Theoretical and Experimental Investigations of the Surge Response of a Vertical Conductor

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Abstract— **This paper describes the theoretical, simulation and experimental investigations of the surge response of a vertical conductor, including the effects of ground surface and without ground surface. One of the authors of this paper derived the formula of the surge impedance in case with ground surface and without ground surface. In this research, these theoretical formulas are examined by the simulation analysis of the vertical conductor using the Numerical Electromagnetic Code (NEC-2) as well as the experimental basis. The arrangement of the current lead wire in the vertical conductor model to be analyzed here is verified with the simulation result of the equivalent circuit model by the Electromagnetic Transients Program (EMTP).**

Keywords— **EMTP, Lightning surge, Numerical electromagnetic field analysis, Tower surge impedance, Vertical conductor.**

I. INTRODUCTION

PREDICTION of lightning surges is very important for the design of electric power systems and telecommunication systems. In particular, tower surge impedance is an important factor in analysis of the lightning performance of transmission lines. Therefore, a number of experimental and theoretical studies on tower surge impedance have been carried out [1]–[11].

The first theoretical formulation of tower surge impedance was proposed by Jordan [1]. The tower was approximated as a vertical cylinder having a height equal to that of the tower, and a radius equal to the mean equivalent radius of the tower. Theoretical formulations of tower surge impedance based on the electromagnetic field theory were proposed by Lundholm *et al.* [2], Wagner and Hileman [3], Sargent and Darveniza [4] and Okumura and Kijima [5], considering effects of the vector potential generated by the injection current into the tower only.

Another experimental value for actual transmission towers was reported by Kawai [6]. He used a direct method to measure tower surge impedance. His experimental results showed that the tower response to a vertical current is different from the response to a horizontal current. Scale-model measurements were reported by Chisholm [7], [8] and Wahab *et al.* [9]. These measurements results showed that the tower surge impedance is strongly influenced by the angle of current injection.

Recently, theoretical work was reported by Ishii and Baba [10]. They estimated the surge response of a tower by numerical electromagnetic field analysis. The calculated results were compared with the field test results [11]. The analysis showed that surge response and surge impedance of the tower depend on the arrangement of the current lead wire.

One of the authors derived the formula of surge impedance with ground surface: $Z = 60\cdot\{ln(2\sqrt{2}h/r_0) - 1.983\}\,(\Omega)$ and without ground surface: $Z = 60\cdot\{ln(2\sqrt{2}h/r_0) - 1.540\}\,(\Omega)$ [12]. The theoretical formula of surge impedance with the ground surface is very close to the well known experimental formula of Hara *et al.* [13]. In this paper, we investigate the surge impedance of the vertical conductor on the basis of experimental and simulation analysis. These analysis results agree satisfactorily with the theoretical values.

II. METHOD OF ANALYSIS

For the present analysis, the Numerical Electromagnetic Code (NEC-2) is employed. It is a widely used three-dimensional electromagnetic modeling code based on the method of moments [14] in the frequency domain, and is particularly effective in analyzing the electromagnetic response of antennas or of other metallic structures composed of thin wires. A vertical conductor system needs to be decomposed into thin wire elements, and the position, orientation and the radius of each element constitute the input data, along with the description of the source and frequencies to be analyzed. In the analysis, all the elements in the systems are treated as perfect conductors. To solve the time-varying electromagnetic response, Fourier transform and inverse Fourier transform are used.

The validity of the computed results when NEC-2 is applied to the analysis of surge response of a vertical conductor has been verified by comparing with experimental results. In the simulation and experimental analysis, a reduced-scale model is chosen in order to make the experiment simple and flexible. However, it is not possible to achieve the same accuracy as with the full-scale model, especially in simulating the direct method, since the geometrical size of the measuring devices is large relative to the whole system.

In the lightning surge analysis with Electromagnetic Transients Program (EMTP), the vertical conductor model has been represented by an equivalent circuit of the transmission-line type since it can be easily interfaced with EMTP. Then, the basic parameters are its surge impedance, the travelling wave propagation velocity, and the attenuation and deformation characteristics of the travelling wave. Of these, the attenuation and deformation characteristics of the travelling wave determine the reflected wave from the base of vertical conductor; that is, the wave tail after its peak in the vertical conductor's potential-rise waveform.

III. THEORETICAL FORMULA OF SURGE IMPEDANCES

One of the author's theory is able to apply widely in case of ground surface and without ground surface. Suppose that the surge electric current invades to the vertical conductor whose height is h and radius is r_0. Then the surge current wave is reflected at the ground of the perfect conductor and returns to the top of the vertical conductor.

Introducing the electric current reflectivity $\beta = 1$ and the magnetic field reflectivity $\gamma(\gamma_i, \gamma_r) = 0$, the theoretical formula of surge impedance which is very close to the well known experimental formula [13] is obtained as follows;

$$Z = 60 \cdot (\ln(\frac{h}{2r_0}) - \frac{1}{4})$$

$$= 60 \cdot (\ln(\frac{2\sqrt{2}h}{r_0}) - 1.983) \qquad (1)$$

However, if it is considered that $\beta = \gamma_i = \gamma_r = 1$, it became

$$V(t) = \frac{c\mu_0 I_0}{2\pi}\left(\ln\frac{(ct+2r_0)}{2r_0} - \frac{ct}{2(ct+r_0)}\right)$$

The above equation can be modified by substituting $ct = 2h$ and assuming $h \gg r_0$ as follows;

$$Z = 60 \cdot (\ln(\frac{h}{r_0}) - \frac{1}{2})$$

$$= 60 \cdot (\ln(\frac{2\sqrt{2}h}{r_0}) - 1.540) \qquad (2)$$

On the other hand, if there is no ground, the following formula is induced [15].

$$V(t) = \int_0^{ct} (-E_i \cdot dl)$$

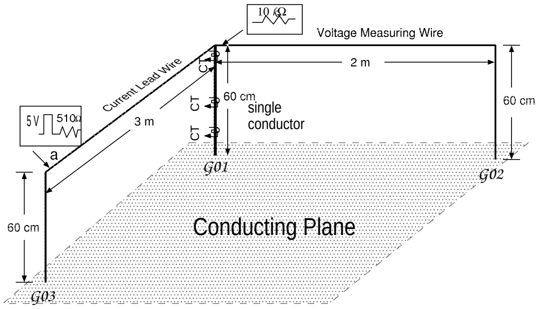
$$= \frac{c\mu_0 I_0}{2\pi}\left(\ln\frac{(ct+2r_0)}{2r_0} - \frac{ct}{2(ct+r_0)}\right)$$

Substituting $ct = 2h$ and assuming $h \gg r_0$ in the above equation, we get

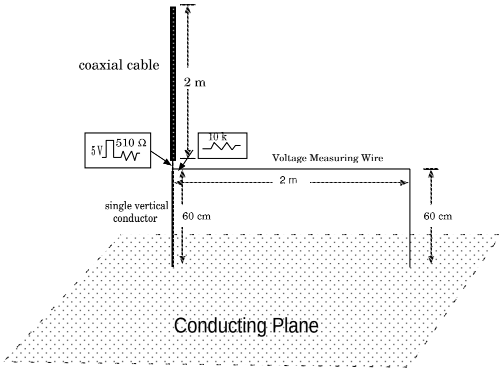
$$Z = 60 \cdot (\ln(\frac{h}{r_0}) - \frac{1}{2})$$

$$= 60 \cdot (\ln(\frac{2\sqrt{2}h}{r_0}) - 1.540) \qquad (3)$$

This formula given by (3) is the same as (2).



(a) With ground surface.



(b) Without ground surface.

Fig. 1. Arrangement of the vertical conductor system.

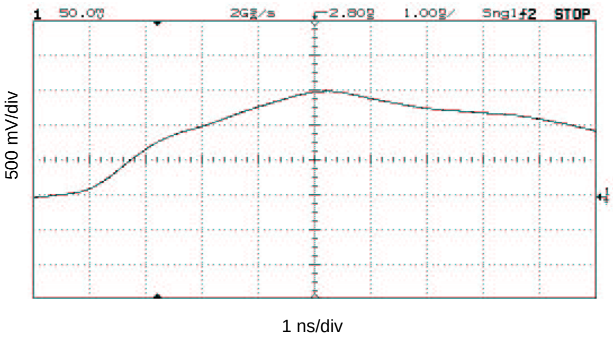
TABLE I

SPECIFICATIONS OF MEASURING EQUIPMENT

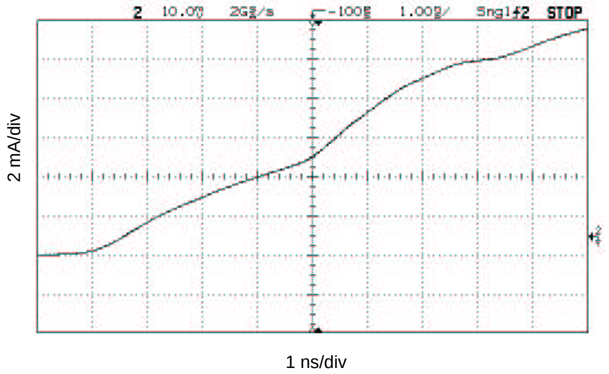
	Equipment	Frequency	Sensitivity
Current sensor	Tektronix CT-1	25kHz∼ 1GHz	5mV/mA ± 3% into 50Ω load.
Voltage sensor	Tektronix P6243	DC∼ 1GHz	10:1
Power supply	Tektronix 1103	40∼ 440Hz	± 5VDC ± 2%
Recording equipment	HP54616B	DC∼ 500MHz	2Gsa/s, 8 bit word
Pulse Generator	HP8131A	DC∼ 500MHz	100mV∼ 5Vpp into 50 ohm

IV. EXPERIMENTAL AND SIMULATION ANALYSIS OF SURGE IMPEDANCE

Fig. 1 shows the reduce-scale model of the vertical conductor system for the simulation analysis. The arrangement of the current lead wire connected to the top of the vertical conductor with the existence of the ground surface and without ground surface are indicated in Fig. 1(a) and 1(b) respectively. Whereas, Fig. 1(b) is also the case of the lightning phenomena caused by the return stroke [16]. The other type of the lightning phenomena caused by a downward travelling current wave can also be examined. For the simulation of this situation, a pulse current generator needs to be placed remotely above the

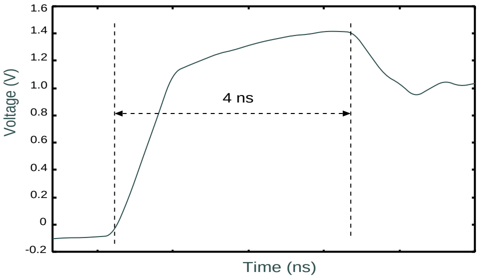


(a) Measured voltage waveform.

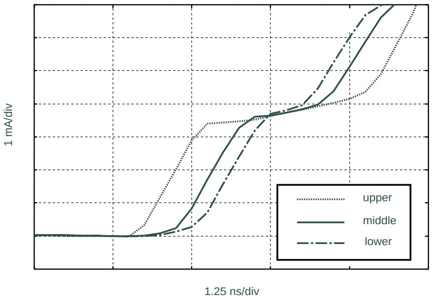


(b) Measured current waveform.

Fig. 2. Experimental results of voltage and current considering with ground surface.



(a) Computed voltage waveform.



(b) Computed current waveforms.

Fig. 3. Computed waveforms of voltage at the top and currents in the various parts of the vertical conductor in case with ground surface.

channel. However, the computed waveforms for this case of current injection, a current wave travelling down the current lead wire, are similar to those of return stroke type.

The experimental arrangement of the vertical conductor system with the existence of ground surface is a little different from the simulation arrangement and is indicated in Fig. 5 of [15]. However, the experimental arrangement without ground surface is similar to Fig. 1(b). A voltage measuring wire of 200 cm in length is placed perpendicular to the current lead wire and is connected to the top of the vertical conductor which is 60 cm in height and radius of 0.05 cm. The ends of the horizontal voltage measuring wire in both cases are stretched down and connected to the ground through matching resistance. This termination condition does not affect the phenomena at the vertical conductor within 17.33 ns.

A step current pulse generator having pulse voltage of 5 V in magnitude, rise-time of 1 ns and pulse width of 40 ns is installed as indicated in both cases which is meant to incorporate the influence of the induction from the lightning channel hitting the vertical conductor. For the simulation analysis, and to save the computation time, the conductors of the system are divided into 10 cm segments. To evaluate the voltage of the top of a structure, 10 kΩ resistance was inserted between the top of the structure and the end of the voltage measuring wire. The voltage at the top of vertical conductor is measured by a voltage probe with high resistance and low capacitance(1MΩ and ¡1pF). The injection current is measured by current transformer. The specifications of the measuring equipment are shown in Table I. The waveform of current flowing through the vertical conductor is also obtained from the experiment and simulation analysis. The system of structures under those analysis was postulated to be on the perfectly conducting ground. Then we calculate the surge impedance which is defined by the ratio of the instantaneous values of the voltage to the current at the moment of voltage peak.

As the pulse applied to the current lead wire according to Fig. 1(b), the current starts flowing through the vertical conductor instantly. However, for the arrangement of Fig. 1(a), the current through the vertical conductor is delayed by the round-trip time of the travelling wave in the conductor. While in both cases, the reflection wave from the ground reaches the top of the vertical conductor at $t = 2h/c$, where c is the velocity of light. And that is why the maximum potential of the vertical conductor will occur at time $t = 2h/c$.

A. *With Ground Surface*

Considering with Fig. 1(a), we want to find the voltages and currents experimentally and with the simulation by the NEC-2. Fig. 2 shows the experimental results of the voltage and current with the ground sur-

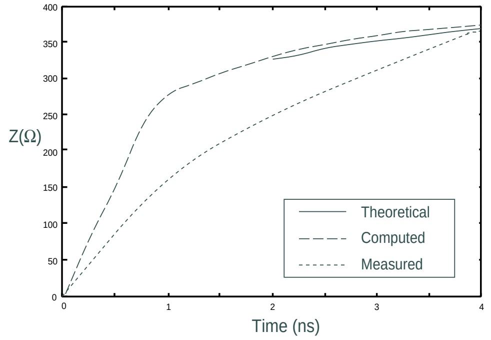
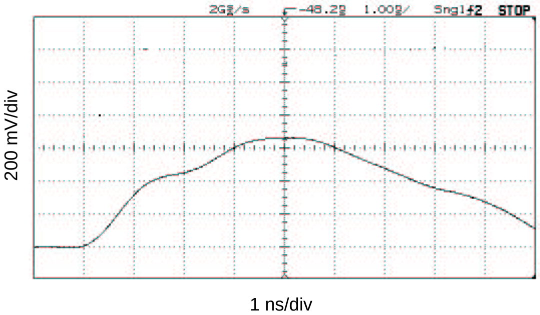
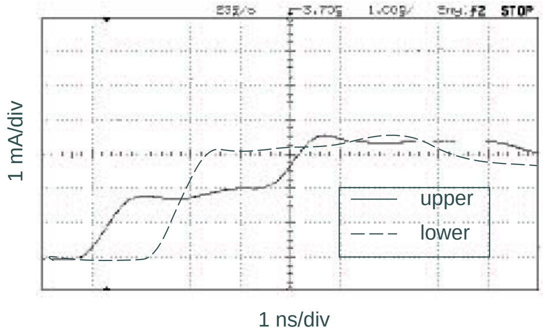


Fig. 4. Surge impedances of the vertical conductor with the ground surface at $0 < t \leq 2h/c$.

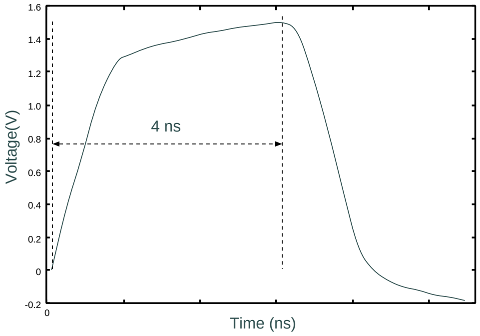


(a) Measured voltage waveform.

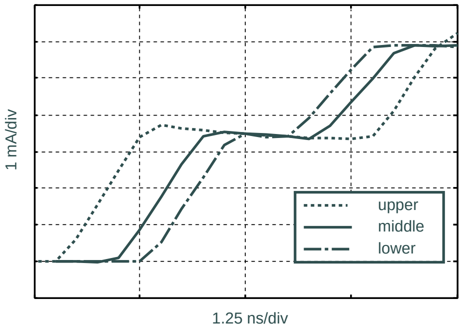


(b) Measured current waveforms.

Fig. 5. Experimental results of voltage and current considering with no ground surface.



(a) Computed voltage waveform.



(b) Computed current waveforms.

Fig. 6. Computed waveforms of voltage at the top and currents in the various parts of the vertical conductor in case with no ground surface.

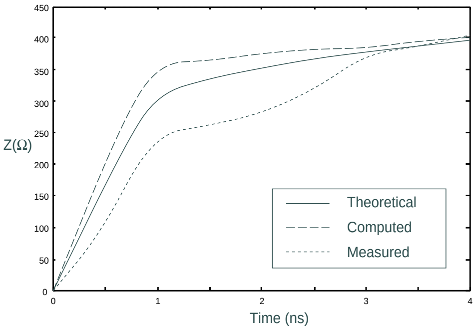
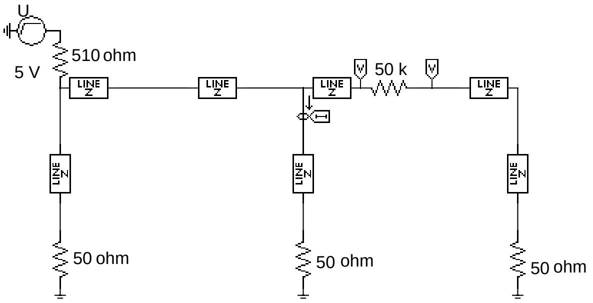
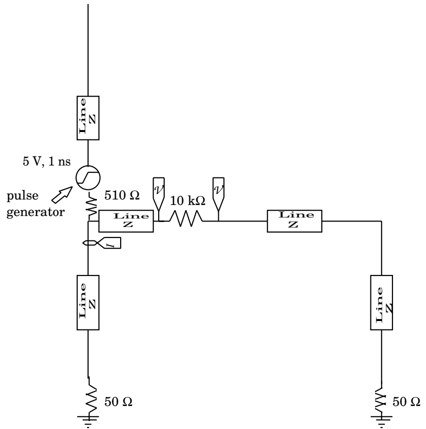


Fig. 7. Surge impedances of the vertical conductor at $0 < t \leq 2h/c$ with no ground surface.

face. Fig. 3 shows the simulation results by the NEC-2. In Fig. 3(a), the influence of the reflected wave from the ground reaches the top of the conductor is observed at $t = 2h/c = 4$ ns exactly which means that the travelling wave is propagating at the velocity of light. Fig. 3(b) shows the computed waveforms of current flowing through the vertical conductor as indicated by the mark 'CT' in Fig. 1(a). As the pulse generator is placed 300 cm from the vertical conductor, the current through the vertical conductor is being delayed approximately 10 ns. The waveforms start rising after 10 ns which can be noticed from Fig. 3. The existence of the ground surface can be observed in Fig. 3(b), where the field produced by the current injected horizontally induce current of small magnitude before the actual surge current flowing through the vertical conductor. These simulation results of currents in Fig. 3(b) obtained by the NEC-2 exactly coincide with the experimental results [15]. Then, we compare the theoretical value of the surge impedance considering the ground surface given by the (1) with the simulation and experimental results of that. Fig. 4 shows that the vertical conductor surge impedances. The theoretical values of surge impedance calculated by using (1) is just after the surge electric current reaches the ground and produce reflected current wave. As we need to know surge impedance at $t = 2h/c = 4$ ns. In these results,



(a) With ground surface.



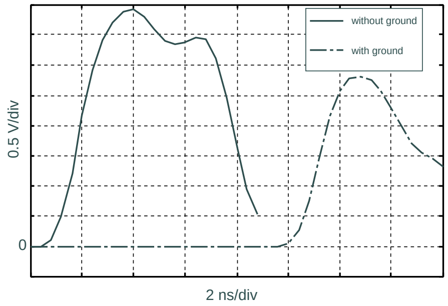
(b) Without ground surface.

Fig. 8. Equivalent circuits of the vertical conductor models.

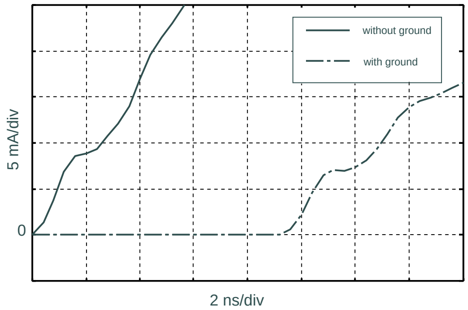
theoretical, computed and experimental values of surge impedances are approaching closely at $t \approx 2h/c$.

B. *Without Ground Surface*

Fig. 5 shows the experimental results of the voltage across the voltage measuring wire and currents through upper and lower parts of the vertical conductor in the absence of ground surface. The simulation results of voltage at the top of the vertical conductor and currents through different parts of it are shown in Fig. 6. However, in this case of analysis, the waveforms of current through the vertical conductor are somewhat different from Fig. 3(b) at the starting region because of absence of the ground surface. Also, the current starts flowing instantly through the vertical conductor without being delayed. Finally, the theoretical, computed and measured values of surge impedances are shown in Fig. 7. Here also we see that the values of surge impedances approach closely at $t \approx 2h/c$.



(a) Voltages across the voltage measuring resistance.



(b) Currents through the vertical conductor.

Fig. 9. Simulation results of voltages and currents by the EMTP at $0 < t \leq 2h/c$.

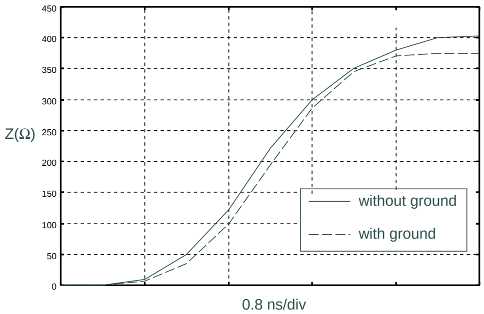


Fig. 10. Simulation results of surge impedances by the EMTP at $0 < t \leq 2h/c$.

V. VERTICAL CONDUCTOR MODELS FOR EMTP ANALYSIS

The Electromagnetic Transients Program (EMTP) is probably the most widely-used power system transients simulation programs in the world today. In this section, the EMTP simulations based on the circuit theory were perfomed for the vertical conductor model with ground and without ground surface. In the circuit model, the line was represented by a distributed R-L-C circuit with the skin effect being neglected. The NEC-2 cannot exactly model the structures of actual towers or vertical conductors, and in addition it cannot directly interfaced with the EMTP. For the EMTP simulations, therefore, it is practical to employ an equivalent circuit of the transmission-line type for representing the vertical conductor system. In developing the model or in determining its parameters, characteristics stated in the preceding sections should be taken into consideration. In this section, vertical conductor models used so far are reviewed with emphasis on their performance in reproduction of measured waveforms of current through the vertical conductor and voltage at the top of it. The surge impedance is then calculated from the ratio of the maximum potential at the conductor top to the current through it at the time of voltage peak. Fig. 8 shows the equivalent circuit representation for the vertical conductor system considering with the existence of ground surface and without ground surface. The dimensions of these circuit models are the same as considered for the simulation and experimental model systems of Fig. 1. The voltage sensors and the current sensors indicated in Fig. 8 represent the measuring points. The surge impedances of the distributed line are used as the input data for the EMTP analysis, and are of different values depending on the height of the conductors. As the analysis with EMTP, it can be easily handled to the horizontal conductor but cannot be handled just as it is to the perpendicular conductor. Therefore, to solve the problem, the perpendicular conductor can be divided into the horizontal conductors as it makes a center level at the axis in each conductor.

Fig. 9 shows the simulation results of voltages and currents with the existence of ground surface and without the ground surface by the EMTP analysis for the equivalent circuit representation of Fig. 8. The solid lines of Fig. 9 correspond to circuit representation of Fig. 8(b) and the chain lines for the Fig. 8(a). The starting time of current flowing through the vertical conductor depends on the position of the pulse generator. As the pulse is injected at 300 cm from the vertical conductor with ground surface as in Fig. 8(a), the currents start flowing after 10 ns of currents through it in case with no ground surface. The occurrence of the reflection can also be observed in Fig. 9. Then the EMTP results of the surge impedances of the vertical conductor with ground surface and without ground surface are shown in Fig. 10.

TABLE II

Surge Impedances of the Vertical Conductor at $t \approx 2h/c$

	With ground	Without ground
Theoretical	368	395
Computed	373	402
Measured	366	406
EMTP	375	403

Theoretical, experimental and computed by the NEC-2 and EMTP results can be summarized in the Table II at $t = 2h/c$ so as to make quantitative evaluation. The surge impedance for the ground surface is naturally much lower than without ground surface that can also be realized by the (1) and (2). The theoretical values of surge impedance agree well with the computed and experimental values.

VI. CONCLUSIONS

The theoretical values of surge impedances are verified by comparing the computed and experimental results on simple structures. The difference is less than about 5%, which is within the accuracy maintained in the analysis. Also, the travelling wave propagates at nearly the velocity of light. The surge characteristics have some influence on the type of the lightning current with the presence of ground surface and without the ground surface. The difference comes from the different electromagnetic field around the vertical conductor influenced mainly by the electric fields associated with the currents propagating the vertical conductor and current lead wire. Also the restriction of the size of the perfectly conducting ground plane and the effect of the voltage probes might cause small difference in the experimental results of voltage and current waveforms.

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