

Application of Some Common Numerical Methods to Simple Radiation and Scattering Problems

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Sample calculations using three commonly used numerical methods are presented for some simple radiation and scattering problems. The three methods are: the method of moments (MM), the finite element method (FEM) and the finite integration technique (FIT). For the most part, standard software packages are used in the comparison:

PATCH – a Sandia Labs code based on the Rao, Wilton, Glisson triangular subdomains

HFSS – High Frequency Structures Simulator by Ansoft Corporation

Microwave Studio – by Computer Simulation Technology (CST)

The problems examined are:

1. The radiation from and impedance of an isolated dipole in free space,
2. An array of three dipoles over a finite perfect electric conductor (PEC) ground plane, and
3. The bistatic radar cross section (RCS) of a PEC plate.

For convenience all dimensions throughout are in meters and a frequency of 300 MHz is used so that 1 m corresponds to 1 wavelength.

Among the issues discussed are methods of exciting the structures, how to compute impedance, and the convergence of the solutions. This is not meant to be a rating of the software packages. Clearly each one has advantages and disadvantages in specific cases, and the engineer must determine the most efficient solution method for the particular problem at hand. A survey of the numerical methods such as this one can serve as a good introductory tutorial of computational electromagnetics (CEM) techniques in a university EM course.

A. Method of Moments

The method of moments is in widespread use in various forms. As applied in most cases, the method of moments is used to convert the E-field integral equation (EFIE) into a set of simultaneous linear equations that can be solved by standard matrix techniques. The subdomains typically used for three-dimensional conducting structures are the triangular surface patches introduced by Rao, Wilton and Glisson [1]. Previous to that, wire approximations were common, such as those employed in the Numerical Electromagnetics Code (*NEC*).

The code *PATCH*, which originates from Sandia Labs, is one of many that use the triangular subdomains [2]. *PATCH* has a limited geometry builder that creates simple objects and combines them to form more complicated bodies. The facet models used here were generated in the computer aided design software *ACADS*. A translator program is then used to convert the *ACADS* “.facet” format into a format accepted by *PATCH*.

An efficient way of modeling a thin wire with circular cross section is to use a flat strip with an equivalent width determined by the wire diameter. A rule of thumb is that the equivalent circular radius is approximately 0.225 of the width, $a_e \approx 0.225w$ [3]. *PATCH* has the ability to essentially break the surface at an edge (even if it is not a physical edge) and apply a voltage source or lumped load across the gap. If a 1 volt source is applied across edge number n , then the surface current crossing the edge J_n (A/m) can be determined from the MM expansion coefficients. Subsequently the current is computed from $I_n = J_n \ell_n$ (A) where ℓ_n is the length of the edge n . The ratio of voltage to current gives the impedance across the edge. Figure 1 shows the thin strip model. The voltage is applied to the center edge.



Figure 1: Example of a strip dipole meshed for the method of moments solution.

Extensive comparisons between the strip model and the *NEC* wire have been made [4]. The impedance and radiation patterns have been found to be essentially identical. For example, for a dipole of length 0.45 m and 0.00125 m in diameter, the impedance computed by *PATCH* is $58.3-j55.9$ versus $61.8-j43.4$ ohms by *NEC*. By slightly reducing the length of the *NEC* wire to 0.44m the agreement is within a couple of tenths of an ohm.

A three element array of dipoles over a finite ground plane is shown in Figure 2. The dipoles are aligned with the z axis and spaced 0.4 m. The height of the dipoles above the ground plane is 0.25 m and the ground plane has dimensions 1.5 m in x and 1 m in z . The dipoles are excited with equal voltages and the H-plane pattern ($\theta = 90^\circ$) is shown in Figure 3 for terminal loads of $R_L = 0, 50,$ and 100000 ohms. The latter case approximates an open circuit condition. There are approximately 3500 edges in the model, with the maximum edge length of 0.063 m, which is less than the 0.1 wavelength rule of thumb for convergence at the -30 dB level.

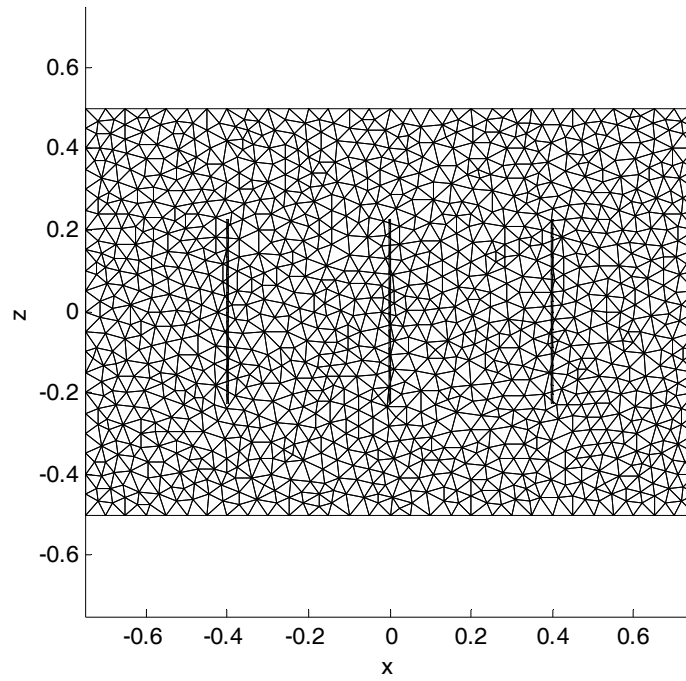


Figure 2: Array of dipoles over a finite ground plane showing triangular subdomains.

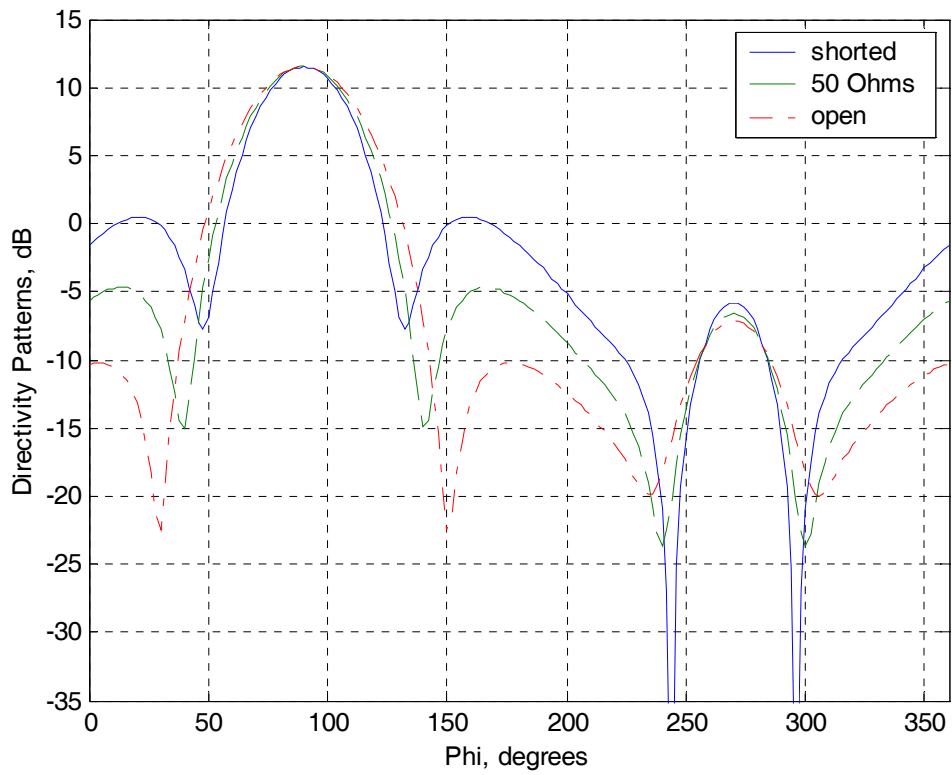


Figure 3: Method of moments patterns for three load conditions.

B. Finite Element Method

The finite element method (FEM) is perhaps the oldest numerical technique used in engineering applications. FEM has been used more extensively for EM problems in recent years due to the development of vector basis functions and effective methods of treating the computational boundaries [5]. The boundary conditions are necessary when applying the FEM to radiation and scattering problems.

Ansoft's High Frequency Structures Simulator (*HFSS*) was used for FEM calculations [6]. *HFSS* has a geometry builder, adaptive meshing, and a powerful post processor. The FEM model of the dipole is shown in Figure 4. The computational "air box" is slightly greater than 0.25 m on all sides (0.25 wavelength is the general rule of thumb for placement of the radiation boundaries from the structures).

Although *HFSS* is capable of modeling the details of the feed, such as the coaxial feed line and balun, the simplest feed arrangement possible was considered for this study. This would be desirable, for example, when an array is being integrated into a platform and the platform effects on the array pattern are of interest. A simple feed model would require less computer memory than a detailed one, thereby freeing up computational resources for more detail in the platform and antenna structure models.

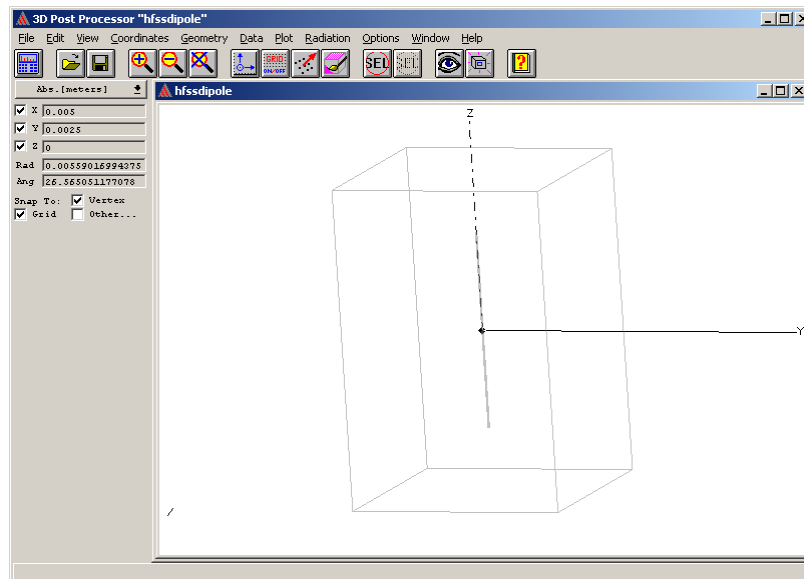


Figure 4: *HFSS* model of the dipole and radiation box.

HFSS has current and voltage gap sources that can be used to excite the dipoles. Lumped *RLC* elements can be applied as boundaries. The *HFSS* dipole model is a circular cylinder of diameter 0.00125 m and a total length of 0.45 m. A gap of 0.01 m is cut at the center and a rectangle spans the gap as shown in Figure 5. A 1 A current source is applied on the rectangle and electric field in the gap computed. To obtain the impedance, the "calculator" functions are used to integrate the field in the gap between the dipole arms. The resulting voltage divided by the current source gives the impedance. Using this technique an impedance of $55.6-j32$ was

obtained for the isolated dipole. (A more direct approach would be to use a lumped gap source type of port, which does not require using the calculator to integrate the field in the gap.)

For the array calculation a 1.5 m by 1 m ground plane was added 0.25 m from the dipoles as shown in Figure 6. The ground plane has a finite thickness of 0.01 m. The lumped RLC boundary is applied to a second rectangle in the gap that is perpendicular to the first rectangle on which the current source was applied (see Figure 7). The H-plane pattern is shown in Figure 8 for several mesh densities, and in Figure 9 for load conditions of $R_L = 0.01$ (approximately shorted), 50, and 100000 (approximately open) loads.

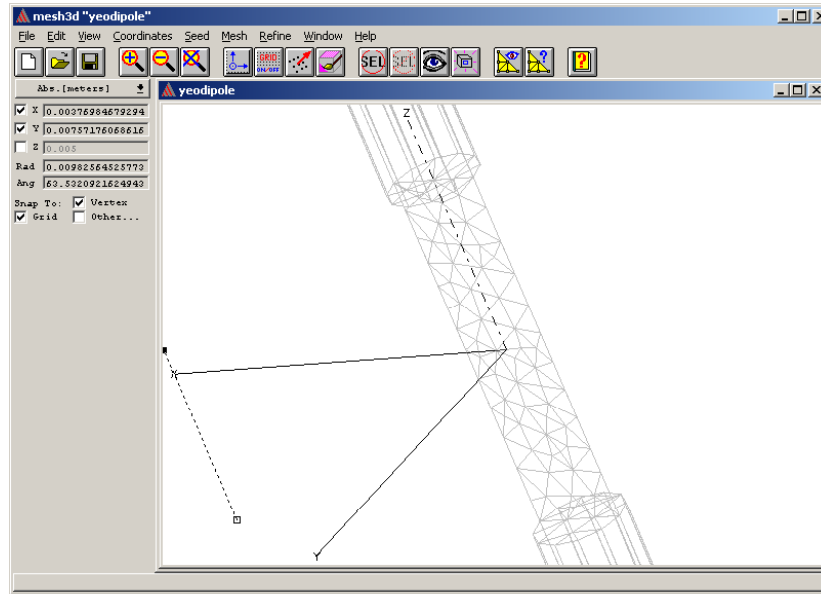


Figure 5: Rectangle in the dipole gap is meshed to compute electric field in the gap.

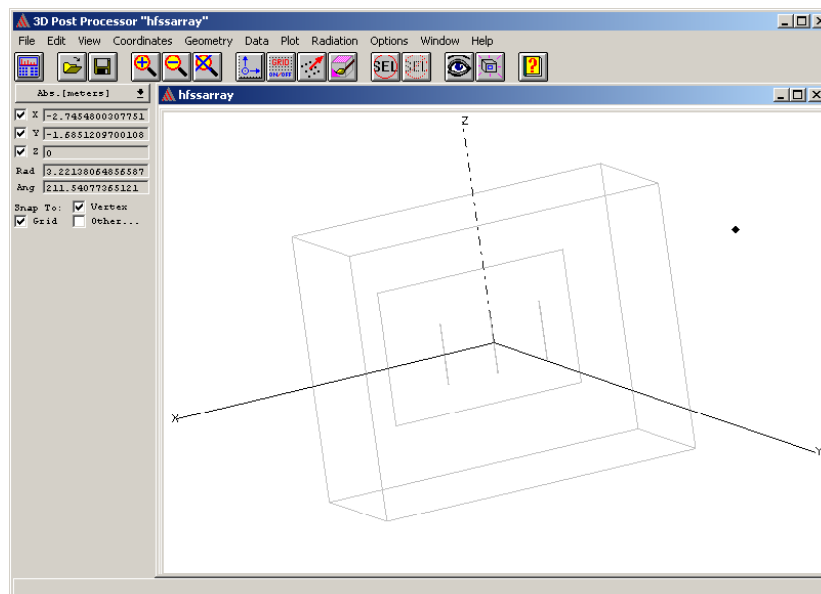


Figure 6: HFSS array model with radiation box.

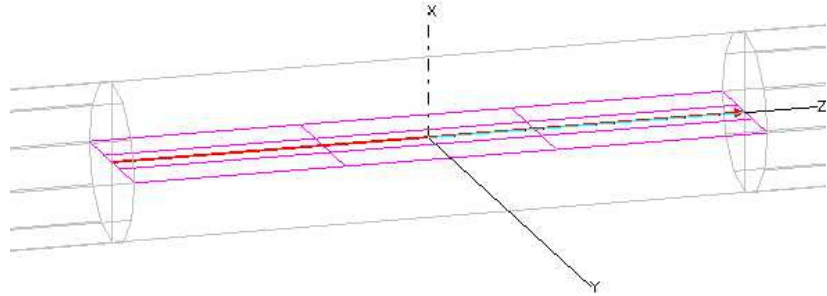


Figure 7: Two orthogonal rectangles in the gap. The lumped RLC boundary condition is assigned to one (highlighted) and a current source to the other.

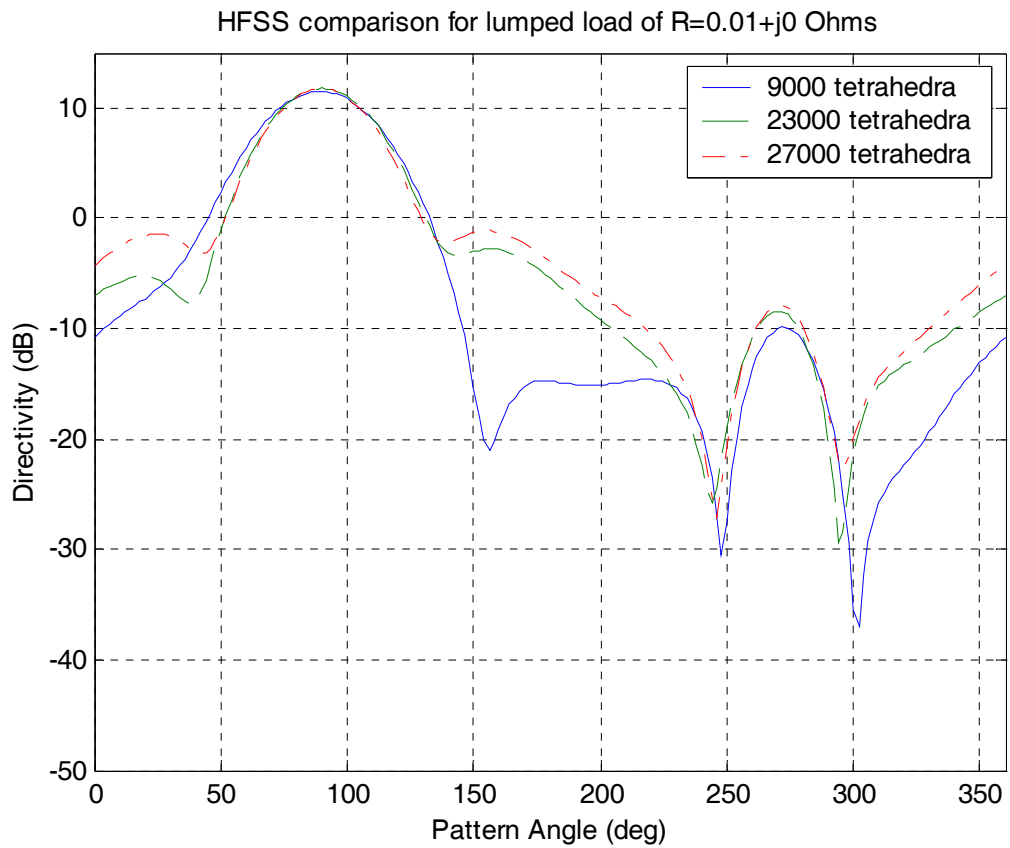


Figure 8: Patterns for various mesh densities.

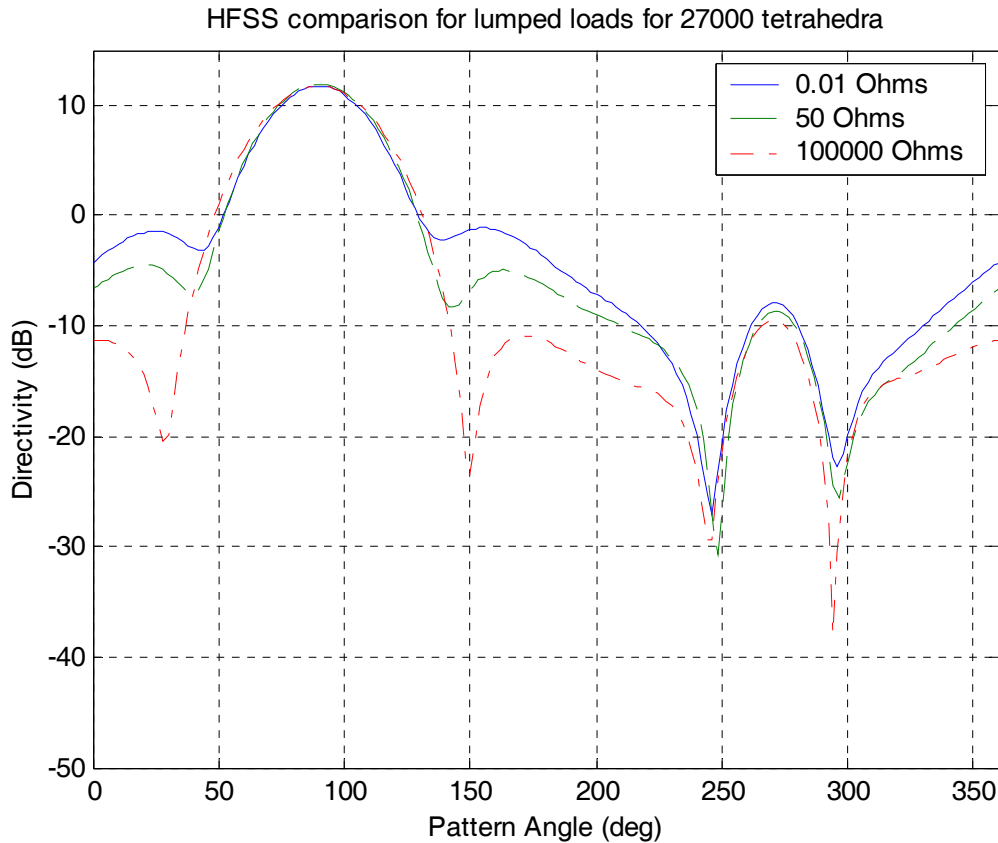


Figure 9: Radiation patterns for three load conditions.

C. Finite Integration Technique

The finite integration technique (FIT) is a means of determining the radiated or scattered field from objects by solving Maxwell’s equations in integral form [7]. Update equations can be derived that give the field at a point in space and time as a function of the field at the same and neighboring points at previous times. Therefore the solution is said to “march in time.”

As with most numerical solutions, the computational region is discretized into appropriate subdomains. For receive or scattering problems, the incident wave is introduced into the computational grid and the fields computed throughout the grid as a function of time. The fields at the boundaries of the computational grid are used to compute equivalent currents, which, in turn, are used in the radiation integrals to compute the far field. The time domain fields can be Fourier transformed to obtain the frequency domain performance.

Microwave Studio by CST [8] is a FIT solver with a geometry builder and post processor. Rectangular meshing is used, but cells need not be filled with only one material. For ease of mesh control a dipole with a square cross section of 0.00125 m on a side was studied, as shown in Figure 10. As in the case of *HFSS*, a simple “Discrete Port” feed source is chosen even though *Microwave Studio* is capable of modeling a more complicated feed. The computational boundary was set about 0.15 m from the structure.

The fields can be plotted anywhere by setting up Monitors. Figure 11 shows a contour plot of the electric field in the region of the gap. The return loss (scattering parameters) and impedance (Figure 12) are computed automatically and available in the “1D Results” section. The impedance at 300 MHz was found to be $53 - j 48$ ohms.

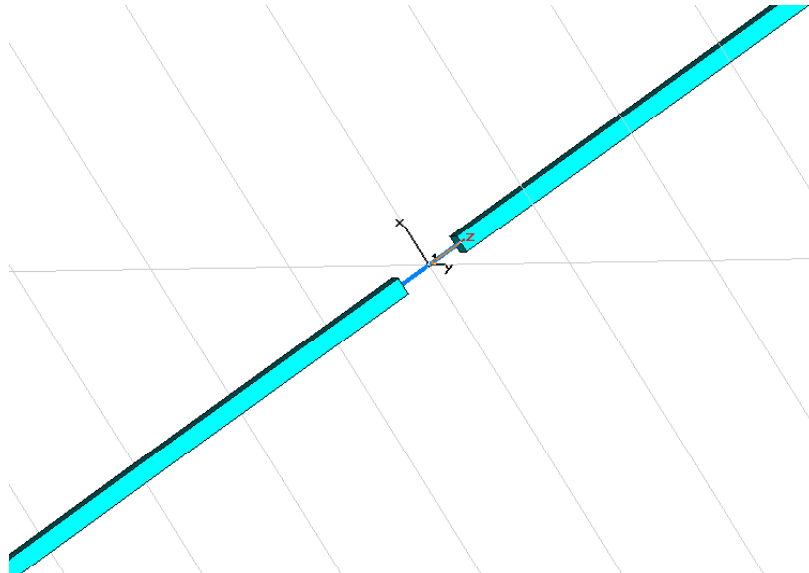


Figure 10: Square dipole model with a gap current source.

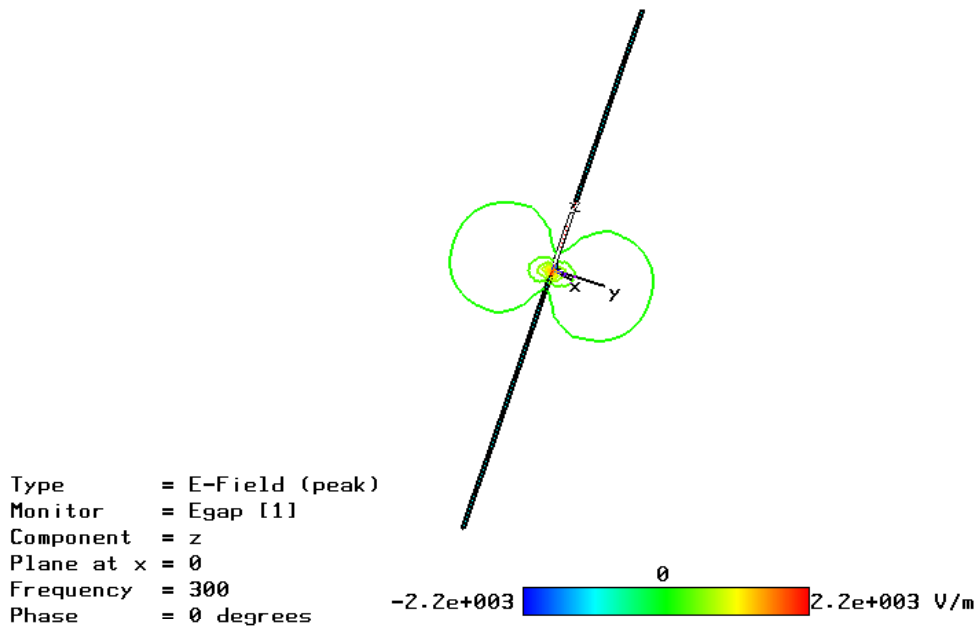


Figure 11: Contour plot of the field in the gap.

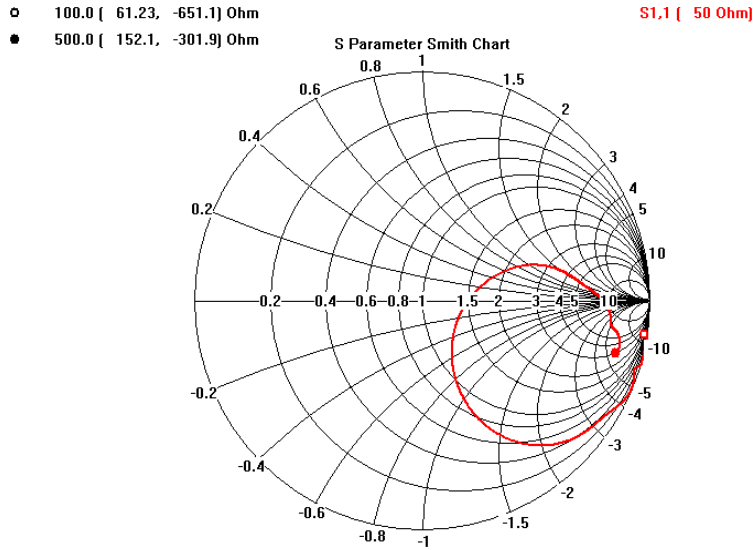


Figure 12: Dipole impedance computed from 100 MHz to 500 MHz.

The *Microwave Studio* array model is shown in Figure 13. The ground plane has a finite thickness of 0.01 m. The array patterns are shown in Figure 14 for three load conditions. A convergence level of -60 dB was requested for the solver, and the final number of meshnodes was about 800000.

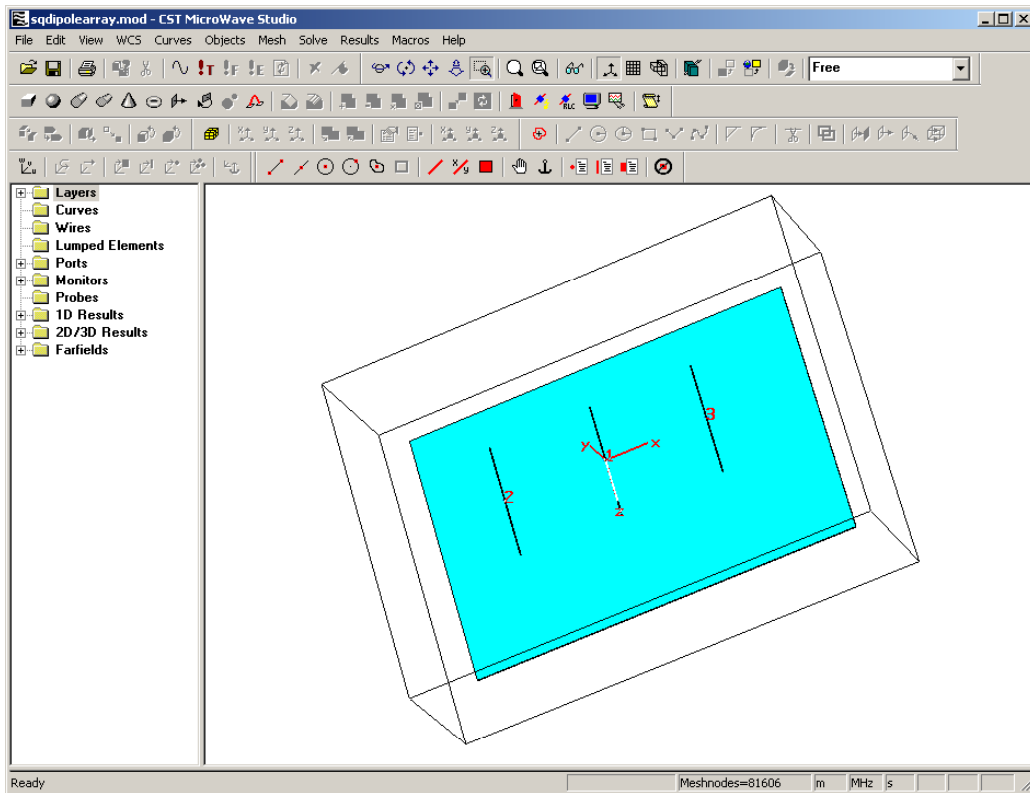


Figure 13: *Microwave Studio* model of the dipole array with ground plane showing the computational boundary.

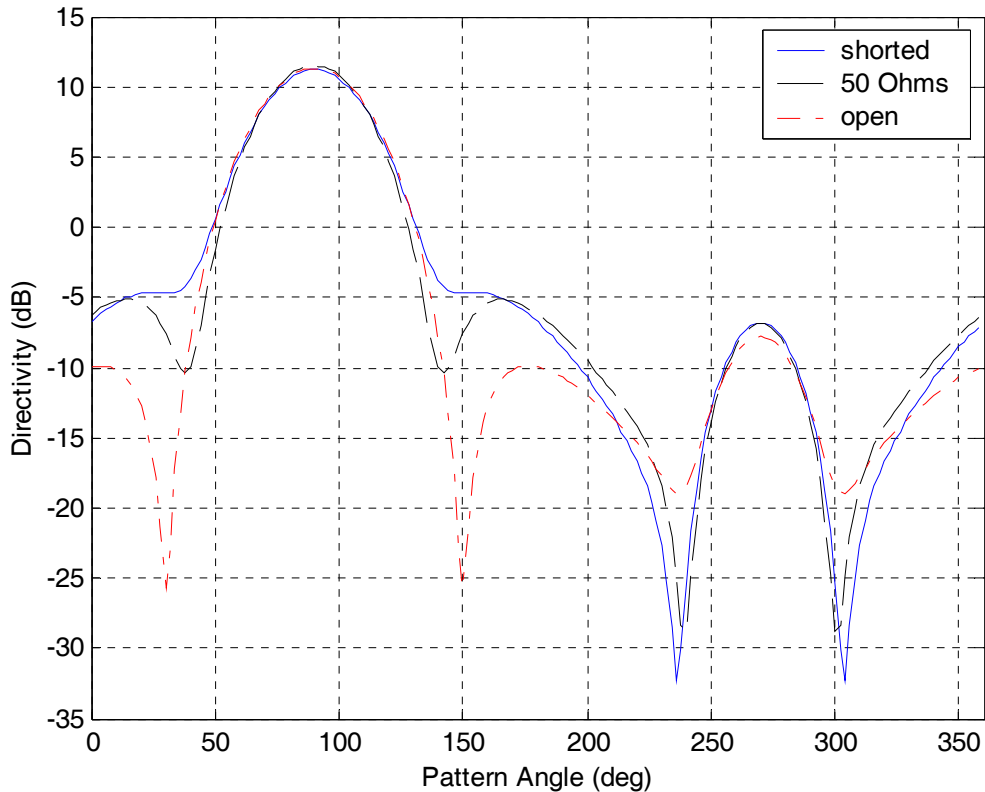


Figure 14: Comparison of directivity patterns for three load conditions.

D. Radar Cross Section

All three of the software packages are capable of solving antenna problems in the receive mode (plane wave incident) rather than the transmit mode. Also, electromagnetic scattering problems can be solved. Figure 15 shows the principal plane ($\theta = 90^\circ$) bistatic RCS of a 3 wavelength square plate for a normally incident a plane wave. For each method, the plate is handled in a manner similar to the array ground plane: for MM it is infinitely thin and for FEM and FIT the ground plane thickness is 0.01 wavelength. The approximate number of subdomains in each case are: 2700 edges (MM), 55,000 tetrahedra (FEM) and 216,000 meshnodes (FIT).

E. Summary and Comments

Three popular numerical methods were used to solve some simple but fundamental radiation and scattering problems. Figures 15 and 16 summarize the results obtained using the three software packages. As expected, the agreement between the results was excellent when the structures are similarly modeled and the solutions converged. A “brute force” approach was used in setting up the models; no effort was made to try and optimize the computational efficiencies of the solutions. For example, in all three cases symmetry planes could be introduced to reduce the number of subdomains. Finite thickness ground planes could be replaced by infinitely thin surfaces with a PEC boundary condition applied. Also, there was no attempt to minimize the number of subdomains for a converged solution. Converged results could probably be obtained

with fewer subdomains in all cases. For these relatively small computational problems, there was no significant difference in the run times. The array calculations were in the range of 20 minutes on a 1.5 GHz PC, and all of the methods were within a couple of minutes of each other.

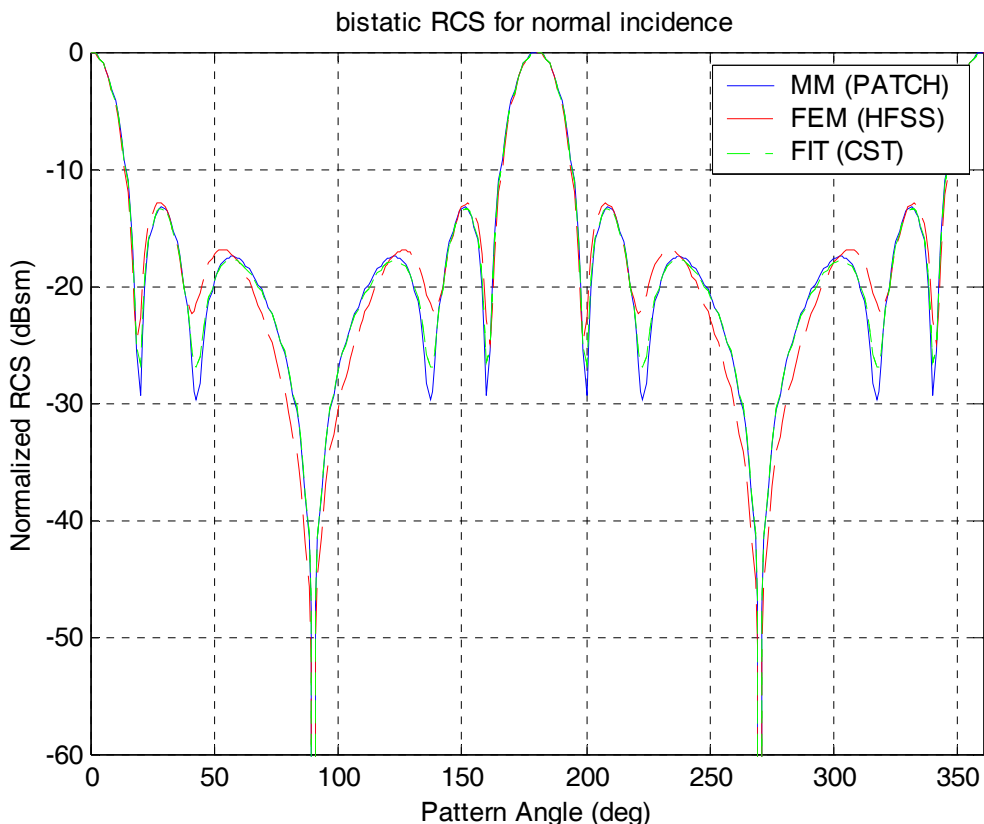


Figure 15: Bistatic RCS of a three-wavelength square plate.

References

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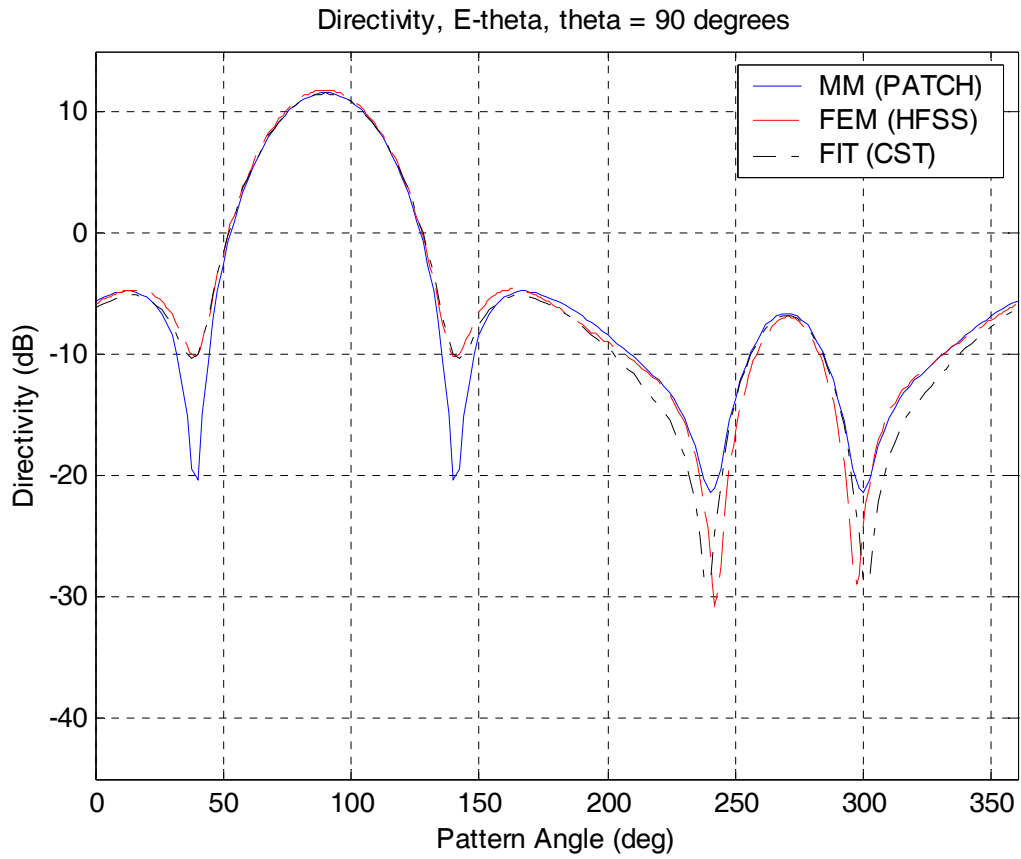


Figure 16: Overlay of the three radiation patterns for the dipole array, 50 ohm load.