Design of Miniaturized Unequal Split Wilkinson Power Divider with Harmonics Suppression Using Non-Uniform Transmission Lines

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Abstract - In this paper, a design of an unequal split Wilkinson power divider (WPD), with high power split ratio, using non-uniform transmission lines (NTLs) is presented. The design is based on using NTLs in each branch of the divider instead of the conventional uniform ones. Besides the achievement of high power split ratio, the size of the designed WPD is reduced. The design procedure is presented for arbitrary design frequency and arbitrary power split ratio. For verification purposes, a 10:1 WPD is designed and fabricated. Good isolation between the output ports, input/output ports matching, and transmission responses are achieved at the design frequency. The experimental and full-wave simulation results show the validity of the designed NTL-WPD. Compared to the conventional design, it is noticed that the proposed divider is more likely suitable for narrowband applications.

Index Terms **-**Non-uniform transmission lines, size reduction, unequal split, Wilkinson power divider.

I. INTRODUCTION

Recently, the design of unequal split Wilkinson power dividers (WPDs) with high power split ratio has attracted much attention and interest. Several different structures have been proposed in the literature to overcome the high characteristic impedance microstrip transmission line required in the WPD with high split ratio. Many papers presented the design of defected ground structures (DGS) and its application in WPDs and branch line couplers [1-6]. In [7], a grooved substrate was proposed for the design of unequal split WPD. The grooves were applied along the strips which required high characteristic impedance which may increase the degree of complexity in fabrication process. A CPW with electromagnetic bandgap was proposed in [8] for designing a transmission line with high characteristic impedance, which was then applied to the design of unequal split WPDs. Nevertheless, the design and realization are even more complex. A 10:1 unequal split WPD using coupled lines with two shorts was presented in [9]. The very thin microstrip transmission line was mitigated using a coupled line with two shorted ends.

In this paper, based on the simple WPD topology proposed in [10], a compact unequal split WPD with high split ratio is designed and fabricated using non-uniform transmission lines (NTLs) theory [11]. The same theory was used in [12] and [13] to design miniaturized dual-frequency WPD and multi-frequency Bagley polygon divider (BPD), respectively. Recently, equivalent circuits for NTLs were proposed in [14].

II. DESIGN OF COMPACT NTLs [11]

In this section, the theory of designing compact NTLs is briefly presented. Figure 1(a) shows a typical uniform transmission line with a length, characteristic impedance and propagation constant of d_0, Z_0 and β_0, respectively, with an ABCD parameters matrix [15]:

$$\begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & jZ_0 \sin(\theta) \\ jZ_0^{-1}\sin(\theta) & \cos(\theta) \end{bmatrix}, \quad (1)$$

where $\theta = \beta_0 d_0$ is the electrical length of the desired uniform transmission line. Figure 1 represents an equivalent non-uniform transmission line of length d, with varying characteristic impedance $Z(z)$ and propagation constant $\beta(z)$. The NTL is designed so that its *ABCD* parameters at a frequency f are equal to those of the uniform transmission line. Moreover, compactness is achieved by choosing the length d to be smaller than d_0.

The general method to design an optimal reduced-length NTLs proposed in [11] is adopted here. First, the NTL is subdivided into K uniform electrically short segments with length of Δz as follows:

$$\Delta z = \frac{d}{K} << \lambda = \frac{c}{f}. \quad (2)$$

The *ABCD* parameters of the whole NTL are obtained by multiplying the *ABCD* parameters of each section as follows:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdots \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \cdots \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix}, \quad (3)$$

where the *ABCD* parameters of the i^{th} segment are:

$$A_i = D_i = \cos(\Delta\theta), \quad (4.a)$$

$$B_i = Z^2\left(\left(i-0.5\right)\Delta z\right) C_i = jZ\left(\left(i-0.5\right)\Delta z\right)\sin\left(\Delta\theta\right),$$
$$i = 1, 2, \ldots\ldots, K. \quad (4.b)$$

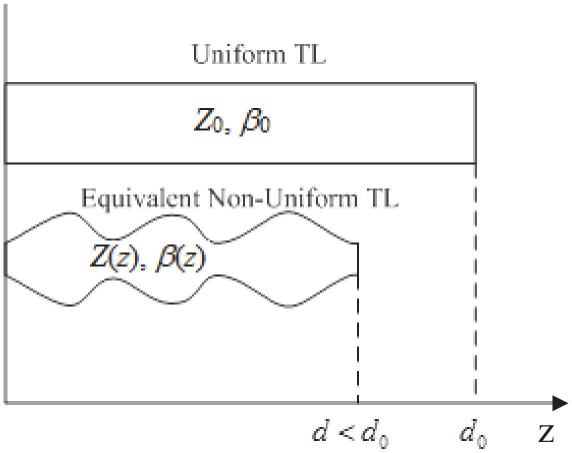


Fig. 1. A typical uniform transmission line (UTL) versus an equivalent non-uniform transmission line (NTL).

The electrical length of each segment is:

$$\Delta\theta = \frac{2\pi}{\lambda}\Delta z = \frac{2\pi}{c} f \sqrt{\varepsilon_{eff}}\,\Delta z. \quad (5)$$

Then, the following truncated Fourier series expansion for the normalized characteristic impedance $\overline{Z}(z) = Z(z)/Z_0$ is considered:

$$\ln\left(\overline{Z}(z)\right) = \sum_{n=0}^{N} C_n \cos\left(\frac{2\pi nz}{d}\right). \quad (6)$$

So, an optimum designed compact length NTL has to have the *ABCD* parameters as close as possible to the *ABCD* parameters of the desired uniform transmission line at a specific frequency. Therefore, the optimum values of the Fourier coefficients C_n's can be obtained through minimizing the following error function [11]:

$$Error = \sqrt{\frac{1}{4}\left(\left|A-A_0\right|^2 + Z_0^{-2}\left|B-B_0\right|^2 + Z_0^2\left|C-C_0\right|^2 + \left|D-D_0\right|^2\right)}. \quad (7)$$

Also, this error function should be restricted by some constraints such as reasonable fabrication and physical matching, as follows:

$$\overline{Z}_{\min} \le \overline{Z}(z) \le \overline{Z}_{\max}, \quad (8.a)$$

$$\overline{Z}(0) = \overline{Z}(d) = 1. \quad (8.b)$$

One should be careful when dealing with such constraints to get the desired performance. The first constraint given in (8.a) guarantees that the resulting non-uniform microstrip line is not too wide, by choosing an appropriate value of $\overline{Z}_{\min}$, and not too thin, by choosing an appropriate value of $\overline{Z}_{\max}$, since the microstrip line width is inversely proportional to its characteristic impedance. The second constraint given in (8.b) guarantees that the widths of the two ends of the resulting non-uniform transmission line will be equal to the width of the uniform ones for matching purposes, and for this constraint to be achieved, the sum of the Fourier coefficients must equal to zero. It is worth mentioning here that the Fourier coefficients are bounded between -1 and 1, i.e., ($-1 \le C_n \le 1$).

So, the goal is to find the Fourier coefficients values (C_n's) that give a non-uniform transmission line that has its ABCD parameters approximately equal to those of the uniform transmission line by minimizing the error function in (7) at a specific design frequency (with the constraints given in (8)). To solve this constrained minimization problem, the MATLAB function "fmincon.m" is utilized.

III. DESIGN OF UNEQUAL SPLIT WPD

Figure 2 shows the schematic of the unequal split WPD that was proposed in [10]. This WPD topology has the merits of having a simple layout, with the dividing ratio k depending on the electrical lengths of its arms rather than the impedances values. In other words, the power-dividing ratio k is a function of ϕ (< π/2):

$$k = \frac{1}{\cos\phi} = \sqrt{\frac{P_2}{P_3}}\,. \tag{9}$$

To achieve a 10:1 dividing ratio, one can obtain ϕ from (9) which gives ϕ=71.57°. So, for a 10:1 WPD operating at 1 GHz and having terminating impedances of Z_0= 50 Ω, the electrical lengths of the uniform transmission lines TL1, TL2 and TL3 are 161.57°, 90°, and 71.57°, respectively, and will share the same characteristic impedance of $\sqrt{2}Z_0 = 70.71\Omega$. For a design frequency of 1 GHz, and considering an FR-4 substrate having a dielectric constant ε_r of 4.6 and a substrate height h of 1.6 mm, the above electrical lengths can be translated into physical lengths of 74.35 mm, 41.41 mm, and 32.94 mm, for the first, second and third uniform transmission lines, respectively. These lengths occupy a large circuit area which will be reduced using NTLs in the next section.

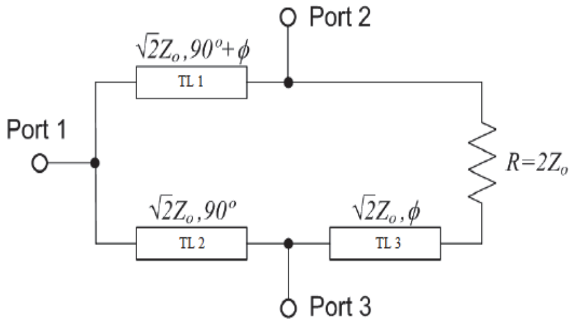


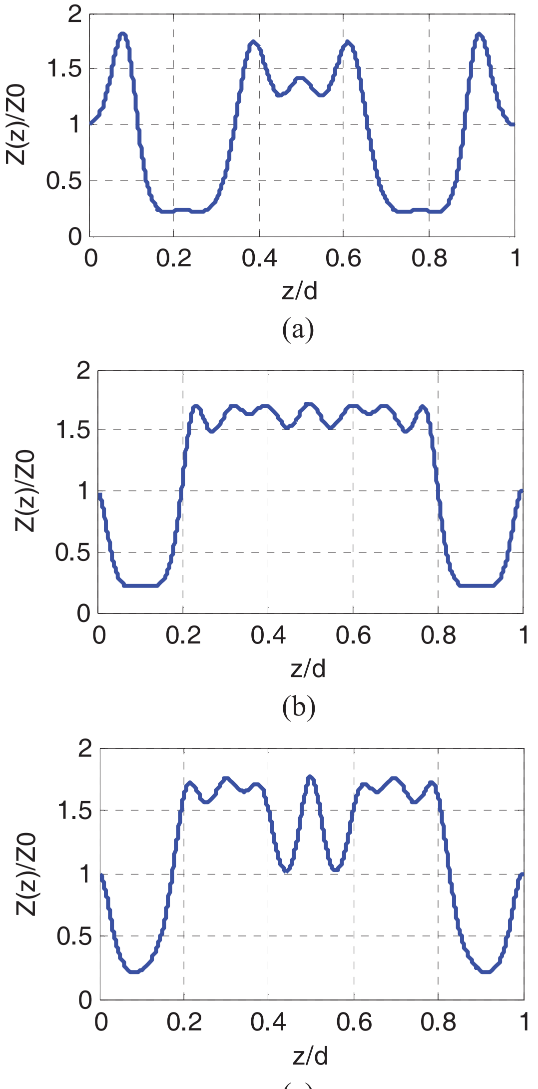
Fig. 2. An unequal split WPD proposed in [10].

IV. DESIGN OF COMPACT UNEQUAL SPLIT NTLs-WPD

An unequal split NTLs-WPD can be realized by substituting each uniform TL in Figure 2 by its equivalent NTL. So, for the uniform transmission line sections of lengths d_{01}= 74.35 mm, d_{02}= 41.41 mm and d_{03}= 32.94 mm, compact NTLs of lengths d_1= 50 mm, d_2= 28 mm, d_3= 25 mm, respectively, have been chosen. The optimization variables K and N are chosen as 50 and 10, respectively. Also, $Z_1(z)$ is bounded between $(0.216 \le \overline{Z}_1(z) \le 1.8)$, whereas $Z_2(z)$ is bounded between $(0.216 \le \overline{Z}_2(z) \le 1.7)$ and $Z_3(z)$ is bounded between $(0.216 \le \overline{Z}_3(z) \le 1.7)$. Figure 3 shows the resulting impedances $\overline{Z}_1(z)$, $\overline{Z}_2(z)$, and $\overline{Z}_3(z)$. The obtained impedances shown in Figure 3 are translated into microstrip line widths variation, as presented in Figure 4. This figure shows that the NTL sections widths are bounded as follows:

$$(0.300\ mm \le W_1(z) \le 15.2\ mm)$$
$$(0.368\ mm \le W_2(z) \le 15.4\ mm)$$
$$(0.335\ mm \le W_3(z) \le 15.2\ mm)$$

Also, the resulting Fourier coefficients and the error values are listed in Table 1.



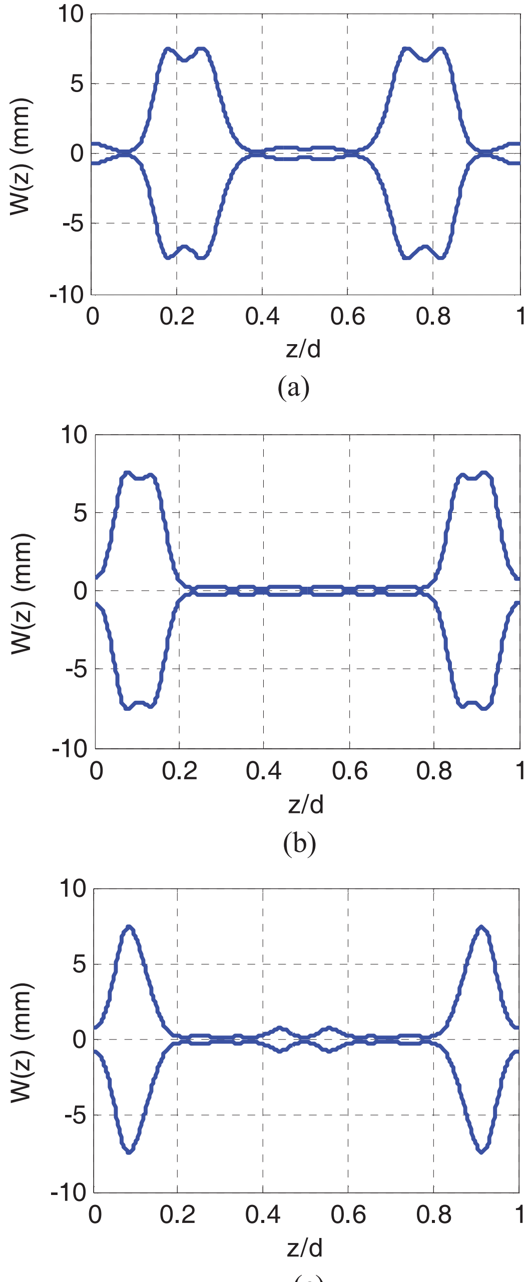
(a)

(b)

(c)

Fig. 3. The normalized impedances for (a) $\overline{Z}_1(z)$, (b) $\overline{Z}_2(z)$, and (c) $\overline{Z}_3(z)$.

For comparison purposes, Figure 5 represents the layout of the conventional 10:1 WPD along with the layout of the proposed NTLs 10:1 WPD. A size reduction of almost 33% is achieved with the use of the NTLs.



(a)

(b)

(c)

Fig. 4. The variation of the microstrip widths for (a) $W_1(z)$, (b) $W_2(z)$, and (c) $W_3(z)$.

V. SIMULATIONS AND MEASURMENTS

The designed 10:1 NTLs-WPD, is first, analyzed using Ansoft Designer [16] (circuit model) by dividing the NTL arms into very short uniform microstrip lines (i.e., a stepped structure with piecewise constant impedance segments). Then, the designed WPD (using the smooth structure as is) is simulated using the full-wave simulators HFSS [16], and IE3D [17]. Moreover, the NTLs-WPD is fabricated and measured using an Anritsu 37369C network analyzer.

Table 1: The values of the Fourier coefficients for the optimized NTL sections

C_n's	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	**Error in eq. 7**
1st section l_1= 50 mm	**-0.3447**	**-0.2034**	**0.9778**	**0.2848**	**-0.3878**	**-0.0880**	**-0.1722**	**-0.2208**	**0.0562**	**0.0516**	**0.0467**	**$1\text{x}10^{-6}$**
2nd section l_2= 28 mm	**-0.1423**	**-0.9494**	**-0.2587**	**0.3139**	**0.4395**	**0.2683**	**0.0491**	**-0.0320**	**0.0737**	**0.1285**	**0.1092**	**$2.6\text{x}10^{-7}$**
3rd section l_3= 25 mm	**-0.0419**	**-0.7070**	**-0.4296**	**0.1486**	**0.3324**	**0.3002**	**0.2542**	**-0.0035**	**0.0761**	**-0.0224**	**0.0928**	**$8\text{x}10^{-8}$**

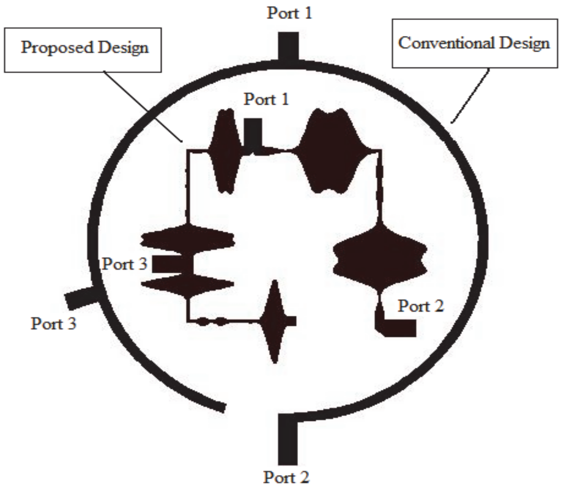


Fig. 5. The conventional WPD layout vs. the proposed NTLs-WPD layout. The 100 Ω lumped resistor is not shown in this layout.

Figures 6-8 show the matching parameters at the input/output ports: S_{11}, S_{22}, and S_{33}, respectively. Ansoft Designer result shows that the input port matching S_{11} is around -37; meanwhile, S_{11} obtained using IE3D and HFSS equals -33 dB around the design frequency. The measured matching parameter S_{11} is -27 dB around 0.94 GHz.

Figures 7 and 8 show good matching at the output ports which is below -20 dB at the design frequency. The measurement results equal -28 dB around 0.9 GHz and -18.5 dB at 0.85 GHz for S_{22} and S_{33}, respectively. The differences between the experimental results and the simulation ones could be due to the use of carbon resistor, as well as fabrication process, soldering, and measurement errors. Figure 9 shows the isolation parameter S_{23}. The simulation and measurement results for S_{23} are accepted at the design frequency.

Figures 10 and 11 show the transmission parameters S_{21} and S_{31}, respectively. As expected, Ansoft Designer results are very close to the ideal ones, i.e., S_{21} close to -0.41 dB and S_{31} close to -10.41 dB at the design frequency. Full-wave simulation results and experimental ones are in good agreement, and show an acceptable behavior around the design frequency (keeping in mind that the loss tangent of the FR-4 substrate used in our design is 0.02).

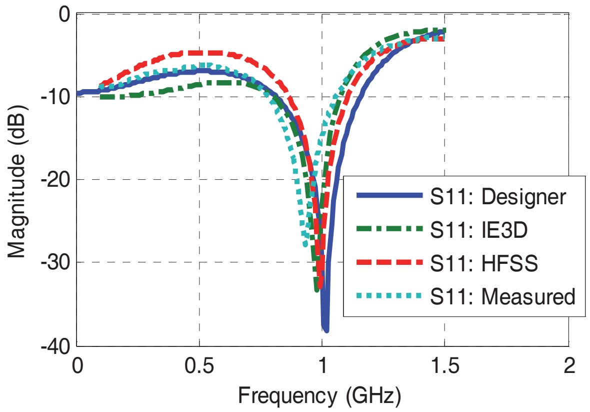


Fig. 6. Matching parameter at port 1.

To demonstrate the odd harmonics suppression for the designed 10:1 NTLs-WPD, Figure 12 shows the frequency response for the proposed NTLs-WPD along with those for the conventional UTL-WPD, in a wider frequency range. It is clearly seen that using the NTLs suppresses the first odd harmonic at 3 GHz. It is worth to point out here that, as shown in Fig.

12(a), the proposed NTLs divider operating bandwidth is narrower than that of the conventional one. Using IE3D results in Fig. 12(a), the 10-dB return loss fractional bandwidth of the proposed divider is about 27.3%, whereas the fractional bandwidth of the conventional one is 97.6%. Thus, the advantages of reducing the overall circuitry area and suppressing the odd harmonics are at the expense of reducing the operating bandwidth. It should be also mentioned that the power handling capability of microstrip lines is restricted by heating caused by ohmic and dielectric losses as well as dielectric breakdown [18]. Step in width, bends, and other discontinuities cause also local concentration of current and thus increase the temperature and decrease the power handling capability. Therefore, it is expected, as given in eq. (10) in [18], that the narrow line width reduces the power handling capability of the structure. On the other hand, the ohmic and dielectric losses depend also on the line length. Since the narrow width sections used in the proposed structure are of limited lengths and the overall structure is small in size with respect to the conventional one, the reduction in power handling capability may not be significant. Figure 13 shows the photograph of the fabricated 10:1 reduced size WPD.

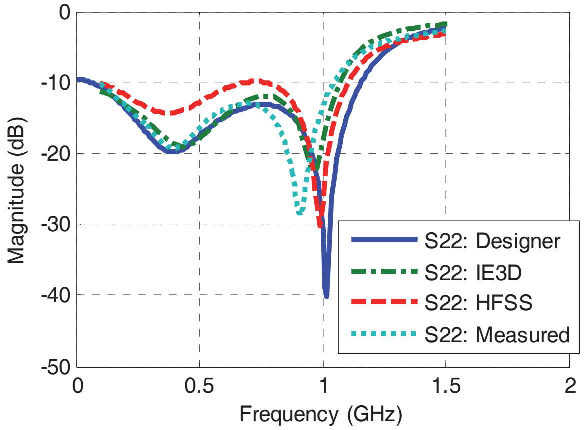


Fig. 7. Matching parameter at port 2.

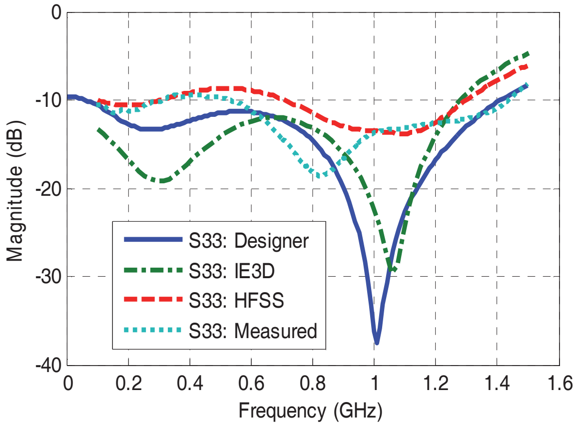


Fig. 8. Matching parameter at port 3.

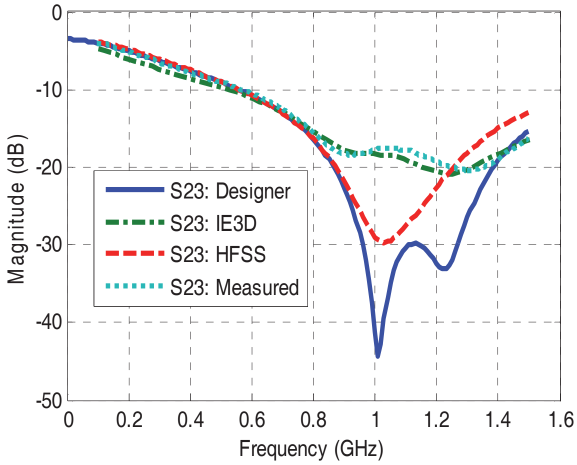


Fig. 9. Isolation parameter S_{23}.

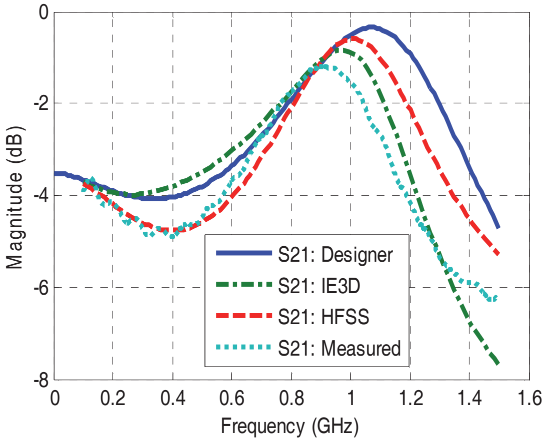


Fig. 10. Transmission parameter S_{21}.

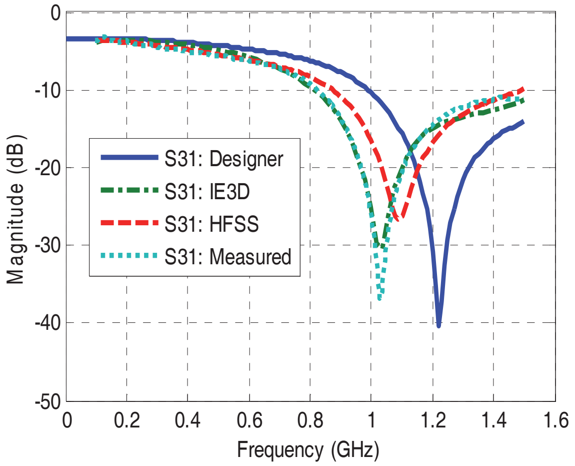
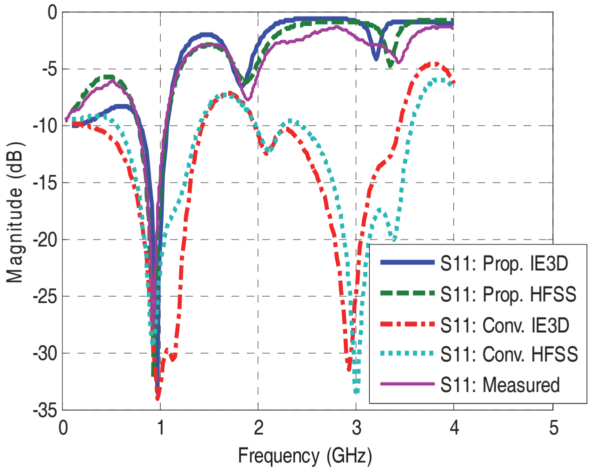
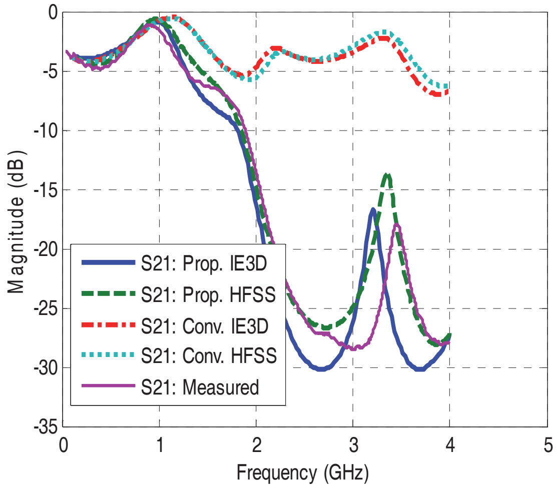


Fig. 11. Transmission parameter S_{31}.



(a)



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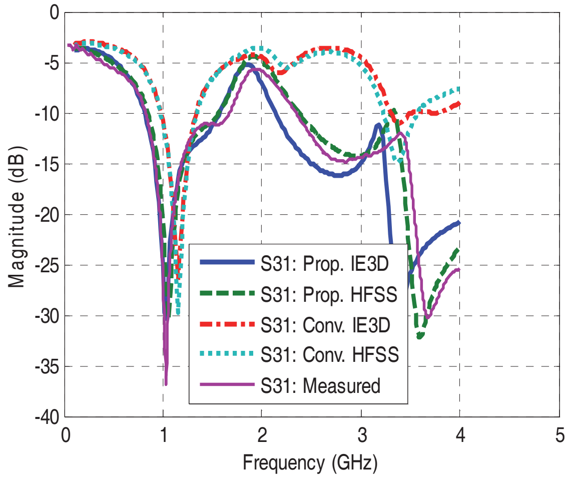

(c)

Fig. 12. Demonstration of odd harmonics suppression (a) S_{11}, (b) S_{21}, (c) S_{31}.

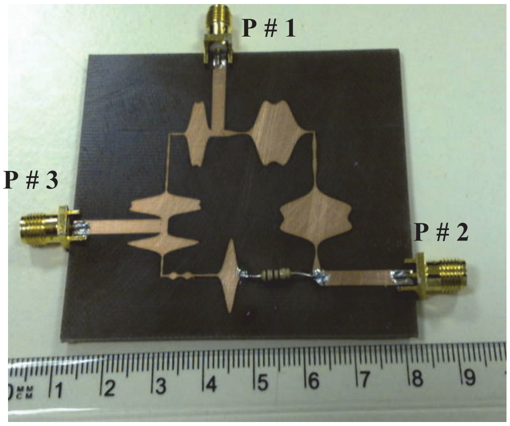

Fig. 13. A photograph of the fabricated 10:1 WPD.

VI. CONCLUSIONS

Based on the theory of non-uniform transmission lines, a compact WPD with a 10:1 split ratio was designed and fabricated. This WPD achieved a size reduction of almost 33% compared to the conventional UTL-WPD. Moreover, the designed WPD suppresses the odd harmonics of the design frequency by enforcing the *ABCD* parameters of the optimized NTLs to be equal to those of the uniform ones at the design frequency only. The agreement between the simulation and experimental results is acceptable, which validates the design procedure.

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