Investigation of Wire Grid Modeling in NEC Applied to Determine Resonant Cavity Quality Factors

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Abstract **–** Numerical computer simulations using the NEC Method of Moments (MoM) code were performed on wire grid models of resonant cavities in order to study how well conductive structures and their surface impedances can be modeled by wire meshes. The resonant cavity quality factor, or *Q,* was examined due to its high sensitivity to surface impedance. Several half-wave coaxial cavities were simulated using various mesh element sizes. The cavities' outer conductor radius was varied to obtain different geometries. The quality factor *Q* was determined from the simulated input impedance spectra. The wire grid model results were compared to well known theoretical and experiment results. Qualitative agreement between simulation, theoretical, and experimental results was achieved for fixed mesh parameters, giving confidence in comparative simulation using the same wire grid meshing parameters. Quantitative agreement of simulation results was achieved through repeated simulation with varying mesh element lengths and extrapolating the simulation results to a conceptual mesh element length of zero. This shows that simulations to determine quantities sensitive to surface impedances can be successfully performed with codes such as NEC.

Key words – Wire grid modeling, method of moments, extrapolation, and surface impedance.

I. INTRODUCTION

A resonant cavity's quality factor, *Q*, is highly dependent on the surface impedance, R_s, of the cavity's interior conducting surface. Numerical simulation of well understood cavities can serve to investigate numerical techniques employed to model conductive surfaces and their impedances. Once shortcomings of a particular numerical modeling technique are determined, they can often be compensated for and hence result in more accurate simulation results. These techniques can then be applied with confidence to the simulation of more complicated resonant structures for which analytical solutions are not readily available. In this paper, the well known quality factor of cylindrical half-wave coaxial cavity resonators was investigated, to determine how well wire grid models can represent conductor surfaces in resonant cavity structures.

II. THEORETICAL BACKGROUND ON QUALITY FACTOR OF HALF-WAVE RESONANT COAXIAL CAVITIES

A basic definition of quality factor, *Q*, for a resonant structure is given in equation (1). For the case of an electromagnetic half-wave coaxial cavity resonator, the energy stored, E_S, can be calculated through equation (2, the integral over the cavity volume, *V*, of the magnetic-field intensity, *H*. The energy dissipated per cycle, E_D, is given by equation (3), the surface integral of the ohmic losses due to the surface current density J_s, over the interior cavity surface area, *A*. The *Q* of a resonant cavity is often normalized with respect to the wavelength, λ, and conductor skin depth, *δ*, as shown in equation (4), and is then referred to as the cavity form factor [1],

$$Q = \frac{2\pi \cdot E_S}{E_D} \tag{1}$$

$$E_S = \frac{\mu}{2}\int \left|\vec{H}\right|^2 \cdot dV , \tag{2}$$

$$E_D = \frac{\pi \cdot R_s}{\omega}\int \left|\vec{J}_s\right|^2 \cdot dA \quad \text{or} \quad E_D = \frac{\pi \cdot \delta \cdot \mu}{2}\int \left|\vec{H}_T\right|^2 \cdot dA , \tag{3}$$

$$Q\frac{\delta}{\lambda} = \frac{2}{\lambda}\frac{\int \left|\vec{H}\right|^2 \cdot dV}{\int \left|\vec{H}_T\right|^2 \cdot dA} . \tag{4}$$

Since the fields in a half-wave long resonant cavity are standing transverse electromagnetic (TEM) waves, the time average magnetic-field intensity, H, which is strictly in the ϕ-direction for this mode, is known to be of the form given in equation (5) where C is a constant and z_0 is the length of the cavity as shown in Fig. 1.

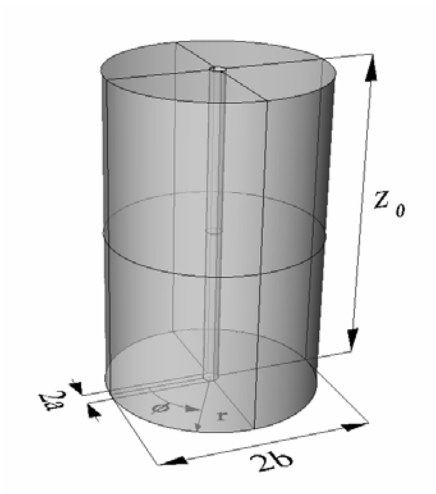

Fig. 1. Coaxial cavity geometry with inner radius, a, outer radius, b, and length, z_0.

$$\vec{H}(r,\phi,z) = C\frac{1}{r}\cdot Cos\left(\frac{\pi\cdot z}{z_0}\right)\cdot\hat{\phi}. \tag{5}$$

Substituting H into equation (4) and evaluating, provides results shown in equation (6) for the form factor of a half-wave coaxial cavity [1]. Note that this quality factor is the unloaded quality factor, Q_u, which does not include losses due to a coupling structure or associated source impedance. If simulations of wire grid representations of such coaxial cavities result in quality factors predicted by theory, then the simulation technique must properly model conductive structures and their surface impedance losses with wire grids.

$$Q_u\frac{\delta}{\lambda} = \frac{\ln\left(\frac{b}{a}\right)}{z_0\cdot\left(\frac{1}{a}+\frac{1}{b}\right)+4\cdot\ln\left(\frac{b}{a}\right)} \tag{6}$$

III. BACKGROUND ON CONDUCTIVE SURFACE MODELING BY WIRE GRIDS

Richmond pioneered modeling of conductive geometries using wire grid representations and this technique has become accepted radiation and scattering problems [2]. Rules of thumb have been developed for modeling conductive structures. A usual requirement for the wire grid models is that the grid element size be "small" with respect to a wavelength. Another commonly used rule is "the equal surface area rule", where the total surface area of the cylindrical wires comprising the grid is made to match the surface area of the conductive object being modeled [3-5]. It has been found that a rectangular wire grid, with the grid axes aligned to electromagnetic polarization, generally gives more accurate simulation results than other types of grids, including triangular grids [5]. A more elaborated set of rules for wire grid simulation of surfaces using the Numerical Electromagnetics Code (NEC) [6], a popular and well tested method of moments code, is discussed in Truman and Kubina [3]. However, these rules are only guidelines. According to Moore and Pizer, some simulations require the wire grid surface area to be up to five times larger than the object's actual surface area in order to match experimental results, so surface impedance seems to not be modeled well in these wire grid simulations [6].

IV. IMPLEMENTATION OF WIRE GRID MODELS FOR COAXIAL CAVITIES

To facilitate the construction of various simulation wire grid geometries for this study's simulations, a commercial computer aided design (CAD) program was employed. Each resulting cavity model mesh was exported to a text file in the open-Wavefront OBJ format [8]. This format specifies the mesh as a series of numbered vertices followed by a series of planar faces or patches with the vertices at the corners of these patches. This text file was then processed into a format compatible with NEC through custom written software. The process generated a wire segment for each edge of each mesh surface patch, while avoiding duplication amongst adjacent patches. The equal area rule was applied, in which the cylindrical surface area of a wire segment was chosen to be the average of the surface areas of the quadrilateral grid patches on either side of the wire. The wire mesh generated used equally sized, primarily square patches, and as such, polarization alignment of the mesh was not completely achieved at the shorted ends of the resonator. The equal area rule resulted in a total surface area of the grid elements of approximately twice the modeled conductor area.

V. MODEL FIELD EXCITATION AND SIMULATION

A small rectangular loop near the base of the cavity wire grid model provided the excitation to the model. This loop was added by manually editing the NEC geometry input file. The input files were then simulated using NEC2++ ver. 1.2.3, a PC implementation of NEC in the C++ programming language [9]. Approximate resonance peaks were found through iterative frequency sweeps. The coupling loop area and the loop's position were adjusted so that the simulation achieved reasonable coupling and the cavity *Q* could be reliably determined. Note that the effect of the coupling structure on *Q* was later removed from the data, and attention was focused on the unloaded quality factor, independent of the coupling structure.

A series of half-wave coaxial cavities were modeled to obtain the simulated impedance spectra. The inner conductor radius, *a*, and the cavity length, *z*, were held fixed arbitrarily, at 1 m and 12 m, respectively. The outer radius *b* was allowed to vary from 2 m to 6 m in increments of 0.5 m. Larger outer radii were not simulated, as 6 m is close to the upper limit for the TEM resonance mode [10]. Some sample cavity models are shown in Fig. 2.

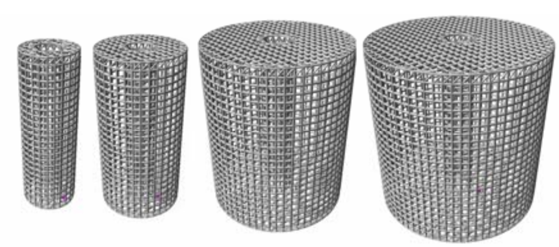

Fig. 2. Sample wire grid cavity models, *a* = 1 m, b = 2 m to 6 m, and *z* = 12 m.

VI. THEORETICAL, SIMULATION, AND EXPERIMENTAL RESULTS

Once simulations were complete, the simulated impedance spectra were used to determine the corresponding unloaded quality factors. The impedance spectra were transformed to reflection coefficients, as they would have been measured with a 50 Ω network analyzer. The loaded (by coupling loop) and unloaded quality factors were then determined by the half power frequency span about the resonance frequency. This was performed on the Smith chart, where the locus of the impedance is known to form a circle in the vicinity of the resonance frequency. A freely available piece of software readily performs these calculations from network the analyzer data. For references on the software and other methods of determining quality factors from impedance data, refer to Ginzton, Kaifez, and Hwan [11][13]. The resulting unloaded quality factors for the half-wave cavities were then normalized with respect to the skin depth, δ, and the wavelength, λ, and compared to theoretically calculated values, as given in equation (6). Several mesh edge lengths were simulated for each cavity. The edge length data was then extrapolated to a conceptual length of zero through a quadratic least squares fit of Q_u. For comparison purpose, experimental cavities (Fig. 3) with correspondingly scaled dimensions were constructed from brass and measured on a network analyzer. The measured data were processed identically to the simulated data to determine the normalized cavity form factors. The results are plotted in Fig. 4.

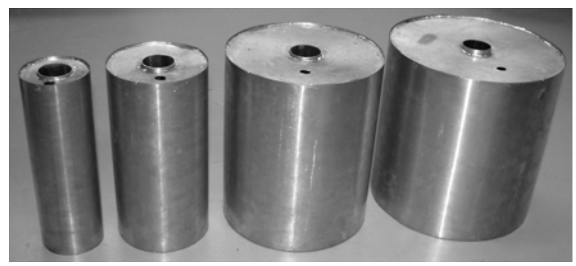

Fig. 3. Experimental cavity models, *a* = 1 in, *b* = 2 in to 6 in, and *z* = 12 in.

VII. DISCUSSION AND CONCLUSIONS

As shown in Fig. 4, mesh size has a considerable effect on the magnitude of the simulated form factors. However, the general shape of the simulated curves for each mesh element size agrees with theory and experiment. A large mesh size seems to result in erratic simulation results especially around *b*/*a* = 5.5 m, which is not reflected in the experimental data. These erratic results must therefore be numerical instabilities rather than excitation of higher resonance modes. As expected, finer mesh sizes result in more accurate and better behaved simulation results, but at the cost of additional computation time. The simulation error in absolute magnitude can be corrected by artificially shifting up the curves, which is equivalent to decreasing the conductivity of the wire grid elements. The required adjustments in conductivity for the 0.5 m and 0.375 m grids, is about 1/5 and 1/4, respectively, and seems to correlate well with Moore and Pizer's suggestion of a simulated area up to five times the actual area [6]. Alternatively, the simulation results for each *b*/*a* can be extrapolated to a conceptual mesh element length of zero. This then results in excellent agreement between

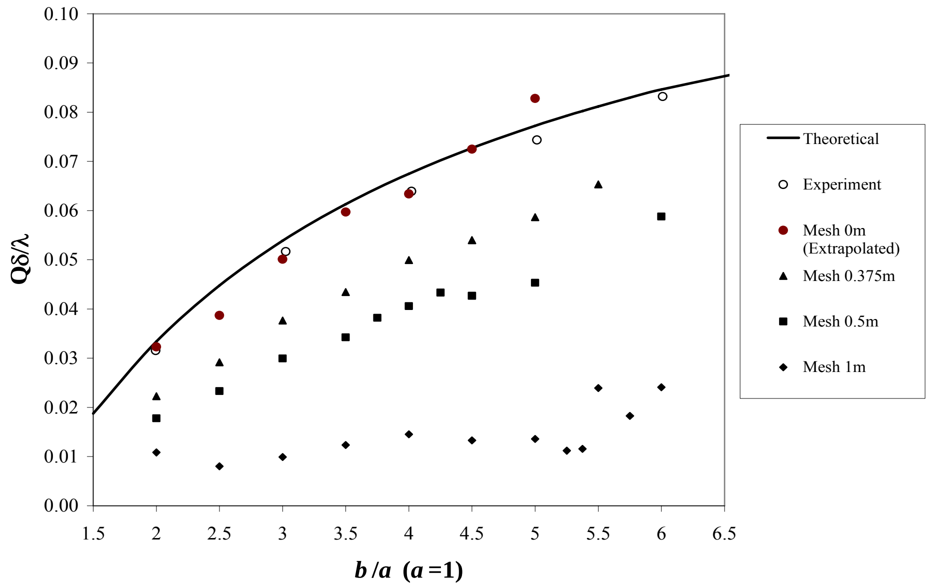

Fig. 4. Comparison of theoretical, simulated and experimental form factors.

experiment, theory and simulation. In practice, experimental data is expected to be slightly below theoretical, due to surface imperfections that lower the overall surface conductivity. Despite modeling errors, such as low level radiation leakage from meshed structures representing closed volumes and inaccurate absolute surface impedance modeling at larger mesh element sizes, trends in the simulation results are retained. This allows for meaningful comparison of other resonant geometries via simulation by using wire grid meshes with identical mesh parameters. As an alternative, the mesh element size can be varied and the results extrapolated to the limiting case of a zero length edge element. This will give quantitatively simulation results with much better accuracy. Wire grid modeling can be a valuable tool, not just for radiation and scattering problems, but even for problems that show sensitivity to surface impedance.

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