APPLICATION OF BEM FOR CALCULATING THE PARAMETERS OF CABLES AND TRANSMISSION LINES

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ABSTRACT. The capacitances of three phases cable and the characteristic impedance of coaxal transmission line with complicated shape of the cross sections are evaluated by using boundary element method. The calculated results obtained by the proposed method are coincident well with the results given by the literatures [2-9].

1 INTRODUCTION

In engineering practice, the capacitances of multi-phases cable are calculated approximately by using curves and tables such as given in reference [3]. The numerical methods are rarely used to calculate the characteristic impedance of the coaxal transmission lines due to the accurate result of the flux density is not easy to obtain and the corner effect can not be dealt with well. Hence many authors intended to find a simple and accurate method to obtain the approximate analytical formulae for calculating characteristic impedance of coaxal transmission lines with complicated cross section.

By comparison with finite difference and finite element methods, the boundary element method is easy to obtain the distribution of the normal derivatives of potential along the boundary of the field region directly. It is profitable to calculate the total flux along the conductor surface. Hence, the parameters of the capacitance of multi conductors with any shape of the cross section can be obtained in a easy way with sufficient accuracy. The capacitances of multi-phases cable and the characteristic impedence of transmission lines with complicated shape of the cross section for TEM mode wave are calculated by the boundary element method in following sections.

2 BRIEF INTRODUCTION OF BOUNDARY ELEMENT METHOD

Based on the weighted residual principle, the boundary integral equation of Laplace's equation is:

$$C_i u_i = \int_{\Gamma} \left(F\frac{\partial u}{\partial n} - u\frac{\partial F}{\partial n}\right) d\Gamma \qquad (1)$$

where u is the function of Laplace's operator, F is the fundamental solution of Laplace's equation, u_i is the function value at any point i of the boundary of the field region, Γ is the boundary of problem region, n represents the normal direction of the boundary. For smooth boundary, $C_i=1/2$ [1]. After using the discretization technique, Eq.(1) is approximated by

$$\frac{1}{2} u_i + \sum_{j=1}^{N} \int_{\Gamma_j} u\frac{\partial F}{\partial n} d\Gamma = \sum_{j=1}^{N} \int_{\Gamma_j} F\frac{\partial u}{\partial n} d\Gamma \qquad (2)$$

where N is the total number of elements along the boundary which is discretized, Γ_j is the boundary of each element.

Suppose the unknown function u is approximated by

$$u = \sum_{i=1}^{n} N_i u_i \qquad (3)$$

where N_i is the shape function. Let

$$\begin{cases} \hat{H}_{ij} = \int_{\Gamma_j} \frac{\partial F}{\partial n} [N] d\Gamma \\ G_{ij} = \int_{\Gamma_j} F [N] d\Gamma \end{cases} \qquad (4)$$

and

$$H_{ij} = \begin{cases} \hat{H}_{ij} & i \neq j \\ \hat{H}_{ij} + \frac{1}{2} & i = j \end{cases} \qquad (5)$$

then Eq.(2) is expressed by a matrix form

$$HU = GQ \qquad (6)$$

Where H,G are coefficient matrices of order N×N, U,Q are column matrices of order N of potentials and its normal derivatives along the boundary. Solve Eq.(5), the normal derivatives of the potential are obtained with the same degree of accuracy as the potential itself.

In Eq.(3) the shape function [N] is depending on the type of discretization element. The fundamental solu- tion of Laplacian is

$$F = \frac{1}{2\pi} \ln \frac{1}{|r_i - r_j|} \qquad (7)$$

where r_i, r_j are positions of field point and source point, respectively.

3 CAPACITANCES OF CABLES WITH ANY SHAPE OF CROSS-SECTION

3.1 Validation of the Method

Two examples are used to examine the accuracy of the method.

Example 1: A coaxial cable with radius of R_1=1cm, R_2=2cm is shown in Fig.1(a).

The inner and outer circles are subdivided into 16 and 24 linear elements. Suppose the imposed voltage is 1V. By using the boundary element method, the field strength at the inner and the outer conductors are o.144535×10^4(V/m) and 0.72298×10^4 (V/m), and the relative are +0.18% and 0.29%, respectively. The capacitance per unit length of the coaxal is 80.37065pf and the relative error of the capacitance is:

$$e_c=\frac{80.37065-80.2607}{80.2607}=0.1\%$$

The accuracy is quite good.

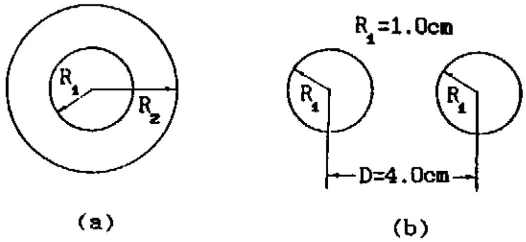

Fig.1(a) Coaxal cylinder
(b) A pair of parallel cylinder

Example 2: Each contour of the cross section of a pair of parallel line as shown in Fig.1(b) is divided by 12 linear elements. By using the boundary element method, the capacitance per unit length of this system is 20.85376pf. The relative error is 1.2%.

3.2 Application:

The partial capacitances of multi-phases cables with different shape of the cross sections are usually calculated by means of figures and tables as introduced in reference[3]. The cross section of a symmetric three-phase cable is shown in Fig.2.

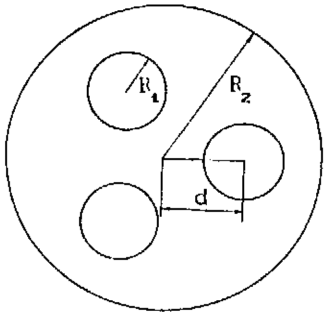

Fig.2 The cross section of a three phases cable ($R=1_1.0cm$ $R_2=4.0cm$ $d=2.0cm$)

Based on the definition of the inductive coefficients between multi conductors and the partial capacitance defined as Eq.(8), the parameters of C_{10}, C_{1k} can be calculated by the above method directly.

$$\begin{cases} Q_1=C_{10}U_{10}+C_{12}U_{12}+\dots+C_{1k}U_{1k}+\dots+C_{1n}U_{1n} \\ \vdots \\ Q_k=C_{k1}U_{k1}+C_{k2}U_{k2}+\dots+C_{ko}U_{ko}+\dots+C_{kn}U_{kn} \\ \vdots \\ Q_n=C_{n1}U_{n1}+C_{n2}U_{n2}+\dots+C_{nk}U_{nk}+\dots+C_{no}U_{no} \end{cases} \quad (8)$$

In Eq.(8), $U_{k2}=U_{ko}-U_{2o}$.

By using the above method, the partial capacitances of the three phase cable shown in Fig.2 are $C_{10}=C_{20}=C_{30}=13.3007pf$ and $C_{12}=C_{23}=C_{31}=9.977897pf$.

Compare the approximate solution obtained from reference [3], the error of C_{10} is:

$$E_{c_{10}}=\frac{13.3007-13.528}{13.528}=1.68\%$$

If the shape of the inner conductor is irregular, for example the sector approximte to elliptic, the partial capacitance can be calculate by the same way.

4 CHARACTERISTIC IMPEDANCE OF TRANSMISSION LINES

The characteristic impedance of coaxal transmission lines with different shape of cross sections both of the inner and the outer conductors are dealt with by many authors by means of analytical and semi-analytical formulations such as those given in references[2,4-9]. In this paper, the boundary element method is successfully used to calculate the characteristic impedance of the coaxal transmission lines constructed by an inner conductor with circular cross section surrounded by an outer conductor with a polygonal cross section and vice versa. For TEM mode transmission lines, the field distribution between conductors satisfies the Laplacian equation. The method introduced in former section is used. The characteristic impedances of the transmission lines with polygonal and circular conductors as shown in Fig.3(a),(b) and Fig.4 are calculated. The comparison of the numerical results calculated by BEM with those obtained from different literatures of these configuration is listed in Table 1 and 2. For Fig.3(a) and (b), the outer and the inner conductors are divided by 36 and 20 linear elements, respectively. For Fig.4, the circle is divided into 20 linear elements and the square is divided into 36 linear elements.

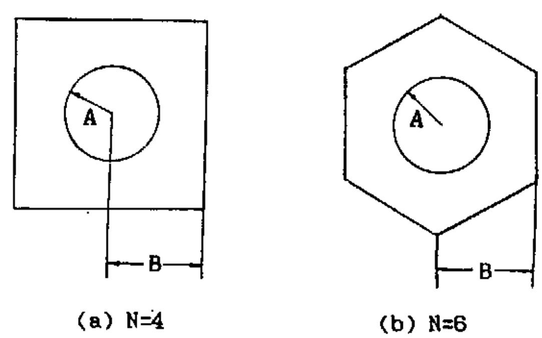

Fig.3 Polygonal transmission line with circular inner conductor

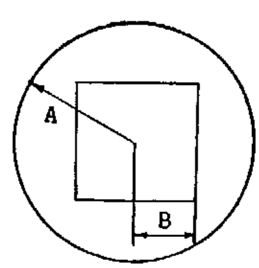

Fig.4 Circular transmission line with square inner conductor

Table 1 Characteristic impedance $Z_o(\Omega)$ of the polygonal transmission line with Circular inner conductor

	(a) N=4					(b) N=6				
A/B	present work	Estevez[2]	Lin[3]	Pan[4]	Riblet[9]	present work	Estevez[2]	Lin[4]	Ma[6]	Pan[8]
.05	183.72	185.21	183.77	184.14		181.44	182.10	182.38	182.79	181.82
.10	142.32	143.16	142.21	142.59		140.04	140.36	140.82	141.20	140.26
.20	100.92	101.02	100.66	101.02		98.64	98.60	99.27	99.61	98.71
.30	76.70	76.41	76.35	76.69		74.43	74.19	74.96	75.28	74.39
.40	59.52	59.02	59.10	59.42		57.24	56.90	57.71	58.02	57.14
.50	46.19	45.61	45.73	46.00	46.09	43.92	43.53	44.34	44.63	43.75
.60	35.29	34.71	34.60	35.00	35.15	32.03	32.64	33.41	33.69	32.80
.70	26.03	25.49	25.55	25.66	25.85	23.81	23.44	24.16	24.44	23.53
.80	17.90	17.44	17.55	17.46	17.68	15.80	15.46	16.16	16.43	15.47
.90	10.37	10.01	10.49	9.97	10.13	8.65	8.31	9.10	9.36	8.27
.94	7.26	6.99				5.93	5.55	6.49	6.75	5.53
.95	6.42	6.19	7.25	6.20	6.25	5.25	4.84		6.12	
.99	2.17	2.36				2.41	1.69	3.39	3.65	1.76
.998	2.49	0.99	4.29	1.07	1.00	1.54	0.68		3.16	

Table 2 Characteristic impedence $Z_o(\Omega)$ of the circular line with square inner conductor

B/A	present work	Lin[5]	Ma[6]	Pan[8]
.05	168.44	169.67	169.72	169.66
.10	127.05	128.11	128.13	128.12
.20	85.66	86.55	86.54	86.56
.30	61.45	62.24	62.21	62.23
.40	44.26	44.97	44.95	44.92
.50	30.87	31.51	31.56	31.38
.60	19.66	20.20	20.62	19.95
.70	9.25	8.85	11.37	7.32

Table 2 shows that if the inner conductor is square, the difference of the impedence between the numerical results and the analytical results looks obvious. However, for B/A=0.05—0.6, the differences are 0.73% — 2.6%. This influence is due to the corner effect of the inner conductor. If the inner conductor is larger then the influence is bigger. But for large ratio of B/A, the coincidence of the result looks well than the result obtained by different analytical methods.

Table 3 The convergence of the method

A/B	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.94	0.95	0.99	0.998
Element No.(36+20)	181.44	140.04	98.64	74.43	57.24	43.92	33.03	23.81	15.81	8.65	5.93	5.25	2.41	1.54
Element No.(30+32)	181.73	140.23	98.73	74.45	57.22	43.86	32.94	23.71	15.68	8.49	5.73	5.03	2.01	1.57

The convergence of the method is proved by using the construction of the transmission line shown in Fig.3(b). The outer and the inner conductors are subdivided by (36+20) and (30+32) linear elements, respectively, the characteristic impedances(Ω) are listed in Table 3. The results show that the value of the calculated characteristic impedance is different if the number of the discretization elements are different but the difference is very small. It shows that the numerical results are reliable.

5 CONCLUSIONS

1. The BEM can obtain more accurate results for the potential derivative along the conductor surface than the finite difference and finite element me thods. Hence it can be used to calculate the capacitance and characteristic impedance of transmission lines with different shape of cross sections.
2. For the polygonal transmission line with circular inner conductor, the numerical results are coincident very well with the analytical results given by different approximate methods of different authors. In this case, the effect of the inner corner of the outer polygonal has no siginificant influence the field distribution. Hence, the corner effect in the numerical method has not present as a problem.
3. Table 2 shows that the accuracy of the characteristic impedance for the circular transmission line with square inner conductor is a little bit lower only if the inner conductor is larger.
4. Table 3 shows that the stablity of the numerical results are good.

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