Numerical Analysis of Impedance of Asymmetric TEM Cell Filled With Inhomogeneous, Isotropic Dielectric

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Abstract–**This paper investigates the effect of vertically offset septum on characteristic impedance (Z_0) of transverse electromagnetic (TEM) cell. The Septum is considered to be of finite thickness for the analysis. Impedance analysis is done initially for symmetric TEM cell with homogenous dielectric using Finite Element Method (FEM) and the numerical results included are compared with results obtained by other authors. A good agreement is established. Numerical analysis using FEM is also done for Z_0 of Asymmetric TEM cell with septum of finite thickness filled with inhomogeneous dielectric. The effect of inhomogenity and offset of septum on Z_0 is discussed. Variation of Z_0 with width of the septum is represented graphically.**

Key Words: TEM cell, FEM, effective dielectric constant

1. INTRODUCTION

The TEM cell is similar in structure to a rectangular coaxial transmission line (RCTL), except for the fact that width of the inner conductor of TEM cell is comparatively larger (Figure 1). These cells are used in the generation of standard electromagnetic fields and to study the effects of electromagnetic radiation on biological objects and electronic systems [1].

The common configuration of TEM cell is, rectangular shaped metallic shielded outer conductors and a flat septum with air dielectric inside the cell. The septum may be located arbitrarily inside the enclosure, parallel to the top and bottom outer conductors. Both the inner and outer conductors are tapered at the ends for impedance matching with the standard 50 ohms coaxial connectors. The central section of the Cell is termed as uniform cross section that propagates uniform TEM fields. A plane wave field environment inside the cell is simulated for electromagnetic interference (EMI) testing of equipments placed between septum and outer conductor.

Some of the problems concerning the design of such cells are the maximization of

- Usable test space.
- Transmitted power through the cells.

Optimum dimensions of these cells are to be determined to solve the above problems specified.

Offseting the inner conductor from the center in vertical direction creates two chambers of unequal sizes that allow the testing of both larger equipment and smaller probes without increasing the overall size of the cell.

Some of the asymmetric configurations are available commercially. GTEM cell is a class of asymmetric cell that can be used for the Electromagnetic Susceptibility measurements in the Ghz range. Rohde and Schwarz have fabricated S-Line that is used in the measurement of Electromagnetic interference and Susceptibility. Its operating range spans from 150 KHz to GHz. It offers test volume comparable to that of anechoic chambers. This S line is characterized by high field strength and field uniformity. These lines have a compact design.

Another type of compact test cell which Rohde and Schwarz presents is an M-line which can be used in the frequency range of 800 MHz to 40 GHz which can be used for measurements in the RF and microwave range.

The TEM cell and the tapered transitions should have Z_0 that matches with the coaxial connectors connected at the end. This ensures minimum standing wave ratio (SWR) and hence maximum power transfer to the cell from the feeder line.

Therefore, for accurate design of such cells and to interpret RF measurements made, Z_0 has to be determined considering finite thickness of septum for its different vertical positions in the cross section.

Rectangular Coaxial Transmission Line was analyzed by [5]-[8] for symmetrically placed inner conductor of finite thickness. A set of design curves for Z_0 of RCTL was given by [6], [7]. Guckel [9] used conformal mapping to find Z_0 of rectangular transmission lines of asymmetrically placed inner conductor with air dielectric inside. Pantic and Mittra [10] performed Quasi TEM analysis on thick transmission lines with multidielectric layers to find Z_0 neglecting the influence of sidewalls. However in TEM cells the effect of sidewalls on Z_0 cannot be neglected, as it influences the RF measurements made inside the cell.

In the light of above observations this paper analyses the effect of finite thickness septum placed in different vertical off set positions, on the characteristic impedance (Z_0) of the TEM cell. In this analysis, inhomogeneous dielectric in the cell is considered. The region above the septum is considered to have air dielectric ($\varepsilon_{r1} = 1$) and the region below the septum is considered to have a dielectric with ($\varepsilon = \varepsilon_{r2}\varepsilon_0$) (Figure 1). The dielectric placed below the septum lends a good mechanical support to the EUT placed inside for RF measurements. The effect of this inhomogenity on the characteristic impedance is also studied. The analysis helps in the design of such cells and to interpret the RF measurements made inside the cell.

Classical method of approaches may fail if the medium is inhomogeneous or anisotropic [12]. So Finite Element Method (FEM) is used to solve the laplace equation to obtain the potential distribution inside the cell. Then Gauss's law is applied to find the charge around the septum, which, in turn, is used to find the capacitance and hence the impedance of the cell.

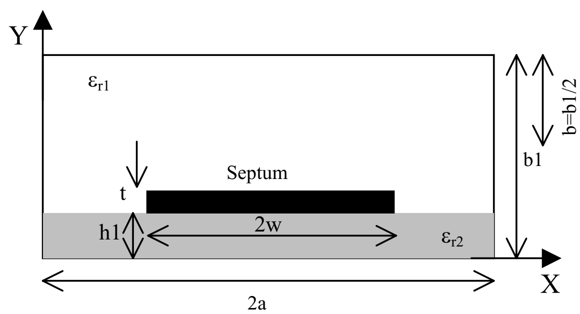


Figure 1. Cross section of asymmetric TEM cell with inhomogeneous dielectric.

2. FEM FORMULATION

To find the potential distribution, Laplace equation

$$\nabla^2 V = 0 \tag{1}$$

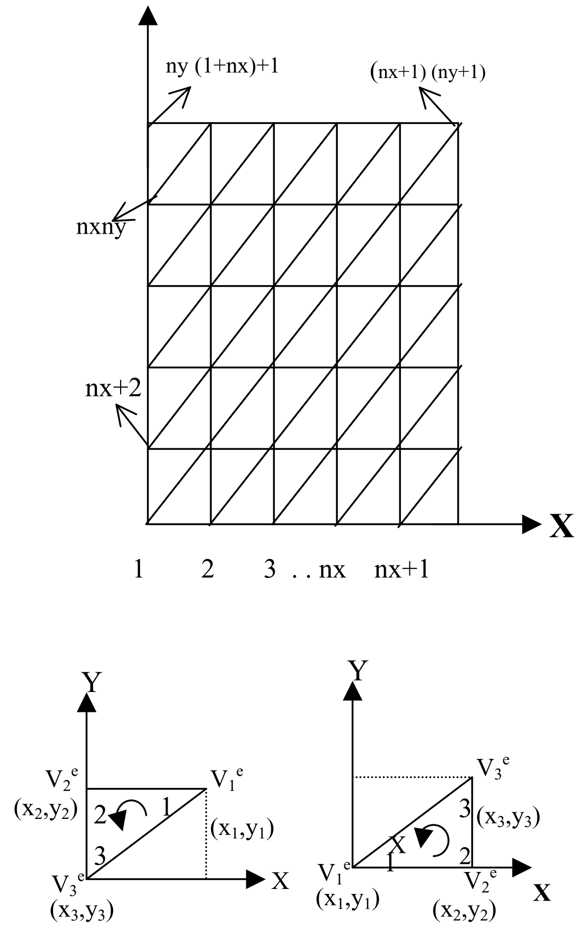
is solved, where V is the potential at (x,y) (Figure 1). The boundary conditions are

$$\left.\begin{array}{l} V(x,0) = 0 = V(x,b1)\ ;\ \text{for } 0 \le x \le 2a \\ V(0,y) = 0 = V(2a,y)\ ;\ \text{for } 0 \le y \le b1 \\ \text{and } V = V_0 \text{ at } \left\{\begin{array}{l} D - W \le x \le D + W \text{ at } y = h1 \\ \text{for zero thickness septum.} \end{array}\right. \\ \qquad\qquad \left.\begin{array}{l} D - W \le x \le D + W \text{ at } h1 \\ h1 - t/2 \le y \le h1 + t/2 \end{array}\right\} \end{array}\right\} \tag{2}$$

for septum of finite thickness.

2.1 Discretisation

Due to uniaxial symmetry of the domain, only one half of the cross section ($0 \le x \le a$, $0 \le y \le b1$) of the TEM cell is considered for the analysis. The domain is subdivided into a set of triangular subdomains called finite elements as shown in Figure 2, with local nodes (1, 2,3) and global node numbering 1,2,3 … (nx + 1), (nx + 1)(ny + 1).



Typical triangular elements: 1,2,3 are local node numbers.

Figure 2. Finite Element Mesh.

2.2 Element Equation

The approximate solution to equation (1) within a typical finite element in the domain is of the form

$$V_{(x,y)} = \sum_{j=1}^{n} V_j^e \, \tau_j^e \, (x,y) \tag{3}$$

where V_j^e is the value of the potential at the jth node (x_j, y_j) of the element and τ_j^e are the lagrange interpolation functions [2]. The polynomial approximation for V within an element is chosen as

$$V^e_{(x,y)} = g_1 + g_2 x + g_3 y\ , \tag{4}$$

where g_1, g_2, g_3 are constants pertaining to local nodes (Figure 2) equation (4) contains three linearly independent terms and it is linear in both x and y considering a typical triangular element the potential V_1^e, V_2^e, V_3^e are obtained using equation (4) as

$$\begin{bmatrix} V_1^e \\ V_2^e \\ V_3^e \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix}. \quad (5)$$

The coefficients g_1, g_2, and g_3 are determined from (5) and substituted in (4) to obtain,

$$V^e = \sum_{j=1}^{3} V_j^e \tau_j^e (x, y) \quad (6)$$

where

$$\tau_j^e = \frac{1}{2A^e} \left(\alpha_i^e + \beta_i^e x + \gamma_i^e y\right) (i = 1, 2, 3), \quad (7)$$

α_i, β_i and γ_i are geometric constants given by

$$\alpha_i = x_j y_k - x_k y_j$$
$$\beta_i = y_j - y_k \quad (8)$$
$$\gamma_I = -(x_j - x_k)$$

$i \neq j \neq k$ and i, j, and k permute in natural order, and A^e is the area of the triangle [2] given by

$$A^e = \frac{\alpha_1 + \alpha_2 + \alpha_3}{2}. \quad (9)$$

The value of A is positive if the nodes are numbered counter clockwise [3]. Equation (6) gives the potential at any point (x, y) within the element. The calculus of variations, an extension of ordinary calculus, is concerned with the theory of maxima and minima [3]. In this problem we are concerned with seeking the minima of an integral expression involving a function of functions or functionals. Moreover we are interested in the necessary condition for a functional to achieve a stationary value. This necessary condition on the functional is generally in the form of a differential equation with boundary conditions on the required function.

In the problem considered, the minimum potential energy requires the potential distribution, which will minimize the stored field energy per unit length. Minimization of the energy then determines the coefficients and thereby implicitly determines an approximation to the potential distribution.

The functional corresponding to Laplace equation is [4]

$$F_e = \frac{1}{2} \int \varepsilon |E|^2 \, dS = \frac{1}{2} \int \varepsilon | \nabla V^e |^2 \, dS \quad (10)$$

where Fe is the energy / unit length associated with the element e; E is the electric field strength over the surface dS; ε is the permittivity of the medium.

From (6), $$\nabla V^e = \sum_{i=1}^{3} V_i^e \nabla T_i \quad (11)$$

Substitute equation (11) in (10)

$$F_e = \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \varepsilon V_i^e \left[\int \nabla \tau_i \cdot \nabla \tau_j . dS\right] V_j^e . \quad (12)$$

The term in the brackets of (12) is defined as

$$K_{ij}^e = \int \nabla \tau_i . \nabla \tau_j \, dS . \quad (13)$$

Writing (12) in Matrix form

$$F_e = \frac{1}{2} \varepsilon \left[V^e\right]^t \left[K^e\right]\left[V^e\right] \quad (14)$$

where the subscript t denotes the transpose of the matrix,

$$\left[V^e\right] = \begin{bmatrix} V_1^e \\ V_2^e \\ V_3^e \end{bmatrix}, \quad (15)$$

and $$\left[K^e\right] = \begin{bmatrix} K_{11}^e & K_{12}^e & K_{13}^e \\ K_{21}^e & K_{22}^e & K_{23}^e \\ K_{31}^e & K_{32}^e & K_{33}^e \end{bmatrix} \quad (16)$$

is the element coefficient matrix.

The matrix element K_{ij}^e of the coefficient matrix may be regarded as the coupling between nodes i and j. Its value is obtained from equations (7) and (13).

2.3 Assembling of All Elements

Having considered a typical element, all such elements in the solution region are assembled. The energy associated with the assemblage of elements is

$$F = \sum_{e=1}^{N} F_e = \frac{1}{2} \varepsilon [V]^t [K] [V], \quad (17)$$

where $$[V] = \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix}, \quad (18)$$

and where N is the number of elements and n is the number of nodes and [K] is called global coefficient matrix [2] with the following properties:

1) It is symmetric.
2) $K_{ij} = 0$ if no coupling exists between nodes i and j, so [K] is sparse.
3) It is singular.

3. SOLVING THE RESULTING EQUATIONS

Laplace equation is satisfied when the total energy in the solution region is minimum i.e.,

$$\frac{\partial F}{\partial V_1} = \frac{\partial F}{\partial V_2} = \ldots\ldots\ldots = \frac{\partial F}{\partial V_n} = 0 \quad (19)$$

$$\text{or} \quad \frac{\partial F}{\partial V_k} = 0 \quad K = 1, 2, ... n . \tag{20}$$

In general $\frac{\partial F}{\partial V_k} = 0$ leads to $\sum_{k,i=1}^{n} V_i \; K_{ik} = 0$, (21)

where n is the number of nodes in the mesh. Writing equation (21) for all nodes we obtain a set of simultaneous equations from which the solution of $[V]_t = [V_1, V_2, \ldots, V_n]$ can be found by the method of iteration. As node m is a mesh with n nodes

$$V_m = -\frac{1}{K_{mm}} \sum_{i=1, i\neq m}^{n} V_i \; K_{mi} \quad , \tag{22}$$

where node m is a free node. Since $K_{mi} = 0$ if node m is not directly connected to node i. Only nodes that are directly linked to node m contribute to V_m in equation (22) can be iteratively applied to all the free nodes where the value of the potential is to be found. The global coefficient matrix was obtained with the help of computer implementation using MATLAB package.

4. THE CHARACTERISTIC IMPEDANCE Zo

The characteristic impedance Z_0 is obtained as follows

$$Z_0 = 1/uC \tag{23}$$

$$\text{where} \quad u = u_o \sqrt{\frac{Co}{C}} = \frac{u_o}{\sqrt{\varepsilon_{eff}}} \tag{24}$$

$$\varepsilon_{eff} = C/Co \quad , \tag{25}$$

where u_0 is speed of light in free space, ε_{eff} is the effective dielectric constant, C = Capacitance of the cell with the dielectric of relative permittivity ε_{r2} below the septum, and Co the capacitance of the cell without the dielectric below the septum (i.e) ($\varepsilon_{r2} = \varepsilon_0$). Thus to find Z_0 for an inhomogeneous medium requires calculation of capacitance / unit length of the structure with and without the dielectric substrate below the septum. If Vo is the potential difference between the inner and the outer conductors, then,

$$C = 2Q/Vo \tag{26}$$

where Q is the charge / unit length. The factor 2 is included since only half of the domain was considered for the analysis. To find Q, Gauss's law is applied [4] to the closed path J enclosing the septum, as shown in Figure (3),

$$Q = \oint D.dI = \oint \varepsilon \frac{\partial V}{\partial n} . dl \tag{27}$$

$$Q = \varepsilon\left(\frac{V_a - V_b}{\Delta y}\right)\Delta x + \varepsilon\left(\frac{V_c - V_d}{\Delta y}\right)\Delta x + \varepsilon\left(\frac{V_u - V_w}{\Delta x}\right)\Delta y + \varepsilon\left(\frac{V_y - V_z}{\Delta x}\right)\Delta y + ... \tag{28}$$

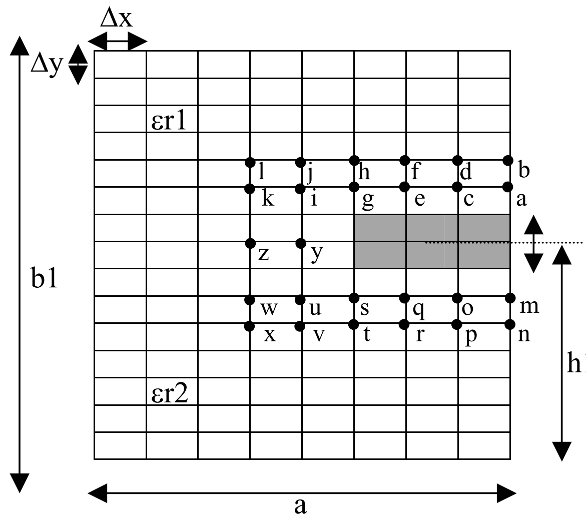


Figure 3. Illustrating path to find Q.

Since $\Delta x = \Delta y$, $Q = \varepsilon_0 \sum \varepsilon_{ri} V_i$ for nodes i on external rectangle b,d,f...r, p,n with corners (such l and x) not counted. $Q = -\varepsilon_0 \sum \varepsilon_{ri} V_i$ for nodes i on inner rectangle a, c, e, ...q,o,m with corners (such as i and u) counted twice where V_i and ε_{ri} are the potential and dielectric constant at i^{th} node. If i is on the dielectric interface $\varepsilon_{ri} = (\varepsilon_{r1} + \varepsilon_{r2}) / 2$. Also if i is on the line of symmetry $Vi = Vi/2$ to avoid V_i twice in equation (28) [4]

$$Co = Q_0 / V_0 , \tag{29}$$

where Q_0 is obtained by removing the dielectric and finding V_i at the free nodes and then using (31) with $\varepsilon_{ri} = 1$ at all nodes. Once Q and Q_0 are calculated C, Co and Z_0 are obtained, $Z_0 = \frac{1}{uo\sqrt{CC_0}}$, (30)

where $u_0 = 3 \times 10^8$ m/sec.

5. RESULTS AND DISCUSSIONS

Uniform Cross section of the TEM cell for two dimensions 2a/b1=1 and 2a/b1=2 and septum thickness t/b1=0.1 is considered for the impedance analysis. Table 1 gives the impedance values (Z_0) of Symmetric TEM cell computed by FEM and its comparison with other methods. The impedance values obtained by FEM are found to be in good agreement with those published by other authors. Tables 2 and 3 gives Z_0 values of asymmetric TEM cell for various vertical offset position of septum for homogeneous and for inhomogeneous dielectric inside the cell.

It is observed that maximum impedance is achieved when the septum is placed at the center of the TEM cell. Offsetting the septum vertically decreases Z_0 of the cell. (Figures 4 and 5). The variation of Z_0 for different dielectric constants ($2.1 < \varepsilon_{r2} < 9.9$) below septum.is shown in Figures 6 to 10. It is shown in figures 8 to 10 the variation of Z_0 with the width of the septum. Effective permittivity of the cell is presented in Tables 2 and 3.It is observed that presence of a different dielectric (other than air) below the septum decreases the value of Z_0.So by proper choice of ε_{r2} and the width of the septum desired values of Z_0 can be obtained by suitably offsetting the septum (to lesser height) so that bigger equipments can be tested.

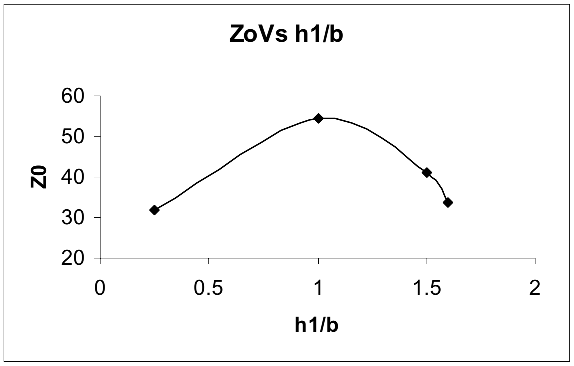


Figure 4. Z_0 versus h1/b.

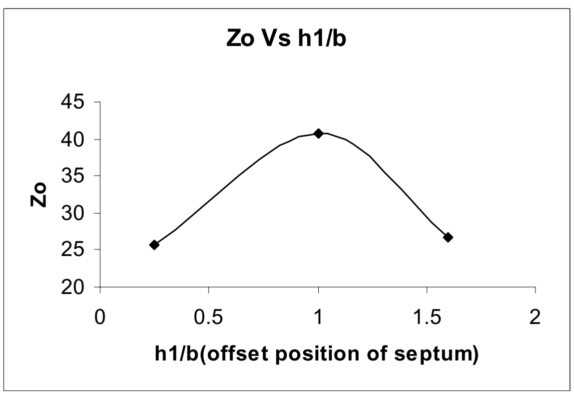


Figure 5. Z_0 versus h1/b.

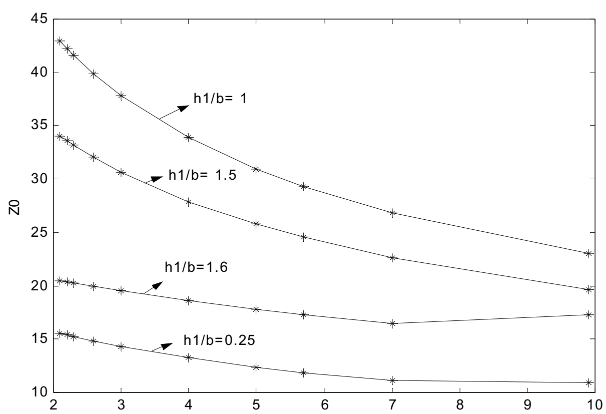


Figure 6. Relative dielectric constant below septum.

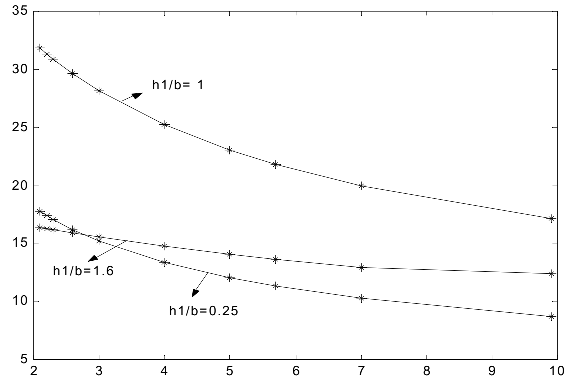


Figure 7. Relative dielectric constant below spectrum.

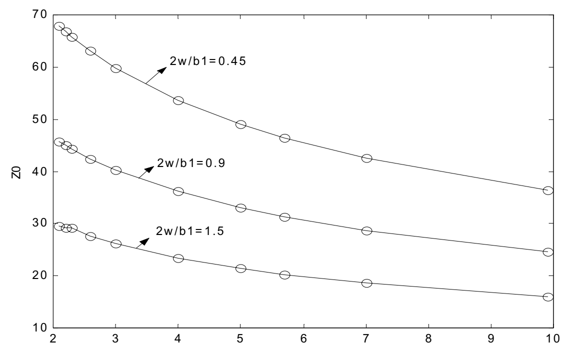


Figure 8. Relative dielectric constant below septum.

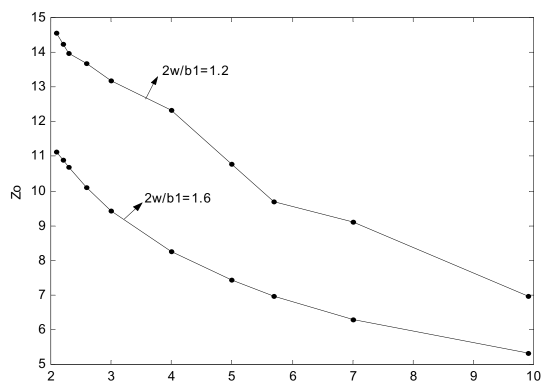


Figure 9. Relative dielectric constant below septum.

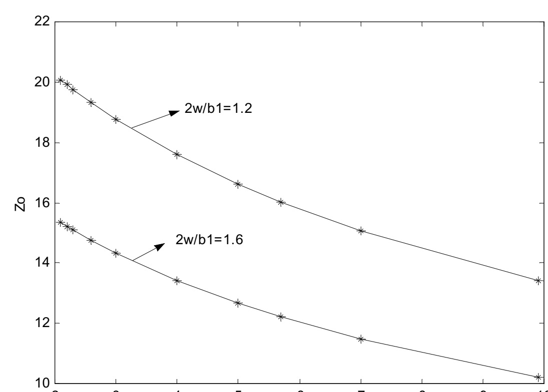


Figure 10. Relative dielectric constant below septum.

Table 1. COMPARISON OF Z_0 VALUES (t/b1=0.1) $\varepsilon r_1 = \varepsilon r_2 = 1$

Dimension		FEM	Chen expression [5]	Cruzen & graver [6]	Metcalf [7]	Getsinger [8]
2a/b1	2w/b1	Z_0	Z_0	Z_0	Z_0	Z_0
1	0.8	45.2	44.7	40.9	40.5	40.9
1	0.7	55.6	57.4	55.3	54.7	55.2
2	1.8	26.6	29.3	27.7	27.7	27.6
2	1.5	36.5	38.7	38.0	38.3	38.1
2	0.9	58.9	58.6	58.6	58.7	58.6
2	0.5	87.9	86.7	86.5	85.9	87.3

Table 2. Z_0 VALUES OF TEM CELL WITH INHOMOGENEOUS DIELECTRIC (2a/b1=1 & t/b1=0.1) $\varepsilon r_1 = 1$

2a/b1	w/a	h1/b	$\varepsilon_{r1}=\varepsilon_{r2}$ Z_0		Relative Dielectric constant below septum (εr_2)									
					2.1	2.2	2.3	2.6	3	4	5	5.7	7	9.9
1	0.8	0.25	25.76	Z_0	17.75	17.4	17.07	16.19	15.21	13.36	12.059	11.34	10.298	8.73
				Eps_eff	1.79	1.86	1.94	2.15	2.44	3.16	3.88	4.39	5.32	5.71
1	0.8	1	40.66	Z_0	31.83	31.34	30.88	29.59	28.11	25.197	23.04	21.82	19.99	17.15
				Eps_eff	1.53	1.57	1.62	1.77	1.96	2.44	2.91	3.25	3.87	5.26
1	0.8	1.6	26.71	Z_0	16.35	16.25	16.15	15.87	15.52	14.73	14.05	13.63	12.93	12.39
				Eps_eff	2.47	2.5	2.53	2.62	2.75	3.05	3.35	3.56	3.95	3.69
1	0.65	0.25	32.01	Z_0	15.55	15.4	15.25	14.84	14.33	13.26	12.397	11.89	11.08	10.92
				Eps_eff	3.72	3.799	3.87	4.1	4.39	5.12	5.86	6.38	7.33	7.56
1	0.65	1	54.27	Z_0	42.93	42.26	41.62	39.87	37.84	33.88	30.95	29.296	26.82	22.99
				Eps_eff	1.54	1.59	1.64	1.79	1.98	2.47	2.96	3.31	3.94	5.37
1	0.65	1.5	41.17	Z_0	34.03	33.61	33.19	32.04	30.68	27.91	25.78	24.54	22.66	19.65
				Eps_eff	1.39	1.43	1.46	1.57	1.71	2.07	2.43	2.68	3.14	4.18
1	0.65	1.6	33.54	Z_0	20.52	20.41	20.296	19.97	19.55	18.62	17.81	17.30	16.46	17.32
				Eps_eff	2.51	2.54	2.57	2.66	2.77	3.05	3.33	3.54	3.91	3.53

Table 3. Z0 VALUES OF TEM CELL WITH INHOMOGENEOUS DIELECTRIC (2a/b1=2 & t/b1=0.1) $\varepsilon_{r1}=1$

2a/b1	2w/b1	w/a	h1/b	$\varepsilon_{r1}=\varepsilon_{r2}$ Z_0		Relative Dielectric constant below septum									
						2.1	2.2	2.3	2.6	3	4	5	5.7	7	9.9
2	1.5	0.75	1	37.57	Z_0	29.6	29.15	29.12	27.52	26.14	23.44	21.43	20.29	18.59	15.95
					Eps_eff	1.53	1.57	1.62	1.76	1.96	2.43	2.91	3.25	3.87	5.25
2	0.9	0.45	1	58.36	Z_0	45.64	44.95	44.28	42.45	40.32	36.15	33.05	31.3	28.68	24.61
					Eps_eff	1.52	1.57	1.62	1.76	1.95	2.43	2.91	3.24	3.86	5.25
2	0.45	0.225	1	86.2	Z_0	67.84	66.797	65.798	63.05	59.87	53.65	49.03	46.43	42.52	36.47
					Eps_eff	1.53	1.58	1.63	1.77	1.97	2.45	2.93	3.27	3.897	5.298
2	1.2	0.6	0.25	20.0973	Z_0	14.55	14.24	13.96	13.69	13.19	12.34	10.78	9.69	9.097	6.96
					Eps_eff	1.91	1.99	2.07	2.16	2.32	2.65	3.48	4.3	4.88	8.35
2	1.6	0.8	0.25	15.36	Z_0	11.13	10.896	10.68	10.09	9.44	8.25	7.42	6.96	6.3	5.32
					Eps_eff	1.91	1.99	2.07	2.32	2.65	3.47	4.29	4.87	5.94	8.32
2	1.2	0.6	1.6	22.09	Z_0	20.07	19.92	19.76	19.32	18.78	17.60	16.62	16.02	15.06	13.43
					Eps_eff	1.21	1	1.25	1.31	1.38	1.58	1.77	1.9	2.15	2.71
2	1.6	0.8	1.6	16.91	Z_0	15.34	15.22	15.1	14.76	14.34	13.42	12.66	12.2	11.47	10.21
					Eps_eff	1.22	1.24	1.25	1.31	1.39	1.59	1.78	1.92	2.18	2.74

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