ON THE COMBINATION OF MMP WITH MOM

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Abstract

The Multiple Multipole Program (MMP) [1] is based on the Generalized Multipole Technique (GMT) [2]. This method lies between a purely analytic solution and the well-known Method of Moments (MoM) [3]. The goal of this paper is to demonstrate how a combination of MMP with MoM can be achieved for obtaining more powerful codes. The combination presented here is based on an implementation of rooftop functions as a new type of basis function in 3D MMP. Since users of this code are often interested in the close near-field, a special implementation of rooftop functions is presented and tested.

1. Introduction

Both MMP and MoM codes expand the electromagnetic field inside a homogeneous domain by a set of basis functions

$$field \approx \sum_k a_k basis_k \qquad (1)$$

In the MoM, this expansion is more or less implicit because one usually expands currents rather than fields

$$current \approx \sum_k A_k Basis_k \qquad (2)$$

Since the electromagnetic field is obtained by integration from the current and charge densities, any basis function *Basis* used in MoM codes can easily be translated into a basis function *basis* for the MMP code, i.e., the combination of MMP with MoM is straightforward.

The most frequently used basis functions of the MMP code are multipoles. Although these functions have many benefits, they are not efficient for the modeling of special structures. For this reason, a library of other basis functions is available in the MMP code. One part of this library is obtained from well-known analytic solutions of Maxwell equations (plane waves, harmonic functions, waveguide and cavity modes). The second part of this library can be obtained from the translation and generalization of MoM basis functions. In [4,9] the MMP implementation of thin-wire expansions and its generalization, i.e., so-called line multipoles are discussed. For modeling thin, ideally conducting sheets, neither thin-wire expansions nor line multipoles are appropriate. The most successful MoM approach for modeling such structures involve the subdivision of the surface of ideal conductors into patches and in the approximation of the surface currents by piecewise linear functions. Rectangular patches with rooftop basis functions for the currents are most popular. The problems associated with the implementation of rooftop functions in the MMP code are discussed in the following sections.

2. MMP Basis Functions

MMP is based on a direct expansion of the EM field. One of its most important strengths is the ability to accurately compute the EM field in the near field and even on the surface of a structure. Multipoles and many other possible basis functions have non-physical properties such as singularities or discontinuities of the EM field. These non-physical properties prevent an accurate numerical solution. Fortunately, they are usually localized in certain points (multipoles), along certain lines (wire expansions, line multipoles) or on more complicated two-dimensional geometric objects (rooftop functions). The MMP remedy for this problem is simple. Each basis function is applied to approximate the EM field in a certain domain. The source of troubles, i.e., the area where the basis function has non-physical properties, must be outside this domain. Consequently, all basis functions used in MMP are strict solutions of Maxwell's equations. Applied to "current patches" this means that the patches are moved away from the surface into the ideal conductor. Since there are no "real" currents at this location, such patches may also be called fictitious sources of the EM field.

It is important to note that the interpretation of fictitious currents generating the EM field of the basis functions can be helpful for constructing new basis functions but this interpretation plays no role in the MMP code itself. Any given distribution of electric and magnetic charges and currents generates an EM field that fulfills the Maxwell equations outside the area of the charges and currents - even when the Maxwell equations are violated inside this area. This is not relevant for MMP because the critical area is outside the domain where the basis function is applied.

3. MMP Rooftop Functions

The approximation of the electric current on the surface of an ideal conductor by piecewise linear functions causes discontinuities of the first derivatives along certain lines on the surface. This causes inaccuracies of the EM field computation at least close to these lines. In MoM codes one is often not directly interested in the EM field. In order to fulfill the boundary conditions of the EM field on the surface of the conductor, one can apply a projection technique with appropriate testing functions. This requires the evaluation of the EM field on the patch itself which simplifies the formulae. Moreover, the testing functions allow one to avoid numerical problems arising from the singularities of the EM field on the borders of the patch.

MMP requires an accurate and efficient computation of the EM field of rooftop functions anywhere in space (except on the patch itself). This computation is not trivial. It is outlined in the following section.

3.1 Electric and Magnetic Currents

Electric charges and currents are considered to be the sources of the EM field. Although magnetic charges and currents are missing in the ordinary Maxwell equations, one can easily introduce them to obtain a symmetric form. This allows one to construct a set of basis functions that is dual to the basis functions obtained from electric currents and charges. Since the "sources" of the EM field of MMP basis functions are fictitious anyway, the existence of magnetic charges and currents plays no role. Introducing magnetic charges and currents is just a trick to obtain another set of basis functions for modeling EM fields. Such basis functions are helpful for certain applications, e.g., apertures. Since the implementation is almost identical with the implementation of basis functions obtained from electric charges and currents, it is not explicitly explained in the following.

4. Efficient Computation of the EM Field of a Rooftop Function near the Patch

4.1 Vector Potential Formulation

For a given electric current distribution $\vec{j}$ one first obtains the vector potential

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\int_{D'}\vec{j}(\vec{r}\,')G(\vec{r},\vec{r}\,')\mathrm{d}V' \qquad (3)$$

where

$$G(\vec{r},\vec{r}\,') = \frac{e^{ik|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} = \frac{e^{ikR}}{R} \qquad (4)$$

is the well-known free space Green's function in three dimensions.

One can directly obtain the EM field from the vector potential using

$$\vec{E}(\vec{r}) = \frac{i\omega}{k^2}\,\mathrm{grad}\ \mathrm{div}\,\vec{A} + i\omega\vec{A} \qquad (5)$$

$$\vec{H}(\vec{r}) = \mu^{-1}\,\mathrm{curl}\,\vec{A} \qquad (6)$$

For an arbitrary current distribution on a flat patch D' one obtains several integrals of the form

$$I(x,y,z) = \iint_{D'} f(x',y')\frac{e^{ikR}}{R^n}\mathrm{d}x'\mathrm{d}y' \qquad (7)$$

Unfortunately, no analytical solution of (7) is known - even for a piecewise linear current distribution on a rectangular patch (see Figure 1). The direct numerical integration (even with adaptive methods) is very inefficient when the order n is large and R has small values, i.e., when the field point is close to the patch where the current is defined. For points far away one can use any simple numerical technique.

4.2 Mixed Potential Formulation

Especially (5) leads to relatively high orders because of the second order derivatives in the operator grad div. The maximum order required in (7) can be reduced by a mixed-potential formulation. In this formulation one first computes the charge density on the patch from the charge conservation law

$$i\omega\rho = \mathrm{div}\,\vec{j} \qquad (8)$$

then the scalar potential

$$\phi(\bar{r}) = \frac{1}{4\pi\varepsilon} \int_{D'} \rho(\bar{r}')\, G(\bar{r},\bar{r}')\, \mathrm{d}V' \quad (9)$$

and finally the electric field

$$\bar{E}(\bar{r}) = -\mathrm{grad}\,\phi + i\omega \bar{A}\,. \quad (10)$$

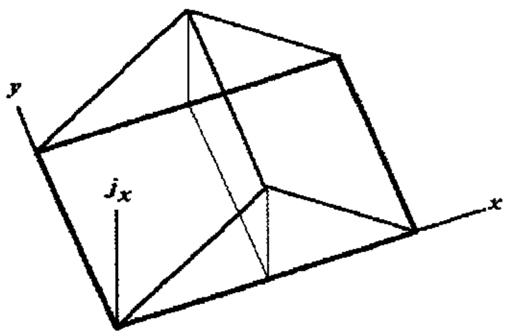

Figure 1: *Rooftop distribution of the x component of the electric current j on a rectangular patch in the x-y plane.*

Although the mixed-potential formulation can reduce the order of the integrals (7), one still has to compute integrals of up to third order when the most simple linear current distribution is assumed.

4.3 Expansion of the Green's Function

The best method [5] that has been found for points close to the patch involves a Taylor series expansion of the exponential function in the Green's function around the origin R=0. This method is similar to the method proposed in [6]. It leads to series expansions of the potentials and of the EM field containing integrals of the following types:

$$K_n(x,y,z) = \iint_{D^*} r^n \mathrm{d}\xi\; \mathrm{d}\eta \quad (11)$$

$$L_n(x,y,z) = \iint_{D^*} \xi\; r^n \mathrm{d}\xi\; \mathrm{d}\eta \quad (12)$$

$$M_n(x,y,z) = \iint_{D^*} \xi\eta\; r^n \mathrm{d}\xi\; \mathrm{d}\eta \quad (13)$$

These integrals can be evaluated analytically by some simple elementary functions and recurrence relations. First of all, the auxiliary integral

$$I_n(x,z) = \int_0^x (\xi^2 + z^2)^{n/2} d\xi \quad (14)$$

is computed recursively with the relation

$$(n+1) I_n(x,z) - n z^2 I_{n-2}(x,z) = x(x^2+z^2)^{n/2} \quad (15)$$

and the following starting values:

$$I_0(x,z) = x \quad (16)$$

$$I_{-1}(x,z) = \mathrm{Arsinh}(x/z) \quad (17)$$

$$I_{-2}(x,z) = z^{-1} \arctan(x/z) \quad (18)$$

$$I_{-3}(x,z) = x/(z^2 \sqrt{x^2+z^2}) \quad (19)$$

For the integrals (11) one has the recurrence relations

$$(n+2) K_n(x,y,z) - n z^2 K_{n-2}(x,y,z) = x I_n(y, \sqrt{x^2+z^2}) + y I_n(x, \sqrt{y^2+z^2}) \quad (20)$$

and the starting values:

$$K_0(x,y,z) = xy \quad (21)$$

$$K_{-3}(x,y,z) = z^{-1} \arctan\left(\frac{xy}{z\sqrt{x^2+y^2+z^2}}\right) \quad (22)$$

Note that orders with $n < -3$ are not needed in our formalism for rooftop patches with piecewise linear current distributions. The integral of order -2 is numerically most difficult. It requires the solution of a transcendental function related to Euler's dilogarithm. Fortunately, this integral is not required here.

The integrals (12,13) are directly obtained from

$$L_n(x,y,z) = \frac{I_{n+2}(y, \sqrt{x^2+z^2}) - I_{n+2}(y,z)}{n+2} \quad (23)$$

$$M_n(x,y,z) = \frac{(x^2+y^2+z^2)^{\frac{n}{2}+2} - (x^2+z^2)^{\frac{n}{2}+2} - (y^2+z^2)^{\frac{n}{2}+2} + |z|^{n+4}}{(n+2)(n+4)} \quad (24)$$

One obviously cannot use (23) for the order -2 and (24) for the orders -2 and -4. For these orders one has:

$$L_{-2}(x,y,z)=\frac{y}{2}\log(\frac{x^2+y^2+z^2}{y^2+z^2})+ \sqrt{x^2+z^2}\arctan(\frac{y}{\sqrt{x^2+z^2}})-z\arctan(\frac{y}{z}) \quad (25)$$

$$\begin{aligned}4M_{-2}(x,y,z)=&(x^2+y^2+z^2)\log(x^2+y^2+z^2)\\&-(x^2+z^2)\log(x^2+z^2)\\&-(y^2+z^2)\log(y^2+z^2)\\&+z^2\log(z^2)\end{aligned} \quad (26)$$

$$4M_{-2}(x,y,z)=\log\frac{(x^2+z^2)(y^2+z^2)}{z^2(x^2+y^2+z^2)} \quad (27)$$

This technique allows one to recursively compute the basis functions with a user-defined accuracy using a minimum of arithmetic operations.

5. Computation of the EM Field of a Rooftop Function away from the Patch

Since the convergence of the Taylor series expansion of the exponential function is poor for large arguments, another technique is required for larger distances. Numerical studies show that the series expansion method is useful and extremely efficient for distances up to one wavelength. It is important to note that the sides of the patch are typically considerably smaller than one wavelength. Therefore, it is not difficult to evaluate the EM field using a purely numerical method.

5.1 Gaussian Quadrature

Gaussian quadrature is a well-known simple algorithm. It has been found that this algorithm is inefficient when the distance of the point where the EM field has to be evaluated to the rooftop patch is smaller than the size of the patch. Gaussian quadrature is less efficient than our "semi-analytic" method discussed in section 4 - provided that the "semi-analytic" method can be applied (which is not true for distances larger than the wavelength) and provided that a high accuracy is desired (which is true in 3D MMP). However, Gaussian quadrature is a good choice for distances larger than one wavelength.

5.2 Adaptive Numerical Integration

When a simple Gaussian quadrature is applied, the user has to set the number of integration points and the order. Both parameters affect the accuracy and the speed.

Adaptive techniques such as adaptive Simpson algorithms have the advantage that the user can define the maximum error. The kernel function must be computed in at least nine points for one-dimensional integrals. Thus, one must evaluate the kernel function in at least 81 points on a rectangular patch. All basis functions of the MMP code are analytic solutions of the Maxwell equations. They are usually computed with a very high accuracy of about 14 digits. This causes an extremely large number of kernel function calls for field points close to the patch.

It has been mentioned that the patch is usually considerably smaller than one wavelength and that the "semi-analytic" algorithm presented in the previous section is very efficient and accurate for points close to the patch. Outside this area, it is not clear whether the Gaussian quadrature is more efficient than the adaptive Simpson algorithm. Moreover, the orders of the Gaussian quadrature required for obtaining a given accuracy are not known in advance. It has been found that the Simpson algorithm is less efficient than Gaussian quadrature outside this area because less than 81 function calls are required in order to obtain an accuracy of about 14 digits with Gaussian quadrature, whereas 81 function calls is the minimum for the adaptive algorithm. Therefore, the adaptive algorithm has been used for finding rules for setting the parameters (orders) of the Gaussian quadrature.

5.3 Approximate methods

It is well-known that a piecewise linear current distribution on a thin wire can be approximated by a piecewise sinusoidal current distribution that allows an analytical treatment. This result can be applied to the current distribution on rooftop patches [7]. It is obvious that the larger the distance from the patch the better the approximation, i.e., this approximation should not be used for computations in the near-field. Although this method can be useful for reducing the computation time for a moderate distance when a moderate accuracy is desired, it has not been implemented in the 3D MMP code where a high accuracy is desired.

At very large distances, the rough approximation of the current on the rooftop patch by a dipole could be useful. However, this method has also been discarded because of the high accuracy desired in the 3D MMP code. In this code we efficiently compute the EM field of the rooftop patch either with the "semi-analytic" method or with the Gaussian quadrature. The "semi-analytic" method is used for points at a distance of less than one wavelength from the center of the patch and the Gaussian quadrature is used for larger distances. Note that the performance of the Gaussian quadrature is better for larger distances. Since the "semi-analytic" method is much faster than Gaussian quadrature in the area where it can be applied, the longest computation times are obtained for points within a distance of one wavelength. It should be pointed out that these statements hold for rooftop patches with a sidelength smaller than one wavelength. For physical reasons, this restriction is usually met.

6. EXAMPLE

For testing the implementation of the rooftop functions in the MMP code, a square plate of finite thickness with round corners and edges is considered in the following (see Figure 2). This example has been studied by Fin Bomholt [8,9] with an array of line multipoles parallel to the edges and with a regular multipole in each corner of the plate. The same problem has been solved by Peter Regli with a slightly improved matching point distribution and with a pure MMP expansion, i.e., with an array of regular multipoles inside the plate.

When one compares the original model of Fin Bomholt with the model of Peter Regli, one finds that Fin's model is 2.5 times faster. According to the internal error checks of the MMP code, the errors of Fin's results are at least 2.7 times higher (the peak errors are more than 10 times higher). Therefore, it is hard to say whether Fin's model with line multipoles instead of Peter's pure MMP expansion is reasonable and efficient. In the following investigation, we will study not only different rooftop expansions but also the influence of the matching points and of additional (supporting) expansions.

6.1 Matching points

The generation of an appropriate set of matching points is tedious with the current version of the input editor, because the refinement requires a completely new construction of all matching points. Therefore one tends to overdiscretized models. Since the matching point sets of Fin Bomholt and Peter Regli have almost the same number of matching points, we need at least one additional set to obtain some information concerning the convergence. A third set of matching points has been generated by simply deleting some of the matching points of Fin's set (see Figure 3). This crude set can only be used for relatively crude models.

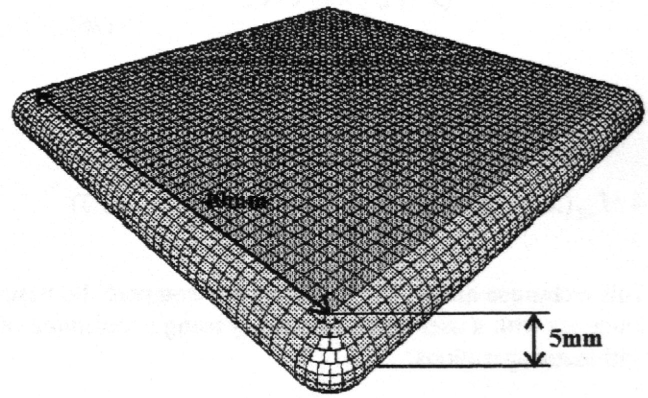

Figure 2: *Square plate of finite thickness with circular edge proposed by Fin Bomholt. Side length of the plane = 40mm, thickness = 5mm. A plane wave is incident perpendicular to the plane, the electric field is parallel to one of the sides of the square. Frequency = 6.6GHz. The matching points of the refined model (by Peter Regli) are indicated by squares.*

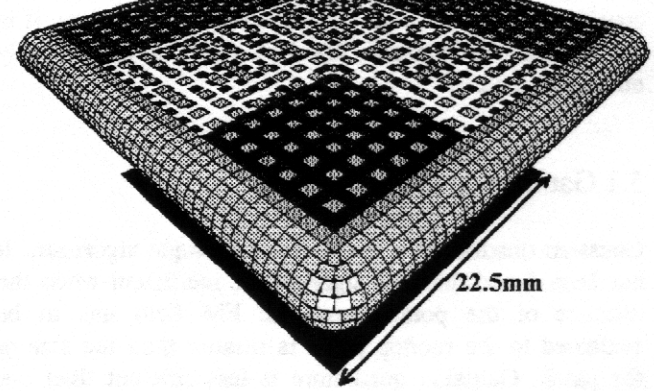

Figure 3: *Model with a reduced number of matching points (indicated by squares). The black square area in the shadow of the plate indicates one of the windows used for testing the close near-field computation.*

Because of the symmetry of our plate with respect to the xy, xz, and yz planes, only one octant needs to be modeled.

In fact, the square plate has even higher symmetry but the actual version of 3D MMP does not allow us to take such symmetries into account. In each matching point one has three boundary conditions (two for the tangential components of the electric field and one for the normal component of the magnetic field). Therefore, the number of equations obtained with M matching points is 3M. Note that the third boundary condition is omitted in most numerical methods because it can cause numerical dependencies. This omission is not necessary in 3D MMP where one works with an overdetermined system of equations [1,2]. The three matching point sets used for our comparison are characterized as follows:

Matching point set	points (M)	rows (3M)
A (Peter Regli)	664	1992
B (Fin Bomholt)	616	1848
C (reduced set B)	343	1029

where "points" is the number of matching points in one octant and "rows" is the number of rows of the resulting symmetry-decomposed MMP matrix, i.e., the number of equations.

The quality of the results depends to a great extent on the expansion that has been used together with the matching point set (see section 6.2). Comparisons of the optimal results obtained with several expansions allows one to reduce the influence of the expansions. The best solutions found with the three sets can be described as follows:

set	average error	peak error	ratio
A	0.47%	1.9%	4
B	0.82%	16.4%	20
C	2.81%	20.1%	7

The errors in this table indicate the mismatching in the matching points relative to the maximum value of the EM field. For more information concerning the MMP error checks see section 6.3 and [1,2]. The ratio peak error / average error is an excellent indicator for the quality of the model. A large ratio indicates unbalanced error distributions that are either caused by an inappropriate matching point distribution or by an inappropriate set of expansions. Since the table is based on the best sets of expansions that have been found for the different sets of matching points, the ratio mainly reflects the quality of the matching point distribution.

Set A has almost as many points as set B. The computation time of set A is only about 8% higher than the computation time of set B when the same expansions are used for both sets. One can conclude from these facts that set B is far from being optimal and that the matching point distribution plays an important role that should not be underestimated. For example, when Peter's pure MMP expansion is combined with Fin's matching point set B, the peak error is almost ten times higher than in Peter's original model.

The matching point distribution of set C is also worse than the one of set A, but set C allows a considerable reduction of the computation time. It is important to note that the best expansions for set A an B cannot be applied to set C because set C does not define enough equations to compute the parameters of these expansions. The expansions for set C have about half as many unknowns as the best expansions for sets A and B. The computation time for the rectangular matrices in the MMP code is almost proportional to the number of matching points and to the square of the number of unknowns. Therefore, the computation of models based on set C is typically up to 8 times faster than the computation of models based on sets A or B. Set C is useful for obtaining quick results.

6.2 Expansions

A pure MMP expansion is known to be inefficient for thin scatterers. The plate proposed by Fin Bomholt is neither thin nor thick. When an MMP expansion is constructed according to the rules for setting multipoles, 8*8=64 multipoles per quadrant are required. This results in 692 unknowns for the pure MMP model of Peter Regli (see Figure 2,4,5). One can easily reduce the number of unknowns by reducing the orders of the multipoles. Because of the fast convergence of MMP this causes a considerable increase of the error. Instead of reducing the number of orders of the multipoles one can reduce the number of multipoles. This causes a less balanced error distribution on the surface of the plate.

Since the same problem exists for wire-like structures, line multipoles were introduced in 3D MMP [4,9]. Line multipoles are a generalization of thin-wire expansions. These expansions offer a simple way to reduce the number of expansions for our test case. Instead of 8*8 regular multipoles one can use 8 line multipoles parallel to the direction of the incident electric field and an additional line multipole along the edge perpendicular to the electric field. To improve the modeling of the field near the corner a single multipole is added. Thus, Fin's model has a total of 10 expansions instead of 64. This does not necessarily mean that the number of unknowns is also reduced. However, the Fin Bomholt's model has only 425 unknowns. Figure 6 shows the results obtained with this model.

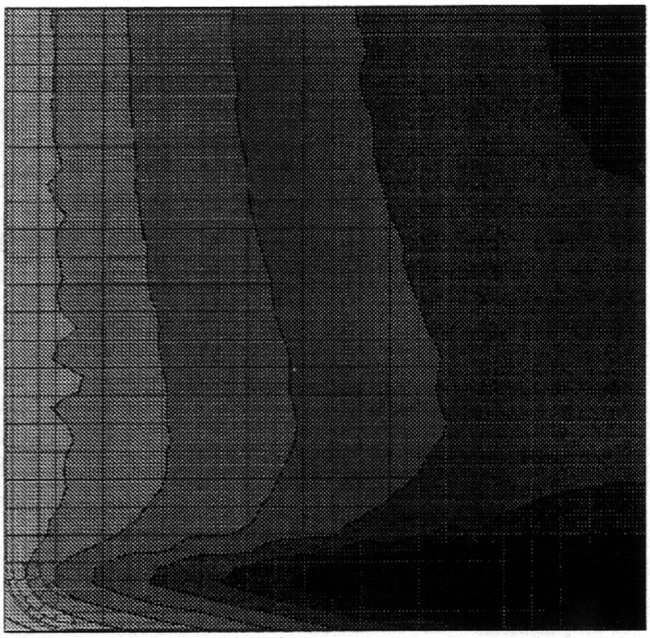

Figure 4: *Time average of the magnetic field density on the bright side of the plate obtained by a pure MMP model (Peter Regli). The incident electric and magnetic fields are parallel to the horizontal and vertical axes, respectively. Dark areas indicate strong fields. The matching points are indicated by gray lines.*

Although the reduction of the number of unknowns in the line multipole model causes a considerable reduction of the computation time, it also causes an increased error (see section 6.1). The resulting lines of constant magnetic field density in Figure 6 obviously reflect the location and orientation of the line multipoles.

It seems to be clear that rooftop expansions are more appropriate for modeling the field along a flat structure such as our plate. However, it is important to note that the plate is not thin at all. A pure rooftop expansion causes large errors especially along the edges and near the corners of the plate. In order to reduce these errors, one can support the rooftop expansions with line multipoles along the edges and regular multipoles in the corners. The resulting model has one rooftop expansion, two line multipoles and one regular multipole instead of 64 regular multipoles. Figure 7 shows the result. Instead of line multipoles along the edges one can also use 15 regular multipoles to support the rooftop expansion (Figure 8). It is important to note that the supporting line multipoles or regular multipoles mainly affect the field near the edges and corners. Nevertheless, they are indispensable for accurate near-field computations.

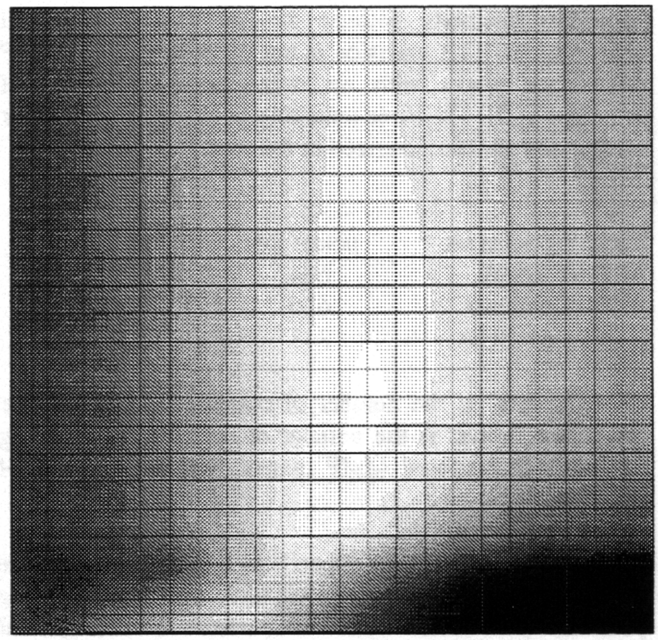

Figure 5: *Time average of the magnetic field density on the dark side of the plate obtained by a pure MMP model (Peter Regli). The incident electric and magnetic fields are parallel to the horizontal and vertical axes, respectively. Dark areas indicate strong fields.*

In our investigation, we applied rooftop expansions with 5*5 and 10*10 patches. In all patches the x or y components of the electric or magnetic currents were used. The most simple model uses the x component of the electric current only, because the electric field of the incident wave points in the x direction. This causes only one unknown per patch, but the resulting errors are quite high. When both components of the electric current are used, one has two unknowns per patch without a significant reduction of the errors. The introduction of magnetic currents is essential for obtaining reasonable results. In our test example, the y component of the electric current and the x component of the magnetic current can be omitted because their contribution to the EM field is very small, i.e., patches with x components of the electric current and y components of the magnetic current are optimal.

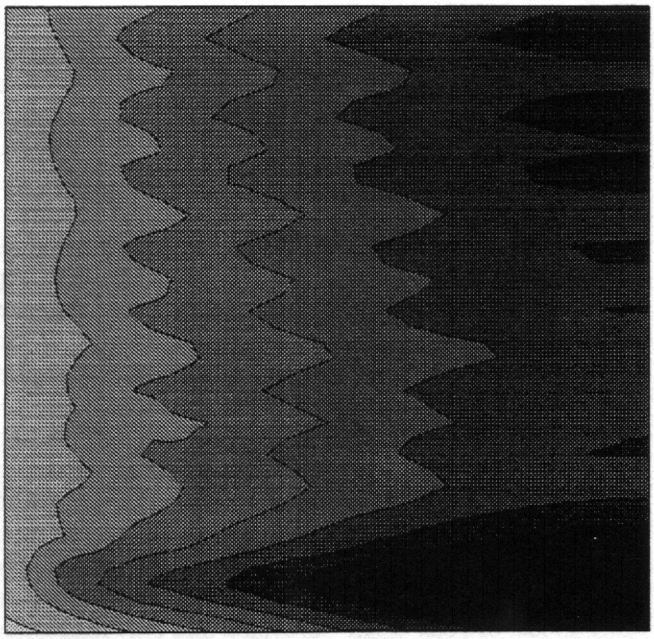

Figure 6: *Time average of the magnetic field density on the bright side of the plate obtained by a model with line multipoles parallel to the edges and a regular multipole in each corner (Fin Bomholt). The incident electric and magnetic fields are parallel to the horizontal and vertical axes, respectively. Dark areas indicate strong fields.*

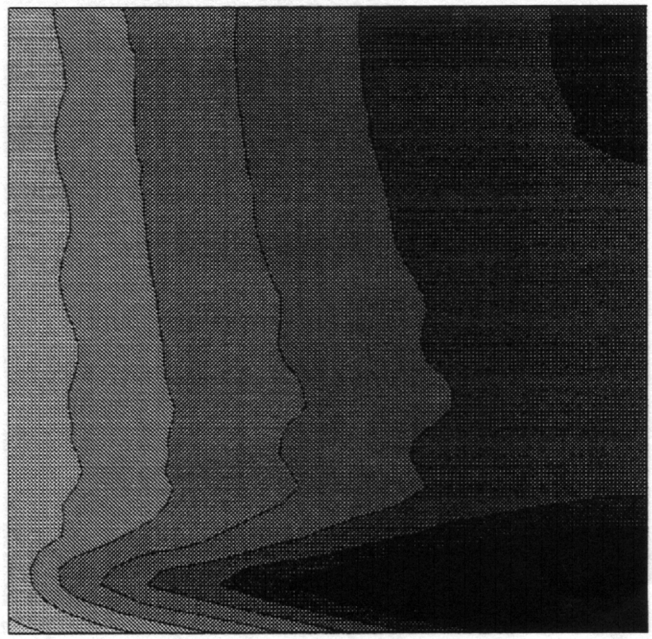

Figure 7: *Time average of the magnetic field density on the bright side of the plate obtained by a model with rooftop patches in the center of the plate, additional line multipoles along the edges and a regular multipole in each corner. The matching point distribution is the same as in figure 6. The incident electric and magnetic fields are parallel to the horizontal and vertical axes, respectively. Dark areas indicate strong fields.*

Rooftop patches in MoM codes are placed on the surface of a scatterer. In 3D MMP, we place the patches somewhere inside the scatterer in order to obtain a smoother behavior of the corresponding EM field. The smoothness is increased with the distance from the surface. At the same time, this causes a larger condition number of the MMP matrix. When a single N*N rooftop expansion is placed in the xy plane in the center of the plate, the size of each side of each patch is 20mm/N and the distance from the surface is 2.5mm. From the rules for setting multipoles, we expect that the ratio side length / distance is between 1.5 and 2.5. Therefore, the rough rooftop expansions with 5*5 patches are almost optimal whereas the finer rooftop expansions with 10*10 patches are suboptimal. The distance of the finer rooftop expansions from the surface of the plate should be reduced. This has another consequence: Instead of a single expansion in the xy plane we require two expansions, one above and one below the xy plane and the number of unknowns is doubled. Since 3D MMP provides an excellent matrix handling which allows one to work with suboptimally placed expansions, excellent results were obtained even with a 10*10 rooftop expansion in the xy plane.

6.3 Error checks

In our investigation we applied three different sets of matching points and 8 different rooftop expansions supported by 1) multipoles, 2) line multipoles and regular multipoles, 3) no additional expansions. This resulted in 72 different models with different speed and accuracy. 3D MMP provides internal error checks based on the mismatching of the boundary conditions in the matching points. The MMP error function [1,2] packs the mismatching of all components of the field in a single scalar number per matching point. Although this function is informative, it can be misleading in some cases. Nevertheless, the average and the peak values of the error functions are helpful for comparing solutions of one and the same problem obtained using different models.

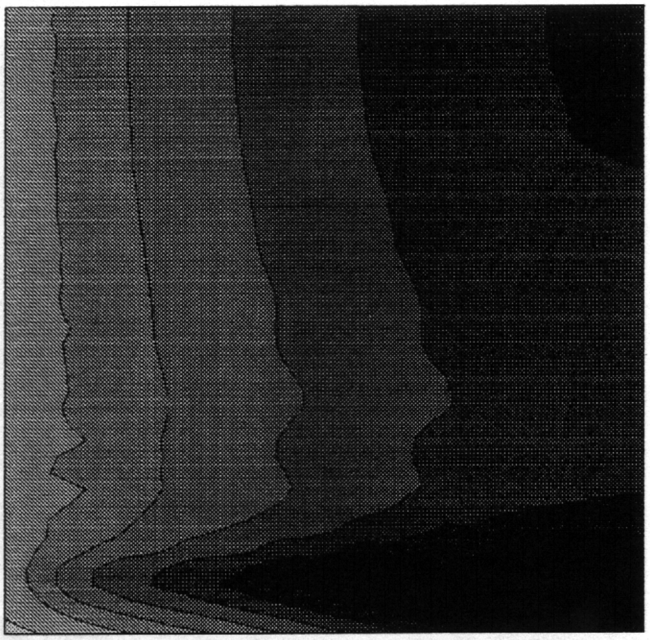

Figure 8: *Time average of the magnetic field density on the bright side of the plate obtained by a model with rooftop patches in the center of the plate and additional regular multipoles along the edges. The matching point distribution is the same as in figure 4. The incident electric and magnetic fields are parallel to the horizontal and vertical axes, respectively. Dark areas indicate strong fields.*

Instead of an internal error check, one can compare the results of a model with the results of a reference model that is considered to be accurate. Peter Regli's pure MMP model of the plate is considered to be quite accurate - according to the internal error checks. Therefore, it has been used as a reference. This external error check has other drawbacks. Models that are close to the reference model are overestimated. When a model is more accurate than the reference model, the error reflects the error of the reference model rather than the error of the model to be tested. From the internal error checks, it seems that the 10*10 rooftop expansion with electric currents in the x direction and magnetic currents in the y direction, supported by multipoles along the edges, is more accurate than the pure MMP expansion. Its peak error is 2% of the maximum field value, i.e., slightly higher than the peak error of the reference (1.9%), but the average of the error is only 0.47% compared with 0.7% of the reference. The average of the difference between the two models is 0.57%.

6.4 Speed, efficiency, and convergence

When a new numerical technique is tested, it is important that it allows one to accurately compute a given problem with the desired accuracy, but it is at least as important that the computation time can be kept as short as possible, i.e., that the method is fast and efficient. Since the computation time depends considerably on the accuracy, we can define the efficiency as speed*accuracy or as 1/(time*error). The different definitions of the accuracy and of the error of a result automatically cause different definitions of the efficiency. We have shown that the accuracy of the results depends not only on the expansion but also on the set of matching points. Therefore, it is not easy to say whether our implementation of rooftop functions is an efficient feature of 3D MMP for modeling Fin Bomholt's plate of finite thickness. It has been found that the efficiency based on the average of the internal MMP error is almost the same for our rooftop model, for the line multipole model of Fin Bomholt, and for the pure MMP model of Peter Regli - provided that only reasonably good models are compared. It is quite clear that rooftop expansions are much more efficient for thinner plates, i.e. Fin's plate is not a good test case for demonstrating the power of rooftop expansions in 3D MMP.

Beside the efficiency, the convergence of a numerical technique is certainly important. One often implicitly assumes that a fast convergence is an attribute of a good numerical technique. Although this might be correct for a pure mathematician who is never satisfied by a limited accuracy, this certainly is not correct in engineering where the requested accuracy is often moderate. Since the efficiency is a function of the accuracy, one usually finds a point with a certain accuracy where two methods with a different convergence have the same efficiency. Below this point, i.e., for a lower accuracy, the method with the slower convergence is more efficient than the method with the fast convergence. Rapidly converging techniques are therefore efficient when a high accuracy is desired.

In complex situations such as computational electromagnetics, the speed and efficiency of a method depends on many parameters and it is often very difficult to estimate the accuracy of a result. Therefore, the investigation of the convergence can be very tricky.

From large series of MMP computations of relatively simple models, we know that the convergence of pure MMP expansions is extremely fast. In the case of the scattering of a plane wave at a sphere, the convergence is at least exponential. Consequently, MMP is very efficient for obtaining accurate results but it is difficult to obtain results with a moderate accuracy within a short computation time. This was confirmed in our example when the orders of the original MMP expansion were slightly reduced. With a reduction of the unknowns from 692 to 439 the average of the internal MMP error was increased from 0.7% to 4.41%.

Statements on the convergence of 3D MMP models with rooftop expansions are even more difficult because we had to support these expansions by additional line multipoles and regular multipoles. However, it seems that convergence of models with rooftop expansions is worse than the convergence of pure multipole models. Therefore, one cannot efficiently obtain extremely accurate results with rooftop expansions, but one can obtain results with a reasonable accuracy with considerably less unknowns than with a pure MMP expansion.

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