NUMERICAL SOLUTIONS OF WAVE PROPAGATION IN DISPERSIVE AND LOSSY TRANSMISSION LINES

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ABSTRACT: A numerical model is proposed for application to dispersive transmission lines. Numerical results are compared with the analytical solution of the dispersive wave equation based on the wave propagation solved by means of finite element method in one dimension case and Newmark-β method in the time domain. This comparison between numerical and analytical solutions validates this numerical method as a suitable method to study wave propagation in dispersive transmission lines. Several practical applications including electromagnetic propagation in a plasma and the transient response of a surge wave in high-voltage transformer windings are presented in this paper.

1. INTRODUCTION

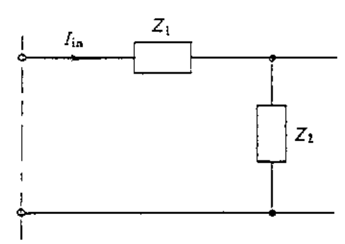
Transmission lines can be used to model a large variety of important applications in addition to the transmission of a signal from one point to another [1]. Physical phenomena such as the propagation of solitons [2]; breakdown process of an avalanche diode [3,4]; high-voltage resistance divider [5]; various plasma physics phenomena [6,7]; propagation in multi-layered earth media [8] have practical applications which have been modeled by transmission lines. While most of these problems can be solved analytically some of them are extremely difficult or impossible to solve by purely analytical methods.

Since the computational power of even quite small computers has rapidly developed, numerical methods are playing an increasingly important role in solving the mathematical equations[9,10]. Such numerical techniques have made wave propagation problems easy to solve using numerical models and results can be represented graphically for rapid understanding [11].

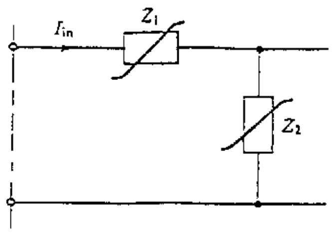
In this research, a one dimensional finite element model is employed for a variety of dispersive transmission lines with a variety of boundary conditions. The emphasis taken in this paper is that the calculation technique itself is very important if accurate and convergent solutions are to be obtained. Calibration factors such as the ratio of lumped capacitors (see fig. 1), velocity of traveling wave, the element size and time step have dependent relationships which can not be ignored. The numerical errors can be controlled by sufficiently fine discretization. Initially the method is validated by comparing numerical solutions with the analytical solutions for the case of the dispersive wave equation for a linear, homogeneous transmission line. In the case of inhomogeneous and nonlinear problems the most suitable method is often a numerical method. Appropriate models for nonlinear dispersive transmission lines are the subject of this paper.

2. A BASIC EQUATION FOR DISPERSIVE TRANSMISSION LINES

Typical transmission lines with dispersive properties can be categorized into linear and nonlinear transmission lines. Fig. 1. shows generalized dispersive transmission lines with both loss and nonlinear characteristics.



a. Linear parameter



b. Nonlinear parameter

Fig. 1. Typical sections of transmission lines with linear and nonlinear parameters

As a start in the numerical formulation of a number of simple cases, the basic analytical solutions of dispersive wave equations for a linear transmission line must be obtained and compared with numerical solutions. In this section the basic solutions of dispersive wave equations are discussed:

2.1 Lossless dispersive transmission line

The lossless dispersive transmission line in Fig. 2 can be described by a set of partial differential equations in instantaneous voltage u and current i as follows [2]:

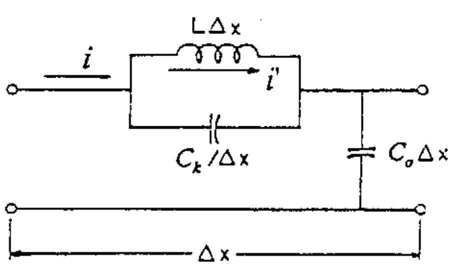


Fig.2. A section of lossless dispersive transmission line

$$-\frac{\partial i'}{\partial x} = C_0 \frac{\partial u}{\partial t} - C_k \frac{\partial^3 u}{\partial t \partial x^2} \qquad (1)$$

$$-\frac{\partial u}{\partial x} = L \frac{\partial i'}{\partial t} \qquad (2)$$

where C_k and C_0 are the series and parallel capacitances per unit length and L is the series inductance per unit length; i is the total instantaneous current, i' is the current in the inductive branch, u is the instantaneous voltage and x is the direction of the transmission line. From (1) and (2), we can derive a dispersive wave equation for the voltage u

$$\frac{\partial^2 u}{\partial t^2} - C_S^2 \frac{\partial^2 u}{\partial x^2} - \lambda_D^2 \frac{\partial^4 u}{\partial t^2 \partial x^2} = 0 \qquad (3)$$

where

$$C_S^2 = 1 / LC_0 \qquad (4)$$

$$\lambda_D^2 = C_k / C_0 \qquad (5)$$

The dispersion relation is obtained by assuming a solution of the form $u \sim u_0 \exp(j\omega t \pm jkx)$.

$$\omega^2 = \frac{C_S^2 k^2}{1 + k^2 \lambda_D^2} \qquad (6)$$

where ω is the angular frequency and k is the wave number of propagation. The dispersion relation is shown schematically in Fig. 3 [13].

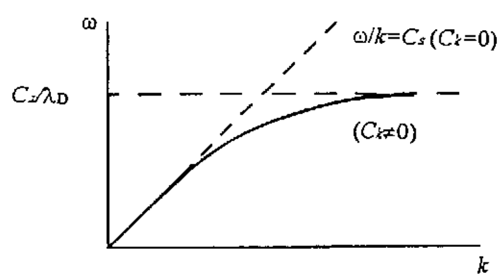


Fig. 3. Dispersion relation of wave equation, where C_S is the velocity of the wave.

In the case of $C_k \rightarrow 0$, λ_D in the third term of eq.(3) tends to zero. The lossless wave equation is retrieved, there is no dispersion, and the initial waveform does not change during propagation. It is the well known wave equation [12]. However, when λ_D (which is called Debye shielding length in plasma physics) is larger than zero, the wave is significantly dispersive and the waveform changes as the wave progresses.

Since eq. (3) is a linear dispersive wave equation, the analytical solutions are obtained using the method of Laplace transformation. Consider the transient response for which the initial conditions of transmission line and its derivative are

$$u(x,0)=0,\ \left\{\frac{\partial u(x,t)}{\partial t}\right\}_{t=0}=0 \qquad (7)$$

and the boundary conditions for the open circuit at terminal N are

$$u(0,t)=1,\ \left\{\frac{\partial u(x,t)}{\partial x}\right\}_{x=\ell}=0 \qquad (8)$$

where l is the total length of transmission line.

With the inverse Laplace transformation of solution using Heaviside's expansion theorem, the exact solution of Eq.(3) for the open boundary condition is given by

$$u(x,t)=1-\sum_{m=1}^{\infty}\frac{16\ell^2\sin\{(2m-1)\pi x/2\ell\}}{\{4\ell^2+(2m-1)^2\lambda_D^2\pi^2\}(2m-1)\pi}\cdot\cos(\omega_{2m-1})t \qquad (9)$$

where

$$\omega_{2m-1}=\frac{(2m-1)\pi}{\{4\ell^2/C_S^2+(2m-1)^2\lambda_D^2\pi^2/C_S^2\}^{1/2}} \qquad (10)$$

In the case of the boundary conditions for the terminal short-circuited, the spatial initial conditions are the same and the boundary conditions are given by

$$u(0,t)=1,\ u(\ell,t)=0 \qquad (11)$$

The exact solution obtained using the same procedure as above, can be written

$$u(x,t)=(1-\frac{x}{\ell})-\sum_{m=1}^{\infty}\frac{2\ell^2\sin\{m\pi x/\ell\}}{\{\ell^2+m^2\lambda_S^2\pi^2\}m\pi}\cdot\cos(\omega_m)t \qquad (12)$$

where

$$\omega_m=\frac{m\pi C_s}{\left[\ell^2+\lambda_D^2 m^2\pi^2\right]^{1/2}} \qquad (13)$$

For both cases,.as λ_D tends to zero, the solutions (9) and (12) reach the well-known solutions of the non dispersive wave equation [12]. The details of the procedure of solving eq. (3) by using the method of Laplace transformation can be found in [14].

2.2 Dispersive transmission line with loss

A natural extension of the dispersive transmission line approach is the inclusion of resistance. The transmission line with loss is shown in Fig. 4, in which linear and homogeneous parameters are assumed [15].

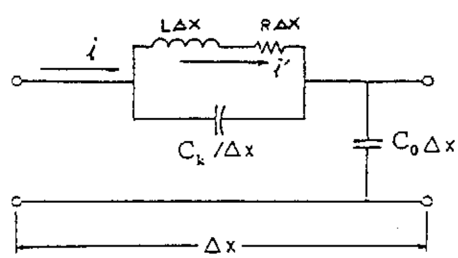


Fig. 4. A section of linear dispersive transmission line with loss

The equivalent circuit corresponding to the following partial differential equation for the voltage u is obtained as

$$\frac{\partial^2 u}{\partial t^2}-C_S^2\frac{\partial^2 u}{\partial x^2}-\lambda_D^2\frac{\partial^4 u}{\partial t^2\partial x^2}+\alpha^2\frac{\partial u}{\partial t}-\alpha^2\lambda_D^2\frac{\partial^3 u}{\partial x^2\partial t}=0 \qquad (14)$$

where $\alpha = (R/L)^{1/2}$. The dispersion relation for this equation is derived as follows:

$$\omega^2 = \frac{C_S^2 k^2}{1 + k^2 \lambda_D^2} \pm j\omega\alpha^2 \tag{15}$$

We consider the transient response for which the initial conditions and the boundary conditions for the open circuited terminal are the same as (7) and (8) respectively. With the inverse Laplace transformation of solution using Heaviside's expansion theorem, we derive the exact solution of Eq.(14).

$$u(x,t) = 1 - \sum_{m=1}^{\infty} A_m e^{-\delta t} \sin\left(\frac{2m-1}{2\ell}\pi x\right) \cdot \cos(\omega_o)t \tag{16}$$

where l is the total length of the transmission line, A_m is

$$A_m = \frac{2j}{-\delta \pm j\omega_o \ell \frac{1}{2}\left[\frac{1+\{R+(-\delta \pm j\omega_o)L\}(-\delta \pm j\omega_o)C_k}{\{R+(-\delta \pm j\omega_o)L\}(-\delta \pm j\omega_o)C_0}\right]^{1/2}} \cdot \frac{1}{\frac{C_0\{R+2L(-\delta \pm j\omega_o)\}}{\{1+C_k(-\delta \pm j\omega_o)(R+L)\}^2}} \tag{17}$$

δ and ω_0 are expressed as follows:

$$\delta = R/2L \tag{18}$$

$$\omega_o^2 = \frac{\{(2m-1)\pi\}^2}{4\ell^2 LC_o + (2m-1)^2\pi^2 LC_k} - \frac{R^2}{4L^2} \tag{19}$$

The case of the boundary conditions for the short-circuited terminal must be considered here, when the initial conditions are the same as that of eq. (11). Using the same procedure as before, the exact solution is

$$u(x,t) = (1 - \frac{x}{\ell}) - \sum_{m=1}^{\infty} A_m e^{-\delta t} \sin\left(\frac{2m}{2\ell}\pi x\right) \cdot \cos(\omega_s)t \tag{20}$$

where

$$A_m = \frac{2j}{-\delta \pm j\omega_s \ell \frac{1}{2}\left[\frac{1+\{R+(-\delta \pm j\omega_s)L\}(-\delta \pm j\omega_s)C_k}{\{R+(-\delta \pm j\omega_s)L\}(-\delta \pm j\omega_s)C_0}\right]^{1/2}} \cdot \frac{1}{\frac{C_0\{R+2L(-\delta \pm j\omega_s)\}}{\{1+C_k(-\delta \pm j\omega_s)(R+L)\}^2}} \tag{21}$$

ω_s is expressed as

$$\omega_s^2 = \frac{(m\pi)^2}{\ell^2 LC_o + m^2\pi^2 LC_k} - \frac{R^2}{4L^2} \tag{22}$$

For both cases, as R tends to zero, the solutions of (16) and (20) will be the same as that the solutions of (9) and (12) which are the dispersive transmission lines without loss.

3. NUMERICAL MODEL OF DISPERSIVE TRANSMISSION LINE

The wave propagation of an arbitrary waveform on a dispersive transmission line is a time-dependent problem. Such problems can usually be simplified and solved as a unidimensional problem in time and space. The finite element method requires that the spatial field can be divided into a number of elements and discretized by means of the Variational method or Galerkin approach [16]. The system matrix equations obtained can be then solved by Newmark-β method or the Runge-Kutta method. If the problems are related to nonlinear properties, then the Newton-Raphson or Relaxation methods can be used to solve the nonlinear system matrix equations.

3.1 Lossless dispersive transmission line

Using the Galerkin method, equation (3) for the one dimensional case can be written as

$$G_i = \int_0^{\ell}\left(\frac{\partial^2 u}{\partial t^2} - C_S^2\frac{\partial^2 u}{\partial x^2} - \lambda_D^2\frac{\partial^4 u}{\partial t^2 \partial x^2}\right)N_i dx = 0 \tag{23}$$

where N_i is a shape function, the matrix equation for a single element is obtained as

$$[M]^e\{\ddot{u}\}^e + C_S^2[S]^e\{u\}^e + \lambda_D^2[S]^e\{\ddot{u}\}^e = \{0\} \tag{24}$$

where $(\cdot\cdot)$ indicates the derivative with respect to time. [M] and [S] are

$$[M]^e = \frac{\ell^e}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{25}$$

$$[S]^e = \frac{1}{\ell^e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{26}$$

where l^e is the length of element. Then we use the Newmark-β method to solve for increasing time steps [17]. The system matrix equation will be written as for the Newmark-β method.

$$(\lambda_D^2[S] + [M])\{\ddot{u}\} + C_S^2[S]\{u\} = \{0\} \tag{27}$$

A comparison between numerical solutions and analytical solutions is given in Fig. 5 where the element number is 20 and the number of nodes is 21. The initial voltage at all *i* nodes *u(i)=0 at* t=0, and the boundary condition at terminal *u(21)=0* for the short-circuited case and ∂u/∂x=0 for the open circuit. To simplify the problem the mutual inductance is not considered. Clearly, the numerical solution is in good agreement with the analytical solution. In addition, it was found that λ_D, the time step Δt and the element length Δx must be related by the conditions to ensure a stable solution.

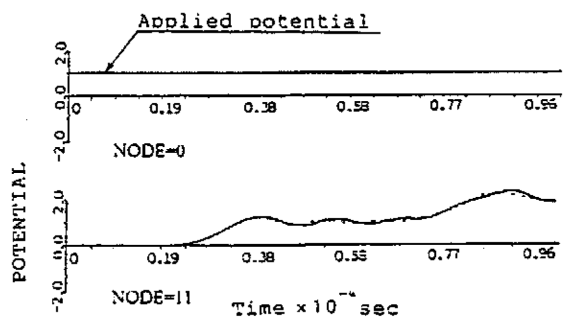
$$\Delta x / \Delta t \rangle C_s \tag{28a}$$

and

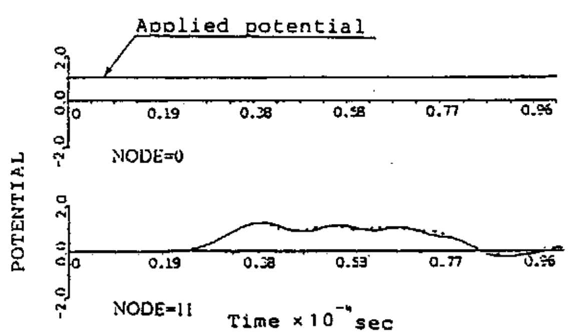
$$\lambda_D \langle \Delta x \langle 3\lambda_D \tag{28b}$$

where Δt is the time step size, Δx is the element length and C_S is wave propagation velocity. λ_D is the Debye shielding length [13] or ratio of lumped capacitors in the section of dispersive transmission line.

The error control of time-dependent problems has been discussed in many numerical method books and articles, but the numerical solution of a dispersive wave equation does not appear in those books and articles. To obtain a convergent solution with minimum error, we found that the normal wave equation only needs condition (28a). For the dispersive wave equation, the numerical solution must not only satisfy (28a), but (28b) as well. The physical meaning of Eq. (28a) is that the wave must not propagate more than one subdivision in space during one time step. Eq. (28b) means that the element length must be chosen between λ_D and $3\lambda_D$ [14]. Since this transmission line is a homogeneous problem, to gain minimum error from (28a) and (28b) the element number was found to be 20 elements.



(a) Terminal is open



(b) Terminal is short circuited

Fig. 5. Comparison of numerical solutions with step wave excitation. The continuous line is the analytical results and the dashed line indicates the numerical results.

The parameters used in calculation are the element number = 20, C_S = 5.68 cm/sec, λ_D=3.29cm, and *l*=95.2cm.

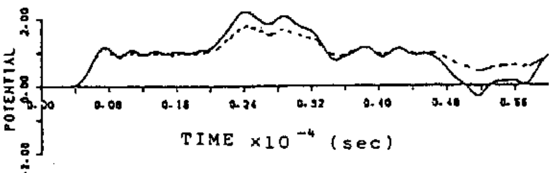
3.2. Dispersive transmission line with loss

Using the same procedure, the single element matrix equation for a dispersive transmission line with loss is obtained as follows:

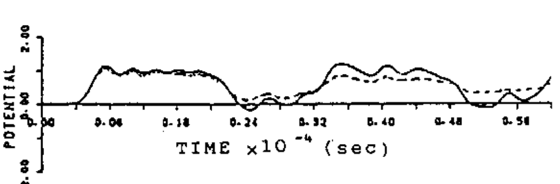
$$[M]^e\{\ddot{u}\}^e + C_S^2[S]^e\{u\}^e + \lambda_D^2[S]^e\{\ddot{u}\}^e$$
$$+\alpha^2[M]^e\{\dot{u}\}^e + \alpha^2\lambda_D^2[S]^e\{\dot{u}\}^e = \{0\} \tag{29}$$

where [M] and [S] have the same details as (25) and (26). Thus, the system matrix equations can be written as eq. (30). The same numerical procedure as above is used to solve the system matrix equations.

$$([M] + \lambda_D^2[S])\{\ddot{u}\} + (\alpha^2[M] + \alpha^2\lambda_D^2[S])\{\dot{u}\}$$
$$+C_S^2[S]\{u\} = \{0\} \tag{30}$$



(a) Terminal is open, the continuous line is for no loss, the dashed line is the loss case.



(b) Terminal is short circuited, the continuous line is for no loss, the dashed line is the loss case.

Fig. 6. Dispersive transmission line with loss. element number=20, Cs=5.68cm/sec, λ_D=52.91cm, α= 175.25sec, and l=95.2cm.

Fig. 6 shows the results of eq.(30) solved by Newmark-β method. The results indicate that the wave is not damped significantly during propagation, even at the present value of parameter (α= 175.25sec) used in the calculation.

3.3 Inhomogenous dispersive transmission line

Sometimes it is of interest to consider inhomogenous problems in a continuous transmission line with finite length. The distributed parameters of the transmission line for each section may be different, but this does not result in a significant difference in the solution. The matrix equation for inhomogenous problems can be written as

$$[M]^e\{\ddot{u}\}^e + \left(\frac{C_k(n)}{C_0(n)}[S]^e\right)\{u\}^e$$
$$+\left(\frac{1}{L(n)C_0(n)}[S]^e\right)\{\ddot{u}\}^e = \{0\} \tag{31}$$

where the parameters of $C_0(n)\cdot\Delta l$, $L(n)/\Delta l$, and $C_k(n)/\Delta l$ are dependent on each section, and n is the section number of an inhomogenous transmission line.

4. NUMERICAL MODEL OF NONLINEAR DISPERSIVE TRANSMISSION LINE

Nonlinear dispersive transmission lines have been used for describing various physical phenomena. In particular, the soliton wave propagating in a plasma is governed by the well studied Kortewg-de Vries (KdV) equation has been investigated [18,19], and the numerical solution of the KdV equation has been proposed by Tadahiko Kawai etc. [20]. In other practical applications in which $L(i')$ and $C_0(u)$ have various characteristics, the KdV equation can no longer be used. One must then find a numerical model which can be used with various nonlinear parameters. The matrix equations of nonlinear dispersive transmission lines for each element are proposed as follows:

4.1 Dispersive transmission line with nonlinear inductance

For simplifying the problem, the lossless dispersive transmission line shown in fig. 7 is considered, where $L(i')$ is a nonlinear inductor dependent on the current or the voltage difference between two points of each section.

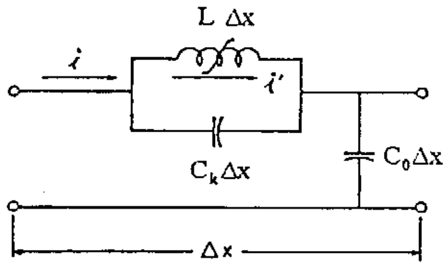


Fig. 7. A section of dispersive transmission line with nonlinear parameter L

The matrix equation for the dispersive transmission line with nonlinear inductance problem can be written as

$$[M]^e\{\ddot{u}\}^e + \left(\frac{C_k}{C_0}[S]^e\right)\{u\}^e + \left(\frac{1}{L(i')C_0}[S]^e\right)\{\ddot{u}\}^e = \{0\} \qquad (32)$$

where L(*i*') is a nonlinear inductance related to current *i*'.

4.2 Dispersive transmission line with nonlinear capacitance

In the same consideration, the lossless dispersive transmission line can be described as fig.8, where $C_0(u)$ is a nonlinear capacitor dependent on the voltage u. The matrix equation for the dispersive transmission line with nonlinear capacitance problem can be written as

$$[M]^e\{\ddot{u}\}^e + \left(\frac{C_k}{C_0(u)}[S]^e\right)\{u\}^e + \left(\frac{1}{LC_0(u)}[S]^e\right)\{\ddot{u}\}^e = \{0\} \qquad (33)$$

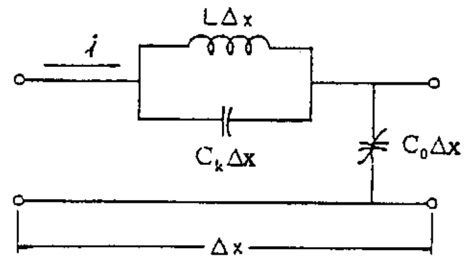


Fig. 8 A section of dispersive transmission line with nonlinear parameter $C_0(u)$

Equations (32) and (33) can be solved by using the Newton-Raphson method or the decelerating relaxation method which has been often used in the nonlinear electromagnetic fields analysis [21,22].

5. APPLICATIONS OF DISPERSIVE TRANSMISSION LINE

As we discussed in the introduction, the dispersive transmission line can be used for modeling various physical phenomena and practical application problems. In this section, dispersive transmission lines are used to investigate wave propagation in a plasma and the transient response of a surge in transformer windings.

5.1 Dispersive wave propagation in plasma

It is well known that the electron plasma waves and ion-acoustic waves are electromagnetic waves which propagate in nonmagnetized plasmas. Since the characteristic frequency of the ion-acoustic wave is lower than the ion-plasma frequency (Ω_{pi}), both electrons and ions participate in this wave motion. The ion-acoustic wave is not strongly damped only when the ion temperature T_i is much lower than the electron temperature T_e. The ion-acoustic wave equation for the perturbed ion density n is defined as follows [23]:

$$\frac{\partial^2 \tilde{n}_i}{\partial t^2} - C_S^2 \frac{\partial^2 \tilde{n}_i}{\partial x^2} - \lambda_D^2 \frac{\partial^4 \tilde{n}_i}{\partial t^2 \partial x^2} = 0 \qquad (34)$$

where C_s and λ_D are the ion-acoustic wave velocity and Debye shielding length respectively.

$$C_s = (T_e / M)^{1/2} \qquad (35)$$

$$\lambda_D = (T_e / 4\pi n_0 e^2)^{1/2} \qquad (36)$$

T_e, M and n_0 are the electron temperature, mass of ion and steady component of the ion density respectively.

Assuming $u = \tilde{n}_i$, the dispersive transmission line can be used to describe the ion-acoustic wave in the plasma. Hence, the C_s and λ_D can be replaced as

$$C_s = (T_e / M)^{1/2} = (1 / LC_0)^{1/2} \qquad (37)$$

$$\lambda_D = (T_e / 4\pi n_0 e^2)^{1/2} = (C_k / C_0)^{1/2} \qquad (38)$$

where $C_0=1/T_e$, $L=M$ and $C_k=1/4\pi n_0 e^2$.

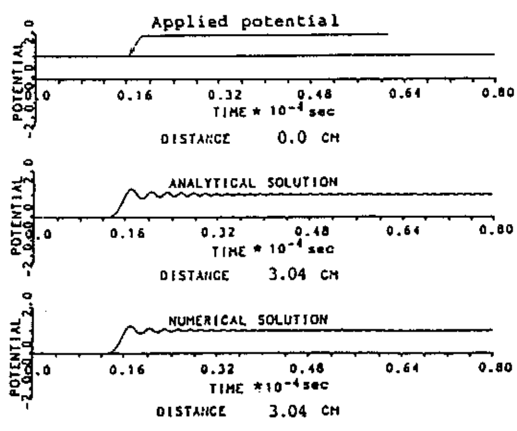


Fig. 9 Step wave response. $\lambda_D=4\times10^{-2}$cm, Cs=$2\times10^{+5}$cm/sec, l=40cm, element number=500.

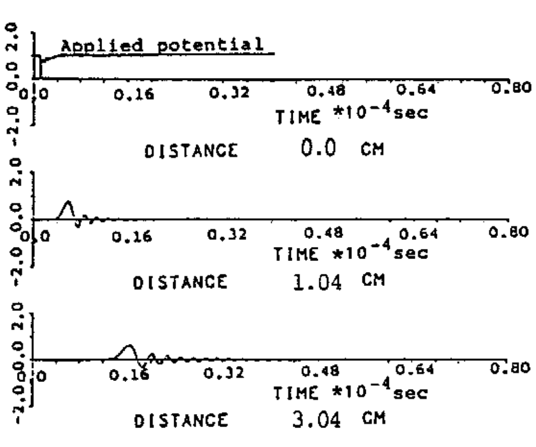
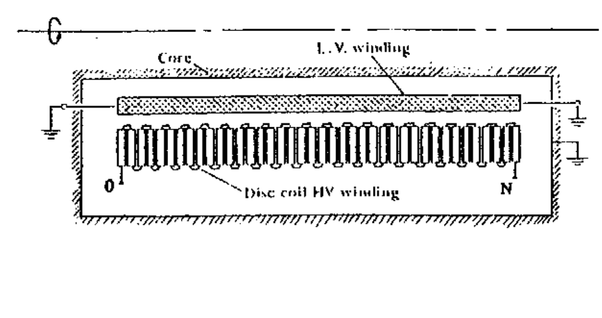


Fig. 10. Pulse response, $\lambda_D=4\times10^{-2}$cm, Cs=$2\times10^{+5}$cm/sec, l=40cm, element number=500.

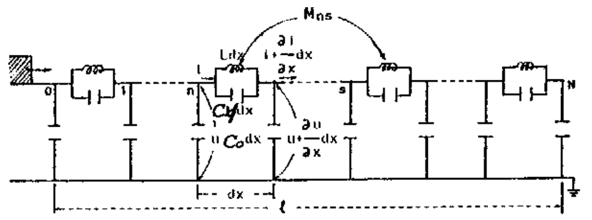
Using the same numerical approach of the previous section, the ion-acoustic wave of the plasma with different input pulses can be obtained [14]. These are illustrated in Fig. 9 and Fig. 10. From Fig.9, we found that the numerical solution has a good agreement with the analytical solution from (12). There are some phase shift between them after several oscillations, which can be considered as a computer processing error. Fig. 10 shows the pulse response in plasma. The comparison between numerical and experimental results has made for the pulse response in plasma [14].

5.2 Dispersive wave propagation in high-voltage (HV) transformer windings

In order to choose a proper winding arrangement and insulation structures in the design of HV transformer windings, the transient voltage stresses to all sub-components of the structure must be known. To investigate voltage oscillations and impulse-voltage stresses in HV transformer windings during impulse test and design reliable insulation structure for the HV transformer windings which can withstand various kinds of transient over voltage, the most convenient and lowest cost method of acquiring transient voltage response data is using a numerical model of the transformer windings and solving for the time function response to applied voltage pulses by means of a suitable numerical analysis. Normally, the impulse response and produced higher transient voltage in the transformer windings can be calculated by using the distributed equivalent circuit of transformer windings as a transmission line has been used as shown in Fig. 11 [24,25]. To simplify the problems the case of the mutual and nonlinear inductance is not considered in this calculation.



(a) Cross-section of a typical 2-winding H.V. transformer

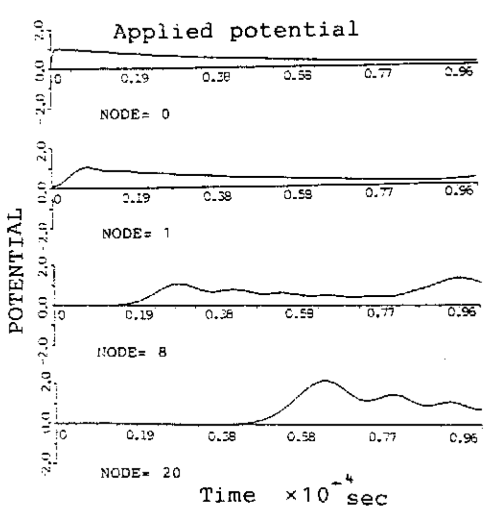


(b) Distributed equivalent circuit of transformer winding

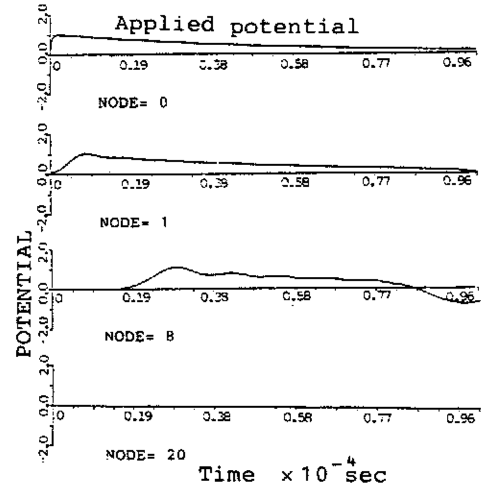
Fig.11 Numerical model of H.V. transformer windings, Element number=20, $L=4.5mH/\Delta l$, $C_o=67.5pF\cdot\Delta l$, $C_k=383.5pF/\Delta l$, and $l=95.2cm$.

The impulse voltage oscillations caused by resonant circuit which is excited by any impulse can be calculated by numerical or analytical method. The voltage oscillations in the transformer windings is a kind of dispersive wave propagation and the frequency of oscillations and amplitude of transient voltages can be calculated by eq. (9) and (12) where the applied impulse is a step wave.

For the other different shapes of impulse voltage (including the IEC standard lightning impulse voltage waveform [26]) and the inhomogenous distributed equivalent circuit of transformer winding, the calculation can be easily done by numerical methods. The numerical results for transient responses from various applied impulses are shown in Fig. 12 [15].



(a) Terminal is open



(b) Terminal is Short-circuited

Fig. 12. Standard lightning impulse voltage response, element number=20, $L=4.5mH/\Delta l$, $C_0=67.5pF\cdot\Delta l$, $C_k=383.5pF/\Delta l$, and $l=95.2cm$.

The effect of dispersion is an oscillating voltage which is up to 1.5 times the input impulse voltage in the case of shorted-circuit and the oscillating voltage will be larger than 2.0 times the input impulse voltage at the opened terminal, if the initial voltage is not zero. Alternatively, for the chopped waveform, there is no significant transient overvoltage along the transformer windings [15].

6. CONCLUSIONS

In this paper, dispersive wave propagation in various transmission line configurations is discussed. Basic analytical solutions with step wave excitation were found and compared to finite element solutions. The results show good agreement. The error control of numerical solution can be worked out by using eq. (28a) and (28b). The numerical model has advantages in dealing with some practical systems excited by different excitation waveforms, especially for inhomogenous, lossy and nonlinear problems. Appropriate numerical models have been proposed. Although while only two applications were modelled by dispersive transmission lines, there are many more interesting phenomena which can be modeled using these techniques.

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