Iterative Solution of Electromagnetic Scattering by Arbitrary Shaped Cylinders

Jamila Selmi[1], Rachida Bedira[1], Ali Gharsallah[1], Abdelhafidh Gharbi [1], Henri Baudrand[2]

[1]Laboratoire de physique de la Matière Molle et Modélisation Electromagnetique,
Faculté des Sciences de Tunis, 2092 ElManar, Tunisia
Selmi_jamila@yahoo.com, Rachida.Bedira@fst.rnu.tn, ali.gharsallah@gmail.com,
Abdel.Gharbi@fst.rnu.tn

[2]Laboratoire LAPLACE, 2 Rue Camichel 31071 Toulouse, France
henri.baudrand@yahoo.fr

Abstract **─** The electromagnetic scattering analysis of geometrically complex structure has played an important role in electromagnetic field theory and practicable applications. The fast and rigorous methods to solve such problem are in great demand. In this work, the Wave Concept Iterative Process (WCIP) combined with a coordinate transformation has been extended to the scattering problems of electromagnetic conducting square cylinder, conducting super-ellipsoid and conducting super-ellipsoid coated with thin dielectric materials. Numerical results illustrate the efficiency of our approach. The radar cross section (RCS) is then reached and compared with the literature.

Index Terms **─** Electromagnetic scattering, radar cross section, coordinate transformation, super-ellipsoid, wave concept iterative process (WCIP).

I. INTRODUCTION

The problems of electromagnetic wave scattering by objects of arbitrary shape placed in free space is very important in many applications, namely, antennas design, remote control and especially in defence applications.

The growing interest in better understanding the electromagnetic scattering of conducting objects and coated conducting objects has been reported in many investigations. As a consequence, many different techniques have been used to resolve these electromagnetic problems, such as the Moment Method (MOM) [1], the Finite Element Method (FEM) [2] and Pattern Equation Method (PEM) [3]. Here another technique is adopted called Wave Concept Iterative Process (WCIP) which takes advantage of the other methods. This method was firstly presented in [4-5], applied to analyse microwave circuits [6-7] and was later successfully applied to a number of diffraction and scattering problems [8-10].The concept of waves has been developed for the evaluation of the electromagnetic scattering by conductor objects and coated conductor cylinders [7-9].

In this paper, the formulation of the scattering problems of arbitrarily shaped cylinders is developed. The first step, called coordinate transformation consists of dividing the scattering surface in the small cells. Theses cells can be placed in a fictitious circular cylinder constituted by different materials. In the second step, to study electromagnetic scattering in the cylindrical coordinate system a WCIP is then applied.

To validate the new iterative formulation, the Radar Cross Section (RCS) for a square perfect electric conductor, metallic super-ellipsoid and a super-ellipsoid coated with dielectric materials are calculated. In the last case, the thickness of the dielectric is supposed very weak, so that iterations are considered identical to those obtained by the consideration of laminated flat coats (layers) [9].

The principle of the WCIP is the expression of boundary and closing conditions in term of waves, (incident $\vec{A}$ and reflected $\vec{B}$), a system of equations relate the incident and reflect waves is deduced from these conditions. This system is resolved by an iterative process. The resolution is stopped when a good precision reached on the required value [6-8].

II. FORMULATION

A. Coordinate transformation

Let us consider an arbitrary shaped object bonded by surface S, Fig. 1. The structure is meshed into elementary cells, and these cells can be placed on a fictitious circular cylinder. In this case, the Cartesian coordinate system becomes inefficient; which is why the cylindrical coordinate formulation is used. To pass to the cylindrical coordinate a common normal at all cells, Fig. 1, is found. The study of electromagnetic interaction between two cells is equivalent to the modelling of the wave scattering on one or more coaxial fictitious circular cylinders, which have a common axis (Oz). This common centre is constructed geometrically by the intersection between the right bisectors of the two cells [9].

These cylinders are composed of two materials: metallic parts defined on the scattering surface and an isolator part (dielectric parts). In order to calculate the electromagnetic scattering by the metallic parts, it is necessary to define the cylindrical basis function $\{|f_n\rangle\}_{n\in IN}$ and the impedance of the n^{th} mode (Z_n) [11]. These parameters depend on the radium ($r_{1/2}$) of the cylinders and the position of the metallic cells. So, the incident waves $\vec{A}$ on the metallic cells are reflected. The reflected waves $\vec{B}$ are reflected by the free space and generate the next incident waves via a reflection operator $\hat{\Gamma}$ [11].

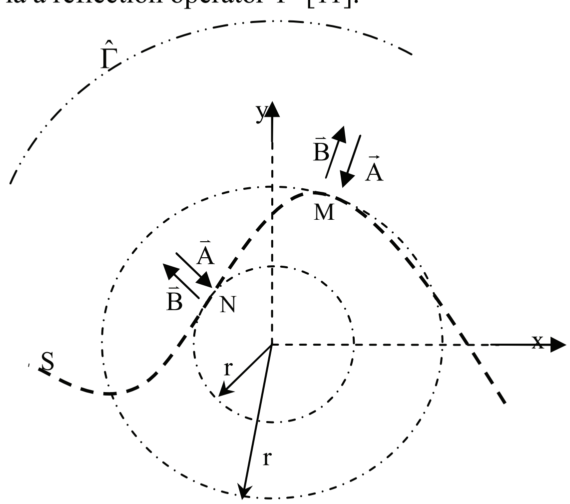

Fig. 1. Meshed surface and coordinate transformation.

B. The wave formulation

The incident $\vec{A}$ and reflected $\vec{B}$ waves are defined by a linear combination of electric field and magnetic one [4-5-8]

$$\vec{A} = \frac{1}{2\sqrt{Z_0}}(\vec{E} + Z_0\vec{J}) \tag{1a}$$

$$\vec{B} = \frac{1}{2\sqrt{Z_0}}(\vec{E} - Z_0\vec{J}), \tag{1b}$$

with, Z_o is an arbitrary impedance. In the following, we shall consider the free space impedance as:

$$Z_o = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \quad \Omega. \tag{2}$$

And the current $\vec{J}$, deduced from the tangential magnetic field $\vec{H}$, is used:

$$\vec{J} = \vec{H} \times \vec{n}, \tag{3}$$

where $\vec{n}$ denoted the normal vector to the surface S defined in each cells.

The Wave Concept Iterative Process is based on the expression of boundary condition, in the case of the scattering by an obstacle, the relationship between $\vec{A}$ and $\vec{B}$ is expressed as:

$$\vec{B} = \hat{\Gamma}\vec{A} \tag{4}$$

$$\vec{A} = R\vec{B} \quad , \tag{5}$$

where $\hat{\Gamma}$ designates the scattering operator associated to the geometry of the target [10] and R is a reflected coefficient defined on the surface;

$$\hat{\Gamma} = \sum_n |f_n\rangle \frac{Z_n - Z_o}{Z_n + Z_o} \langle f_n| \quad . \tag{6}$$

$\{|f_n\rangle\}_{n\in IN}$ is a complete modal basis function, expressed as a solution of the Helmholtz equation in the cylindrical coordinates illustrated by the following equation:

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f_{nm}}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f_{nm}}{\partial\theta^2} + \frac{\partial^2 f_{nm}}{\partial z^2} - k^2 f_{nm} = 0. \tag{7}$$

The separation variable method is used and the solution of this equation is given by:

$$\langle \rho, \theta, z | f_{nm} \rangle = \alpha_{nm} B_n(k_\rho \rho) e^{jn\theta} e^{jk_m z}. \tag{8}$$

In this work, the independence on z coordinate is taken, so the expression of the basis function is reduced:

$$\left|f_n\right\rangle = \frac{1}{\sqrt{2\pi r}} e^{jn\theta} . \qquad (9)$$

Z_n is the impedance of the n^{th} target mode [11] and Z_0 is the parameter introduced in equation (2).

C. The iterative process

The primary electromagnetic fields $(\vec{E}_0, \vec{H}_0)$ are introduced in the spatial domain, the relation ship between the incident and reflected is expressed by the previous equations (4) and (5). These equations are defined in two different domains. The Fourier Transform (TF) and its inverse (TF^{-1}) insure the links between these domains [12].

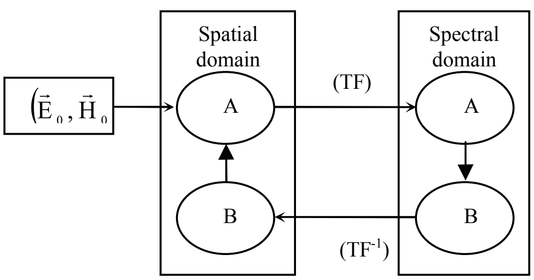

Fig. 2. The iterative scheme.

D. The wave formulation of a dielectric coated layer

By taking into account of the scattering problems of an obstacle coated by a thin dielectric layer, an arbitrary shaped conducting object bonded by surface S_2 coated with a dielectric covering with external surface S_1 , Fig. 3.

The dielectric covering S_1 are homogenous with electrical permittivity ε and magnetic permeability μ. Their expressions are $\varepsilon = \varepsilon_r \varepsilon_0$ and $\mu = \mu_r \mu_0$, where ε_r and μ_r are respectively the relative permittivity and permeability one.

To describe the WCIP and to reach the unknown electric and magnetic fields the interface condition in term of waves at (S_1, S_2) are reviewed and the boundary condition on the environment of the scattering target are written.

Figure 4 shows the multiple transmissions, reflection process for the electromagnetic waves. The theory of the WCIP to analyse electromagnetic

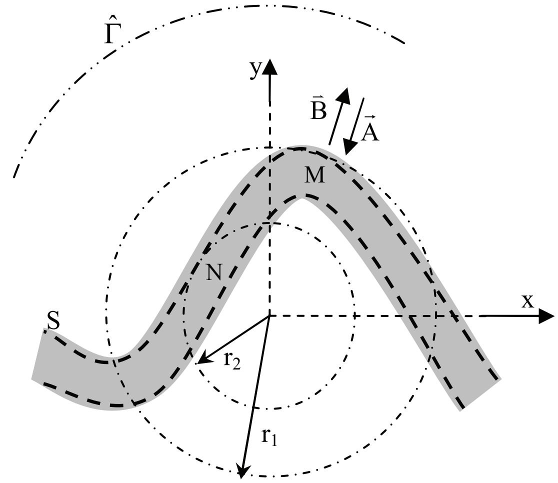

Fig. 3. Dielectric layer coating metallic object.

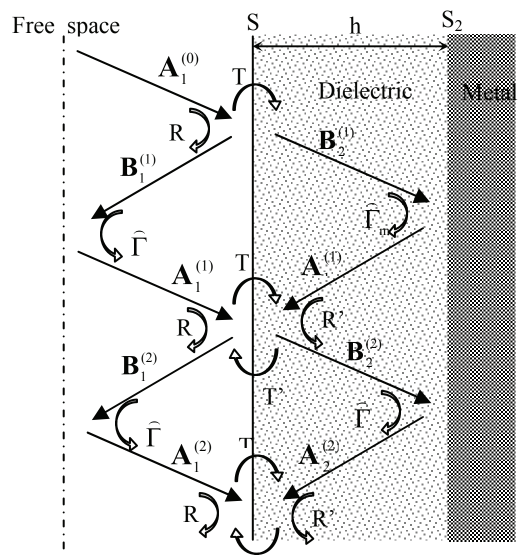

Fig. 4. Dielectric layer coating metallic object.

scattering problems by a treated metallic object is developed previously works [8-9]. Referring to this theory the iterative process explained in different media can be expressed as:

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix}^{(P)} = \begin{pmatrix} R & T \\ T & R' \end{pmatrix} \begin{pmatrix} \hat{\Gamma} & 0 \\ 0 & \Gamma_m \exp(-2jk_2 h) \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}^{(P-1)} + \begin{pmatrix} R & R \\ T & T \end{pmatrix} \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} \qquad (10)$$

$A^{(p)}$ respectively $B^{(p)}$ are the incident and the reflected waves respectively at the p^{th} iteration (p=1,2,3………) with R and R' (respectively T and T') the reflection (respectively the transmission) coefficient defined as [13-14].

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{11}$$

$$R' = \frac{Z_1 - Z_2}{Z_2 + Z_1} \tag{12}$$

$$T = \frac{2Z_2}{Z_2 + Z_1} \tag{13}$$

$$T' = \frac{2Z_1}{Z_2 + Z_1}\quad , \tag{14}$$

where Z_1 and Z_2 are the intrinsic impedances of the free space and the dielectric layer, Γ_m=-1 is the reflection coefficient on the metallic surface, exp(-$2jk_2h$) is the attenuation term in the dielectric layer which is thickens h and $\hat{\Gamma}$ is the reflection operator expressed on equation (6) and defined by Harrington [11].

III. APPLICATIONS AND RESULTS

A. Metallic square cylinder

In order to test the efficiency of the coordinate transformation and the ability of the wave concept with an arbitrary shape, a perfect conductor cylinder with a square section is analysed. The structure is shown in Fig. 5, using TM wave. The dimensions of the square are a=2λ and the frequency is *f*=1GHz; this geometrical parameters are chosen to be the same as in reference [14].

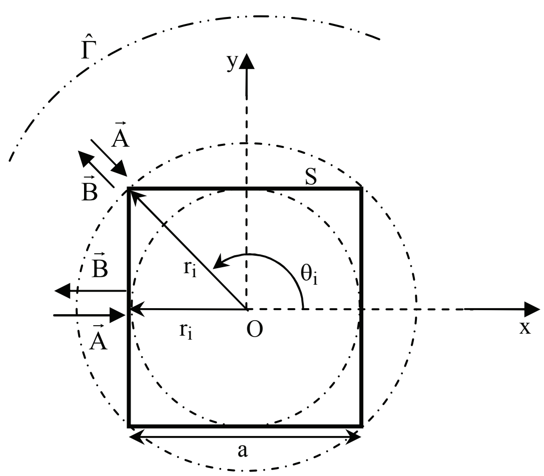

Fig. 5. Conducting square section.

The WCIP leads to the scattering tangential electric field value and the current density at the surface in the side of the out space. The Radar Cross Section is calculated using the far-field transformation, defined by equation (15) [14]:

$$RCS = \lim_{\rho\to\infty}\left(2\pi\rho\frac{|E^s|^2}{|E^i|^2}\right), \tag{15}$$

where ρ is the distance from the target to observation point, E^s and E^i are the scattering and the incident electric fields respectively. To evaluate the scattering operator of the square geometry, the coordinate transformation has been firstly used. Second, the surface current density using the iterative process in the cylindrical coordinates system is then calculated. Finally, the radar cross section, and the numerical results are shown in the following figure.

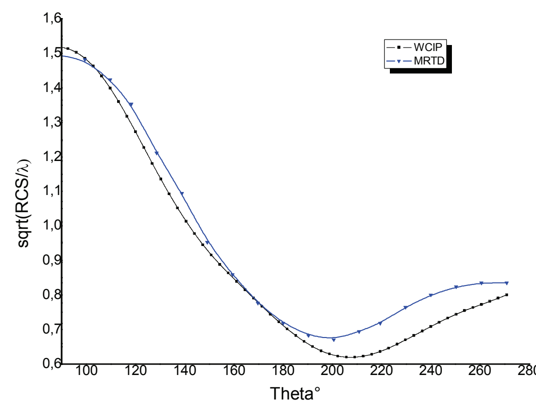

Fig. 6. The normalized RCS of a square geometry.

The results given by WCIP are closely with those found in a Multi Resolution Time Domain MRTD method [14], except in a few points between 210° and 260°. This difference can be explained by the use of some approximation used by the MRTD method and the coordinate transformation applied in the present theory.

A square cylinder is a simple structure to study, using the Waves Concept Iterative Process in case of the electromagnetic scattering some difficulties are founded; The WCIP is based on a definition of the normal at all points of the diffraction surface, and the normal is not defined at corners of the square. That why a regular structure has not any corner called a super-ellipsoid is carried out.

B. Electromagnetic scattering of a super-ellipsoid

In this section, the electromagnetic scattering field by a metallic super-ellipsoid and a super-ellipsoid coated with a dielectric layer are analysed. However, for practical application, general geometrical structures are covered by thin dielectric materials in order to protect the antennas or to find optimum radar cross section.

The super-ellipsoid surface is formed by rotation of a curve $\left(\frac{x}{a}\right)^{2n}+\left(\frac{y}{b}\right)^{2n}=1$, n=8. The structure is illustrated in the following figures. Referring to the theory, when $2n>>1$, the super-ellipsoid can be approximated by a rectangle [15]. In this case, the previous coordinate transformation and the iterative method are combined and applied to analyse scattering by surface treated targets [9].

The scattering parameter calculated here is the radar cross section illustrated by equation (15) which is obtained by evaluating the scattered fields.

It is be noted that the length of the inner conducting super-ellipsoid and the thickness of the dielectric layer are expressed as a fraction of the wave length for a wave number, $ka=7.4$, $kb=12.6$, $h_1=\frac{a}{100}$, $h_2=\frac{b}{100}$ and the relative permittivity of the dielectric layer is $\varepsilon_r=4$.

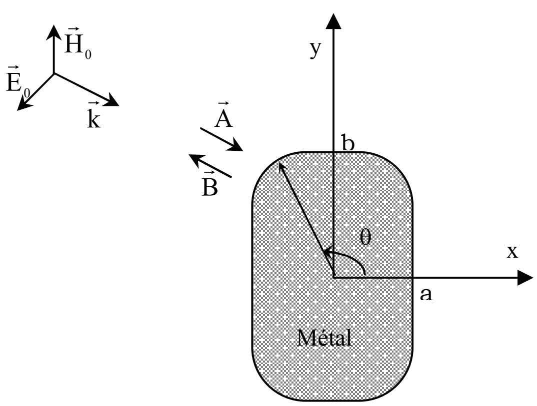

Fig. 7. Metallic super-ellipsoide.

In order to test the efficiency of present combined theory, a process has been implemented in Matlab. The numerical results reach the convergence when the difference between two consecutive iterations is about 10^{-10}. In this case, the convergence of the WCIP is achieved about 25 iterations. The following figure shows the evolution of the surface current density versus the iteration number.

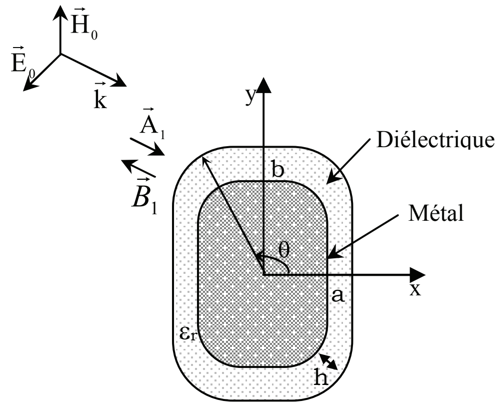

Fig. 8. Super-ellipsoid coated with dielectric material.

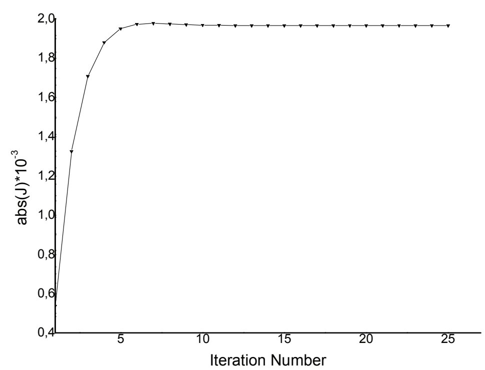

Fig. 9. The normalized current density versus the iteration number.

To illustrate the advantage of covered bodies with dielectric materials, the evolution of Radar Cross Section and the comparison between a metallic super-ellipsoid and the super-ellipsoid covered with dielectric materials are treated, Fig. 10.

It is clear that the effects of dielectric materials is diminution of magnitude of the Radar Cross Section, so diminution of current density distribution generated on the scattering surface which proves the efficiency of dielectric covering conductor bodies.

To illustrate the efficiency of the present iterative formulation, a simulated program is developed, all the calculation up to the Radar Cross

Section. So, the scattering operator of the super-ellipsoid coated with dielectric layer is evaluated. Then, the surface current density and the tangential electric fields are calculated using the iterative process. Finally, the radar cross section is found. The numerical results giving by the WCIP are compared with reference [3], as showing in the following Fig. 11.

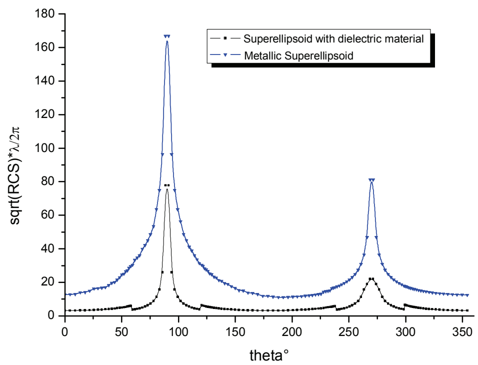

Fig. 10. The RCS of a metallic super-ellipsoid and a super-ellipsoid coated with a dielectric materials.

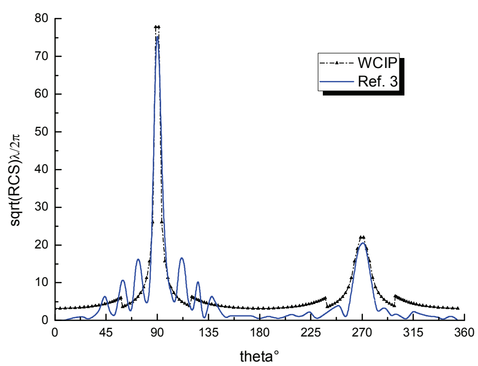

Fig. 11. The RCS of a metallic super-ellipsoid coated with a dielectric materials.

Figure 11 represents the evolution of the radar cross section of a super-ellipsoid coated by a dielectric layer. As expected, the results given by WCIP are in good agreement with those previously published in reference [3], especially at peaks located at 90° and 270° respectively, except that no oscillations have been found in the present work on both sides of these peaks as reported by Pattern Equation Method PEM. To explain this phenomenon, it is assumed that the attenuation term in the dielectric layer as if it was in transmission lines; that is to say the present approach neglect the effect of curvature on the attenuation term. The present study proves that the WCIP takes the advantage of simplicity; it consists of computing a simple recurrent relationship. Furthermore, the iterative method has only small memory requirements. So, all these results have been computed in a personnel computer.

V. CONCLUSION

The WCIP combined with geometry transformation has been extended to the solution of the electromagnetic wave scattering problems by conducting bodies and other conducting bodies coated with thin dielectric layer.

In this research, the coordinate transformation and the presented iterative method are used to calculate the Radar Cross Section of conductor square cylinder, conductor super-ellipsoid and super-ellipsoid coated with a dielectric material. A good agreement in comparison to the MRTD results for the metallic square example and the PEM results for the covered super-ellipsoid are found. The comparison of the scattering characteristics of conductor super-ellipsoid and super-ellipsoid coated with dielectric materials has been done and proved the effect of the dielectric.

The numerical results obtained here show the power of the giving theory. Other works are in progress to extend the iterative method using the geometry transformation to resolve more complicated shapes of electromagnetic waves scattering.

REFERENCES

[1] R. F. Harrington, *Field Computation By Moment Methods*, MacMillan, 1968.

[2] P. P. Silvester and R. L. Ferrari, *Finite Elements For Engineering*, Cambridge University Press, 1990.

[3] A. G. Kyurkchan, D. B. Demin, and N. I. Orlova, "Solution based on the pattern equation method for scattering of electromagnetic waves by objects coated with dielectric materials," *Journal of Quantitative Spectroscopy & Radiative Transfer,* pp. 192-202 , 2007.

[4] R. Bedira, A. Gharsallah, A. Gharbi, and H. Baudrand, "An iterative process based on the concept of waves for electromagnetic scattering problems," *Electromagnetics*, vol. 22, pp. 129-140, 2002.

[5] C. C. H. Tang, “Back-scattering from dielectric coated infinite cylindrical obstacles,” Journal of Applied Physics, vol. 28, no. 5, pp. 628–633, 1957.
[6] A. Gharsallah, A. Gharbi, and H. Baudrand, “Efficient analysis of multiport passive circuits using the iterative technique” Electromagnetics, pp. 73-84, 2001.
[7] M. Azizi, H. Aubert, and H. Baudrand, “A new iterative method for scattering problems,” Proc of the Euro Microwave Conference, vol. 1, pp. 255-258, 1995.
[8] R. Bedira, A. Gharsallah, L. Desclos, A. Gharbi, and H. Baudrand, “The wave concept iterative process: scattering of a conducting target coated by thin dielectric layer,” IEEE Antennas and Propagation Society International Symposium, vol. 2, pp. 98-101, 2002.
[9] A. Mami, R. Bedira, A. Gharsallah, A. Gharbi, and H. Baudrand, “Iterative solution of scattering by surface-treated targets,” Int. J RF and Microwave CAE, vol. 13, no. 3, pp. 206-214, 2003.
[10] N. Raveu, O. Pigaglio, G.Prigent and H. Baudrand, “Improvement in the wave concept iterative procedure through spectral connection,” IEEE EuMW, October 2007.
[11] R. F. Harrington, Time-Harmonic Electromagnetic Fields, McGraw-Hill, New York, 1961.
[12] M. Yeddes, A. Gharsallah, A. Gharbi, and H. Baudrand, “Characterization of microstrip ring with a narrow gap by an iterative method,” International Journal of Microwave Science and Technology, 2008.
[13] S. -G. Pan and I. Wolff. “Scalarization of dyadic spectral Green’s function and network formalism for three dimensional full-wave analysis of planar lines and antennas,” IEEE Trans MTT, vol. 42, pp. 2118-2127, 1994.
[14] C. A. Balanis, Advanced Engineering Electromagnetics, Wiley 1989.
[15] Q. Cao, Ylnchaochan and P. K. A. Wal, “MRTD electromagnetic scattering analysis,” Microwave and Optical Technology Letters, vol. 28, no. 3, pp. 189-195, February 2001.
[16] http://www.mathopenref.com/ellipse.html.

Jamila Selmi was born in Kairouan, Tunisia. She received a master‘s degree in solid state physics from the Faculty of Sciences of Tunis, Tunisia, in 2004. She is currently working toward the Ph.D. degree in electromagnetics at the Faculty of Sciences of Tunis.

Rachida Bedira was born in Téboulba, Tunisia, in 1975. She received the Ph.D degree in 2006 from the Faculty of Sciences of Tunis, Tunisia. Since 2001, she has been with the Department of Physics at the Faculty of Sciences of Tunis, Tunisia. Dr. Bedira’s current research interests include modeling of electromagnetic scattering problems, antennas and microwaves components using liquid crystal material.

Ali Gharsallah is a Professor of electrical engineering. Since 1991 he has been with the Faculty of Sciences of Tunis. His current research interests include antennas, multilayered structures and microwave integrated circuits.

Abdelhafidh Gharbi is a professor with the Faculty of Sciences of Tunis. He is the Director of Research Laboratory of Liquid crystal. His current research interests include microwave components using liquid crystal material.

Henry Baudrand is a Professor at the Ecole Superieure d'Electronique ectrotechnique, Informatique, ENSEEIHT, of Toulouse, France. His current research interests are the modeling of passive and active circuits and antennas. He co-signed over 100 publications in journals and 250 communications in international conferences. He is Fellow Member of IEEE Society, Member of ''Electromagnetism Academy'' and Senior Member of IEE society. He was President of URSI France commission B for 6 years (1993–1999), and President of IEEEMTT-ED French chapter. He is awarded Officier des Palmes académiques, and Director Honoris causa of lasi University.