# Design, Full-Wave Analysis, and Near-Field Diagnostics of Reflectarray Antennas

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***Abstract*** **─** This article presents a detailed procedure for design, modeling, full-wave simulation, and near-field diagnostics of reflectarray antennas, through a case study of a Ka-band reflectarray antenna with 437 elements. A comparison between analytical approaches and full-wave simulations for reflectarray radiation analysis is also presented, illustrating the necessity of full-wave approaches for an accurate analysis. Furthermore, the phase shift provided by the phasing elements, in the real reflectarray configuration, is analyzed by using the near-field data from the full-wave simulation. This provides a means to diagnose a designed reflectarray, and identify phasing elements that are not providing the required phase shift. The effectiveness of the diagnostic approach is demonstrated through numerical examples.

***Index Terms*** – Far-field, near-field, radiation pattern, and reflectarray.

## I. INTRODUCTION

Reflectarray antennas combine some of the best features of reflectors and array antennas, and create a hybrid design, which is well suited for high-gain applications [1-8]. The flat aperture of the reflectarray antenna consists of phase changing elements, which mimic the parabolic reflector curved surface, and create the collimated beam. In comparison with an array antenna, the feed network is replaced by a space feed, which is simple, and moreover eliminates the distribution losses associated with large arrays [1]. With the rapid advancement of printed circuit technology, reflectarray antennas can offer a low profile, low mass, and low cost solution for high-gain antennas in deep space communication systems.

Different approaches for analysis of reflectarray radiation pattern have been developed over the years [1-5]. These numerical approaches provide a fast method to compute the radiation pattern of the reflectarray antenna with a good accuracy; however, several approximations are made in the analysis. With these approaches usually a good agreement between measured and simulated results is observed in the pattern shape, but in most cases there are some discrepancies in side-lobe level and cross-polarization levels [5]. In general, an accurate analysis of a reflectarray antenna radiation performance requires a full-wave simulation; however, this is quite challenging. The electrically large size of the reflectarray antenna aperture, combined with hundreds of elements with dimensions smaller than a wavelength, demands an efficient full-wave technique. Considering the planar geometry of the reflectarray antenna aperture, a surface meshing approach will be more appropriate for this problem. As such, the method of moment (MoM) technique will be more advantageous than other full-wave techniques such as finite element or finite difference, which require volume meshing.

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In this paper we present a detailed procedure for design, modeling, and full-wave simulation of a reflectarray antenna through a case study of a 437-element Ka-band design. In addition, the near-field data is obtained by the full-wave simulation, and the performance of the phasing elements is studied in the real reflectarray configuration. This illustrative field visualization can serve as a useful diagnostic tool for antenna engineers to observe the performance of each element on the reflectarray aperture and potentially correct the phasing elements that are not providing the necessary phase shift.

## II. REFLECTARRAY ANTENNA SYSTEM DESIGN

Designing a reflectarray antenna is usually carried out in several stages, and as such, it can be viewed as a system design. The main stages in this process are: designing the aperture, feed positioning, phasing element design, and designing or selecting a feed antenna. The geometry of the reflectarray antenna system is given in Fig. 1.

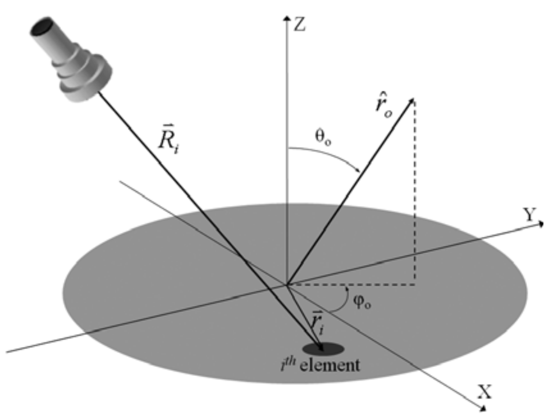



Fig. 1. Basic geometry of a reflectarray antenna system.

### A. Aperture design and feed positioning

In aperture type antennas such as reflectarrays, the antenna gain is proportional to the electrical size of the aperture [9]. Therefore, if the required gain is specified for the antenna, one only needs to determine the aperture size that can achieve such a gain level. Aperture efficiency however plays a major role here, and to achieve high aperture efficiency [2], several factors must be taken into account in the design. Here we will outline the design process for a 32 GHz Ka-band reflectarray for a 30 dB gain.

A circular geometry is selected for the reflectarray aperture, since it can attain higher aperture efficiency in comparison with rectangular geometries. The diameter of the aperture (D) is set to be 12.5 λ at the design frequency of 32 GHz, which has a maximum directivity of 31.88 dB. With this relatively small aperture, an offset feed would be necessary to avoid blockage effects, thus a tilt angle of 25° is selected for the feed. The next task is to determine the feed position. In reflector antennas this is usually specified by the F/D ratio. One generally has two options for the design. If a specific feed is to be used, one has to determine the optimum F/D. On the other hand, one can choose the F/D, and design a feed antenna that achieves the optimum radiation performance. This is quite advantageous, since increasing F/D improves several characteristics of the reflectarray. In any case, the optimum values are determined based on efficiency analysis [10]. Here we chose the latter approach. An F/D of 0.75 was selected for the system, and the task was to determine the optimum feed radiation pattern that achieves maximum aperture efficiency.

The radiation pattern of the feed is usually modeled as a $\boldsymbol{cos^q}(\theta)$ function with no azimuth dependence, therefore one only needs to determine the optimum value of $\boldsymbol{q}$ [11]. The optimum value of $\boldsymbol{q}$ was determined to be 6.5, which corresponds to an aperture efficiency of 74.14 %. These results are given in Fig. 2. Here $\eta_i$, $\eta_s$, and $\eta_a$, are the illumination, spillover, and aperture efficiencies, respectively.

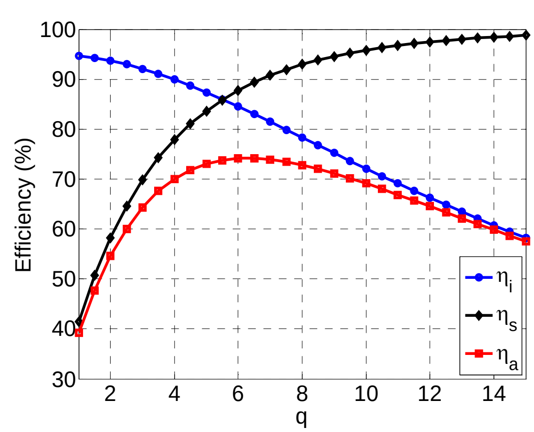



Fig. 2. Aperture efficiency of the reflectarray antenna as a function of feed cosine q power.

As discussed earlier, an offset system is used for the feed here. To minimize beam squint in offset reflectarrays, the beam direction should be equal to the offset feed angle, thus the beam direction is set to 25° [12]. This projected aperture size reduction (cos 25°) corresponds to a loss of 0.43 dB. Combined with the losses due to aperture efficiency (74.14 % or 1.3 dB), the estimated gain of the aperture designed here is 30.15 dB, which is quite close to the desired value.

### B. Phasing element design

The phase shift distribution on a reflectarray aperture is designed to compensate for the spatial delay associated with the feed antenna, and provide a progressive phase shift on the aperture that points the collimated beam at a specific direction. Mathematically, the required phase shift of the $i^{th}$ element on the aperture in Fig. 1 is given by,

$$\psi_i = k(R_i - \bar{r}_i \cdot \hat{r}_o) + \psi_0 . \qquad (1)$$

The phase constant $\psi_0$ indicates that a relative phase rather than an absolute phase is required in reflectarray design. The key concept however is how these individual elements are designed to scatter electromagnetic waves with the desired phases. Different methods have been demonstrated over the years to control the reflection phase of the elements, which can be categorized into three general groups: 1) phase/time delay lines, 2) variable size elements, and 3) rotated elements. For our design here we use the variable size technique, where a small shift in the resonant frequency of an element is introduced by changing the dimension that has the effect of changing the phase of the reflected field.

The reflectarray aperture is excited with a feed antenna that in general, may be positioned at an arbitrary angle and distance from the reflectarray, but is assumed to be far enough so that the incident field can be approximated by a plane wave. As such, the conventional approach for analysis and design of reflectarray phasing elements is to use a plane wave excitation. The infinite-array approach is often used for analysis of reflectarray elements, which takes the mutual coupling between the elements into account by means of the periodic boundary conditions [13]. While it is implicit that some approximation is made in this element analysis approach, i.e., ignoring the quasi-periodic nature of the reflectarray elements, as long as the variation of the element geometry is not significant between adjacent elements, this is quite acceptable in most cases [1, 2].

The phasing elements used in our design are variable size square patches, and the unit-cell size is 4.7 × 4.7 mm$^2$. The substrate has a thickness of 0.508 mm, and the dielectric constant is 2.2. The patch sizes vary from 1 mm to 4.5 mm with a resolution of 0.1 mm. The reflection phase responses of the elements are obtained using the commercial software Ansoft Designer [14], which is based on the MoM. Note that for reflectarray element designs, the reference plane for reflection phase calculation is the top surface of the element. It is worthwhile to point out that while full-wave simulation of the elements is usually very fast for simple geometries such as the single-layer patch element designed here, if necessary one may limit the number of simulated element dimensions and use a fitted curve for the element design. The reflection phase response of variable size elements usually forms an S-curve; therefore it can also be approximated with an inverse tangent function. The function used for the interpolation is

$$\psi = -b_l \frac{180}{\pi} \tan^{-1}\left(a_l (L - L_0)\right) + c_l . \qquad (2)$$

For this design $a_l$ = 4.814, $b_l$ = 1.868, $c_l$ = -14.72, and $L_0$ = 2.722, where these parameters were evaluated using the curve fitting toolbox in Matlab [15].

Both simulated and interpolated reflection phase response of the elements, under normal incidence excitation, are given in Fig. 3. Also the simulated reflection phase response for 30° oblique excitation is given for comparison, where it can be seen that a normal incidence approximation is quite acceptable for this angular range.

### C. Feed antenna design

Based on the efficiency analysis presented earlier, a feed antenna with an azimuthally symmetric radiation pattern and $\boldsymbol{q}$ = 6.5 is required for this reflectarray system. Typically horn antennas are used as reflector feeds [11]; however, the conventional pyramidal and conical horn antennas cannot achieve a symmetric radiation performance. One of the most fundamental methods to achieve a symmetric radiation pattern with a horn antenna is to excite higher-order modes in the horn waveguide that is done by introducing a step change into the diameter of the horn. Therefore, the Potter horn antenna [16] was selected as the feed, where the waveguide steps generate the $TM_{11}$ mode. Combined with the fundamental $TE_{11}$ mode, this will generate azimuthally symmetric fields at the horn aperture, which in turn corresponds to azimuthally symmetric radiation patterns for the horn. A cross sectional geometry of the Potter horn antenna is given in Fig. 4.

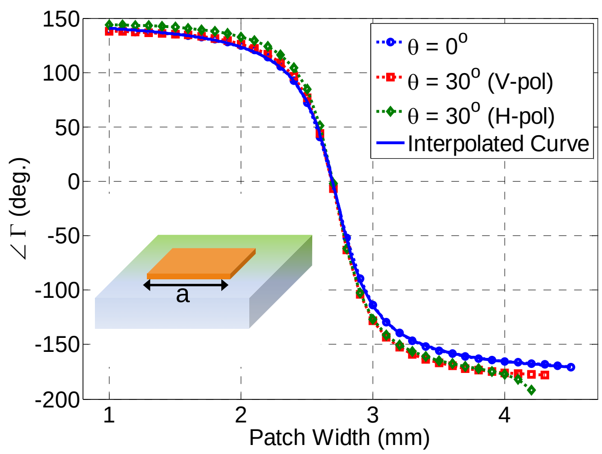



Fig. 3. Reflection phase versus patch size for the reflectarray elements.

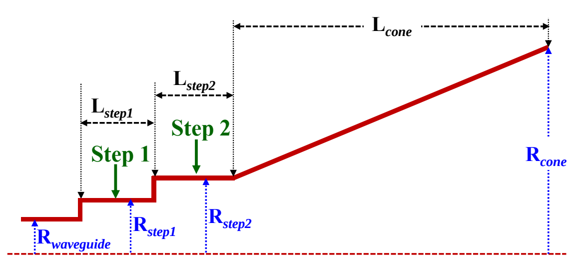



Fig. 4. Cross sectional view of a Potter horn antenna.

To achieve a symmetric radiation pattern with a Potter horn antenna, one has to determine the required portion of power that is transferred to the $TM_{11}$ mode, thus the conventional design approach is to use a mode matching technique [17]. While this approach will ensure a symmetric radiation pattern, it will generally not allow one to control the shape of the pattern. On the other hand, several design parameters are available in a Potter horn antenna; therefore another design approach is to tune these parameters to achieve the desired radiation performance. In our design, full-wave simulation of the horn antenna was done using FEKO [18], and the particle swarm optimizer [19] was used to tune the parameters, and achieve the desired pattern. In total, 7 parameters were optimized for this design. These are the radius of the waveguide feed ($R_{waveguide}$), the radius and length of two waveguide steps ($R_{step1}$, $L_{step1}$, $R_{step2}$, $L_{step2}$), and the radius and length of the cone ($R_{cone}$, $L_{cone}$). At each fitness evaluation during the optimization, the radiation pattern is computed at a number of discrete points chosen to match the required $\cos^q(\theta)$ pattern and achieve a symmetric pattern in both *E*- and *H*- planes. The radiation pattern of the optimized horn antenna is given in Fig. 5, while the optimized dimensions of the Potter horn are given in Table 1 at 32 GHz.

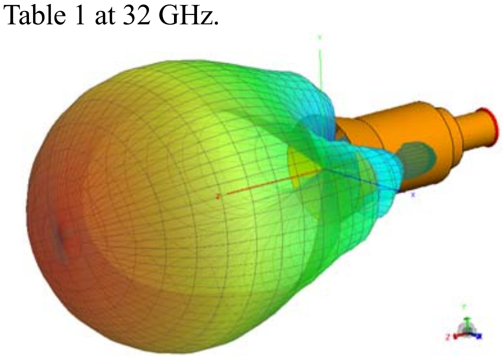


Fig. 5. Radiation pattern of the Potter horn antenna.

Table 1: Optimized dimensions of the Potter horn antenna (λ = 9.37 mm).

| **Parameter** | **Dimension (λ)** |
|---|---|
| $R_{waveguide}$ | 0.323 |
| $R_{step1}$ | 0.571 |
| $L_{step1}$ | 0.386 |
| $R_{step2}$ | 0.763 |
| $L_{step2}$ | 1.539 |
| $R_{cone}$ | 1.009 |
| $L_{cone}$ | 0.848 |

As discussed earlier, the important consideration in our design here was not only to achieve a symmetric pattern, but also to match it with the ideal $\cos^{6.5}(\theta)$ model. A comparison between the ideal model and the optimized horn pattern is given in Fig. 6, where it can be seen that the optimized horn antenna completely matches the required pattern up to 40º, which is sufficient for the excitation of the reflectarray antenna.

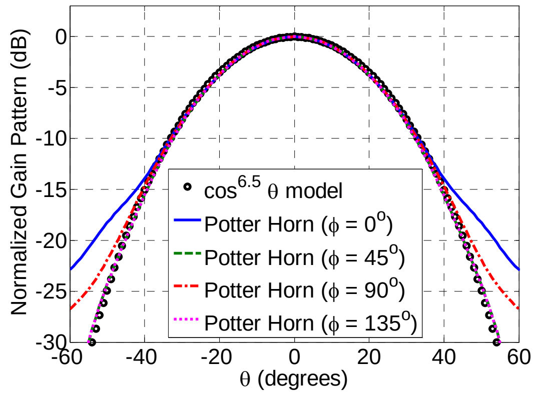


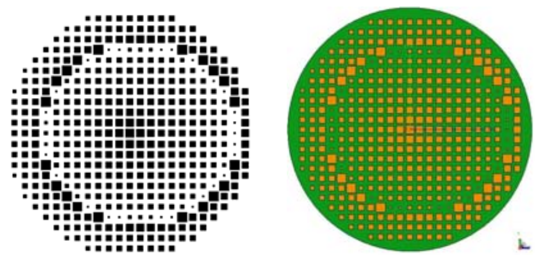

Fig. 6. Comparison between the radiation pattern of an ideal feed and the Potter horn antenna.

## III. FULL-WAVE SIMULATION OF THE REFLECTARRAY ANTENNA

### A. Full-wave simulation using FEKO

The next stage in the design is modeling the entire reflectarray antenna system. As discussed earlier, the electrically large size of the reflectarray antenna aperture, combined with hundreds of elements with electrically small dimensions, makes the full-wave simulation a challenging task. Here we use the commercial software FEKO v.6.1 for full-wave simulation of the reflectarray antenna.

The Ka-band reflectarray antenna has a circular aperture with a diameter of 12.5 λ at the design frequency of 32 GHz. The feed horn is positioned at $X_{feed}$ = -37.1 mm, $Y_{feed}$ = 0 mm, $Z_{feed}$ = 79.6 mm based on the coordinate system in Fig. 1, and is pointing toward the geometrical center of the array. The patch dimensions are designed to generate a beam in the direction of $(\theta, \varphi)$ = (25°, 0°). In total 437 square patch elements are to be placed on the aperture. Modeling the feed horn, substrate layer, and ground plane is straight-forward; however modeling the patches requires further attention. Considering the large number of variable size patches in a reflectarray antenna, it would be efficient if one imports the patch elements with a geometry file, which contains the location and dimension of each patch. For this design, a dxf file was created using Matlab© and imported into FEKO. The mask of the reflectarray phasing elements, and the geometry of the reflectarray system modeled in FEKO are shown in Fig. 7. For this design 434,450 unknown basis functions need to be calculated by the FEKO method of moments (MoM) solver. Considering the large number of unknowns, the multilevel fast multi-pole method (MLFMM) solver in FEKO was selected for this simulation. In total, the full-wave simulation here required 22.25 GB of memory with a CPU time of 19.94 hours on an 8 core 2.66 GHz Intel(R) Xeon(R) E5430 computer. The simulated radiation pattern of the reflectarray antenna is shown in Fig. 8.



(a) (b)

Fig. 7. (a) Mask of the reflectarray phasing elements and (b) model of the reflectarray antenna in FEKO.

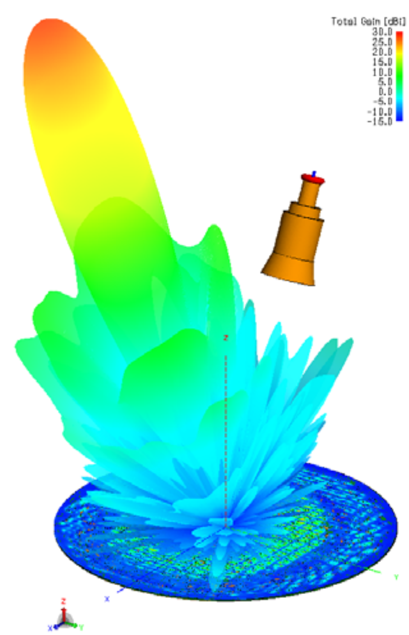



Fig. 8. Simulated radiation pattern of the 437 element Ka-band reflectarray antenna.

The main beam is correctly scanned to 25° off-broadside, which indicates that the phasing elements are providing the necessary phase shift on the reflectarray aperture. The computed maximum gain of the reflectarray antenna is 28.22 dB, which corresponds to an aperture efficiency of 43.05 %. The side-lobe and cross-polarization levels are -23.5 dB and -28.6 dB, respectively. It should be pointed out that the full-wave simulation takes into account all approximations in reflectarray element design as well as the edge diffraction effects. Therefore the full-wave simulation here can provide a good measure to observe the performance of the reflectarray elements in the real reflectarray application environment.

**B. Comparison with analytical solution**

Different analytical approaches are available to calculate the radiation characteristics of the reflectarray antennas. However in general, several approximations are made in these methods. Therefore, some discrepancies are usually observed in comparison with full-wave or measured results. This is primarily due to element design approximations, mutual coupling, and edge diffraction effects, which are not taken into account in these methods. Furthermore, for smaller reflectarrays the approximations in the analysis are less accurate, and as such they show a larger discrepancy with the measured results. Nonetheless, the analytical approaches can serve as a fast and comparable method to verify the full-wave simulation results presented in the previous section.

For the study here we used the aperture field approach [4], where the radiation pattern of the reflectarray antenna is calculated using the tangential fields on the reflectarray aperture. To compare the radiation patterns of the two approaches, we study the radiation patterns in the 2-D planes that best capture the radiation features of the antenna, i.e., the principal planes (P.P.), which are defined according to [20]. For this configuration, P.P.1 is the *xz*-plane and P.P.2 is the *yz′*-plane in the *xyz′*-coordinate system. This *xyz′*-coordinate system is obtained by rotating the *xyz*-coordinate system, 25° about the *y*-axis. The radiation patterns in the principal planes are shown in Fig. 9. It can be seen that a good agreement in the co-polarized radiation pattern shape is observed between the analytical and full-wave approach. In addition, the main beam direction, beam-width, side-lobe, and cross-polarization level in the main beam area show a close agreement. The discrepancies however, which are mainly observed outside the main beam areas, are due to the approximations in the aperture field approach. This study also reveals that analytical approaches have limited accuracy, and when accurate radiation pattern computation in the entire 3-D space is required, full-wave simulation of the reflectarray system is necessary.

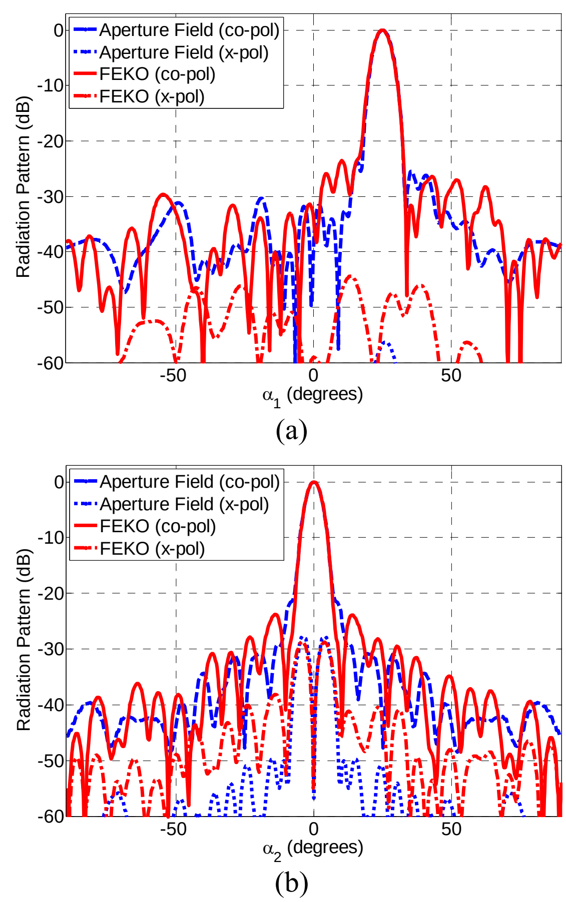


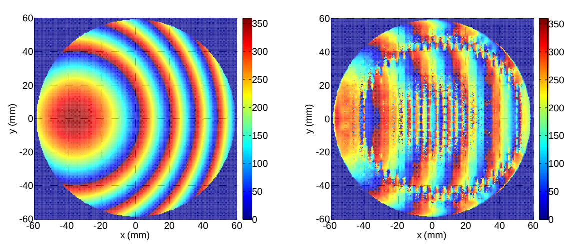

Fig. 9. Radiation pattern of the reflectarray antenna: (a) P.P.1 and (b) P.P.2.

It is worthwhile to point that the aim of the study here was to analyze the accuracy of the reflectarray radiation pattern computed using analytical approaches, thus comparison with full-wave simulations is more advantageous than measurements, since measurement results are susceptible to both fabrication and measurement errors. However, it is worth mentioning that our analytical approach has been confirmed by successful fabrication and experimental verification of several reflectarrays, as demonstrated in [20, 21].

## IV. NEAR-FIELD ANALYSIS AND ELEMENT DIAGNOSTICS

### A. Phase shift of reflectarray elements

A detailed procedure and a successful design of a Ka-band reflectarray antenna were presented in the previous sections. One of the notable advantages of a full-wave simulation is that in addition to calculating the far-field radiation performance, it can also provide the electromagnetic field quantities in the near field of the reflectarray system. For a reflectarray antenna this can be quite advantageous, since as discussed in section II, several approximations are made in the element design and in general, performance evaluation of the elements would require a near-field analysis. It should be noted that the near-field study here requires analysis of the scattered field. Therefore in addition to the full-wave simulation in section III, which provides the total field, one must also perform another simulation for the feed antenna alone, to obtain the incident fields. The phase of the incident and total electric fields on the reflectarray aperture are given in Fig. 10. It should be noted that since the feed horn antenna is *x*-polarized, the phase is only given for the $\boldsymbol{E}_x$ component.




(a) (b)

Fig. 10. Phase of the electric field on the aperture: (a) incident and (b) total.

The aim of this study is to observe the phase shift provided by the phasing elements of the reflectarray antenna. From the total and incident fields, the scattered fields are calculated using

$$\vec{E}_{scattered} = \vec{E}_{total} - \vec{E}_{incident} \,. \tag{3}$$

The phase of the scattered electric fields (for the $\boldsymbol{E}_x$ component) on the reflectarray aperture is given in Fig. 11. The phase shift provided by the reflectarray elements can then be calculated as

$$\text{Element Phase Shift} = \angle\vec{E}_{scattered} - \angle\vec{E}_{incident} \,. \tag{4}$$

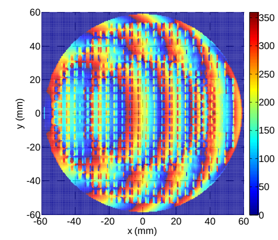
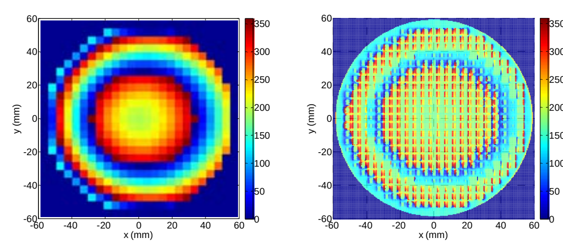



Fig. 11. Phase of the scattered electric field on the aperture.

This provides a useful visualization to determine if the phasing elements are designed correctly and are providing the necessary phase shift. Comparison between the elements ideal phase shift and the quantized phase shift resulted from the full-wave simulation of the reflectarray is given in Fig. 12. These results clearly indicate that for this design, the phasing elements on the reflectarray aperture are generating a phase shift that creates the collimated beam. In terms of performance diagnostics, if any errors are made in the phasing element design and placement, this study will be able to detect the elements that are not providing the necessary phase shift.




(a) (b)

Fig. 12. Phase shift on the reflectarray aperture: (a) ideal phase shift and (b) phase shift obtained from full-wave simulation.

### B. Random phase error and diagnostic

To better illustrate this near-field diagnostic technique, we also designed a reflectarray antenna with randomly distributed patch dimension error in a small segment on the aperture. The error, with a maximum of 1 mm, is randomly distributed in a circle at the top right quadrant of the array. It is worthwhile to note here that in practice these errors could be due to fabrication, wear, or even design errors. The mask of the reflectarray phasing elements, and the phase shift on the aperture are shown in Fig. 13.

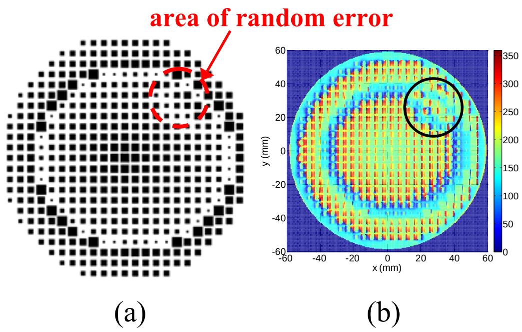



Fig. 13. (a) Mask of the reflectarray antenna with randomly distributed error and (b) phase shift on the aperture of the reflectarray antenna based on full-wave simulation.

It can be seen that, while it is quite difficult to observe these small dimensional errors in Fig. 13 (a), the computed phase shift on the aperture can clearly show the elements that are not providing the necessary phase shift, thus illustrating the advantages of full-wave near-field diagnostics proposed in this paper. It is worthwhile to point out that while in the study here only 30 elements, i.e., less than 7 % of the total number of elements, exhibit phase error, the radiation performance also shows notable degradation. A comparison between the radiation patterns of both designs is given in Fig. 14.

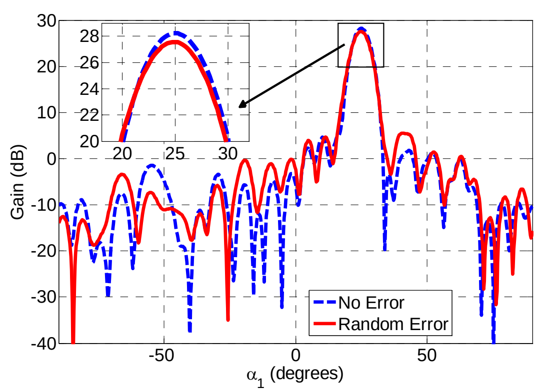



Fig. 14. Radiation pattern of the reflectarray antenna.

It can be seen that in addition to some beam deterioration, the maximum gain of the antenna has also been reduced from 28.22 dB to 27.56 dB. Furthermore, while the gain reduction here is about 0.66 dB, if such a phase error shall be observed in a strongly illuminated region, i.e., the geometrical center of the array, it would correspond to a greater loss.

## V. CONCLUSION

A detailed procedure for design, modeling, and full-wave simulation of reflectarray antennas was presented. This was demonstrated through a case study of a 437-element Ka-band reflectarray antenna with comprehensive details on aperture, phasing element, and feed antenna design. In addition, comparison between analytical solution and full-wave simulation is also presented, which clearly illustrates the necessity of full-wave approaches for an accurate radiation analysis of reflectarray antennas. Moreover, the performance of each individual element of the reflectarray antenna was obtained in the real reflectarray configuration through the computed near-field. This is a useful tool for diagnosing and providing corrections when defected elements are observed.

## ACKNOWLEDGMENT

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