ABOUT THE STUDY IN THE TIME DOMAIN OF JUNCTIONS BETWEEN THIN WIRES

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ABSTRACT

DOTIG1 is a computer code developed for the study of the interaction of arbitrary electromagnetic signals with thin-wire structures, in the time domain. It calculates the current distribution induced on the structure by solving the electric field integral equation using the moment method. The numerical procedure used to develop the program and different possibilities for treating junctions are briefly described.

To obtain an accurate solution for the current induced on the thin-wire structures it is very important to pay attention to the zones at which the wires intersect. Thus, different junction treatments were tested for several simple structures. Following some convergence criteria the current distributions were compared to a reference solution and also, by way of Fourier transform, with results obtained using some well known frequency-domain codes.

1. INTRODUCTION

Recent developments in high-resolution radar and electromagnetic-pulse technology (EMP) have generated interest in their interaction with structures. This can be studied either by using time-harmonic analysis followed by Fourier inversion or solving directly in the time domain using a time-step algorithm [1]. The time domain approach not only offers computational advantages but sheds light on the problem in a way that is not possible using frequency-domain techniques [2].

The first step for such an analysis is to calculate the electric current induced on the surface of the structures. As they are normally very complex, a common technique is to approximate the surface geometry by a set of joined wires. To obtain an accurate numerical solution for the structure current distribution, particular attention must be paid to the zones at which the wires are joined together [3].

In this paper different treatments for the study of the junctions are presented in order to compare how stable and accurate the solution is obtained. The treatments were applied to: a cross of wires, a stepped-radius wire, a net (Fig. 13) and a dodecahedron (Fig. 16), when illuminated by a transient electromagnetic field. The currents induced on the wires were calculated in the time domain by solving the electric field integral equation (EFIE) using the moment method. The results obtained using the different junction treatments were compared, by Fourier transform, with those obtained from the computer code NEC-2 [4], working in the frequency domain, and for the dodecahedron with the AWAS code [5] as well.

2. NUMERICAL METHOD

DOTIG1 computes, in the time domain, the currents induced on a structure modelled by interconnected wires when it is excited by an arbitrary electromagnetic signal. The method employed consists of resolving the electric field integral equation (EFIE) using the moment method. This equation for a thin wire is [6],[7]:

$$\hat{s}\cdot\vec{E}^{i}(s,t)=\frac{\hat{s}}{4\pi\varepsilon_0}\cdot\int_{C(s')}\left[\frac{\hat{s}'}{c^2R}\frac{\delta}{\delta t}I(s',t')+\frac{\vec{R}}{cR^2}\frac{\delta}{\delta s}I(s',t')-\frac{\vec{R}}{R^3}q(s',t')\right]ds' \qquad (1)$$

where $\hat{s}$ and $\hat{s}'$ are tangent vectors to the wire axis of contour C(s') at position s(r)=s and s(r')=s'. I(s',t') and q(s',t') are the unknown current and charge distributions at source point s' at retarded time t'= t-R/c and $\bar{E}^i$ is the field applied to the observation point $\bar{R} = |\bar{r}-\bar{r}'|$ (See Fig. 1). The charge q(s',t') can be expressed in terms of I(s',t') by the equation of continuity.

Using the point matching form of the moment method, integral eq. (1) was transformed into the group of equations (in matrix notation) [8],[9]

$$\tilde{E}^{s}{}_{j}+\tilde{E}^{i}{}_{j}=\tilde{Z}\tilde{I}_{j} \qquad (2)$$

which allows us to calculate the current I_j at time t_j from the elements E^s_j of the tangential electric field scattered by currents of previous times and the elements E^i_j of the tangential applied electric field at the observation point and at time step t_j. Matrix $\tilde{Z}$ is a matrix of interaction, the elements of which are time-independent. They depend on the geometry and on the electromagnetic characteristics of the structure alone [10].

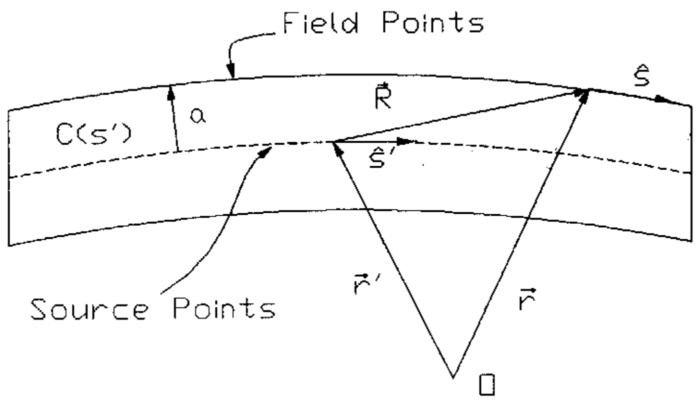

Fig. 1 Geometry of the thin wire

The base function used was a nine-point lagrangian interpolation, which in the case of a single wire can be written

$$I_{i,j}(s',t') = \sum_{l=-1}^{+1} \sum_{m=-1}^{+1} B_{i,j}^{(l,m)} I_{i+l,j+m} \tag{3}$$

where $I_{i,j}(s',t')$ is the value of the intensity current at position s' and time t' inside one of the space-temporal intervals i-j into which the wire was divided. $B_{i,j}^{(l,m)}$ are the coefficients of the interpolation that depend on s' and t', and $I_{i+l,j+m}$ are the values of the current at the space-temporal interval i+l-j+m nearest to the i-j interval [8],[10]-[11].

Fig. 2 shows how the space interpolation is carried out for an isolated wire. The current at a point s' in the i-segment is interpolated to the values of the currents at the centers of its neighbouring spatial intervals (i-1, i and i+1) (fig. 2a). The current on the end segments is interpolated to zero just at the wire ends (fig. 2b) [12].

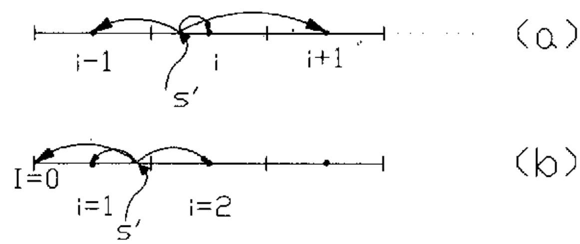

Fig.2 Spatial interpolation for a single wire.
a) Not end segments, b) End segments.

3. DESCRIPTION OF THE DIFFERENT MULTIPLE-JUNCTION TREATMENTS.

When dealing with structures that have several interconnected wires, the assumption that the values of the currents at the ends meeting at the junction are zero is invalid, so the current on the segments closest to the junction point cannot be interpolated to zero and the numerical method described in section 2 cannot be applied [12].

For treating junctions we have modified the method, trying several schemes which are summarized in Fig. 3; the main differences between them are the way of discretizing the wires and how the interpolation function is handled near the junction [13]:

T1: **Overlapping-segments treatment:**

The first method employed was the translation to the time domain of a technique that had been used successfully in the frequency domain [12]. The method consists of replacing the actual configuration of N wires that meet at the junction (see Fig. 3a) by a set of electrically unjoined but overlapping wires (Fig. 3b). It is easy to prove that the way chosen to represent the overlapping wires satisfies Kirchoff's first law at the junction. This way is that when N wires meet at any junction, wire i overlaps one segment onto wire i-1, except that wire 1 is not overlapped

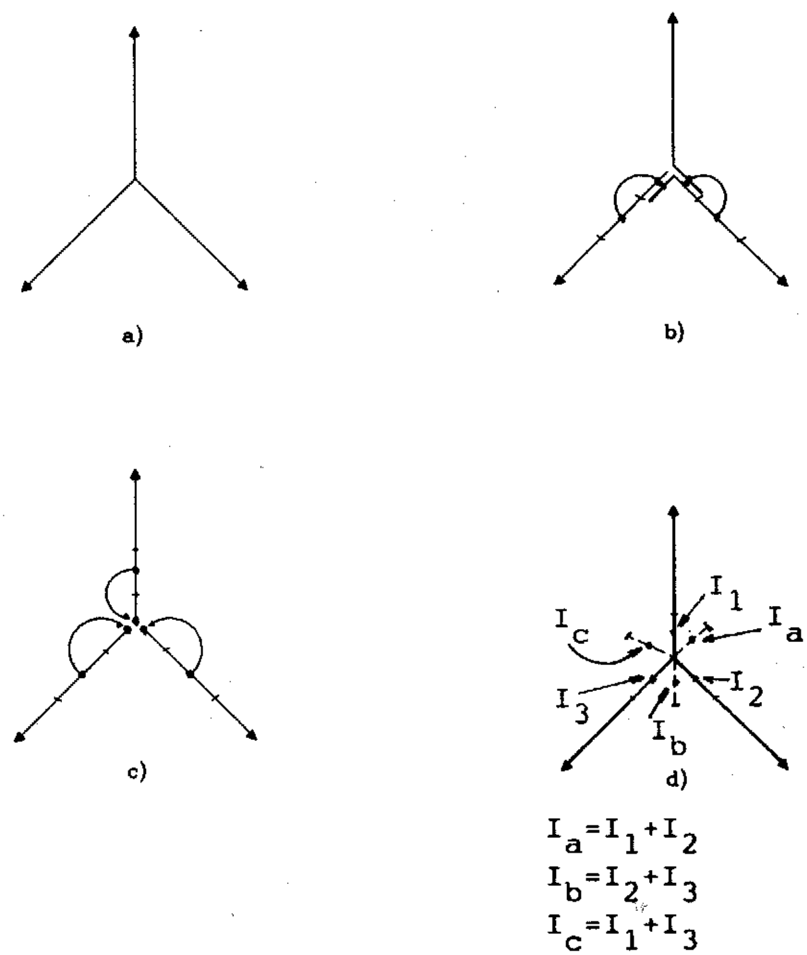

Fig. 3. Different junction treatments. a) Real structure. b) Overlapping-segments treatment. c) Half-segments treatment. d) Sum treatment.

onto wire N. A wire is then considered to end not at a junction but at the end of the overlap, where the current is set to zero. This way each wire can be treated as if it were isolated.

The interpolation of the current on a segment will be to the sum of the current on that segment and the extra overlapping segment if it exists, as shown by the arrows in the diagram (Fig. 3b).

As matrix $\tilde{Z}$ in eq. (2) depends only upon the geometry of the structure, the application of the moment method to a segment and its overlapping segment leads to exactly the same equation. For a junction of N wires the system of equations (2) has N-1 identical equations (the number of overlaps). These extra equations were removed and substituted by the Wu and King condition for the charge at the junction [14], which for thin wires can be expressed as the match of the derivative of the current over space at each wire at the junction. Therefore, a linear system is obtained with the same number of unknowns as equations.

T2. **Half-segments treatment** (Fig. 3c):

Each wire was discretized, locating at each end, segments of half the length of the other segments of the structure. The current on those half segments is interpolated backwards to the currents on segments of the same wire, as shown in Figure 3c.

The Kirchoff's first law and the Wu and King condition described in the T1 treatment are imposed adding extra equations to the ones that come from the moment method. In this case the method leads to a linear system of equations with one equation more than the number of unknowns. The system is solved by multiplying eq.(2) by Z transpose, which minimizes the square error.

T3. **Sum treatment** (Fig. 3d):

The multiple junction of N wires is treated by interpolating the current from one segment across the junction to the center of a "ghost segment" with current equal to the sum of the N-1 currents of the other segments at the junction.

It can be seen that the Wu and King condition is implicit in this junction treatment, but not the Kirchoff's first law, which has been imposed by adding an extra equation to eq. (2) and solving as described in T2.

4. RESULTS

The different junction treatments described in section 3 were applied to several simple structures in order to study the accuracy of the solution obtained and the efficiency of the method employed.

The first structure analyzed was a cross-wire structure (see Fig. 4). The currents induced on it were calculated when it was illuminated by a gaussian pulse of the form:

$$\vec{E}=\exp(-g^2(t-t_{max})^2)\,\hat{z} \tag{4}$$

with $g=5*10^9\ s^{-1}$ and $t_{max}=4.28*10^{-10}$ s. The radius of all the wires was 0.00222m.

In Fig. 5 the current distribution is presented versus time at a point on the first wire situated 4.6 mm from the junction point (Point O).

In order to compare how convergent the different junction treatments are with respect to the number of segments in which the structure is divided, two error criteria were calculated: the root mean square current I_{rms} and the normalized root mean square current error e_{rms} [15]. These variables are defined by:

$$I_{rms}=\sqrt{\frac{1}{N_s N_t}\sum_{i=1}^{N_s}\sum_{j=1}^{N_t}|I_{i,j}|^2}$$

$$e_{rms}=\sqrt{\frac{N_s^r N_t^r}{N_s N_t}\,\frac{\sum_{i=1}^{N_s}\sum_{j=1}^{N_t}|I_{i,j}-I_{i,j}^r|^2}{\sum_{i=1}^{N_s^r}\sum_{j=1}^{N_t^r}|I_{i,j}^r|^2}}$$

where N_s is the number of segments into which the structure was divided and N_t the number of time intervals calculated in each case. N_s^r, N_t^r and $I^r_{i,j}$ correspond to a reference solution that was obtained dividing the wires into a sufficiently large number of segments.

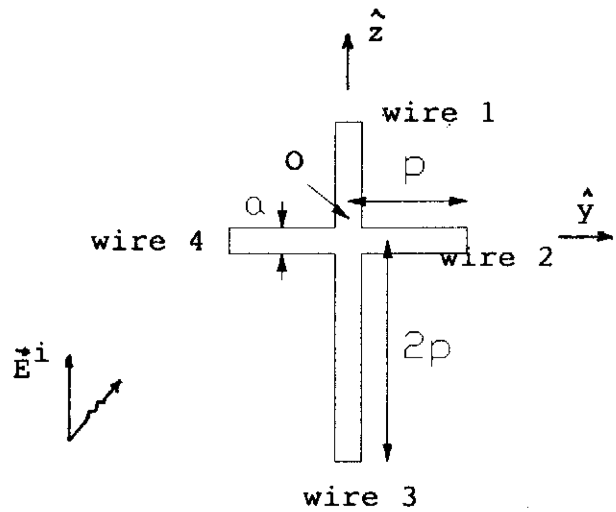

Fig.4 Cross-wire structure. p=0.11 m, a=0.00444 m. Length of the wires: (wire 1= wire 2= wire 4= p; wire3= 2p).

Figs. 6 and 7 show respectively I_{rms} and e_{rms} versus the number of segments. The rapidity with which a technique leads to a stable solution can be estimated from the I_{rms} plot. It is desirable that the curves should approach a constant as soon as possible. As can be seen in Fig. 6, the I_{rms} obtained using T1 and T3 approach a constant more quickly than T2. The e_{rms} plot in Fig. 7 also shows that T1 and T3 behave better than T2, that is, these approaches need fewer segments than T2 to arrive to solution with $e_{rms} \sim 0$.

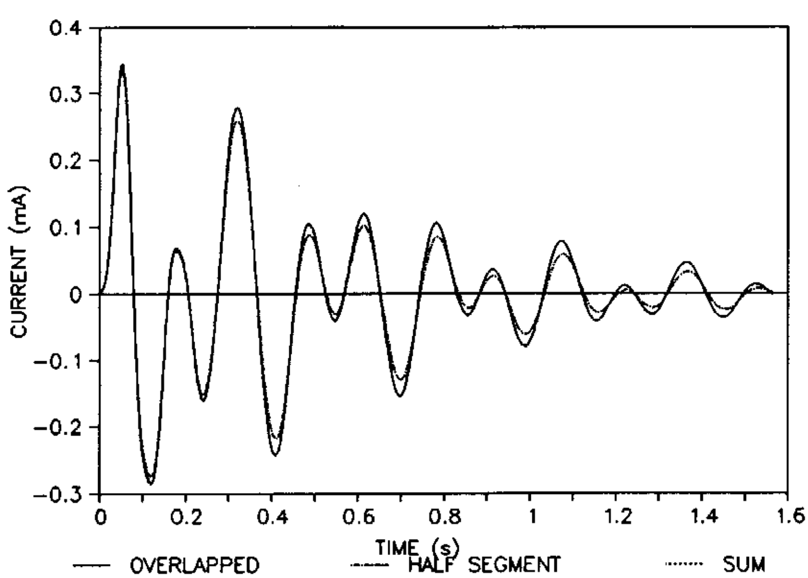

Fig. 5. Current distribution versus time at point O on the cross wire structure. Overlapped and Sum give the same results.

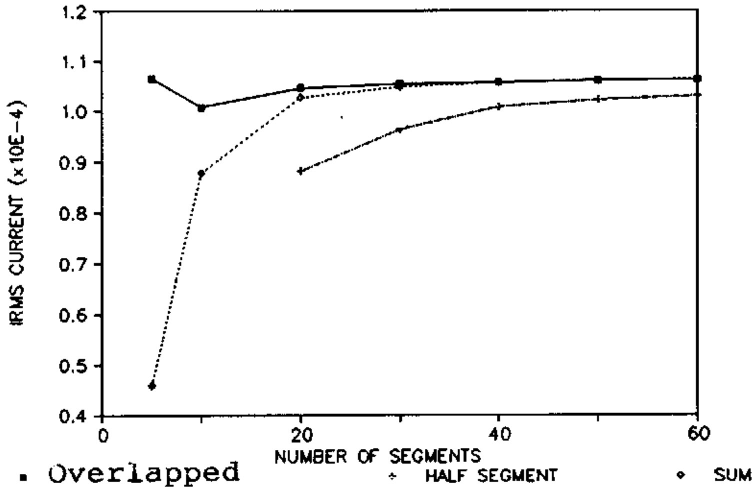

Fig. 6 Root mean square current versus the number of segments into which the cross-wire structure was divided.

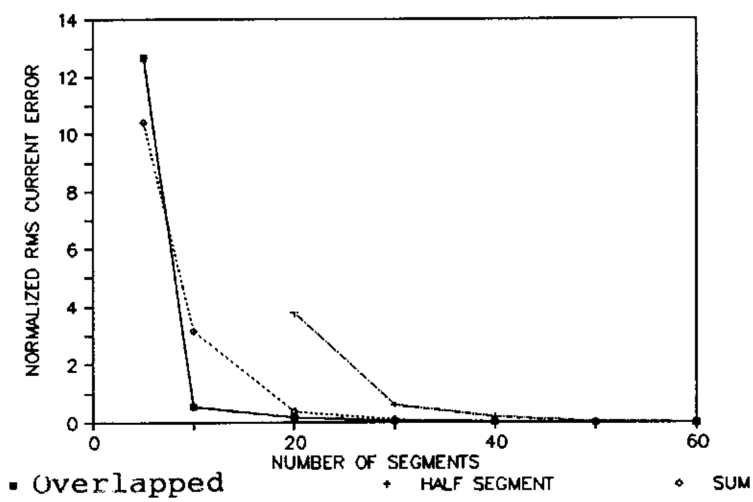

Fig.7 Normalized root mean square current error versus the number of segments into which the cross structure was divided.

In order to compare our time-domain results with the frequency domain results given by the computer code NEC-2 [4] we Fourier transformed them and divided by the transform of the incident field. Fig. 8 shows the Fourier transformation of the current at a point 4.6 mm from the junction versus frequency. The results obtained with T1 and T3 agree closely with those obtained from NEC.

The same analysis was made for the case of the stepped-radius wire (considering it as a junction of two wires with different radii) drawn in Fig. 9, when illuminated by a gaussian pulse

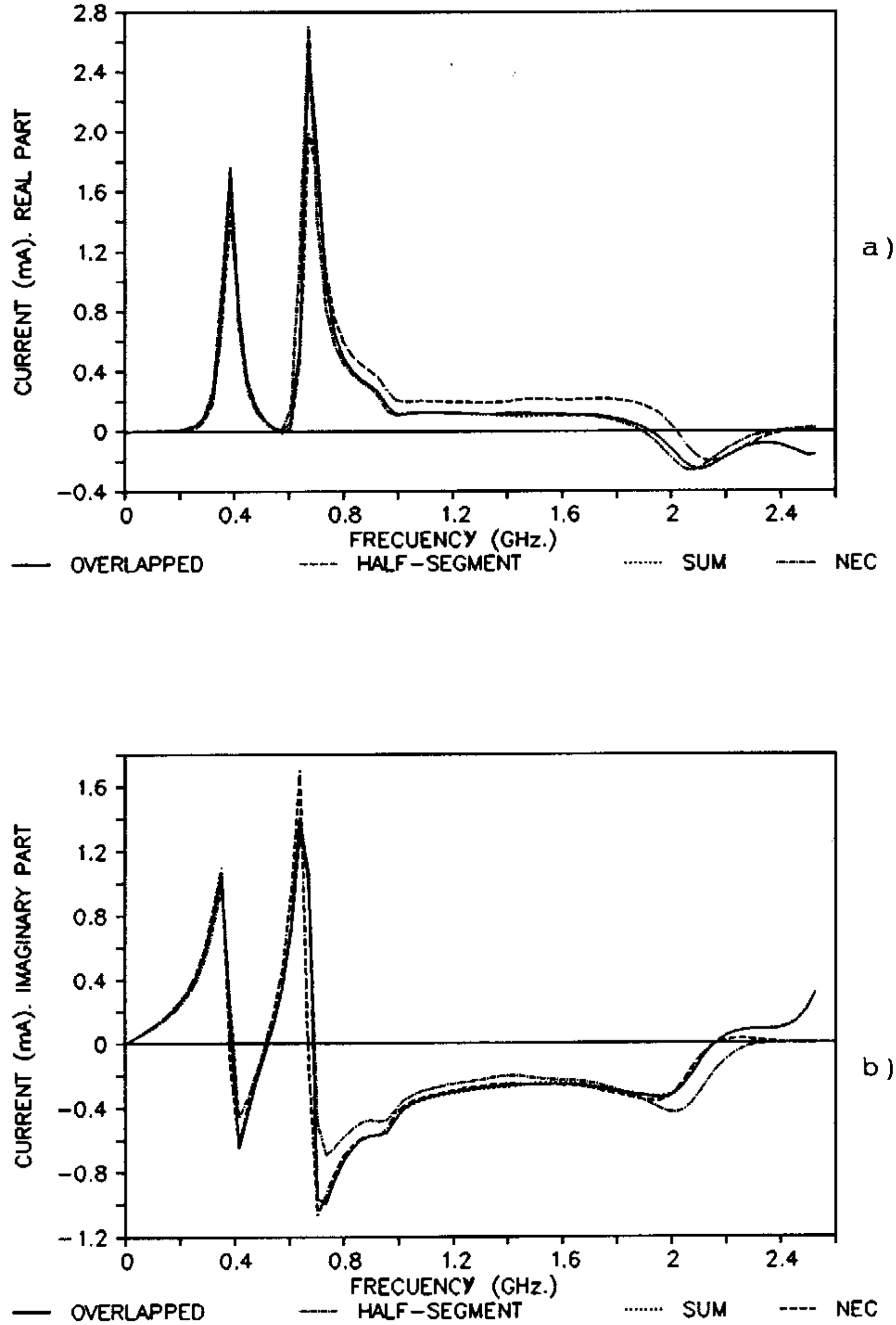

Fig.8 Current at point O on the wire-cross structure versus frequency. a) Real part. b) Imaginary part. (Overlapped and Sum give the same results).

with $g=5*10^9$ s^{-1} and $t_{max}=9.2*10^{-10}$ s. Figs. 10, 11 and 12 show respectively the current versus time at point O, the e_{rms} error versus the number of segments and the comparison of the Fourier transform of the time-domain results with the ones obtained by NEC. As can be seen, in this case all the treatments behaved similarly.

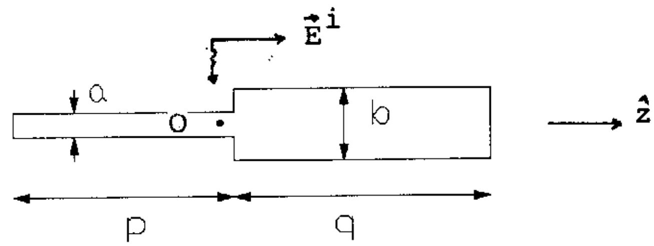

Fig. 9. Stepped-radius wire geometry. p= 0.3m, q= 0.2m, a= 0.002m and b= 0.004m.

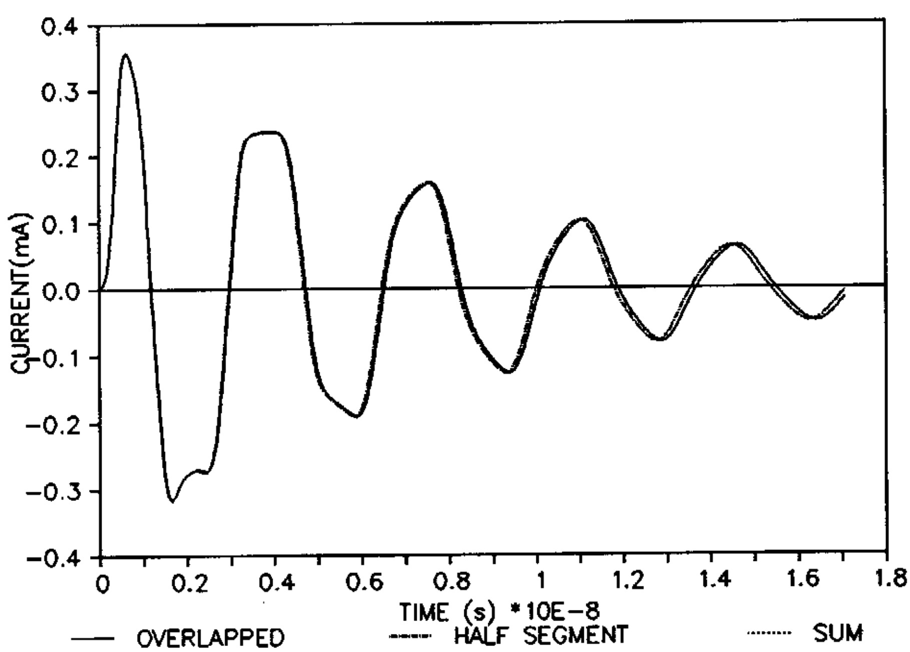

Fig.10 Current distribution versus time at point O on the stepped-radius wire. (Overlapped and Sum give the same results).

The following structures studied were:

1.- The net of twelve wires represented in Fig. 13.

2.- A dodecahedron structure (Fig. 16).

In both cases the electric field incident was proportional to the temporal derivative of eq. (4) polarized in the x-direction and propagating in the z-direction:

$$\vec{E}^{i}=2g(t-t_{max})\exp(-g^2(t-t_{max})^2)\hat{x} \tag{7}$$

with $g=1*10^9$ s^{-1} and $t_{max}=4.6*10^{-9}$ s for the net and $g=8*10^8$ s^{-1} and $t_{max}=5.7*10^{-9}$ s for the dodecahedron. This incident field was chosen because it has not zero-frequency component and, therefore, does not excite the circulating current that would exist on structures with closed-circuit parts [10].

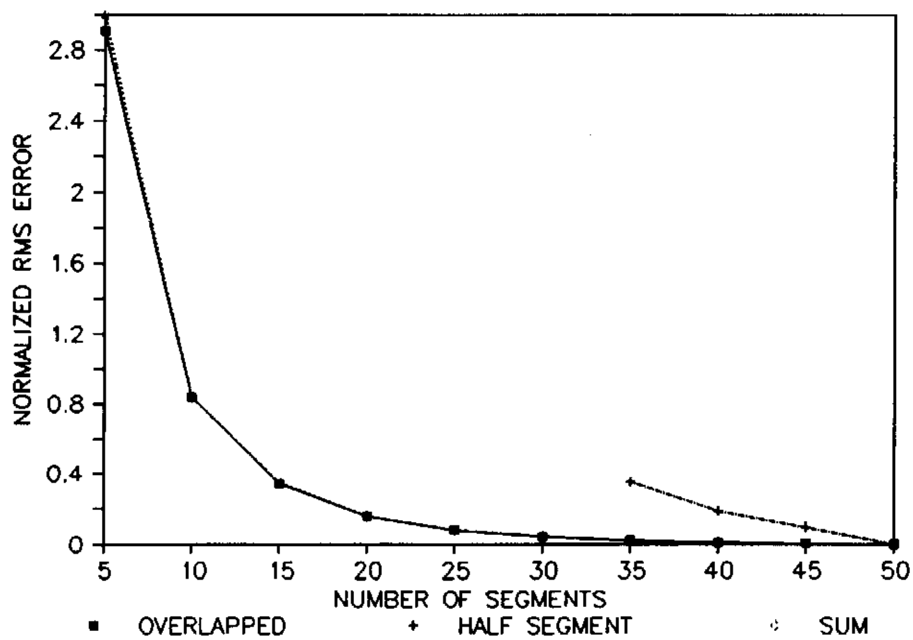

Fig. 11 Normalized root mean square current error versus the number of segments into which the stepped-radius wire was divided. (Overlapped and Sum give the same results)

As the T2 treatment was found to give less accurate results when compared with NEC and to need more segments to arrive at a stabilized solution, for these two more complex structures a comparison was only made between the T1 and T3 treatments.

In Figs. 14 and 17 the current distribution is represented versus time at point O on both structures, and the magnitude of the Fourier transformation of these figures was compared with NEC and plotted in Figs. 15 and 18. The results for the dodecahedron structure were also compared with the ones obtained from the computer code AWAS [5]. In both cases the results arrived at with the T1 treatment (overlapping segments) were found to agree with NEC and

AWAS more closely that the ones resulting from T3.

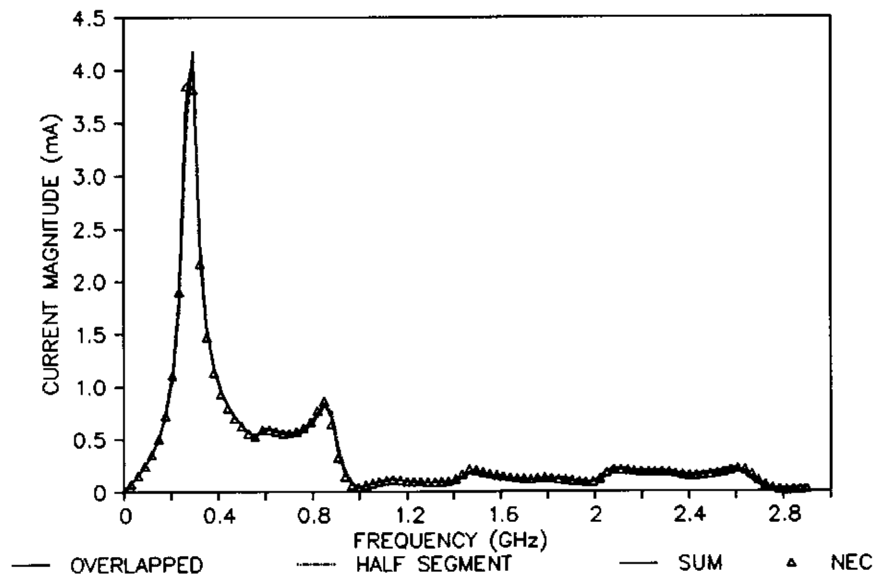

Fig.12 Current magnitude at point O on the stepped-radius wire versus frequency. (All the treatments agree closely).

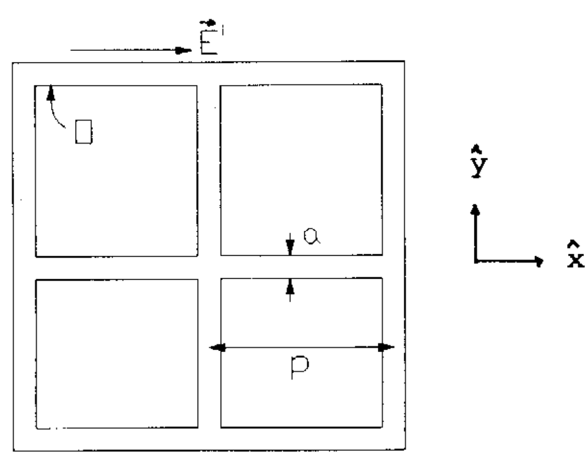

Fig. 13. Geometry of the net structure. p=0.2m, a=13.48mm.

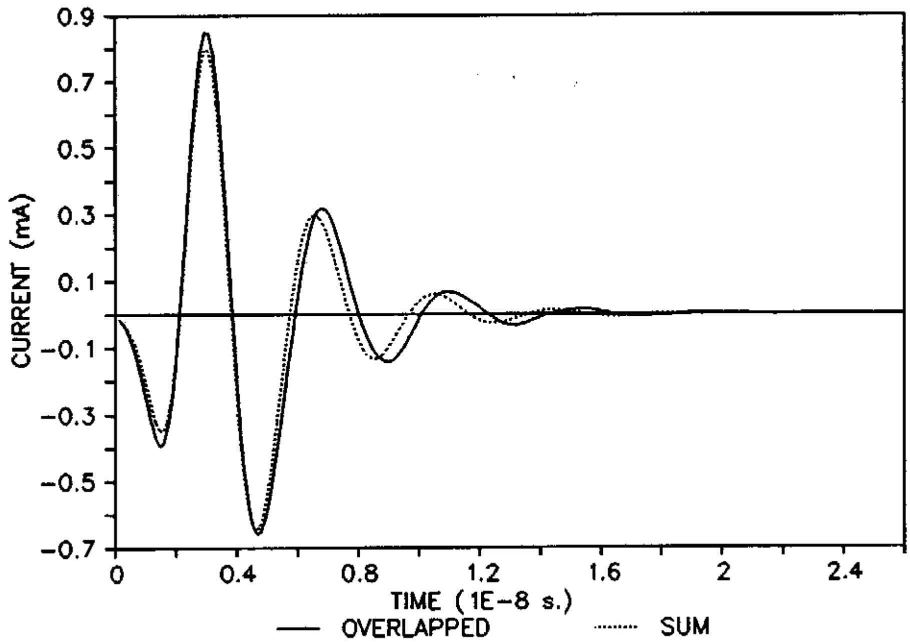

Fig. 14 Current distribution versus time at point O on the net structure.

5. CONCLUSIONS

Different possibilities for treating the multiple junction problem in the time domain have been compared. They have been called:

T1. Overlapping-segments treatment.

T2. Half-segments treatment.

T3. Sum treatment.

The three junction treatments proposed were specifically applied to the study of the interaction of a gaussian pulse with:

1.- A cross-wire structure.

2.- A junction of two wires with different radii.

3.- A net of twelve interconnected wires.

4.- A dodecahedron structure.

To analyze the convergence of the method with the number of segments, the root mean square current and the root mean square error of the current induced on a point on the structure were calculated.

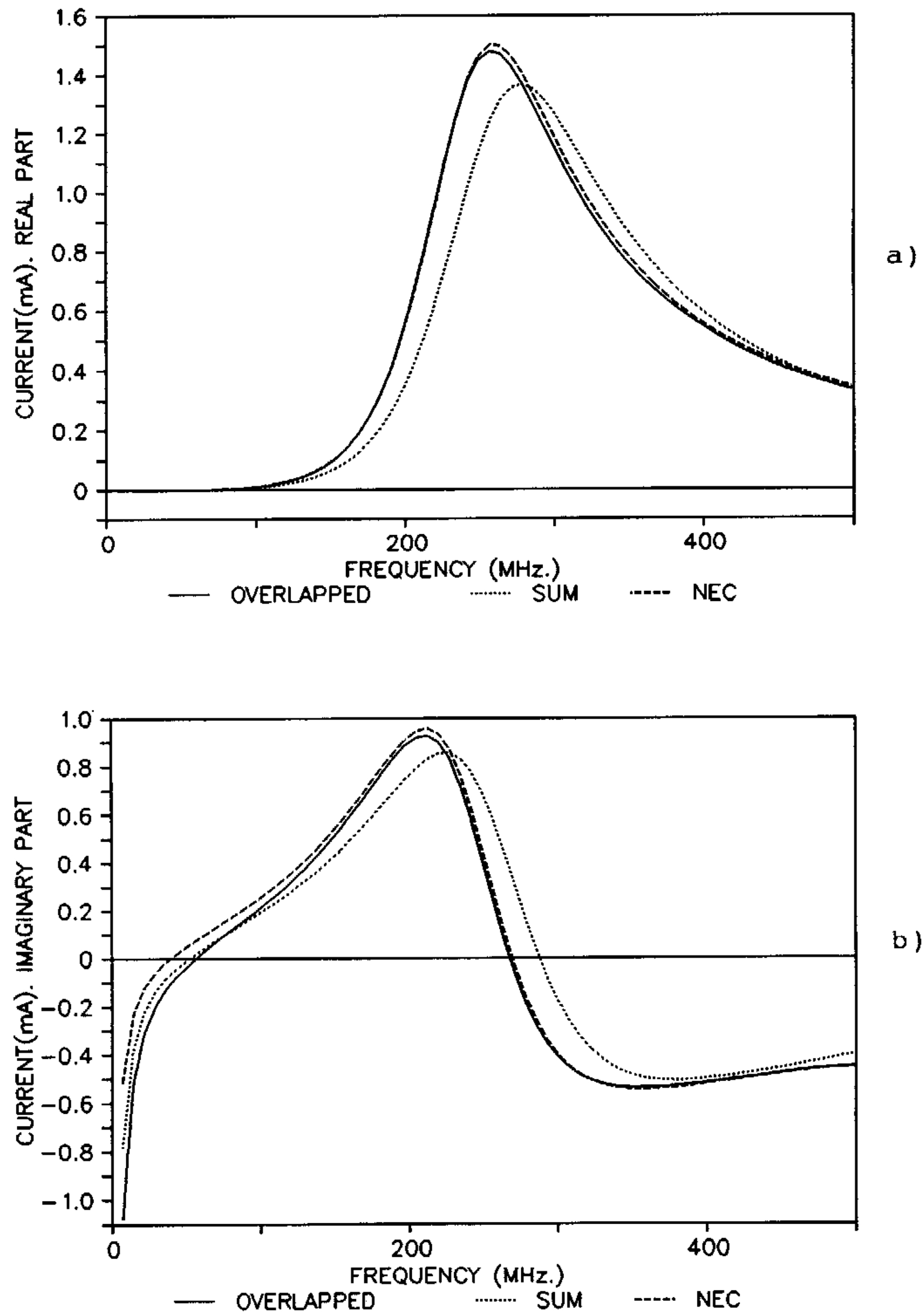

Fig. 15. Current at point O on the net structure versus frequency. a) Real part, b) Imaginary part.

To ascertain their accuracy we applied the Fourier Transform to our time domain results and compared them with some well known codes that work in the frequency domain (NEC and AWAS).

In the light of our criteria, the overlapping procedure seemed to be that which provided the most accurate solution, requiring the structure to be divided into the least number of segments. This treatment was, therefore, the one finally included in the computer program we have developed for studying the interaction of EMP with thin-wire structures, the DOTIG1 code.

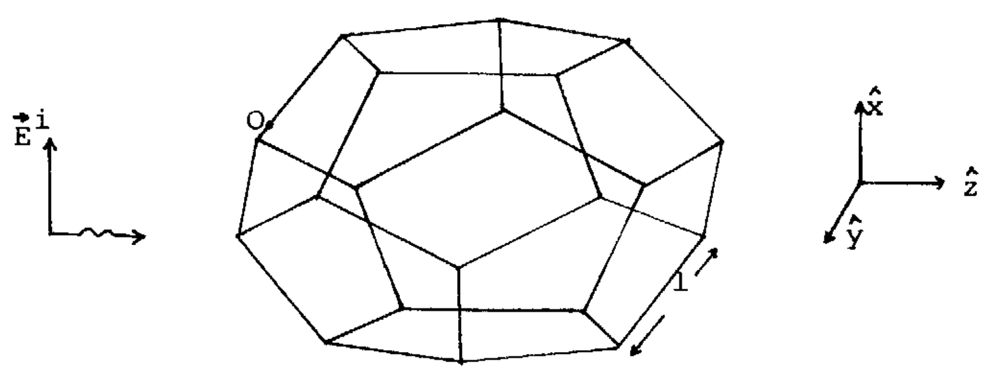

Fig. 16 Geometry of the dodecahedron structure. l= 0.5m. Wire radii = 0.008m.

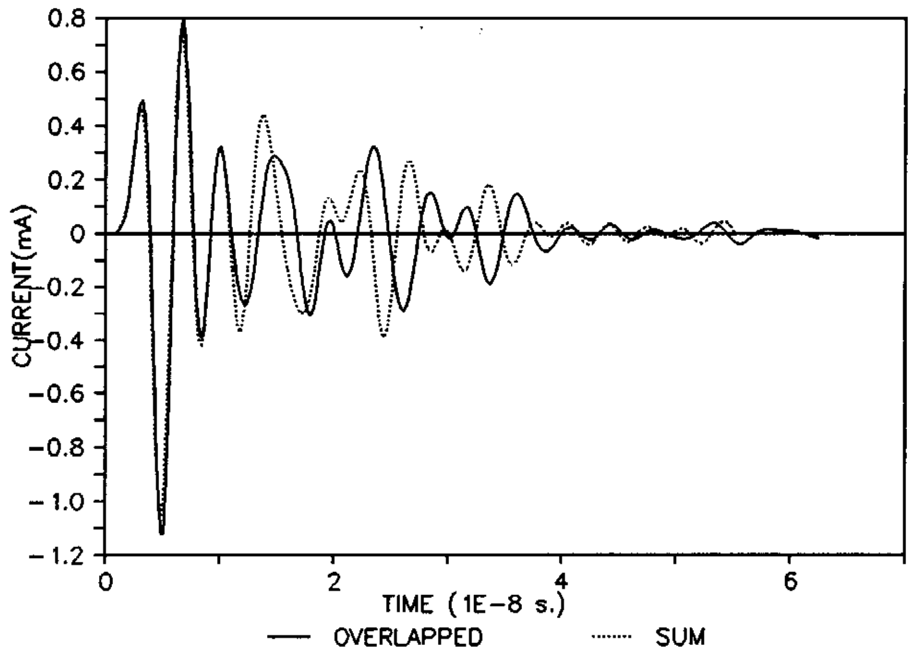

Fig. 17 Current distribution versus time at point O on the dodecahedron.

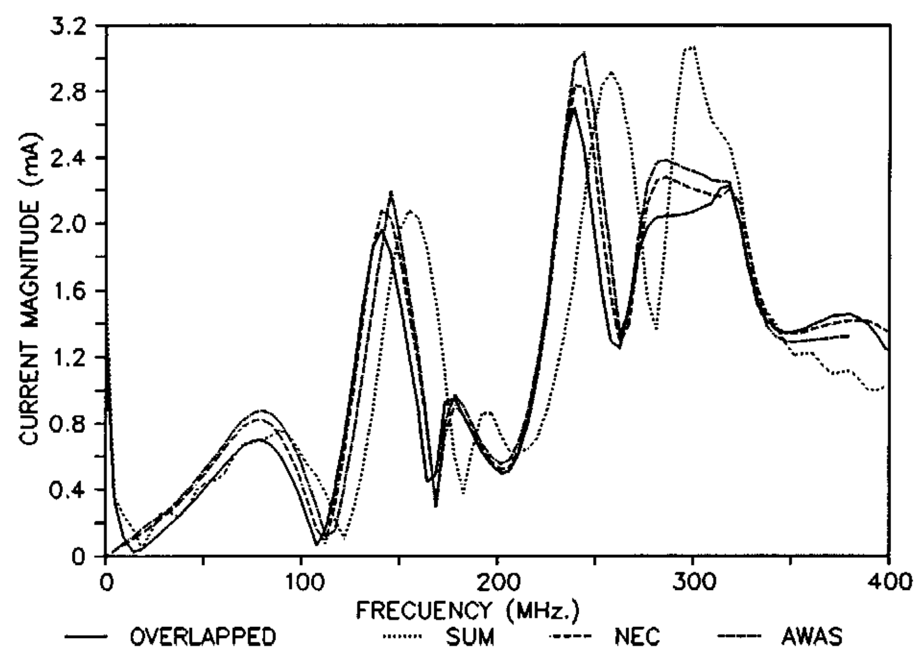

Fig. 18. Current magnitude at point O on the dodecahedron structure versus frequency.

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