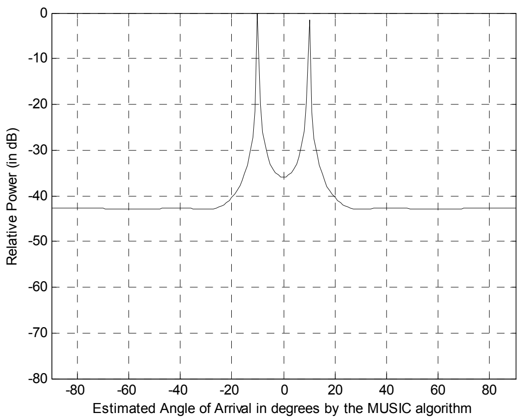
The effect of changing different parameters on the performance of MUSIC is investigated in this section of the paper. Some of these parameters are related to the size of the sensor array in terms of the number of sensors forming the array and the physical separation between them. Other parameters are related to the signal environment. These parameters include the number of incident signals, their angular separation, number of samples taken as well as the *SNR* (signal to noise ratio).

B.1. Sensor Array Parameters

The performance of MUSIC improves as more elements are used in the sensor array especially in the case of closely spaced incident signals. This is demonstrated in Fig. 6 which shows the improvement when the number of elements of a sensor array, used to detect 2 signals incident at angles +10° and –10°, is increased from *N*=3 to *N*=6 elements. This improvement is seen in the form of sharper spectral peaks at the directions of the detected signals.

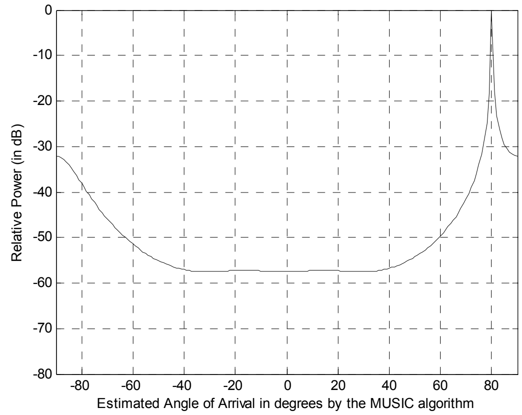


(a) N=3

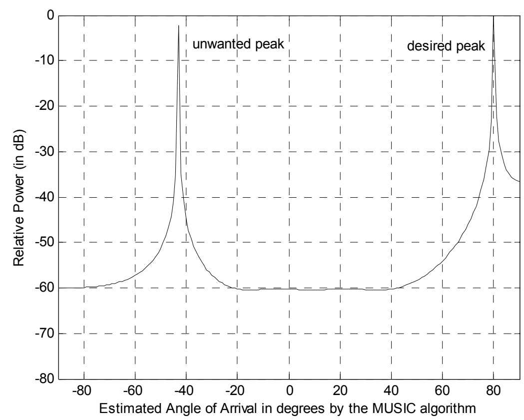


(b) N=6

Fig. 6. Effect of increasing the number of elements of the sensor array on the MUSIC spectrum (d=0.5λ, SNR=10 dB and K=100).



(a) d=0.5λ



(b) d=0.6λ

Fig. 7. Effect of increasing the spacing of elements of the sensor array on the MUSIC spectrum (SNR=10 dB and K=100).

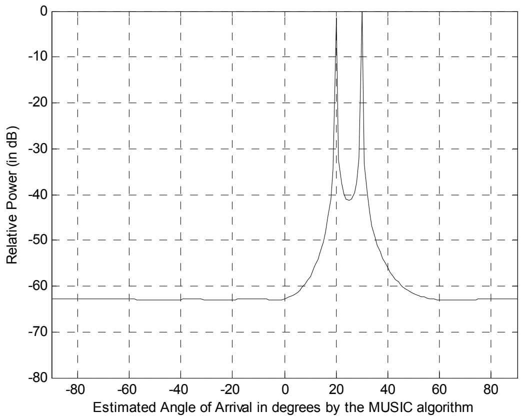
The effect of changing the spacing between the elements of the sensor array was also investigated for a sensor array of two elements used to detect one signal incident at an angle θ=80°. It was observed that MUSIC is capable of detecting an incident signal successfully, without generating unwanted peaks in the angular spectrum, as long as the element spacing does not exceed 0.5λ, as evident from Fig. 7(a). However, using larger values for the element spacing results in unwanted peaks appearing in the MUSIC spectrums, as evident from Fig. 7(b). A further study of the reason behind this degradation in performance is presented in [26]. Since most of the DOA estimation algorithms ignore the effect of mutual coupling between the elements of the sensor array, an element spacing of 0.5λ is recommended. It is to be noted that the existence of mutual coupling [27–30] has to be taken into consideration when designing the physical antenna array elements with spacing less than 0.5λ.

B.2. Signal Environment Parameters

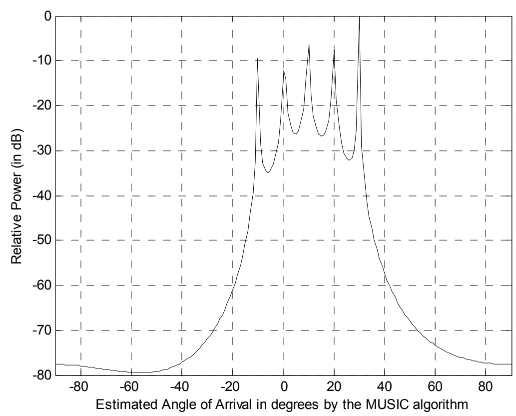
Experimental results show that the performance of MUSIC degrades as more signals are incident on the sensor array. This is illustrated in Fig. 8 which shows the degradation of the MUSIC spectral peaks as the number of signals incident on a six-element sensor array increases from M=2 to M=5. To overcome this, the number of elements of the sensor array must be increased [18, 19].

The spatial correlation between the incident signals, in terms of their angular separation, was also investigated. It was found that MUSIC produces sharper peaks, with a lower noise floor, as the angular separation between the incident signals increases. Fig. 9 demonstrates the improvement in the performance of MUSIC when the angular separation between two signals incident on a sensor array of three elements is increased from 10^o to $60^{o.}$

Moreover, it was found that the performance of MUSIC improves as more snapshots (or sapmples) are taken from the incident signals as illustrated in Fig. 10.



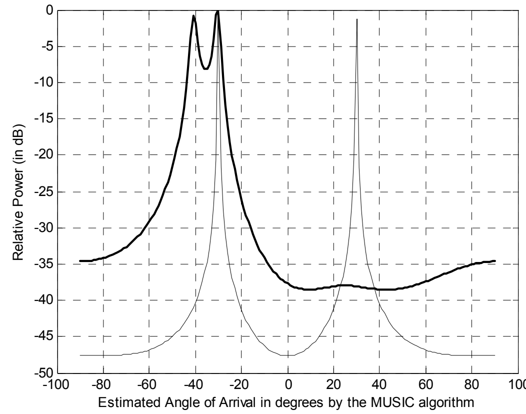
(a) Angles of arrival = $+20^o$, and $+30^o$



(b) Angles of arrival = -10^o, 0^o, $+10^o$, $+20^o$, and $+30^o$

Fig. 8. Effect of increasing the number of incident signals on the MUSIC spectrum (d=0.5 λ, SNR=20 dB and K=100).

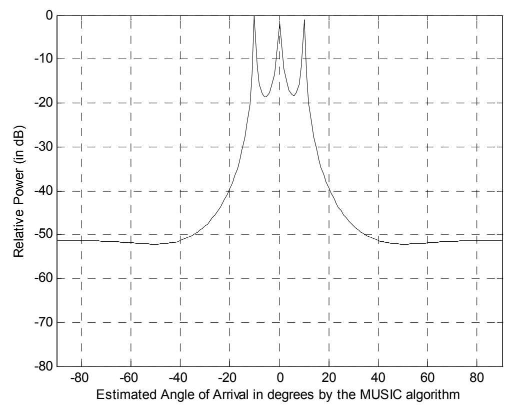
Finally, the effect of SNR has been investigated and results are depicted in Fig. 11. As one would expect, it is clear that sharper peaks are resolved at the directions of the incident signals as the value of the SNR is made larger.



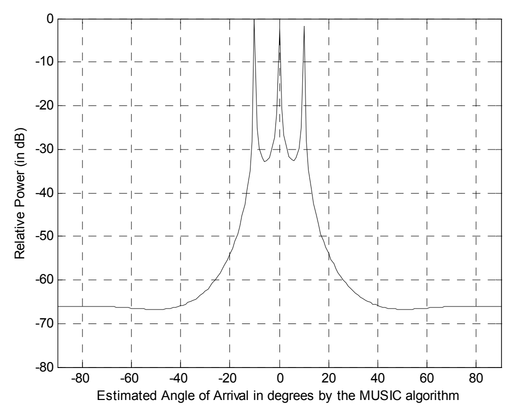
__ angular separation = 60^o

--- angular separation = 10^o

Fig. 9. Effect of increasing the angular separation between the incident signals on the MUSIC spectrum (N=6, d=0.5λ, SNR=10 dB, and K=100).

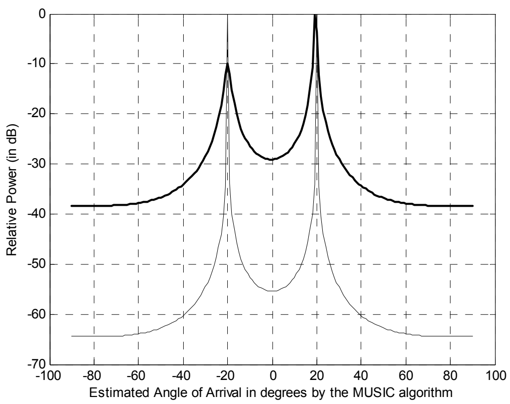


(a) K=10



(b) K=100

Fig. 10. Effect of increasing the number of snapshots taken from the incident signals on the MUSIC spectrum (d=0.5λ, SNR=10 dB, N=5, and M=3).



__ *SNR* = 20 dB, --- *SNR* = 10 dB

Fig. 11. Effect of increasing the *SNR* value on the MUSIC spectrum (d=0.5λ, K=100, N=3, and M=2).

V. DETECTION OF CORRELATED SIGNALS: SPATIAL SMOOTHING

As mentioned in Section IV, the signal covariance (or correlation) matrix $\mathbf{R}_{ss}$ is a full-rank matrix (i.e., non-singular) as long as the incident signals on the sensor array are uncorrelated, which is the key to the MUSIC eigen values decomposition. However, if the incident signals become highly correlated, which is a realistic assumption in practical radio environments, matrix $\mathbf{R}_{ss}$ will lose its non-singularity property and, consequently, the performance of MUSIC will degrade severely [31]. A technique known as Spatial Smoothing (SS) [32, 33] can be used to remove the correlation between the incident signals by dividing the antenna array into subarrays as described in what follows.

A. Forward Spatial Smoothing (FSS)

The basic idea of this method is to decorrelate the incident signals by dividing the sensor array into overlapping smaller subarrays and introducing phase shifts between them [34]. Fig. 12 demonstrates the FSS method when used to partition an N=6 elements sensor array into L=4 overlapping subarrays, each of size p=3 elements.

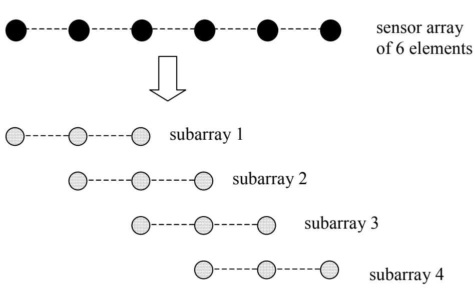


Fig. 12. FSS applied to a six-element sensor array.

The vector of the received signals at the k^{th} forward subarray is expressed as:

$$\mathbf{u}_k^F(t) = \mathbf{A}\,\mathbf{D}^{(k-1)}\mathbf{s}(t) + \mathbf{n}_k(t)\,, \tag{14}$$

where (k–1) denotes the k^{th} power of the diagonal matrix $\mathbf{D}$ given by:

$$\mathbf{D} = \text{diag}\left\{ e^{-j\frac{2\pi}{\lambda}\sin\theta_1}, \ldots, e^{-j\frac{2\pi}{\lambda}\sin\theta_M} \right\}. \tag{15}$$

The spatial correlation matrix $\mathbf{R}$ of the sensor array is then defined as the sample mean of the covariance matrices of the forward subarrays:

$$\mathbf{R} = \frac{1}{L}\sum_{k=0}^{L-1}\mathbf{R}_k^F\,. \tag{16}$$

It is to be noted that the division of the sensor array into forward subarrays must satisfy the following conditions [19]:

$$L \geq M \;\Rightarrow\; N - p + 1 \geq M\,, \tag{17}$$

$$N > p > M\,. \tag{18}$$

It can be clearly seen from (18) that p_{min}=M_{max}+1. By substituting this in (17), the maximum number of correlated signals that can be detected by FSS becomes N/2 compared to N–1 uncorrelated signals that can be detected by conventional MUSIC.

B. Forward/Backward Spatial Smoothing (FBSS)

It has been proven that it is possible to increase the number of detected correlated signals to 2N/3 by using a set of forward and conjugate backward subarrays simultaneously [35]. Fig. 13 illustrates the FBSS method when used to partition an N=6 elements sensor array into L=4 overlapping forward subarrays and L=4 overlapping backward subarrays, each of size p=3 elements. In this method, the vector of the received signals at the k^{th} backward subarray is expressed as:

$$\mathbf{u}_k^B(t) = \mathbf{A}\,\mathbf{D}^{(k-1)}cong[\mathbf{D}^{(N-1)}\mathbf{s}(t)] + \mathbf{n}_k(t)\,. \tag{19}$$

The spatial correlation matrix $\mathbf{R}$ of the sensor array is then given by:

$$\mathbf{R} = \frac{\mathbf{R}^F + \mathbf{R}^B}{2} \tag{20}$$

sensor array
of 6 elements

where $\mathbf{R}^F$ is the average covariance matrix of the forward subarrays vectors $\mathbf{u}^F$ given by (14) and $\mathbf{R}^B$ is the average covariance matrix of the backward subarrays vectors $\mathbf{u}^B$ given by (19).

C. Results and Discussion

First, the need for using spatial smoothing techniques for the detection of correlated signals is demonstrated. A simple case of two correlated signals which are incident at angles θ= –30° and –60° on an N=5 element

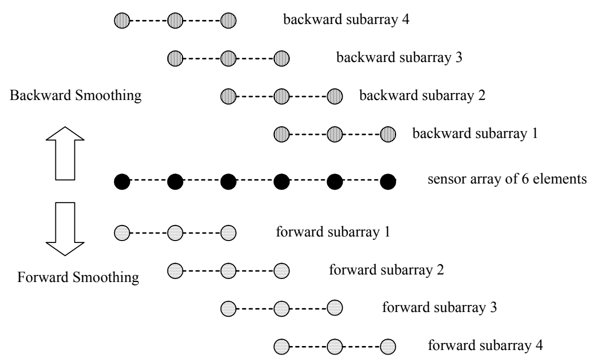


Fig. 13. FBSS applied to a six-element sensor array.

sensor array is considered. Fig. 14 shows that conventional MUSIC fails to detect the two correlated signals as evident from the absence of spectral peaks at the incident angles. These spectral peaks are clearly resolved when FSS-MUSIC is employed with p=3.

Although spatial smoothing techniques can be used to detect correlated signals, the existence of correlation reduces the number of signals that can be detected by the N-elements linear array. As mentioned previously, conventional MUSIC can successfully detect up to N–1 uncorrelated signals while FSS-MUSIC allows the detection of only up to N/2 correlated signals, and FBSS-MUSIC can detect up to 2N/3 correlated signals. Fig. 15 compares the performance of the two spatial smoothing techniques for detecting M=6 correlated signals incident on a linear sensor array comprised of N=9 elements. It can be clearly seen that FSS-MUSIC fails in such a case due to the fact that the number of incident signals is greater than N/2. However, FBSS-MUSIC is able to detect all the incident signals due to the fact that the number of incident signals does not exceed 2N/3.

Moreover, conditions (17) and (18) discussed previously highlight the constraints on the size p and number of subarrays L that should be used in order to ensure successful detection of correlated signals. Fig. 16 demonstrates the failure of FSS-MUSIC in detecting M=3 correlated signals by an antenna array of N=6 elements if the main antenna array is divided into L=2 overlapping subarrays of p=5 elements each. This is simply due to the fact that the number of subarrays L is less than the number of correlated signals M, which violates condition (17). Fig. 16 also demonstrates the success of the same smoothing technique if the main antenna array is divided into L=3 overlapping subarrays of p=4 elements each. Obviously, spatial smoothing works in this case since condition (17) is now satisfied.

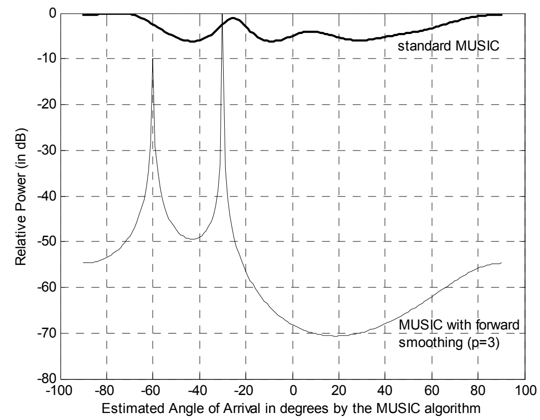


Fig. 14. DOA estimation using conventional MUSIC and FSS-MUSIC (K=100 and SNR=20 dB).

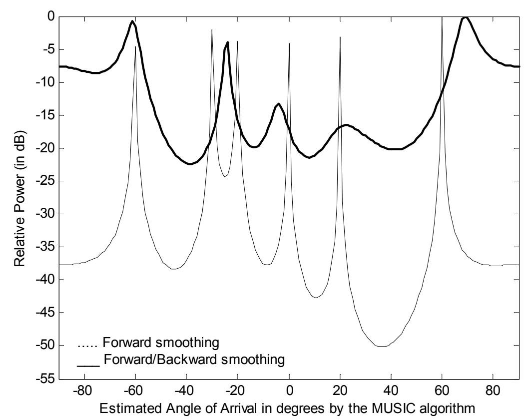


Fig. 15. DOA estimation using FSS-MUSIC and FBSS-MUSIC (θ = –60°, –30°, –20°, 0°, +20°, +60°, and p=7).

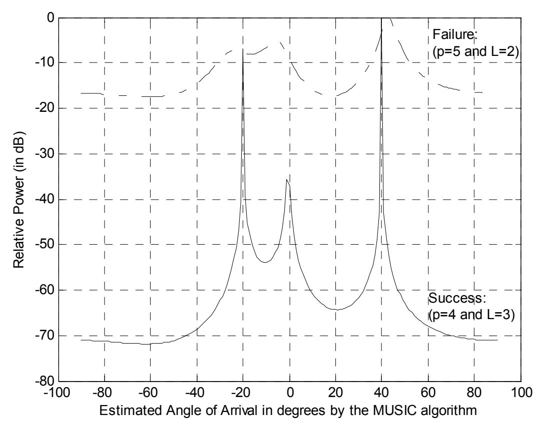


Fig. 16. Performance of FSS-MUSIC for different values of p and L (θ = –20°, 0°, and +40°, K=100, SNR=10 dB).

The size p and hence the number of the subarrays L are the key parameters which determine the computational complexity of spatial smoothing techniques. The acceptable values of these two parameters are determined by the number of correlated signals to be detected. Fig. 17 demonstrates the computational complexity of conventional MUSIC compared to FSS-MUSIC and FBSS-MUSIC when detecting two signals incident on a sensor of N=20 elements. If the two incident signals are uncorrelated, then conventional MUSIC can be used and there is no need to divide the sensor array into overlapping subarrays. However, if the two incident signals become correlated, then spatial smoothing techniques must be used. Fig. 17 shows that dividing the sensor array into larger subarrays reduces the number of subarrays to be processed and hence reduces the computational time. It is also obvious from Fig. 17 that FBSS-MUSIC requires more computational time compared to FSS-MUSIC due to the fact that the smoothing process is carried out in two directions instead of one direction only. Finally, it is to be noted from Fig. 17 that the difference in computational time between the two smoothing techniques reduces when fewer subarrays (i.e., smaller L) are processed.

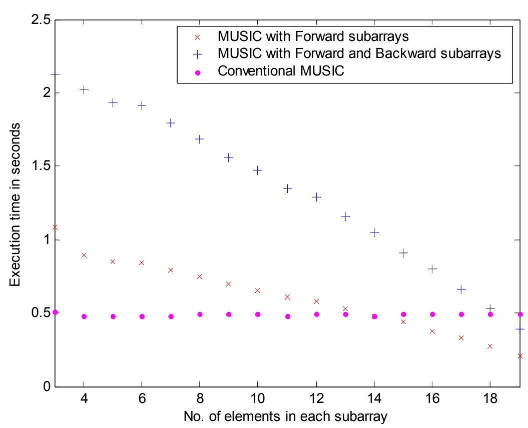
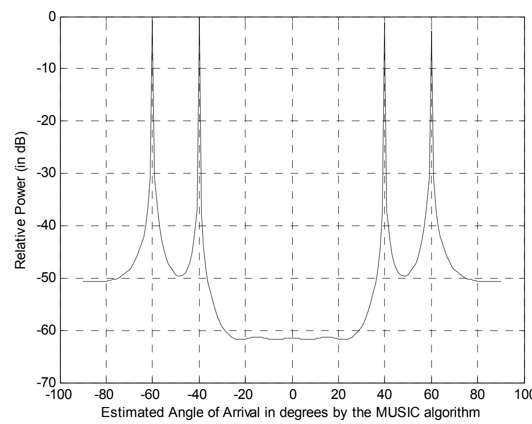


Fig. 17. Computational time of conventional MUSIC and spatial smoothing techniques (K=500, SNR=20 dB).

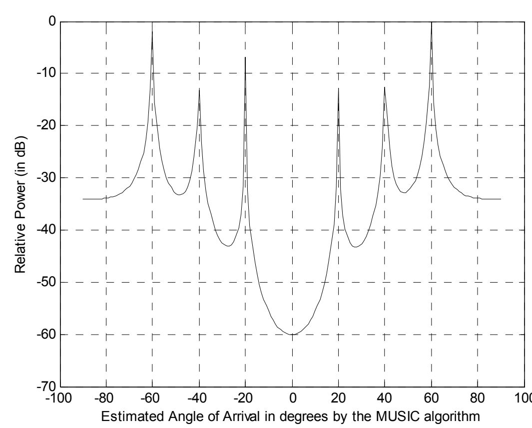
VI. DOA ESTIMATION IN A MULTIPATH ENVIRONMENT

A. Standard Methods

The question that may arise now is: "what is the maximum number of signals that can be detected if some of the incident signals are correlated while others are not?". In this case, conventional MUSIC cannot be used as it fails for detecting even two correlated signals regardless of the number of elements of the sensor array. Hence, spatial smoothing techniques must be used with a minimum number of array sensors equal to $N=2M_c+M_u$ in the case of FSS-MUSIC [19]. Here, M_c is the number of correlated signals and M_u is the number of uncorrelated signals. In the case of FBSS-MUSIC, the minimum number of array sensors becomes $N=(3M_c/2)+M_u$ [19]. In both cases, spatial smoothing should be carried out with respect to $L= M_c$ subarrays, with $p= M_c+1$ elements in each subarray. Fig. 18 illustrates the performance of both, conventional MUSIC and FSS-MUSIC for detecting M_c=2 correlated signals incident at angles –20° and +20°, and M_u=4 uncorrelated signals incident at angles –60°, –40°, +40°, and +60° on a sensor array of N=8 elements. It can be clearly seen from Fig. 18(a) that conventional MUSIC detects only the four uncorrelated signals, whereas in Fig. 18(b) all six signals have been detected using FSS-MUSIC.



(a) MUSIC detects correlated signals only



(b) FSS-MUSIC detects correlated and uncorrelated signals

Fig. 18. DOA estimation using conventional MUSIC and FSS-MUSIC in a multipath environment.

B. Proposed Method: Covariance Differencing and Iterative Spatial Smoothing

As highlighted in Section V, the performance of MUSIC shows severe degradation when trying to detect highly correlated signals as encountered in multipath propagation environments. To overcome this, spatial smoothing must be used in conjunction with MUSIC. However, the spatial smoothing process adds computational load to the DOA estimation process as well as reducing the number of signals that can be detected by the sensor array. In this subsection, a computationally efficient high-resolution algorithm, which is based on covariance differencing and iterative spatial smoothing, is proposed. Unlike standard MUSIC, which estimates the DOAs simultaneously, the proposed algorithm estimates them sequentially, in two stages, yet at a much lower computational cost. Fig. 19 shows a flowchart of the proposed method. It comprises two stages through which the uncorrelated and correlated signals are separated and processed independently.

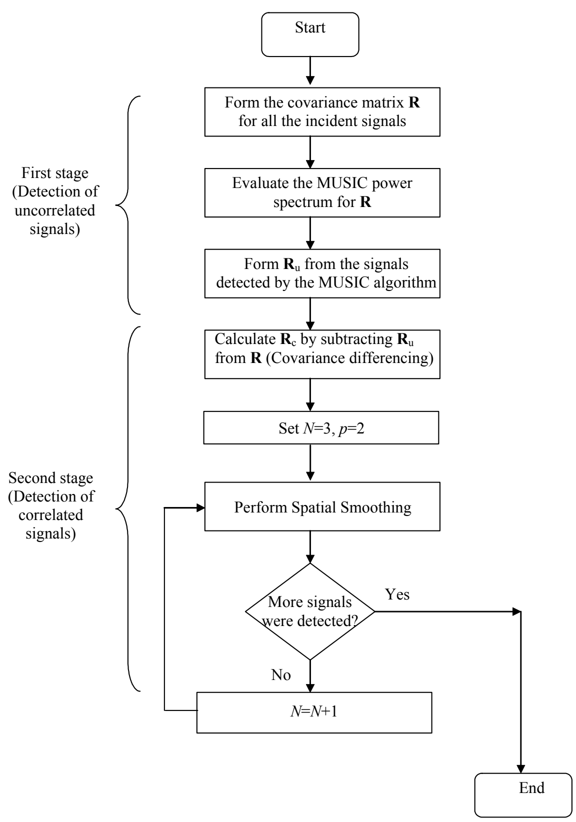


Fig. 19. Flowchart of the proposed algorithm.

In the first stage, standard MUSIC is applied to the overall signal covariance matrix **R** of all the incident signals. The DOAs that are resolved in this stage, if any, are used to form a new covariance matrix $\mathbf{R}_u$, corresponding to the uncorrelated signals only. In the second stage, a new covariance matrix $\mathbf{R}_c$, containing information on the correlated signals only is formed by covariance differencing of **R** and $\mathbf{R}_u$. Thus:

$$\mathbf{R}_c = \mathbf{R} - \mathbf{R}_u \,. \tag{21}$$

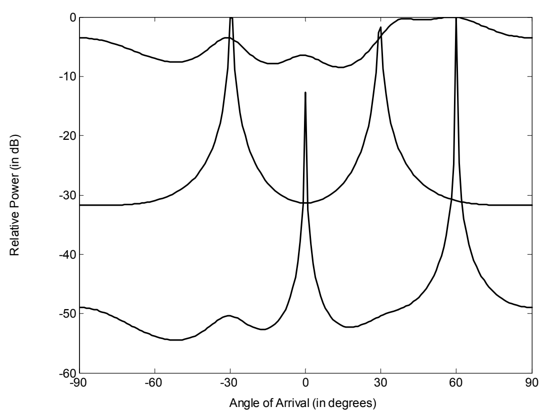
Spatial smoothing is then performed iteratively with respect to the new covariance matrix $\mathbf{R}_c$. In the first iteration of applying spatial smoothing, a sensor array of N=3 elements divided into L=2 overlapping subarrays each of size p=2 elements is used. This arrangement is capable of detecting two correlated signals only. If no peaks appear in the MUSIC angular spectrum, then this indicates that there are more correlated signals embedded in the spectrum of $\mathbf{R}_c$ and that the number of elements used in the sensor array was not sufficient to detect them all. Hence, in the second iteration, the number of elements is increased to N=4 so that the sensor array may be divided into L=3 overlapping subarrays each of size p=2 elements capable of detecting three correlated signals. The iteration process continues until the peaks corresponding to the DOAs for the correlated signals appear in the MUSIC angular spectrum.

The lower computational cost of the proposed method compared to the standard DOA estimation method is due to the fact that spatial smoothing is applied iteratively utilizing smaller sensor arrays and fewer overlapping subarrays. As already mentioned, the standard method with FSS requires the use of $N = 2M_c + M_u$ sensors to successfully detect all incident signals, whereas the proposed method with iterative FSS uses only $N = 2M_c$ sensors when $M_c > M_u$, and $N = M_c + M_u + 1$ sensors when $M_u \geq M_c$. Similarly, the standard method with FBSS requires the use of $N = 3/2\, M_c + M_u$ sensors to successfully detect all incident signals, whereas the proposed method with iterative FBSS uses only $N = M_c + M_u + 1$ sensors for any combination of correlated and uncorrelated signals. Therefore, it can be concluded that the proposed method also offers a hardware saving by reducing the size of the sensor array required to detect a given number of correlated and uncorrelated signals.

C. Results and Discussion

The simulation results shown in this part of the paper are taken for the case of a linear array of sensors equispaced at half the carrier wavelength. Simulations were carried out using an *SNR* value of 20 dB with K=200 snapshots taken from the incident signals. Fig. 20 shows the MUSIC spectrum obtained using the proposed method. The solid line shows that two uncorrelated signals at angles θ=0° and θ=+60° were successfully detected using the first stage of the proposed method. The dotted line shows the spectrum corresponding to covariance differencing which yields the covariance matrix of the correlated signals, $\mathbf{R}_c$. The dashed line shows that after applying iterative spatial smoothing on $\mathbf{R}_c$, two correlated signals at $\theta = -30^o$ and $\theta = +30^o$ have been successfully detected.

Fig. 21 compares the computational time of the standard and proposed methods with FSS as the number of correlated signals M_c is varied. It can be seen that the proposed method is highly efficient as it offers a saving of up to 75% in computational time, when compared with the standard method. Similarly, Fig. 22 shows this saving if the proposed method is used with iterative FBSS compared to the standard method. A saving of up to 77% in computational time can be seen for this particular example.



__ Spectrum of **R** due to the first stage of proposed method
… Spectrum of $\mathbf{R}_c$ due to covariance differencing
--- Spectrum of $\mathbf{R}_c$ due to iterative spatial smoothing

Fig. 20. MUSIC spectrum with two uncorrelated signals at 0^o and $+60^o$, and two correlated signals at -30^o and $+30^o$.

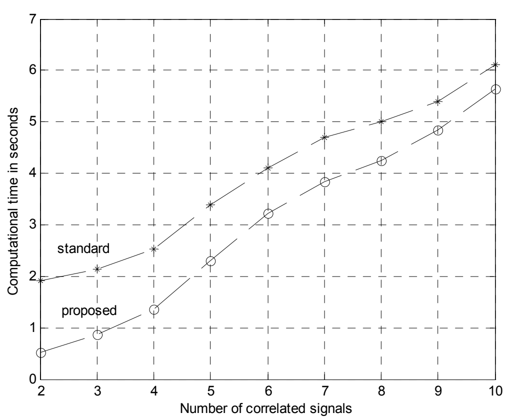


Fig. 21. Computational time of the standard and proposed methods using FSS as the number of correlated signals M_c increases (N=30, K=200, M_u=2).

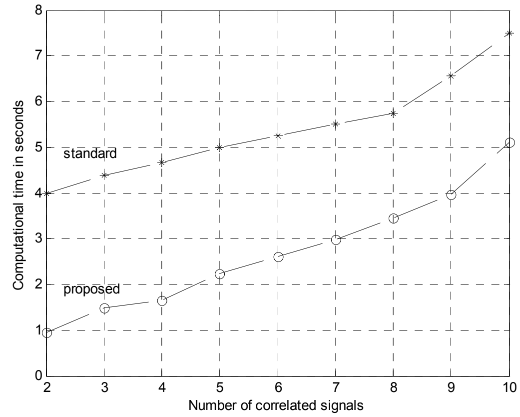


Fig. 22. Computational time of the standard and proposed methods using FBSS as the number of correlated signals M_c increases (N=30, K=200, M_u=2).

Besides the saving in computational time, the proposed method offers also hardware saving in the form of fewer elements in the sensor array, as shown in Table 1.

Table 1. Comparison between the standard and proposed methods in terms of the number of sensor array elements required to detect correlated and uncorrelated signals.

Number of signals		Required Number of elements		
Uncorrelated M_u	*Correlated* M_c	*Standard FSS-MUSIC*	*Proposed*	
			with FSS	*with FBSS*
3	2	7	6	6
3	3	10	7	7
3	4	11	8	8
3	5	13	10	9
3	6	15	12	10
3	7	17	14	11
4	2	8	7	7
4	3	10	8	8
4	4	12	9	9
4	5	14	10	10
4	6	16	12	11
4	7	18	14	12
5	2	9	8	8
5	3	11	9	9
5	4	13	10	10
5	5	15	11	11
5	6	17	12	12
5	7	19	14	13

VII. CONCLUSIONS

The performance of the MUSIC DOA estimation algorithm was investigated for a set of parameters related to the sensor array and signal environment. The performance of MUSIC was shown to improve by using more elements in the sensor array, more samples of the incident signals, as well as an increased *SNR*. Increasing the angular separation between the incident signals also leads to an improvement in the MUSIC power spectrum in the form of sharper peaks at the directions of the detected signals. Extensive simulations demonstrated that an element spacing of $\lambda/2$ results in optimum performance of the sensor array.

The use of spatial smoothing to detect correlated signals was also demonstrated. A comparative study of the performance of two spatial smoothing methods has been carried out. These methods are based on dividing the sensor array into overlapping subarrays which are used to remove the correlation between the incident signals. Each subarray must consist of a number of elements that exceeds the number of correlated signals. It was found that the forward/backward spatial smoothing technique (FBSS) requires more computational time when compared to the forward spatial smoothing (FSS) technique due to the fact that the smoothing process is carried out in two opposite directions instead of one. It is to be noted that the existence of correlation reduces the number of correlated signals that can be detected by the N-elements sensors array to $N/2$ in the case of FSS, and to $2N/3$ in the case of FBSS.

A computationally efficient method for DOA estimation in a multipath environment has been proposed. This method is based on covariance differencing and iterative spatial smoothing. Simulation results demonstrated the savings offered in computational time and hardware in comparison with the standard method. This makes the proposed method more suitable for real-time DSP implementations.

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