

Characteristic Basis Function Method (CBFM) - An Iteration-free Domain Decomposition Approach in Computational Electromagnetics

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Abstract— In this paper we review a novel Domain Decomposition (DD) approach, called the Characteristic Basis Function Method (CBFM), which tackles large-scale electromagnetic problems by generalizing the concept of principle of localization that forms the cornerstone of asymptotic methods. The paper shows that the problem of having to deal with large matrices that arise in the conventional formulation of large problems with the Method of Moments (MoM) can be obviated, by dividing the original large problem into a number of smaller sub-problems that are more manageable to handle. However, unlike the conventional DD approaches that typically rely upon iteration algorithms to account for the inter-coupling between the subdomains, the CBFM tackles the problem with direct solvers instead. It is possible to do this in the context of CBFM, because it reduces the original large system matrix into one whose size is orders of magnitude smaller, and is appropriately called the “reduced matrix.” Furthermore, an important salutary feature of CBFM is that the algorithm is naturally parallelizable, an attribute that distinguishes it from many other CEM solvers, and makes it well suited for parallel platforms that have become ubiquitous in recent years. This, in turn, enables us to take advantage of the power of these platforms and to solve, numerically efficiently, large problems that were well beyond our reach in the past. The paper also shows that the basic concepts of CBFM are quite general, and they not only apply to MoM, but can also be tailored for both FEM and FDTD.

Index Terms— Characteristic Basis Function Method (CBFM), Domain Decomposition,

Method of Moments (MoM), Finite element Method (FEM), Finite Difference Time Domain (FDTD).

I. INTRODUCTION

Solutions of large problems that are described by a large number of Degrees of Freedom (DoFs) are of considerable interest in Computational Electromagnetics. Presently, iterative techniques, coupled with fast matrix-vector multipliers, form the backbone of leading Method of Moment (MoM) solvers, for instance, the Fast Multipole Method (FMM) and similar techniques [1-4] that are used for such problems. Historically, the asymptotic methods, such as the GTD [5] dominated the electromagnetics scene for several decades, and were the only viable options available to us when we wanted to tackle problems, whose dimensions were very large when compared to the wavelength. However, it is well known that while the asymptotic methods can handle smooth, perfectly conducting objects with relative ease—regardless of how large they may be—the methods do suffer from several inherent limitations, especially when dealing with arbitrarily shaped, inhomogeneous, and multiscale objects. Some of these limitations are: the GTD diffraction coefficients are available only for a limited number of canonical geometrical shapes, such as PEC wedges and smooth surfaces with large radii of curvature; resonant structures, for instance re-entrant cavities, or Frequency Selective Surfaces (FSSs), are not amenable to convenient analysis via asymptotic methods, regardless of their size; and, multiscale and material inhomogeneities are always problematic because they do not lend themselves to asymptotic analysis owing to the fact that the wave phenomena in

these structures do not satisfy the ansatz upon which the asymptotic methods are based. Attempts to modify the GTD, either by using techniques such as the Physical Theory of Diffraction PTD [6], or by hybridizing it with rigorous numerical methods [7], have met varying degrees of success. This is because either these approaches do not fully overcome the fundamental limitations of the GTD—alluded to above—or they are not sufficiently robust.

In this paper we review a novel Domain Decomposition (DD) approach, called the Characteristic Basis Function Method (CBFM) [8-10], which tackles large-scale electromagnetic problems by generalizing the concept of principle of localization that forms the cornerstone of asymptotic methods. For instance, in Physical Optics (PO) we simply use the local property of the surface in the neighbourhood of a “bright point” to compute the field reflected from the surface, instead of solving a large dense matrix equation arising in MoM, which assumes that each of the subdomain basis functions fully couples to all other similar basis functions in a global sense. One consequence of this is that, for large problems, the system matrix arising in MoM is large as well as fully populated, and burdens both the CPU time and the memory. As shown in this paper, this problem can be obviated, however, by dividing the original large problem into a number of smaller sub-problems that are more manageable to handle. However, unlike the conventional DD approaches that typically rely upon iteration to account for the inter-coupling between the subdomains, the CBFM tackles the problem with direct solvers instead. It is possible to do this in the context of CBFM, because it reduces the original large system matrix into one whose size is orders of magnitude smaller, and is appropriately called the “reduced matrix.” Furthermore, an important salutary feature of CBFM is that the algorithm is naturally parallelizable, an attribute that distinguishes it from many other CEM solvers, and makes it well suited for parallel platforms that have become ubiquitous in recent years. This, in turn, enables us to take advantage of the power of these platforms and to solve, numerically efficiently, large problems that were well beyond our reach in the past. We also mention that the basic concepts of CBFM are quite general, and they not only apply to MoM, but can

also be tailored for both FEM and FDTD. Although CBFM was originally developed for MoM solution of microwave circuit problems [9], and has been applied to quasi-static problems in the context of FEM [11], we restrict our attention in this paper solely to scattering and radiation problems.

The organization of this review paper is as follows. In Sec. 2 we present the details of CBFM for MoM problems to lay the foundations of the method. We show how we can use the concepts of domain decomposition and high-level or macro basis functions to significantly reduce the size of the MoM matrix, which can then be solved directly, without resorting to iteration. Next, in Sec. 3, we present two numerical examples of the application of CBFM. Following this, in Sec. 4, we describe the basic steps of parallelization of the CBBOR (Characteristic Basis Body of Revolution) code. The next section, Sec. 5, discusses an efficient technique for handling locally modified objects in the context of CBFM. Adaptations of the CBFM concept to FEM and FDTD, are presented in Secs. 6 and 7, respectively, which have their own unique features. Finally, we refer to some recent developments in CBFM and present some summary conclusions in Sec. 8.

II. CBFM FOR MOM PROBLEMS

Let us begin by describing the principle of localization as it is incorporated in CBFM. Suppose we have an arbitrary scatterer illuminated by a plane wave, as shown in Fig. 1.

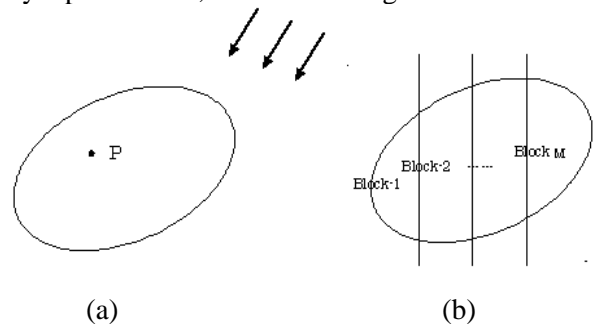


Fig. 1. (a) Arbitrary scatter illuminated by a plane wave; (b) Dividing original object into blocks.

Let us consider the behavior of the induced current at a point P , which resides on the surface of the object. Then, we could approximate the current at P by invoking the PO, which says that

this current can be approximately expressed as $2\hat{n} \times \bar{H}^i$, provided the principle radii of curvature of the surface are large. If the point P is located on an edge, we can use the GTD to find the field scattered from P , again by using the properties of the edge in the vicinity of P , and making suitable approximations if the geometry there is not one of the canonical shapes for which we can derive the diffraction coefficients. (Note: If we desire to find the approximate induced current near P , we have to employ the PTD instead.) We can either move the point P around to determine the induced current on the entire body by following the above procedure, or we can simply go directly to the far scattered field by using the ray theory approach, rather than by integrating the induced current on the scatterer. The procedure, just described, is an extreme limit of localization, which works well at very high frequencies, provided we are dealing either with smooth bodies, or with those that have canonical edge geometries and are amenable to ray type of analysis. If, however, these conditions are violated by the geometry of the object, then the ray theories do not provide a systematic way to generalize the analysis, or enable us to handle arbitrary objects whose geometries may have multiscale features. The CBFM generalizes the above concept, by first using DD to break up the original object into “blocks,” and then viewing entire subdomains as the local regions upon which macro basis functions are defined in a manner explained below. Following this, the interactions between the blocks are accounted for not via the use of iteration, as is the case in conventional DD procedures, but by rigorously incorporating them in the formulation by using the Galerkin’s method to generate a reduced matrix, as is further explained below. The procedure described above is very general, and is capable of handling arbitrarily shaped objects, which can even be multiscale.

Let us now describe the CBFM in a little more detail. Its formulation starts with the conventional MoM procedure, whereby the mixed potential integral equation is discretized into a matrix equation:

$$\mathbf{Z} \cdot \mathbf{I} = \mathbf{V}, \quad (1)$$

where \mathbf{Z} denotes the conventional MoM impedance matrix; \mathbf{I} is the unknown current vector; and, \mathbf{V} is the excitation voltage vector.

Typically, the desired solution \mathbf{I} is represented in terms of the RWG basis functions using a discretization of $\lambda/10$; hence, the number of unknowns grows rapidly with the increase in the size of the object being analyzed. This, in turn, forces us to resort to iterative techniques, which often offer the only viable option for handling large scatterers. However, the CBFM circumvents this problem by working with high-level basis functions \mathbf{I}_i ($i=1, \dots, N$), called the Characteristic Basis Functions (CBFs), and representing \mathbf{I} as:

$$\mathbf{I} = \sum_{i=1}^N c_i \mathbf{I}_i, \quad (2)$$

where c_i ’s denote the weights of these currents. Note that each \mathbf{I}_i would have non-zero entries only at the positions belonging to a sub-block. When \mathbf{I}_i is normalized, the value of c_i provides a measure of the coupling effects between the currents induced on the blocks. This observation is useful in determining N , the total number of characteristic bases that would be needed to yield an accurate solution to the problem at hand.

The primary bases are computed by solving a series of smaller matrices, arising from the application of the MoM procedure to the sub-blocks, using a spectrum of plane wave excitations as incident waves. This is done by anticipating the fact that we would be solving the scattering problem for multiple excitations anyway, and arguing that we might as well formulate the reduced matrix in a way such that, once generated, it can be re-used for different excitations without modification. This is also a powerful feature of the CBFM, which can solve the multiple incidence problems very efficiently, once the reduced matrix has been L-U factored. This is in contrast to iterative techniques, which must start the procedure from the beginning each time the RHS is changed. (Note: A slightly modified version of this procedure is used in Microwave circuits, where the secondary (see [9]) and even tertiary basis functions have been proposed, instead of just the primaries.)

As mentioned earlier, we begin by dividing the geometry of the object to be analyzed into blocks, for instance M in number (see Fig. 1b). Next, we derive the primary characteristic basis functions by illuminating the isolated blocks with plane waves, say N_{PWS} in number (see Fig. 2), which impinge upon the object at intervals of θ and φ , say every 20 degrees, for two orthogonal polarizations.

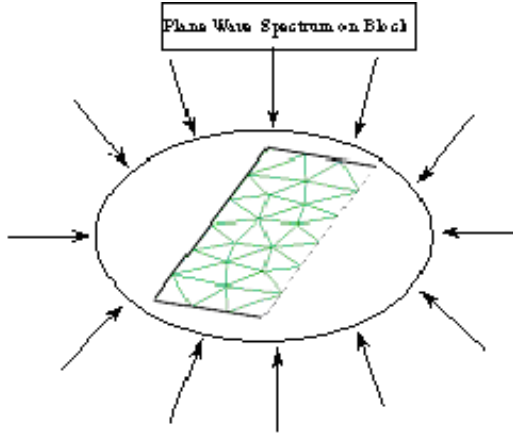


Fig. 2. Spectrum of Plane Waves incident on a single block.

We can be flexible in choosing the number of these incident waves, and can also include a part of the invisible range of the spectrum—if desired—since the SVD will downselect the number of basis functions to remove the redundancy and will retain only as many as needed to represent the unknown current with a certain degree of accuracy, determined by the level of the SVD threshold we set. In addition, the decomposition of the object into blocks is also somewhat arbitrary, and there is no limitation on the number and size of the blocks. The upper size is bounded by the available RAM needed for the unknowns in the self-blocks that are solved to generate the CBs. Typically, the block size ranges from a few hundred to a few thousand sub-domain type of unknowns. As pointed out earlier, the advantages of following this procedure is that it enables us to solve for multiple excitations using the same reduced matrix with a significant time-saving, since only the RHS of the reduced system needs to be computed for a new excitation.

For the sake of illustration, we consider a thin plate which is divided into 25 blocks, shown in Fig.3. Although, in general, the blocks can have different sizes, we assume that they have approximately the same dimension N_b in terms of number of unknowns. To mitigate the problem of spurious edge effects in the CBs, introduced by the truncation, we extend all blocks by a fixed amount (typically 0.2λ to 0.4λ) in all directions, except when the boundaries are free edges.

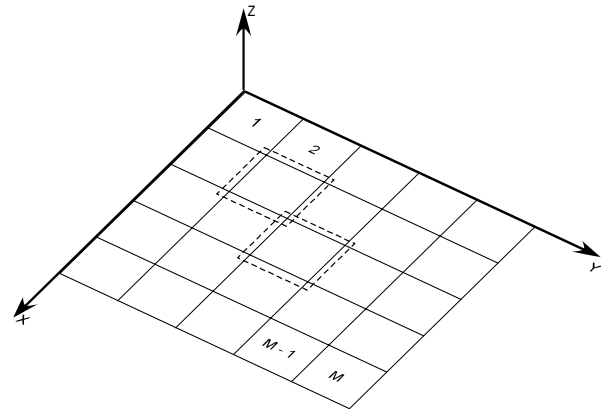


Fig. 3. Geometry of a PEC plate divided into 25 blocks. Extended blocks are duplicated by dashed lines.

The final step is to generate the reduced $KM \times KM$ MoM matrix for the KM unknown complex coefficients c_k by following the usual Galerkin procedure in the context of the Method of Moments. Once the induced surface current distribution for the entire structure has been derived, the electrical parameters such as RCS, scattered field, etc., can be computed in the usual manner.

The most computationally intensive steps of the proposed method are associated with the generation of the primary CBs and the construction of the reduced matrix reduction, though the latter task can be speeded up by invoking the symmetry of the matrix.

Once we have generated the reduced matrix, we proceed to factorize it. Note that generation of CBs is one of the time-consuming and memory-demanding tasks. It requires the filling the self-impedance matrix Z_{ii} for the extended block and its factorization in an LU form. Since the CBs are independent of the incident angles, this factorization needs to be performed only once, and the resulting primaries can be reused for multiple incident angle directions. This implies that the final reduced matrix (5) is independent of the excitation, and this fact enables us to handle a problem involving multiple excitations by only solving the reduced system for the new RHS (excitation). Moreover, we can store the reduced matrix on the hard disk, and reuse it whenever we need to analyze a new excitation. Furthermore, if the geometry within a particular block is modified,

only the CBs belonging to this block need to be recomputed.

The technique described above realizes a saving in the CPU running time and RAM requirement compared to the conventional MoM technique. The memory requirement is now proportional to the square of the self impedance matrix of the extended block, and this is different from that in the conventional MoM where the storage requirement is related to the square of the dimension of the entire impedance matrix. Moreover, we realize a consistent saving in the execution time, which reduces to $O(M(Nbe)^3)$ instead of $O(N^3)$.

As mentioned earlier, the generation of CBs is one of the most computationally intensive tasks in CBFM. We will now briefly discuss two techniques for reducing this time, both of which are physics-based.

The first approach [12] is best suited for geometries that are relatively smooth — though they may have edges — but are not multiscale. In this approach we simply use the P.O. solutions for different incident angles as the characteristic basis functions, totally bypassing the matrix solution that we would employ in the conventional procedure for CB generation. An added advantage of using this approach is that these PO/CBs no longer suffer from the spurious edge effects — even after the SVD — as do the CBs generated by matrix methods. We point out that even if the geometry under consideration has true edges, we can still accommodate this situation by using relatively small-size blocks near the edges. This is because a superposition of these basis functions can still pick up the edge behavior of the current, despite the fact that none of the individual basis functions possess this behavior.

The second method [13], which is more general than the one we just described above, works with a sparsified version of the matrix for the subdomain that we need to solve to generate its CBs. In this approach, we simply use a threshold value to discard the matrix elements that fall below this value. This operation obviously sparsifies the matrix, and not only enables us to employ a sparse-LU factorization scheme, but to also reduce the storage requirement significantly.

In the next section we present some numerical examples to illustrate the application of CBFM to scattering problems.

III. NUMERICAL RESULTS DERIVED WITH CBMOM (CHARACTERISTIC BASIS METHOD OF MOMENTS)

3.1 Plane wave scattering by a 4λ radius PEC Sphere

To validate the accuracy of the method we will compare the CBFM solution with the analytical one for a PEC sphere of radius 4λ , at a frequency of 300 MHz. The object is excited by a normally incident ($\theta=0$, $\phi=0$) theta-polarized plane wave. The discretization is carried out by using triangular patches with a mean edge length of 0.1λ , resulting in a problem with 85155 unknowns. The geometry is divided into 16 blocks with an average size of 8000 unknowns. Each block is extended by $\Delta=0.4\lambda$ in all directions, and analyzed for a spectrum of plane waves incident from $0^\circ \leq \theta < 180^\circ$ and $0^\circ \leq \phi < 360^\circ$, with $N_\theta = N_\phi = 20$. These results in a total of 800 CBs; but, after the SVD, only 310 are retained in each block. The 85155×85155 MoM matrix is then reduced to only 4925×4925 , which is solved directly.

As we mentioned earlier, the construction of the CBs can be speeded up, with little loss of accuracy, by using a sparsified version of the self-blocks—that retain only the near-region interactions—rather than working with the full versions of these blocks. It should be realized that we are only generating the basis functions at this point and, hence, they themselves need not be the rigorous solutions of the self-block problems, so long as they span the solution space, they need not strictly be solutions of the original self-blocks. To validate this concept, we have analyzed the problem at hand by using both dense and sparse matrix approaches. The use of the latter allows us to reduce the computational cost by a factor of approximately 4. The bi-static E- and H-plane RCS results are presented in Figs. 4(a) and 4(b), respectively, using the dense and sparse approaches, as well as the MIE series. The agreement is seen to be good for all scattering directions, including the grazing angles.

Next, we turn to some example problems involving bodies of revolution (BORs), to which the CBFM has been successfully applied. The BOR geometry offers the advantage that we can

factor out the azimuthal variation and thereby reduce a 3-D/BOR problem into a number of 2-D ones. Thus we can easily solve problems with very large dimensions — that are hundreds if not thousands of wavelengths. In Fig. 5 we show a finite-length cylinder, which is subdivided into two blocks to illustrate the partitioning scheme.

Figure 6 shows the flowchart of the CBF_BOR code, which combines the CBF algorithm with the BOR code. The above code includes five steps: (i) Data entry--input data, input (generate) structure, define indices for vectors; (ii) Call “Split structure” subroutine--split the original structure into a number of smaller blocks; (iii) Call “Generate CBFs” subroutine: the CBFs for each mode of each block are obtained and stored; (iv) Call “Generate reduced matrix” subroutine--compute the reduced matrix for each mode; (v) Call “Solve reduce matrix” subroutine--solve the reduced matrix system for each mode and sum the results for all modes to derive the final results.

3.2 Numerical Example of CBF_BOR Code

We now present some numerical examples obtained by using the CBF_BOR code for the test geometry of a PEC sphere. The relevant parameters are:

- Radius of the sphere = 6.755 wavelengths
- Overlap region between blocks = 1 wavelength
- Number of incident angles for generating the CBFs = 100
- Threshold for SVD = 1000 (ratio of largest to smallest singular values retained).

Comparisons with the results obtained by using the non-CBF version (single block) code are included for the purpose of validation of the CBF result, and demonstrating that it is not sensitive to the number of blocks in which the object is divided. Both the induced current distributions and the far field results are presented in Figs. 7a through 7d for the threshold value of 1000.

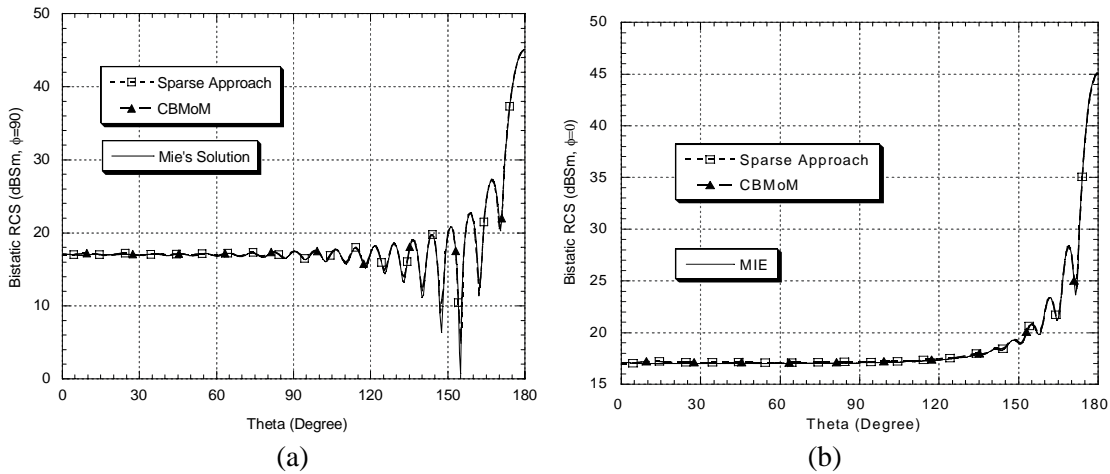


Fig. 4. RCS of radius PEC sphere at 0.3GHz: (a) E-plane; (b) H-plane.

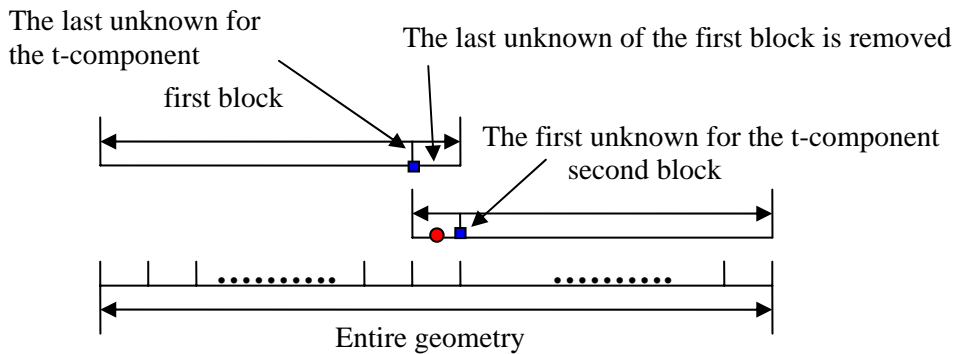


Fig. 5. BOR geometry of a finite cylinder and the partitioning scheme for two blocks.

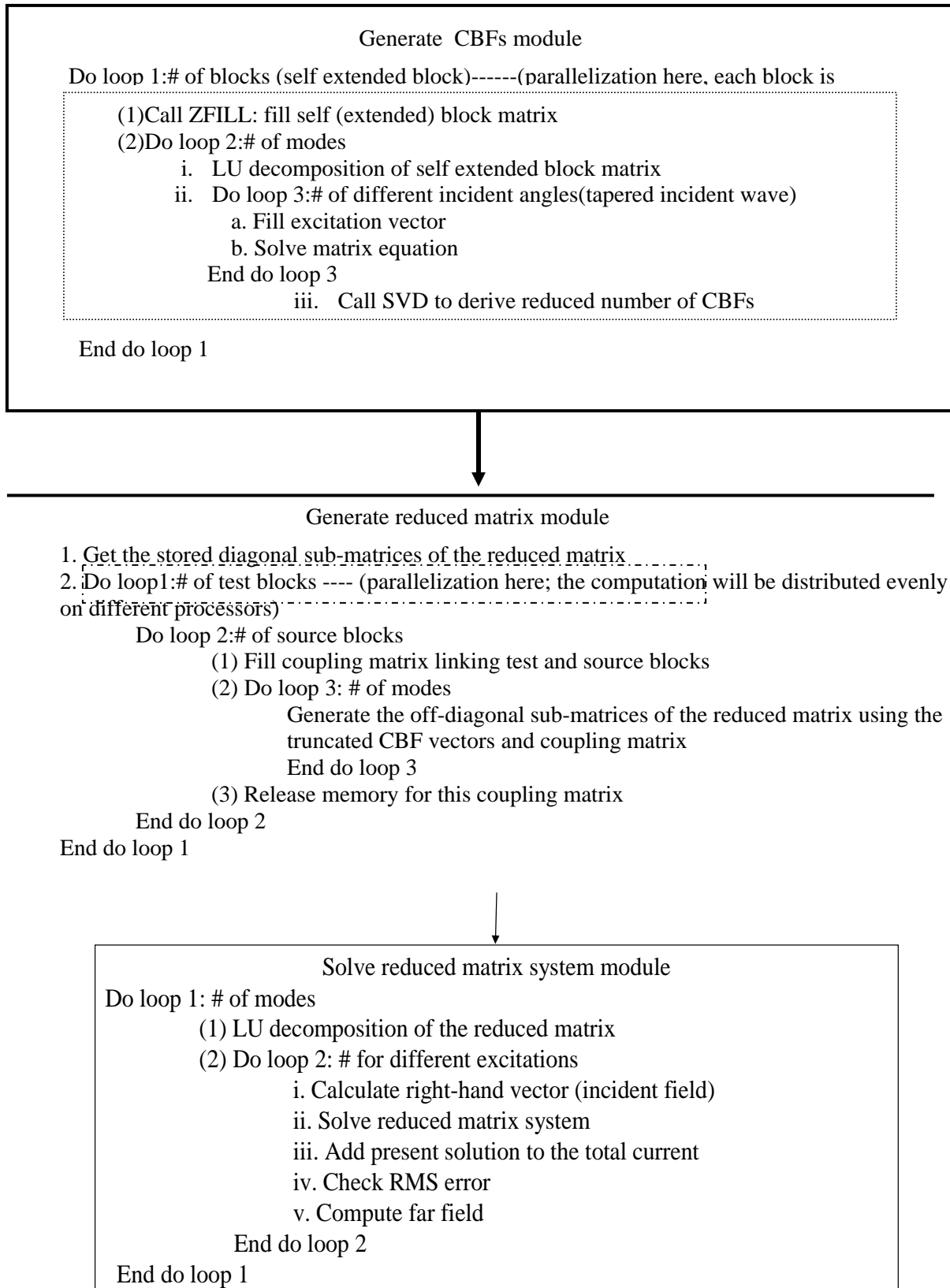


Fig. 6. Flowchart for the CBF_BOR code.

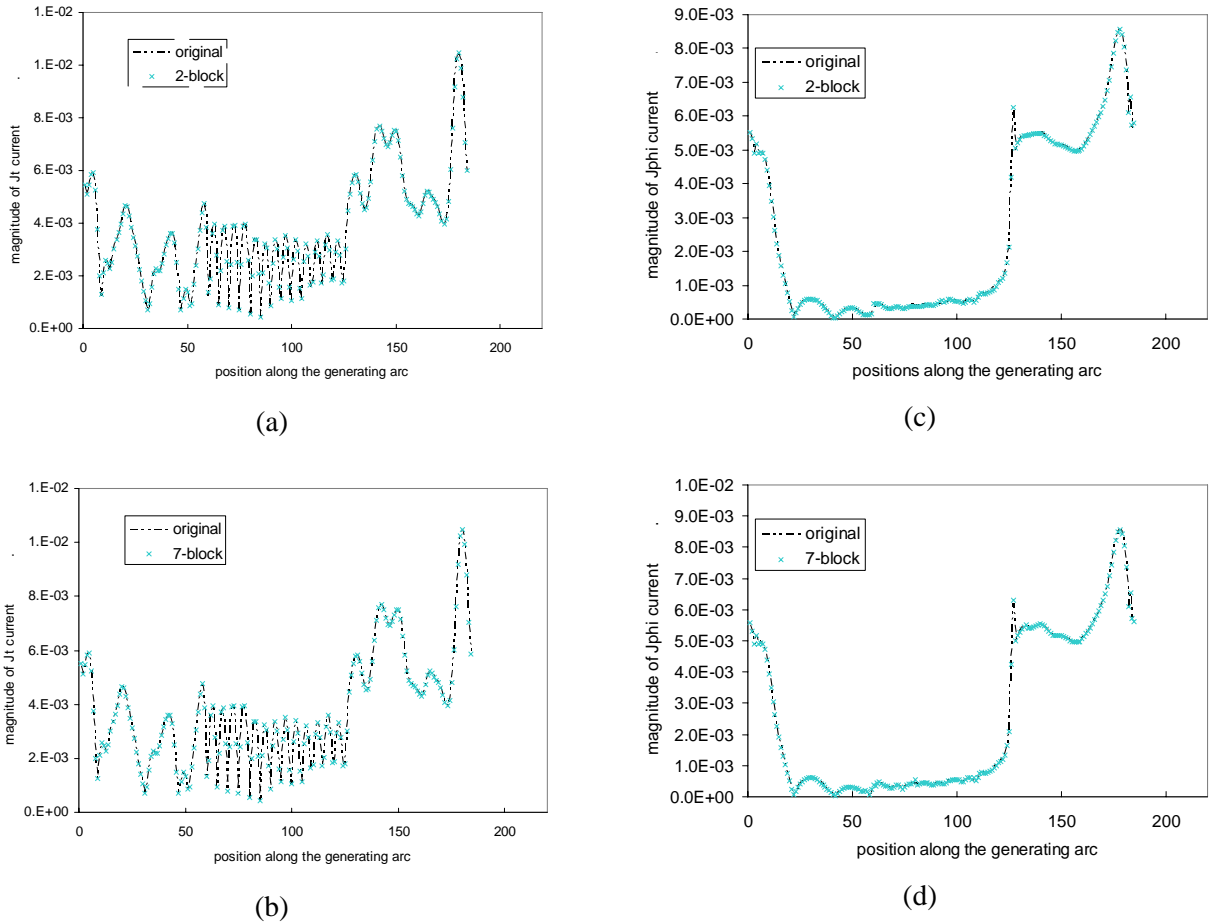
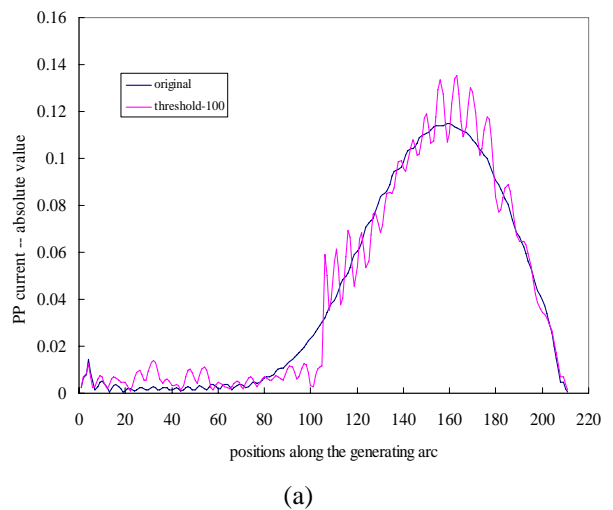
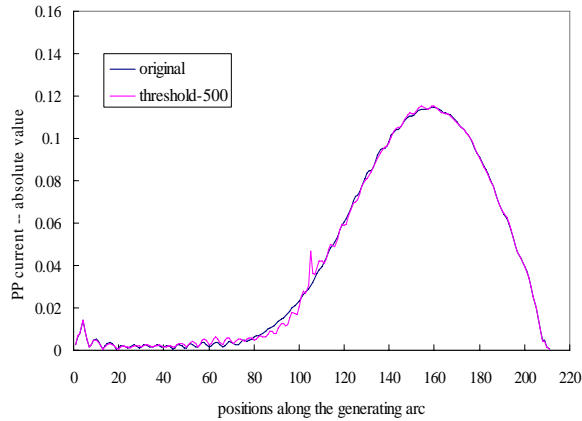


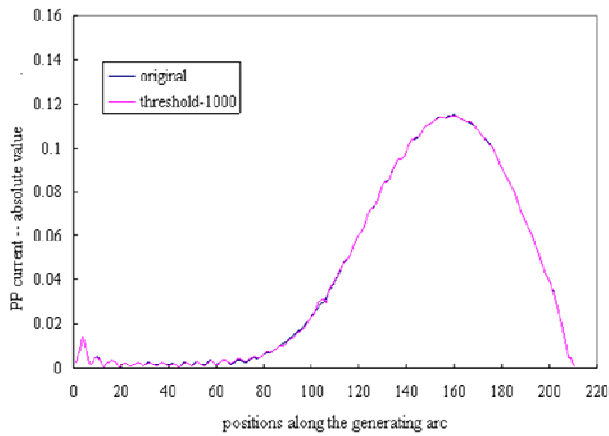
Fig. 7. The J_t current on the cylinder in Fig. 5 due to theta excitation. Axial incidence (a) 2-block, (b) 7-block. The J_{phi} current on the cylinder in Fig. 5 at $\phi=90$ due to θ excitation. Axial incidence (c) 2-block. (d) 7-block.

Next, we illustrate the effect of changing the SVD threshold on the accuracy of the results. In Fig. 8 we show the results of progressively varying this threshold value, beginning from 100 and moving upward, steadily improving the accuracy of the current distribution in the process. We notice from these plots, which compare the direct solution with those obtained by using the CBF/BOR code, that a value of 1,000 marked in the figures as original is adequate for achieving numerical convergence. Of course, the far-field is more forgiving, and we can probably use a somewhat lower value for the threshold, without compromising the accuracy of the far field pattern noticeably, as may be seen from Fig. 9.



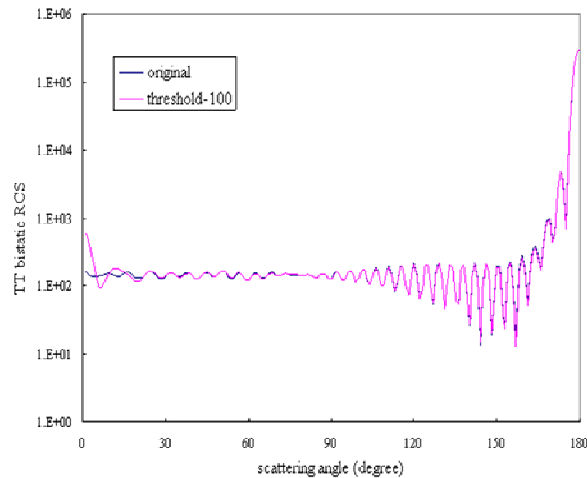


(b)

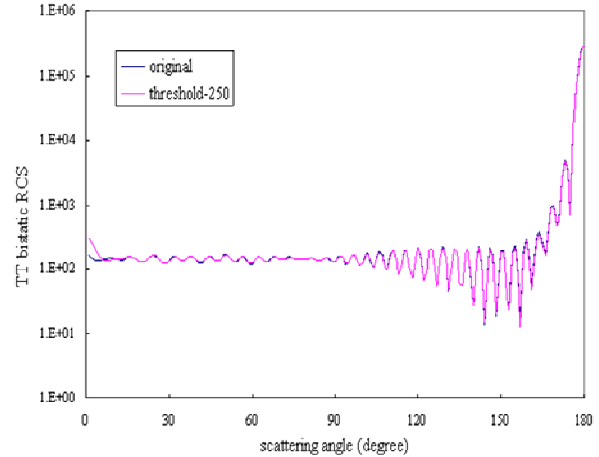


(c)

Fig. 8. The ϕ - ϕ current on a sphere along the generating arc for axial incidence. (a) Threshold value for SVD is 100. (b) Threshold value for SVD is 500. (c) Threshold value for SVD is 1000.



(a)



(b)

Fig. 9. The θ - θ bistatic RCS of a sphere. (a) Threshold for SVD is 100. (b) Threshold for SVD is 250.

Obviously, lowering the threshold increases the number of CBs we retain and, hence, the size of the reduced matrix and the associated computational burden. Numerical experiments involving a variety of different problems have shown that a good choice for the SVD threshold is the value we have mentioned above, namely 1000.

Table 1: First five letters of the Greek alphabet

Threshold Value	Name
100	33
250	43
500	51
1000	54
2000	55
5000	58

We will now describe the parallel implementation of CBF/BOR, which involves four steps, as detailed below.

IV. STRATEGY FOR PARALLEL IMPLEMENTATION OF CBBOR

As mentioned earlier, one of the important attributes of the CBFM is that the algorithm is easily parallelizable. Although we do not present the details of the parallelization algorithm in this review work, we outline just the basic steps here. First, we recognize that the generation of the CBs

can proceed independently, on separate processors, since these processors do not need to communicate with each other at this step. Once the CBs have been generated, the next step is to construct the reduced matrix by using the Galerkin approach, which entails matrix-vector multiplications involving the CBs, and the self- as well as off-block matrices. A bulk of these processes can also be readily parallelized, because, at no point we need to deal with the entire MoM matrix concurrently.

We now briefly review the parallelization process for the CBF_BOR code which basically includes four steps:

- (i) generating and splitting geometry;
- (ii) generating CBFs;
- (iii) generating reduced matrix;
- (iv) solving the reduced matrix system.

The parallelization is implemented in parts (ii) and (iii), with computation burden evenly distributed among different processors.

1) Generating and splitting the geometry

All processors participate in the tasks of geometry generation and splitting, so that they will have the needed parameters and geometry data, obviating the need for communication among processors.

2) Generating CBFs

If the number of blocks is less than the number of available processors, each processor will perform the computation for one block. In each processor, the extended self-block matrix is filled and the matrix system is solved to generate the CBFs for this block. The diagonal sub-matrix of the reduced matrix for this block is then obtained by using the self-block matrix and CBFs. The memory for the self-block matrix is then released. Next, the CBFs for this block are broadcast to every processor. The diagonal sub-matrix of the reduced matrix is sent to only one master processor. Figure 10 illustrates the implementation of this procedure. Of course, if the number of blocks is larger than the number of available processors, then each processor would perform the computation for several blocks

3) Generating the reduced matrix

The generation of the reduced matrix entails the linking of the test and source blocks to obtain the off-diagonal elements of the reduced matrix, and a parallel implementation of this process is carried out in accordance with Fig. 11 Each processor computes the coupling matrices in a row. The off-

diagonal sub-matrices of the reduced matrix in one row are then obtained by using these coupling matrices and the CBFs matrices for all blocks on this processor. These off-diagonal sub-matrices are then sent to the master processor.

4) Solving the reduced matrix system

The reduced matrix system is solved only on the master processor. The far field calculation and data output operations are also handled by this processor.

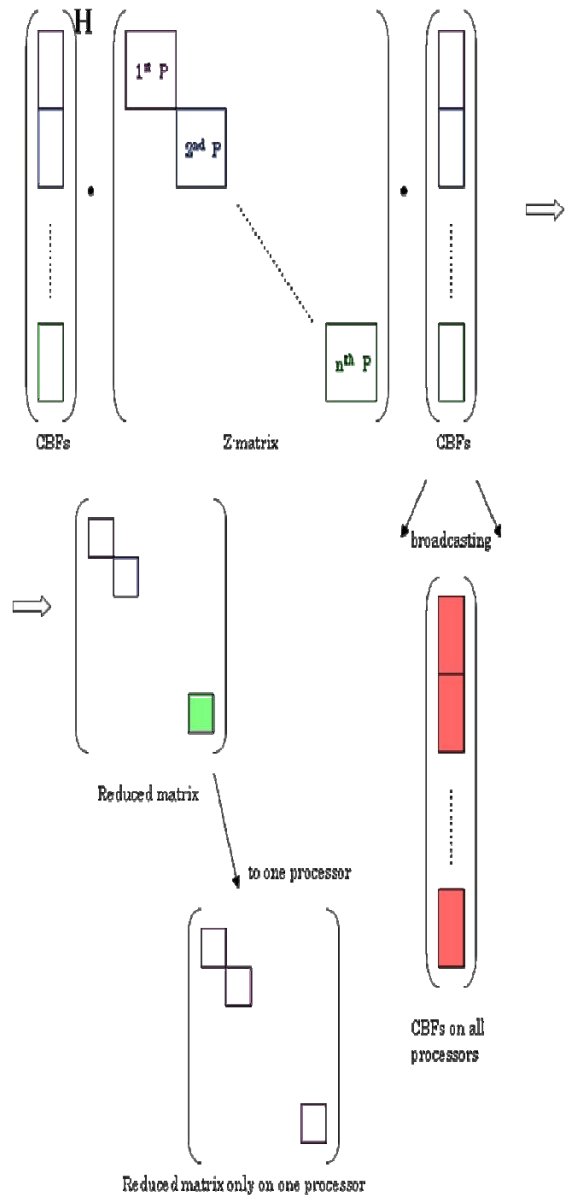


Fig. 10. Parallel implementation for the generation of CBFs for BOR problem.

V. LOCALLY MODIFIED CBFM

Recall that the most computationally intensive task in CBFM algorithm is associated with the generation of the CBFs, which entails the filling and LU factorization of the MoM impedance matrix associated with the various blocks. To generate the high-level basis functions for a given block, the CBFM localizes the problem to the particular macro-domain of that block; hence, it does not need to account for the interaction among the various blocks at this stage. This feature offers great flexibility when modelling targets whose geometries are modified only locally, for instance to reduce (or enhance) its RCS. Consequently, this feature of CBFM has an important advantage over the conventional MoM or FMM, which must analyze the modified problem a new essentially from the very beginning even if only a small part of the geometry is altered. In contrast to the conventional methods, the CBFM only needs to re-work the block that is being modified to obtain a new set of CBFs for it, while bypassing this step for the other blocks whose macro-bases are stored in a file, and are reused to generate the reduced matrix. Furthermore, we can achieve consistent time-saving even during the process of generating the reduced matrix, because only the matrix elements that link the modified block to the rest of the structure need be re-computed.

Below we delineate the steps involved in the implementation of the CBFM to handle a locally-modified object, which is depicted, symbolically in Fig. 12. The steps for modifying the blocks are presented in Fig. 13.

As an example we consider a plate problem which is locally modified, as shown in Fig.14, by introducing a hemispherical “bump” on the right side of the plate.

In Fig. 15 we present the bistatic RCS results for both cases, computed by using the conventional and LM/CBFM algorithms. We note from the above plot that the two results are virtually indistinguishable.

VI. CHARACTERISTIC BASIS FUNCTION FOR FEM

To the best of our knowledge, a CBFM-type of procedure that leads to a reduced matrix generated by using characteristic basis functions—tailored to individual—sub-domains has not been employed

in the past for the Finite Element-Method (FEM), until quite recently. Space does not permit us to discuss the procedure in minute details in this review paper on CBFM, and we refer the reader to [14] where they can be found. Here we only show a representative example of scattering by a sphere, and point out some important difference between the implementations of CBFM in MoM and FEM.

Figure 16(a) illustrates the CBFM procedure implemented in FEM. Let us assume that we decompose the original scattering problem into two blocks, as shown in Fig. 16(b). To generate the CBs for these domains we use fictitious dipoles residing on the surface of the object (see Fig. 16(c)), which is a sphere in this example though it could be arbitrary in general.

The incident fields at the interfaces of the sub-domains are radiated by the dipoles in the absence of the scatterer. Note that this step is different from that followed in the MoM implementation of CBFM.

Next, as before, we apply the SVD procedure to retain only the non-redundant CBFs. The third step is to compute the CBFs in the various sub-domains; by using the interface-based CBFs—which we just derived in the previous step—for the incident fields. Following this, we apply the SVD procedure once again to select the post-SVD CBFs which we wish to retain. Finally, we construct the “reduced” matrix, using the Galerkin’s procedure, similar to that followed in CBMoM (Characteristic Basis Method of Moments).

We will now present the results for a test example, that of a sphere, which has been solved on a parallel platform. We remark here -without going into the details- that, in common with the MoM, CBFEM also tends itself to convenient parallelization. As shown in Fig. 17, the computational domain for the problem geometry in Fig. 16 is divided into 16 blocks.

The number of elements we have after using a uniform $\lambda/10$ discretization is 241,765 and the associated DOFs number of edge is 313,958. The final reduced matrix is only 16,016 and the result for the bistatic RCS, computed by using the above matrix, are shown in Fig. 18. The above are also compared with those derived by using the Mie series, as well as via an alternate numerical approach called the Backward-forward Domain Decomposition method [15].

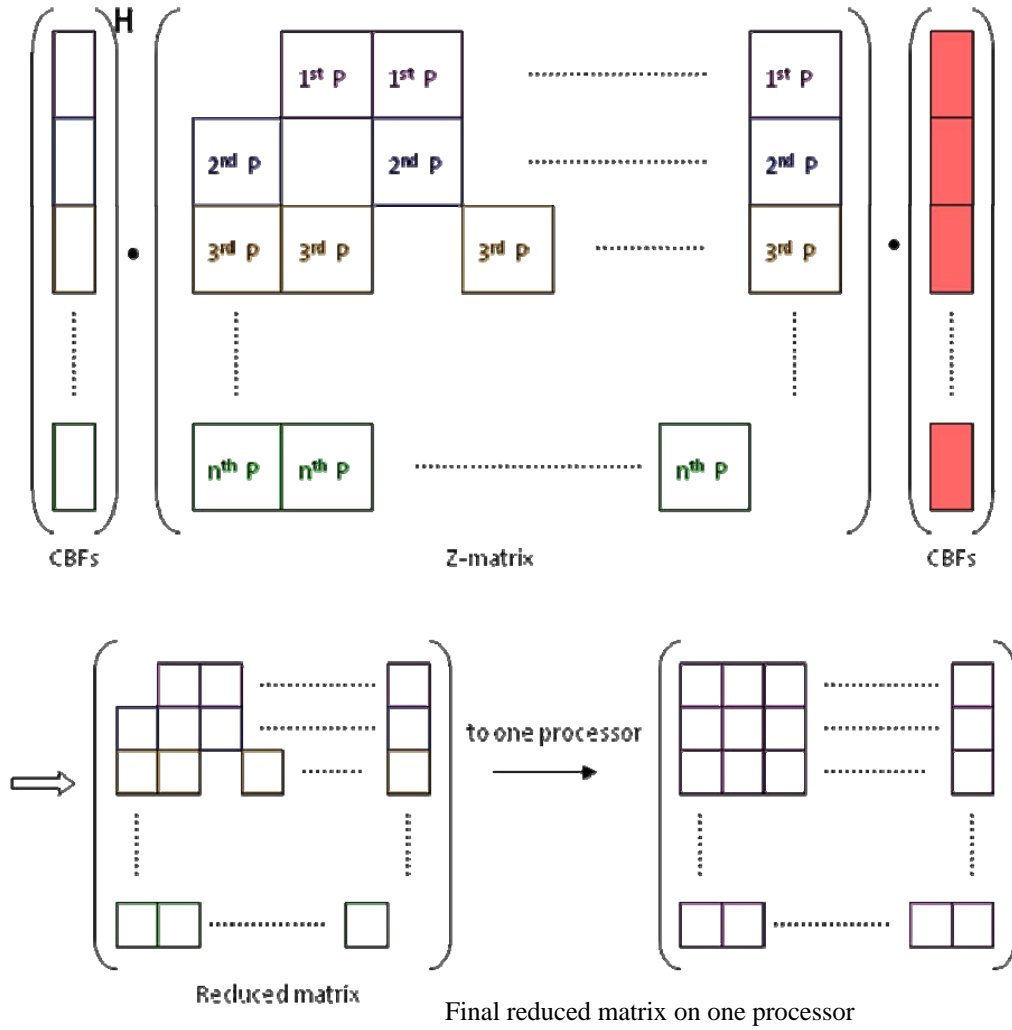
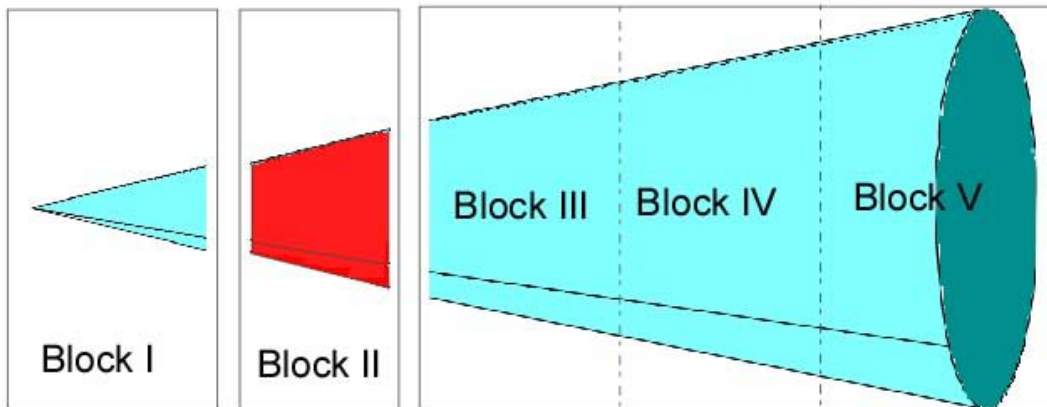


Fig. 11. Parallel implementation of reduced matrix generation.



Modified geometry: Block II is being modified

Fig. 12. Geometry of an object, one of whose (Block-II) is being modified.

Load Project:

- 1) Read block file (each block has an option field)
- 2) Option value must be 1->Redo, 0->Already done
- 3) If all option fields are equal to 1 then a new excitation is being analyzed
- 4) All new block's definitions, not previously present, must be placed last

Split Structure:

- 1) If the block has already been analyzed, then load geometrical info only
- 2) If the block has been modified, then obtain the new mesh

Self Block Analysis:

- 1) If the block has already been analyzed, then load the number of post-SVD CBFs
- 2) If the block has been modified, then obtain the new set of CBFs

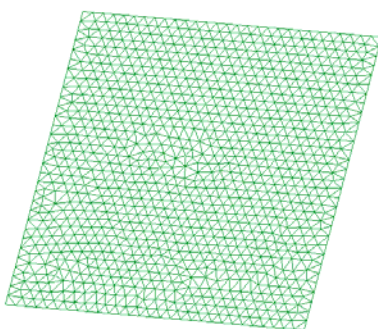
Reduced-RHS Generation:

- 1) Compute the reduced right-hand sides anew. The user might have added some new excitations

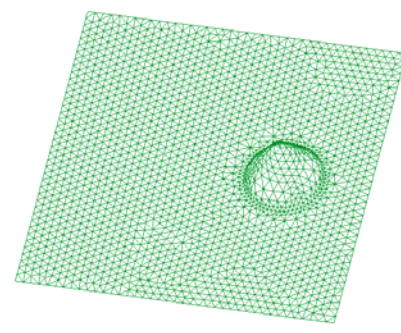
Reduced-Matrix Generation:

- 1) If all blocks have the option field equal to 1, then load the previously computed reduced-matrix
- 2) If some blocks have been modified, then computed the relative row and column in the reduced matrix. Note: Since we compute the reduced-matrix by forcing symmetry, particular care must be exercised to avoid duplicated jobs.
- 3) If some blocks have been modified, then load the previous reduced-matrix and modify it with the new entries. The modified block(s) can be placed in the middle of the reduced-matrix and the number of post-SVD used to generate these blocks can be different from that in the previous simulation. This results in a different size for the reduced-matrix and its indices are shifted.

Fig. 13. Steps for CBMOM modification.

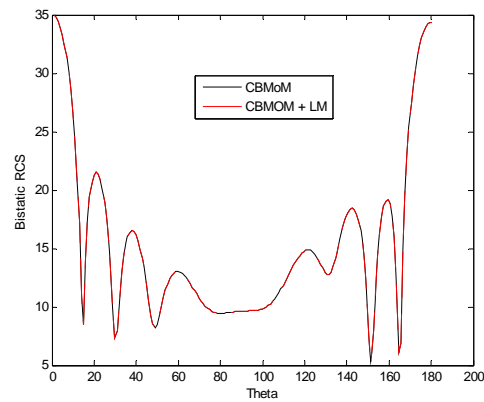


(a)

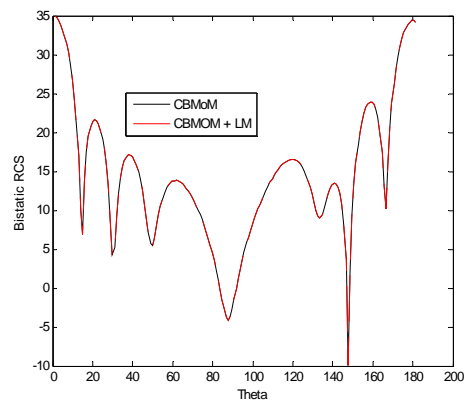


(b)

Fig. 14. (a) Plate Geometry. (b) Local Modification of the plate.



(a)



(b)

Fig. 15. Bistatic RCS of the locally modified plate compared by using the conventional and locally modified CBFMs.

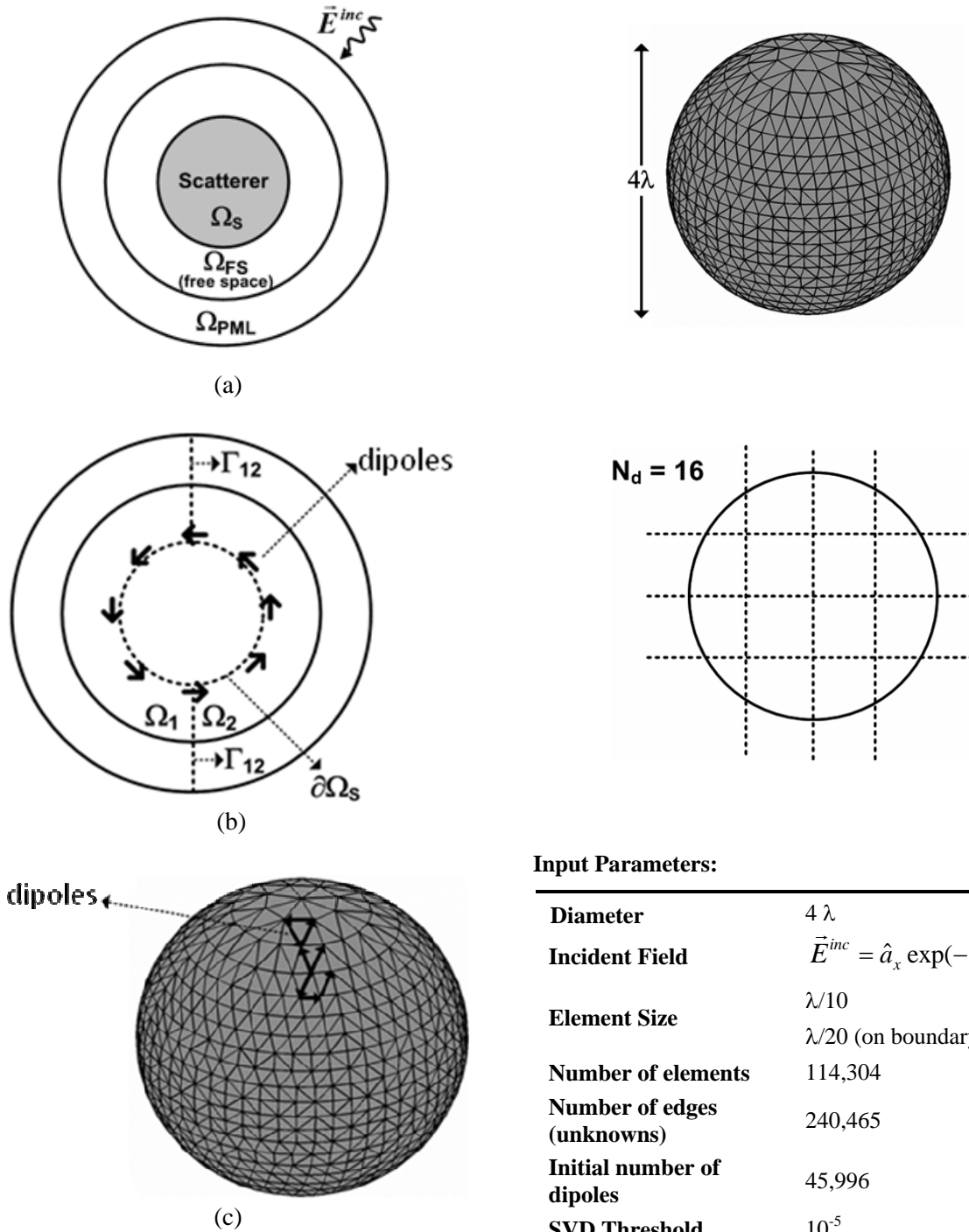


Fig. 16. (a) Original scattering problem of scattering by a 4λ diameter sphere. (b) CBFEM approach. (c) Dipole positions on the surface of the sphere.

Input Parameters:

Diameter	4λ
Incident Field	$\vec{E}^{inc} = \hat{a}_x \exp(-jkz)$
Element Size	$\lambda/10$ $\lambda/20$ (on boundary)
Number of elements	114,304
Number of edges (unknowns)	240,465
Initial number of dipoles	45,996
SVD Threshold	10^{-5}
Number of subdomains (N_a)	16
Number of interfaces	45

Analysis Parameters:

	Pre-SVD		Post-SVD	
	Number of CBFs in each domain	Ω_1	1,733	1,221
Ω_2		1,907	1,515	
Ω_3		1,914	1,502	
Ω_4		1,739	1,228	
Ω_5		2,128	1,348	
Ω_6		1,371	989	
Ω_7		1,383	994	
Ω_8		2,108	1,370	
Ω_9		2,120	1,327	
Ω_{10}		1,369	972	
Ω_{11}		1,383	971	
Ω_{12}		2,108	1,337	
Ω_{13}		1,729	1,224	
Ω_{14}		1,914	1,517	
Ω_{15}		1,918	1,517	
Ω_{16}		1,739	1,229	
Size of the final reduced Schur matrix (Total number of CBFs on interfaces)		14,135		

Fig. 17. Relevant parameters for the CBFEM solution of the 4λ diameter sphere.

VII. DOMAIN DECOMPOSITION IN FDTD

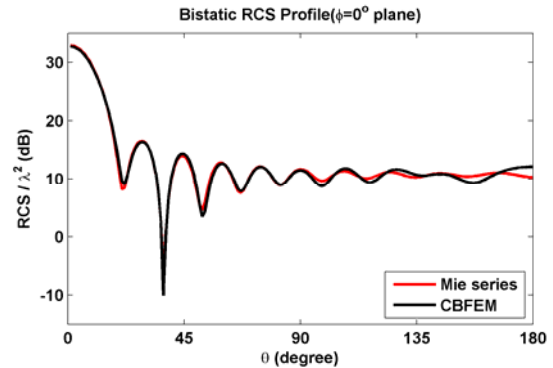
Finally, we will present a novel approach [16] to solve large problems using the domain decomposition in the context of FDTD. The strategy here differs significantly from that utilized in MOM or FEM, because it is unique to the time domain alone. We should point out that this is approach, which is tailored for large problems, is not suitable for problems in which there exist significant multiple interactions between the subdomains, as for instance when the object size is relatively small, of course such an object can be solved directly, without resorting to domain decomposition.

We begin by subdividing the geometry of the large object into smaller blocks, as in the past, but only in one dimension, say x , as shown in Fig. 19. We make sure that there is an overlapping or buffer region between two adjacent sub-domains (see Fig. 19). Next, we begin to analyze the

problem from one end, say the left, and proceed in much the same way as we would in a TDR (time domain reflectometer). Specifically, we track the E and H fields as they traverse from left to right, including only the local interactions within a sub-domain and not the mutual effects between two sub-domains at this point. (To do this, we terminate the interface of the sub-domain on the right with a PML.) We then proceed to interface-1, and record the time domain signatures of the tangential fields at the interface with the wave impinging upon it from the left. We then use this information to excite the sub-domain to the right, which is again terminated with a PML on its right. We repeat this procedure until we reach the end.

In many large problems, such as for instance in the case of two large antenna arrays separated by a large distance, we can terminate the procedure after just one pass, once we have reached the end. This is because the coupling between the two arrays is relatively weak and, hence, the level of the signal reflected from the second array is very small, i.e., essentially negligible.

However, we can have a different scenario, such as the one depicted in Fig. 20, where we wish to estimate the coupling of unwanted signals into a room with electronic devices, e.g., computers. In this case, we find that it becomes necessary to account for the reflected signal, at least through one round-trip, in order to improve the accuracy of the results. The results for the fields, computed by using the DD/FDTD approach, are shown in Figs 21a and Fig. 21b, where they are also computed with the direct solution (see Fig. 21b). We note that the comparison is quite good.



(a)

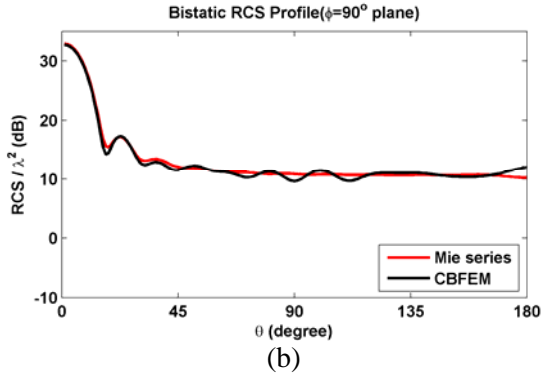


Fig. 18. Comparison of CBFEM and Mie series for the 4λ sphere problem. (a) $\Phi=90^\circ$. (b) $\Phi=0^\circ$.

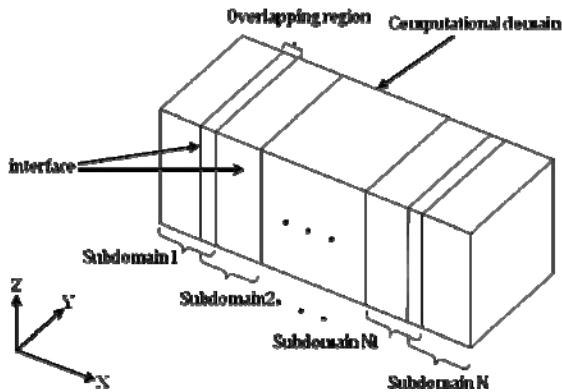


Fig. 19. Domain decomposition scheme in the context of the FDTD.

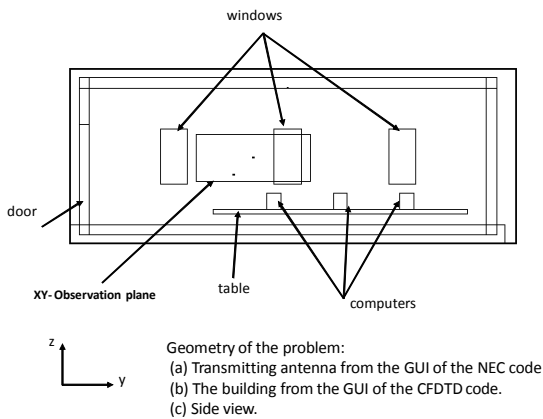
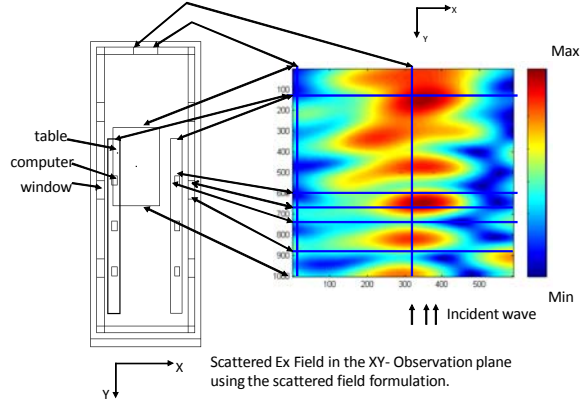
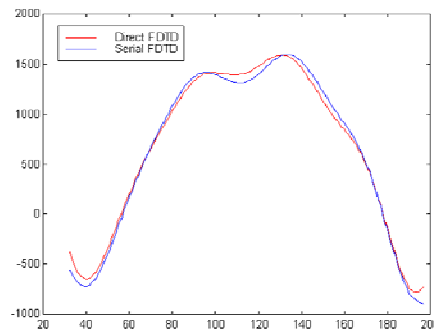


Fig. 20. Geometry of the problem.



(a)

Field distribution at the center line of the observation plane



The serial FDTD result is computed using two reflections

(b)

Fig. 21. (a) Scattered E_x Field in the XY-Observation plane. (b) Field distributions at the center line of the observation plane Computed by using two reflections In the Domain Decomposition sc.

We observe that this type of approach can be quite useful for modelling the problem of remotely tracking the movements of targets in a room with brick walls, which can easily fall into the “large” problem category and, hence, is likely to benefit from the application of the DD/FDTD analysis.

VIII. CONCLUSION AND FUTURE WORK

In this paper, we have reviewed a technique, based on the domain decomposition approach, for solving large problems, unlike the conventional domain decomposition schemes, which rely on iterative procedures, and hence often suffer from convergence problems. The characteristic basis function method (CBFM) yields a reduced-size

matrix, which is solved directly. Yet another important attribute of the technique is that it is general in nature, and can be tailored to apply to integral equation method as well as to FEM and FDTD algorithms. Finally, the CBFM is naturally parallelizable, which enables us to solve even larger problem in an efficient manner.

The method itself is still evolving and is currently an active research topic. Space does not permit us to discuss the various embellishments of the method, and we only provide a bibliography [17-53] that lists some of the contributions on the subject that have inspired the preparation of this paper.

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