Dispersion Analysis of a Negative Group Velocity Medium with MATLAB

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Abstract **–** The dispersive behavior of a waveguide loaded with a metamaterial slab is investigated by means of common mathematical tools demonstrating the existence of a superluminal propagation region. More specifically, MATLAB has been used to investigate the influence of the propagating pulse shape on the possibility to achieve a propagation characterized by a negative group delay. Results achieved in this way have also been validated by means of a Finite Difference Time Domain code.

Keywords: Anomalous dispersion, left-handed media, negative group velocity, gaussian pulse, and MATLAB.

I. INTRODUCTION

Metamaterials (MM) are an appealing new frontier of electromagnetic research, attractive for a wide range of applications. More specifically, media with negative values of the constitutive parameters (ε<0, μ<0), firstly investigated by Veselago [1], have recently attracted a great interest in the scientific community. Among these, the following classes of materials can be identified:

- *Epsilon-Negative* (ENG): media with a negative electrical permittivity and a positive magnetic permeability;
- *Mu-Negative* (MNG): media with a positive electrical permittivity and a negative magnetic permeability;
- *Double Negative* (DNG): media with both the electrical permittivity and the magnetic permeability less than zero (conversely, conventional media with positive values of these parameters are defined Double Positive -DPS).

In the last years, theoretical and experimental studies have demonstrated that the unusual dispersion characteristics of these media induce interesting phenomena, such as: reversed refraction, reversed Doppler effect, backward or superluminal propagation [1-8].

In this paper we focus on the possibility to observe the superluminal propagation phenomenon in these artificial structures; specifically we investigate the dispersive behavior of a MM slab-loaded waveguide (WG) [6-8] demonstrating the existence of a negative group velocity region.

Furthermore, two useful strategies of analysis are presented: a numerical method and an approximated analytical approach.

It is proved that a useful preliminary analysis can be easily performed by means of common mathematical tools such as MATLAB [8]. It is also demonstrated that by using the effective medium theory a more detailed analysis can be approached in an efficient way by means of a Finite Difference Time Domain (FDTD) [9] code based on the Auxiliaries Differential Equations (ADE) method to deal with dispersive media.

Both approaches have been used to investigate the influence of the propagating pulse shape on the Negative Group Velocity (NGV) phenomenon.

The paper is structured as follows: first, the superluminal propagation phenomenon is briefly introduced in section II, and then the dispersion characteristic of a WG loaded by a MM-slab is given in section III.

Later on sections IV and V describe the analytical approach and the FDTD code here proposed, whilst in section VI we report the results obtained for the propagation of modulated signals in a MM-slab-loaded WG. Finally some conclusions are drawn in section VII.

II. SUPERLUMINAL PROPAGATION

For a small-bandwidth signal propagating in a medium characterized by an effective propagation constant γ_{eff} ($\gamma_{eff} = \alpha_{eff} + j\beta_{eff}$), many different kinds of velocity can be defined. To introduce them it can be useful to expand the phase propagation constant in a Taylor series,

$$\begin{aligned} \beta(\omega) &\approx \beta(\omega_0) + \left.\frac{d\beta(\omega)}{d\omega}\right|_{\omega_0} \Delta\omega + \\ &+ \frac{1}{2}\left.\frac{d^2\beta(\omega)}{d\omega^2}\right|_{\omega_0} \Delta\omega^2 + \ldots = \\ &= \beta_0 + \beta'_0\,\Delta\omega + \frac{1}{2}\beta''_0\,\Delta\omega^2 + \ldots, \\ \Delta\omega &= (\omega - \omega_0). \end{aligned} \tag{1}$$

where the angular frequency ω_0 is the carrier frequency of the propagating signal, whose time characteristic can be written as,

$$\widetilde{\mathbf{E}}(t,\omega_0) = E_u(t,\omega_0)\hat{\mathbf{u}} = A(t)\cos(\omega_0 t)\hat{\mathbf{u}} = \\ = \Re\mathrm{e}\{A(t)\exp(j\omega_0 t)\}\,\hat{\mathbf{u}}\,. \tag{2}$$

Related to the zero- and first-order term of equation (1), we can introduce:

– *The phase velocity*, which is the propagation velocity of the sinusoidal signal appearing in equation (2),

$$v_{p,eff}(\omega_0) = \left(\frac{\omega}{\beta_{eff}}\right)_{|\omega_0} = \frac{\omega_0}{\beta_0}\,. \tag{3}$$

– *The group velocity* (v_P), which is the velocity by which the $A(t)$ peak travels,

$$v_{g,eff} = \left[\frac{d\beta_{eff}}{d\omega}\right]^{-1} = (\beta')^{-1}\,. \tag{4}$$

The medium dispersion properties are related to the second-order term of equation (1), commonly known as 'second-order dispersion': it is equal to zero for non-dispersive media, whilst it is greater or less than zero respectively for a normal or an anomalous dispersion medium.

The term 'superluminal' refers to propagation with values of v_g/v_p negative, or greater than the speed of light in vacuum (c).

It is well known that in some media (a possible example being a hollow waveguide) v_p can be superluminal; the phenomenon can be easily explained by considering that v_p has not a physical meaning, indeed it represents the velocity of propagation of a perfectly monochromatic wave of light which is not a real entity.

More surprising it appears the phenomenon of superluminal values of v_g, due to the existence of media with anomalous dispersion regions.

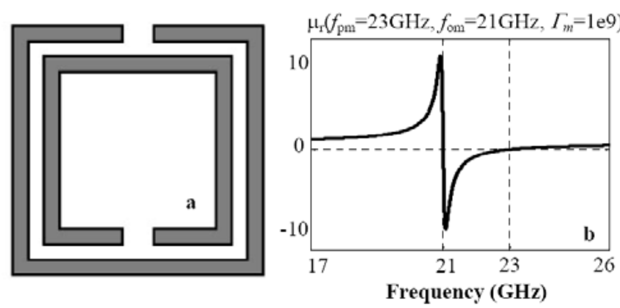
As observed by Sommerfeld and Brillouin [10], the misunderstanding lies in identifying the signal peak velocity with the carried information velocity, so that superluminal values of v_g seem to be inconsistent with Einstein's relativity theory. In [10], by considering a Lorentz medium, they showed that a relativistically causal propagation is exclusively connected to the velocity by which the signal switching-on instant travels (the so called front velocity) which must be limited exactly by c.

These topics are still much debated. Sommerfeld's reasoning has been confirmed by a large number of theoretical and experimental results demonstrating that the superluminal v_g phenomenon can be observed in several artificial structures [3-5,11-13], such as the DNG medium made of alternating layers of wire arrays and Split Ring Resonators (SRRs) arrays (SRR-wire medium) [14]. In the following, it will be showed that similar observations can be developed for a waveguide loaded by a MM-slab and, in order to investigate how the propagating signal time shape acts on the superluminal propagation phenomenon, the results obtained for the propagation of amplitude modulated signals will be reported and discussed.

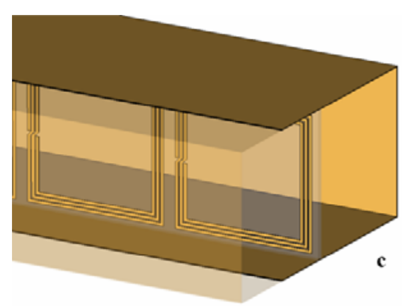
III. DISPERSION ANALYSIS OF A WAVEGUIDE LOADED BY A MM SLAB

A. The SRR Particle

A typical Split Ring Resonator (SRR) is shown in Fig. 1; it consists of two concentric rings interrupted by a gap, and was firstly proposed by Pendry [15] as elementary building block of a medium with negative values of the magnetic permeability (MNG medium). The SRR is a strongly resonant structure, whose resonant behavior is excited by an external time-varying magnetic field perpendicular to the ring surface, inducing currents that produce a magnetic field that may either oppose or enhance the incident field, thus resulting in positive or negative effective permeability.



(a) (b)



(c)

Fig. 1. (a) The Split Ring Resonator particle. (b) The Split Ring Resonator magnetic effective relative permeability. (c) The analyzed structure: a waveguide loaded with a Mu-Negative-slab consisting of an array of spiral resonators on a dielectric substrate.

Nowadays this particle has been studied and experimentally characterized extensively in the literature, demonstrating that an array of SRRs on a dielectric substrate exhibits MNG behavior around the SRR's resonant frequency. The corresponding effective magnetic permeability is given by [4, 16],

$$\mu_{r,eff}(\omega)=\left(1-\frac{\omega_{pm}^2-\omega_{om}^2}{\omega^2+j\omega\Gamma_m-\omega_{om}^2}\right) \tag{5}$$

where Γ_m is the magnetic damping constant, whilst ω_{pm} and ω_{om} are respectively the SRR magnetic plasma and resonance frequencies. They determine the frequency range in which the SRR array behaves as an effective homogeneous (MNG) medium (see Fig. 1(b)).

Furthermore, in order to simplify the design at high frequency or to enhance the SRR magnetic response, in the last years, several modified resonator structures have been proposed, such as: the single ring, the spiral resonator, etc. [17,18].

B. SRR-Slab Loaded WG

One of the more attractive applications for MNG media has been suggested in [6-7], where an array of SRRs on a dielectric substrate has been used as loading slab of a hollow metallic waveguide (WG) to achieve useful stop-band or pass-band behaviors [6-7] (see Fig. 1(c)).

The dispersion equation of the dominant TE_{10} WG mode becomes,

$$\beta_{TE,10}=\beta_0\sqrt{\mu_{r,eff}\left(1-\frac{\omega_c^2}{\omega^2}\right)}\triangleq\beta_0 n_{eff} \tag{6}$$

where β_0 is the free-space phase propagation constant, ω_c is the WG cutoff frequency; consequently, depending on the values assumed by the SRR parameters, a pass-band/stop-band behavior can be generated below/above ω_c.

By using the effective refractive index of the MM-loaded WG, introduced in equation (9), we have,

$$v_p=\frac{\omega}{\beta_{eff}}=\frac{c}{\Re e\{n_{eff}\}},$$

$$v_g=\Re e\left(\frac{d\beta_{eff}}{d\omega}\right)^{-1}=\Re e\left(n_{eff}+\omega\frac{dn_{eff}}{d\omega}\right)^{-1}, \tag{7}$$

$$\frac{\tau_g}{d}=\Re e\left\{n_{eff}+\omega\frac{dn_{eff}}{d\omega}\right\}.$$

being $\tau g/d$ the normalized group delay.

Figure 2 compares the normalized phase propagation constant and group delay of the MM-loaded WG with those corresponding to the SRR-wire medium analyzed in [4] with ω_{pe}=ω_c and Γ_e=0 (the realistic values reported in [4] have been used for the SRR medium, whilst ω_c has been fixed to 40 GHz which is the value assumed for ω_{pe} in [4]): an anomalous dispersion region, characterized by simultaneously negative values of v_g (Negative Group Velocity-NGV) and v_p, can be noticed around the SRR resonant frequency. Furthermore, according to [6], a pass-band with backward propagation characteristic can be also observed below the WG cutoff frequency.

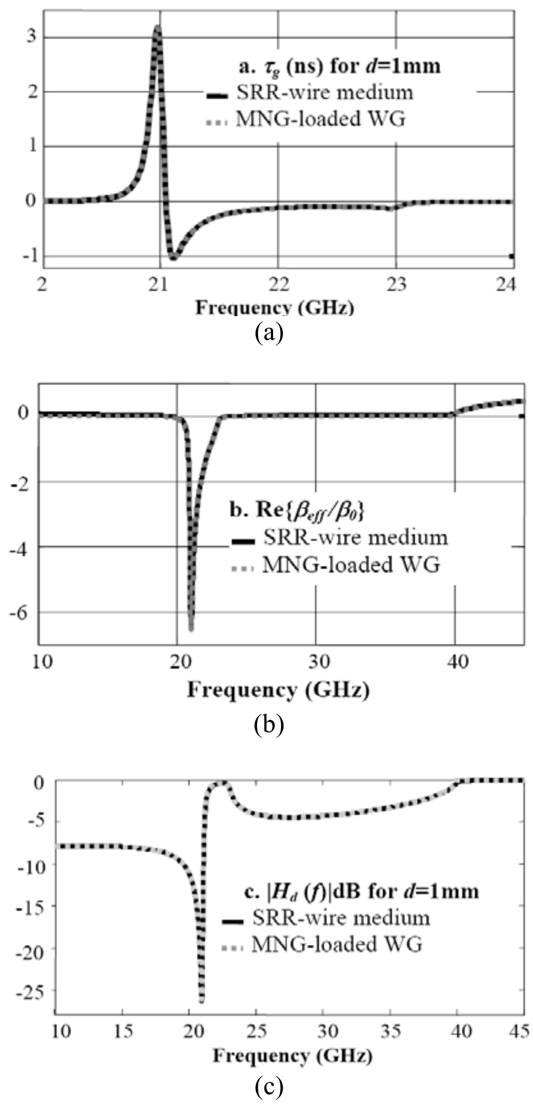


Fig. 2. Comparison between the Split Ring Resonator-wire medium analyzed in [4] and the waveguide loaded by a Mu-Negative (MNG) slab (f_{pm}=23GHz, f_{om}=21GHz, f_c=40GHz). (a) Group delay calculated for d=1mm, (b) propagation constant, and (c) attenuation calculated for d=1mm.

IV. ANALYTICAL APPROACH

The proposed approach refers to a MM-slab, characterized by an effective refractive index and phase propagation constant,

$$n_{eff} = \sqrt{\varepsilon_{r,eff}}\,\sqrt{\mu_{r,eff}} = n'_{eff} - jn''_{eff}\,,$$
$$\beta_{eff} = \frac{\omega}{c} n_{eff} = \beta'_{eff} - j\beta''_{eff}\,. \tag{8}$$

For a plane wave, with angular frequency ω_0, impinging on the slab front-face with an angle θ_i, the slab transfer function is given by [4],

$$H_d(\omega) = \frac{T_{12}T_{21}\exp\{-j\beta_{eff}\cos(\theta_t)d\}}{1-R_{12}^2\exp\{-j2\beta_{eff}\cos(\theta_t)d\}} =$$
$$= |H_d(\omega)|\exp\{+\phi(\omega)\} \approx \exp\{-j\beta_{eff}\,d\} \tag{9}$$
$$\left\{\sin(\theta_t) = \sin(\theta_i)\frac{n_1}{n_{eff}}\right\}.$$

Being T_{ij} and R_{ij}, respectively the Fresnel's transmission and reflection coefficients at the slab interface with respect to the surrounding medium (with a refractive index n_1).

In the case of a normal incidence (i.e., θ_i =0), we have,

$$H_d(\omega) = |H_d(\omega)|\exp\{-j\beta_{eff}\,d\}. \tag{10}$$

We assume a linear polarization and an amplitude modulation for the incident field, so that in the time domain it can be expressed as in equation (2), where the time envelope, $A(t)$, is assumed to be a slowly varying function. Employing $H_d(\omega)$, and assuming that $\phi(\omega)$ can be expanded around ω_0 as a Taylor series arrested to the second-order term,

$$\phi(\omega) \approx \phi(\omega_0) + \left.\frac{d\phi(\omega_0)}{d\omega}\right|_{\omega_0}\Delta\omega +$$
$$+\frac{1}{2}\left.\frac{d^2\phi(\omega_0)}{d\omega^2}\right|_{\omega_0}\Delta\omega^2 \triangleq \tilde{\phi}(\omega). \tag{11}$$

The signal transmitted by the slab can be determined by using the approach proposed in [19]; indeed, by using the Direct/Inverse Fourier transformation (DFT/IFT) we have,

$$E_u(t,d) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \mathrm{E}_u(\omega,\omega_0)|H_d(\omega)|e^{+\{\phi(\omega)+j\omega t\}}d\omega \approx$$
$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \mathrm{E}_u(\omega,\omega_0)|H_d(\omega)|e^{+\{\tilde{\phi}(\omega)+j\omega t\}}d\omega. \tag{12}$$

Being $\mathrm{E}_u(\omega,\omega_0)$ the Electric Field Fourier Transform (FT). The main difference, with respect to the analysis performed in [19], is that, due to the resonant behavior of the MM effective magnetic permeability given in equation (5), in the case under analysis the hypothesis of transfer function with nearly constant amplitude is not applicable.

Considering that the transfer function of a real slab must have a Hermitian symmetry,

$$H_d(-\omega) = H_d(\omega)^* \Rightarrow IFT\{H_d(\omega)\} \in \Re e \tag{13}$$

equation (12) becomes,

$$E_u(t,d) = IFT\{\,FT\{\,E_u(t,d=0)\,\}H_d\,\} =$$
$$= \frac{2\Re e}{\sqrt{2\pi}}\left\{\int_0^{+\infty} FT\{E_u(t,d=0)\}H_d\,e^{+\{\tilde{\phi}(\omega)+j\omega t\}}d\omega\right\} = \tag{14}$$
$$\triangleq 2\Re e\{IFT^+\{\,FT\{E_{u,0}(t)\}\tilde{H}_d(d)\,\}\}.$$

By discretizing the input signal and substituting the FT/IFT with the Discrete Fourier Transform (DFT)/Inverse Discrete Fourier Transform (IDFT), we have,

$$\begin{array}{ccc}
 & E_z(t, f_0, d=0) & \\
 & \text{discretization}, N\text{ points} & \\
 & \downarrow & \\
\sum_{K=1}^{N} E_z(kT_c, f_0, d=0) & , & H_d(f) \\
DFT, N\text{ points} & & DFT, N/2\text{ points} \\
\downarrow & & \downarrow \\
\sum_{K=1}^{N} \mathrm{E}_z\left(\frac{k}{N}f_c, f_0, d=0\right) & , & \sum_{K=1}^{N/2} H_d\left(\frac{k}{N}f_c, d\right) \\
 & \downarrow & \\
 & \left\{\sum_{K=1}^{N/2} \mathrm{E}_z\left(\frac{k}{N}f_c, f_0, d=0\right)\times H_d\left(\frac{k}{N}f_c, d\right)\right\} & \\
 & IDFT, N\text{ points} & \\
 & \downarrow & \\
 & \sum_{K=1}^{N} E_z(kT_c, f_0, d) &
\end{array} \tag{15}$$

where f_c is the sampling frequency and T_c the sampling time (f_c=1/T_c) which must be fixed according to Shannon's theorem. The response to any input signal can be now calculated by using a common and efficient mathematical tool, such as, for instance, MATLAB [19].

In this way, the effect of the dispersive behavior of a MM on finite bandwidth signals can be easily evaluated. Figure 2(c) shows the amplitude of the transfer function corresponding to the SRR-loaded WG (the SRR and WG parameters are the same assumed in the previous section). We can see that the NGV region corresponds to an absolute minimum of H_d.

This is in agreement with the analysis developed in [20], demonstrating that any causal medium with a linear refractive index must exhibits superluminal propagation regions centered at the frequency corresponding to an absolute maximum of the medium absorption.

In the following, equation (15) will be employed to study the propagation of amplitude-modulated signals in this anomalous dispersion region.

V. THE FDTD CODE

In order to validate the results obtained by using the analytical approach presented in the previous section we use a proprietary FDTD tool in a Total Field /Scattered Field (TF/SF) formulation using Mur's II order boundary conditions [9]. The TF/SF formulation allows the evaluation of the slab response to a modulated signal in a 1-D environment, reducing considerably the computational time.

Furthermore, by modeling the MM-loaded WG as an effective homogeneous medium with a relative effective magnetic permeability given by equation (5) and a relative effective electric permittivity given by,

$$\varepsilon_{r,eff} = \left(1 - \frac{\omega_c^2}{\omega^2}\right). \tag{16}$$

The analysis with the FDTD code has been approached in an efficient way by using the Auxiliary Differential Equation method to deal with time dispersion [9]. More specifically the following phasor polarization currents have been employed to simulate the propagation of a plane-wave electric field, with linear polarization, normally impinging on the MM-loaded waveguide,

$$\begin{aligned} J_p(\omega) &= \frac{j\omega\varepsilon_0\omega_{pe}^2}{\left(-\omega^2\right)} E(\omega), \quad M_p(\omega) = \\ &= j\omega\mu_0 \left(\frac{\omega_{pm}^2 - \omega_{om}^2}{\omega_{om}^2 + j\omega\Gamma_m - \omega^2} \right) H(\omega). \end{aligned} \tag{17}$$

The electric polarization current (J_p) given in equation (17) is related to the dispersion equation of the dominant TE_{10} WG mode (i.e., to $\varepsilon_{r,eff}$), whilst the magnetic polarization current (M_p) allows to account the SRR-slab (i.e., it is related to $\mu_{r,eff}$).

VI. RESULTS

Referring to Fig 1(c), in this section we assume that a signal such that given in equation (2) is applied at the input port of a MM-slab loaded WG. By fixing the WG length equal to d, equation (15) and the FDTD code have been used to calculate the time-domain characteristic of the propagating signal at the output port. In the following we report some results obtained by assuming that the time envelope of the input signal is a real function. More specifically, the shapes of the propagating signal have been fixed in order to investigate the assertions made in [14], where the NGV phenomenon has been attributed to asymmetrical energy absorption from the propagating signal: Crisp pointed out that the attenuation experienced by a propagating pulse depends on the time derivative of its time envelope. Consequently, in the following, the case of signals with a trailing and leading portion characterized by an exponential, a sinusoidal and a constant time derivative have been considered (see Fig. 3). Furthermore, as evident from Figs. 3 and 4, the simulated signals exhibit different types of discontinuities, allowing evaluating how the 'well-behaved' property of the propagating pulse influences the NGV phenomenon.

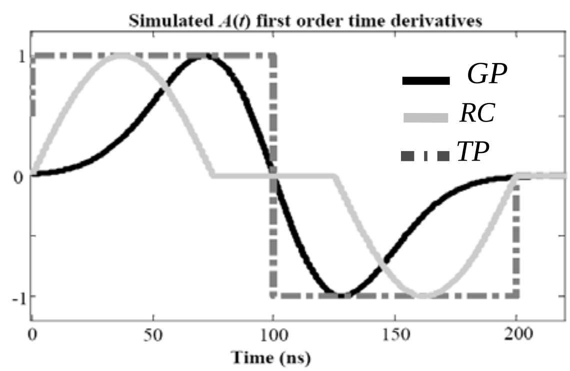


Fig. 3. Normalized first order time derivatives of the time envelope of the simulated signals.

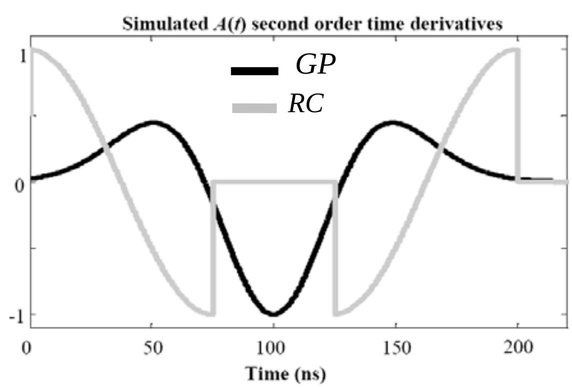


Fig. 4. Normalized second order time derivatives of the Gaussian and Raised Cosine function.

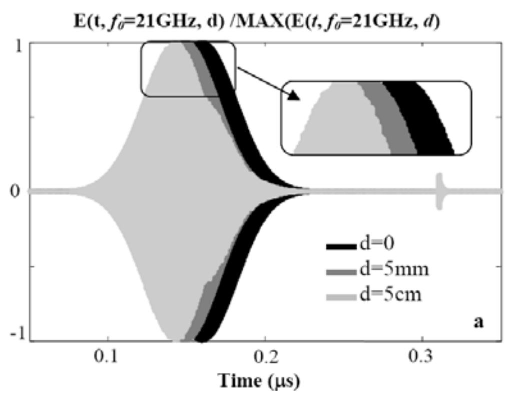
A. Gaussian Pulse

The propagation of a small bandwidth Gaussian Pulse (GP) in an NGV medium is a well known topic [11-14, 21]. It has firstly theoretically investigated by Garrett and McCumber [21], which demonstrated that under some easily verified hypotheses the GP propagates at superluminal group velocity preserving its shape; starting from Faxvog's [11] results, the phenomenon has been confirmed by several experimental observations.

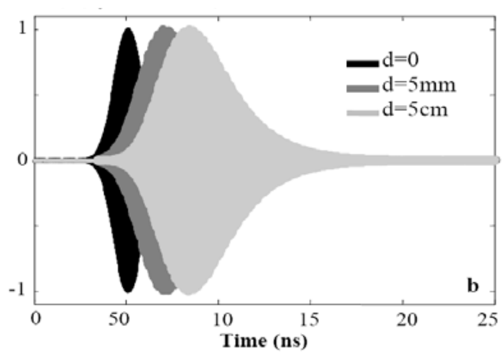
Consequently, in order to validate both the analytical approach and the FDTD code presented in the previous sections, we start our analysis assuming that the input signal time envelope is a (GP) function,

$$A(t) = GP(t,\sigma) = S\exp\left\{-\frac{(t-\tau)^2}{\sigma^2}\right\} \quad . \tag{18}$$

The results obtained by solving equation (15) with MATLAB are coincident with those obtained by using the FDTD code; they are reported in Fig. 5.



(a)



(b)

Fig. 5. Results obtained by using equation (14) for the propagation of a Gaussian Pulse (GP) in a waveguide loaded with a Split Ring Resonator slab (f_{pm}=23GHz, f_{om}=21GHz, f_c=40GHz): (a) GP characterized by σ=30ns, τ=0.16μs, (b) GP characterized by σ=1ns and τ=15ns.

Figure 5(a) refers to a small-bandwidth GP (σ=30ns and τ=0.16μs) modulating a carrier signal at f_0=21GHz. The solution time required on a Pentium 4-2.8 GHz with the MATLAB approach was equal to 27 seconds (s).

As expected the pulse peak experiences a negative group delay during the propagation (i.e., the peak of the output pulse precedes that of the input pulse).

With reference to an absorbing medium with a Lorentzian inhomogeneous line shape, a similar phenomenon has been attributed by Crisp to an energy absorption which, with respect to those corresponding to a constant amplitude light beam, is greater in the case of a rising amplitude (positive time derivative), and smaller in the case of a decreasing amplitude (negative time derivative).

In Fig. 5(b) the results are given for the same analysis obtained for a broadband GP (σ=1ns and τ=15ns, solution time on a Pentium 4-2.8 GHz equal to 25s). In this case our calculation predicts that the pulse peak travels at a positive group velocity.

B. Raised Cosine

The group velocity concept is related to the peak of the propagating pulse; consequently, in order to deeply investigate the NGV phenomenon, the case of a propagating signal with a not-defined amplitude peak has been also considered in our analysis.

More specifically, we studied the propagation of a Raised Cosine (RC) signal,

$$A(t) = RC(t) =$$

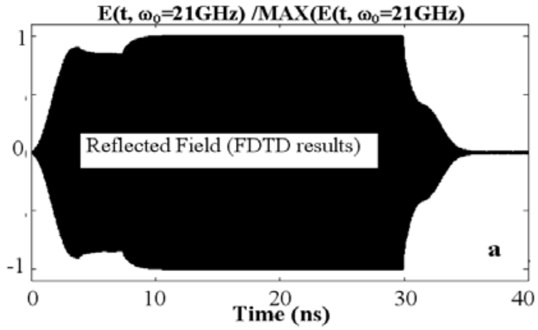
$$= \begin{cases} S \quad , & |t'| \le (1-\rho)2T \\ Sf(t) \quad , & (1-\rho)2T \le |t'| \le (1+\rho)2T \, , \\ 0 \quad , & elsewhere \end{cases} \tag{19}$$

$$f(t) = \left\{1 - \sin\left[\frac{\pi}{\rho}\left(\frac{|t'|}{4T} - \frac{1}{2}\right)\right]\right\}.$$

From equation (19) it is evident that $A(t)$ has a leading and trailing portion with a sinusoidal time derivative (see Fig. 3), and that a constant amplitude portion is also present in its time characteristic.

As in the GP case, two RC signals have been simulated (referred in the following as RC1/2): RC1 is a small bandwidth signal, so that its frequency characteristic is within the anomalous dispersion region of the MM-loaded WG, whilst RC2 is a broadband signal: its spectrum has a significant amplitude in both regions of anomalous and normal dispersion corresponding to NGV and backward propagation.

As noted from Figs. 6 and 7, where the results obtained with MATLAB are reported, in both cases the propagating signal experiences a strong distorsion: the signal energy concentrates at the switch-on/off instants and at the instants corresponding to the transition from the sinusoidal to the flat portion of the RC pulse. From Fig. 4 it is evident that these instants correspond to jump discontinuities for the second-order time derivative of the RC function.



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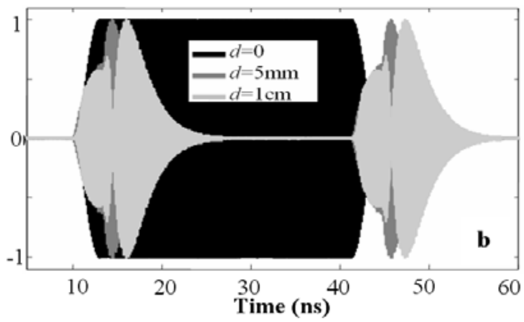


(b)

Fig. 6. Results obtained for the propagation of a Raised Cosine function (ρ= 0.1, T=7.8ns, f_0=21GHz) in the Split Ring Resonator-loaded waveguide (f_{pm}=23GHz, f_{om}=21GHz, f_c=40GHz). (a) Reflected field calculated by using the ADE-FDTD-TF/SF code. (b) Time characteristic obtained with MATLAB for different values of *d*.

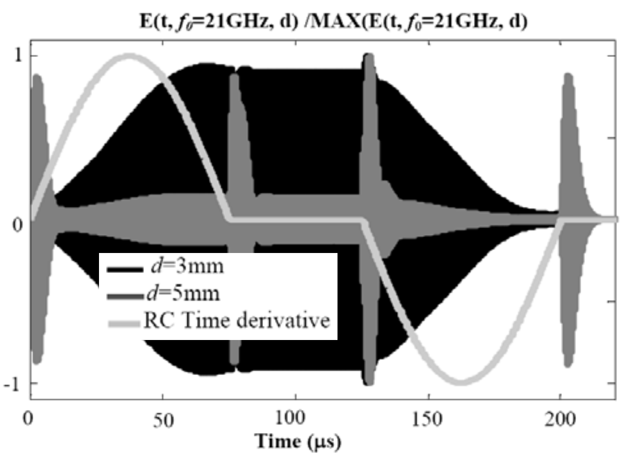


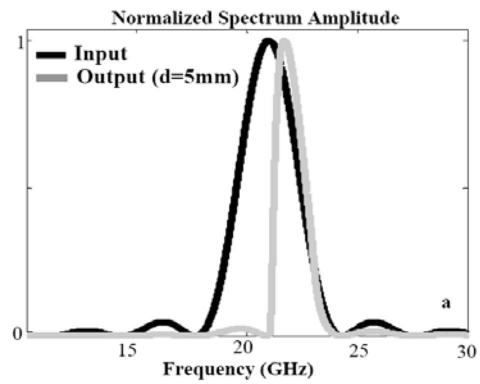
Fig. 7. Results obtained by using equation (14) for the propagation of a Raised Cosine (RC) function in the Split Ring Resonator loaded waveguide (f_{pm}=23GHz, f_{om}=21GHz, f_c=40GHz). The RC parameters are: ρ=0.6, T=31.25ns, f_0=21GHz.

C. Triangular Pulse

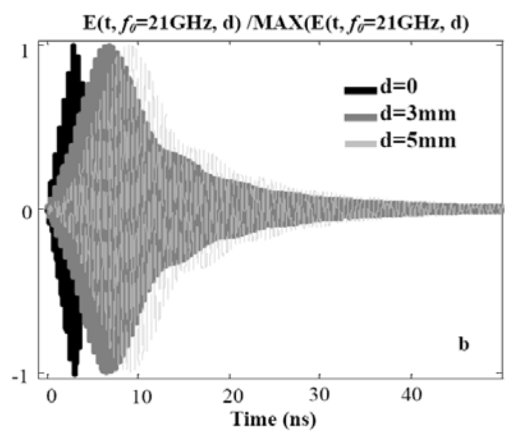
To conclude, we analyze the case of a triangular function as modulating signal,

$$A(t) = TP(t) = \begin{cases} \left(\dfrac{t}{T}\right) & , \quad t \in [0,T] \\ \left(\dfrac{2T-t}{T}\right) & , \quad t \in [T,2T] \end{cases} \tag{20}$$

From equation (20) we can see that in this case the leading and trailing portion exhibit a constant derivative respectively equal to *-1/T* and to *1/T* (see Fig. 3). The results obtained for this broadband TP are given in Fig. 8, they are similar to the one obtained for the GP pulse: due to the asymmetric attenuation experienced by the TP spectrum amplitude (see Fig 8(a)), the output signal is distorted.



(a)



(b)

Fig. 8. Results obtained for a Triangular Pulse characterized by T=0.3ns and f_0=21GHz: (a) comparison between the spectrum amplitude of the input signal and those of the output signal; (b) time domain characteristics obtained with MATLAB for different values of *d*.

In Fig. 9 the results are given for a *TP* with T=100ns. We can see that, as in the small-bandwidth *RC* case, the signal experiences a strong distortion: the *TP* energy concentrates at the instants corresponding to a jump discontinuity for the first-order time derivative of the *TP* function. Furthermore, from Fig. 9(a), we can see that the attenuation experienced by the *TP* spectrum amplitude is strongly asymmetric.

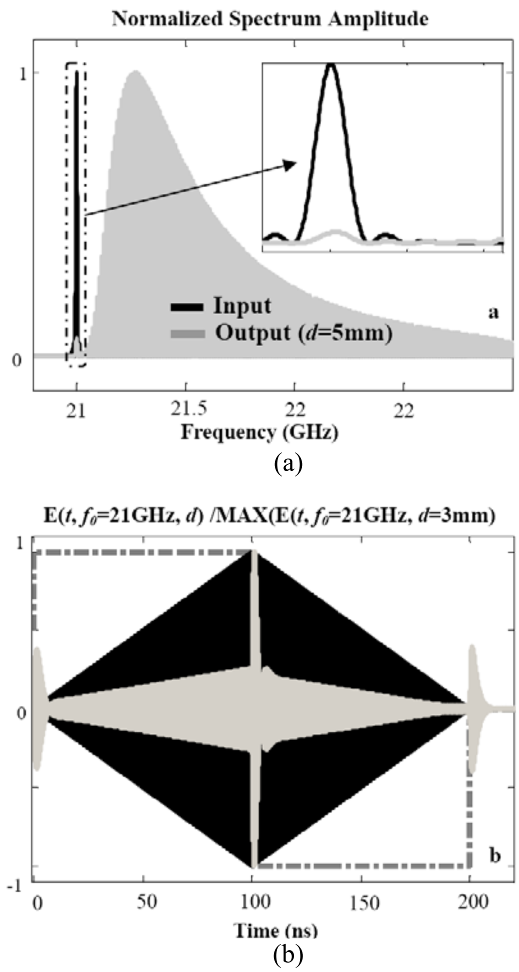


Fig. 9. Results obtained for a Triangular Pulse characterized by T=100ns and f_0=21GHz: (a) comparison between the spectrum amplitude of the input signal and those of the output signal, (b) time domain characteristics obtained with MATLAB for d=3mm (the broken line is the time derivative of the input signal time envelope).

VII. CONCLUSIONS

In this paper, two strategies for the analysis of a waveguide loaded with a Mu-Negative material have been proposed:

– *An approximated analytical approach implemented with MATLAB*: reported results demonstrate that it allows to immediately evaluate the effects of the dispersive behaviour of an effective homogeneous medium on a finite bandwidth propagating signal representing a useful strategy for the so called dispersion engineering;

– *An FDTD code*: based on the effective medium theory and on the Auxiliary Differential Equations method, a very efficient formulation of the problem under analysis has been suggested.

Both approaches have been used to investigate situations of superluminal propagation. More specifically, we firstly theoretically demonstrate the existence of an anomalous dispersion region in the waveguide dispersion characteristic, then, we analyze the effects of this negative velocity behavior on amplitude modulated signals.

Three different time envelopes have been considered:

– The well behaved Gaussian (GP) function;
– The Raised Cosine (RC) function which presents jump discontinuities in the second order time derivative;
– The Triangular (TP) function which presents jump discontinuities in the first order time derivative;

In all cases a broadband and a small-bandwidth signal have been considered. As expected the small-bandwidth *GP* experiences a negative group delay preserving its shape, whilst the *RC* and *TP* signal are strongly distorted after the propagation: the propagating signal energy concentrates at the instants corresponding to the jump discontinuities respectively in the second and first order time derivative. The phenomenon is due to the strongly asymmetric attenuation experienced by the spectrum amplitude of the propagating signals. The relevant consequence is that only the well behaved Gaussian Pulses, characterized by single-lobe spectrum amplitude, experience a negative group delay, preserving its shape during the propagation.

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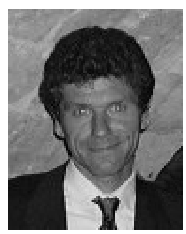

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