Relationship Between the Path Loss Exponent and the Room Absorption for Line-of-Sight Communication

[1] C. W. Trueman, [1, 2] D. Davis, and [2] B. Segal

[1] Department of Electrical and Computer Engineering, Concordia University, Montreal, Canada
trueman@ece.concordia.ca, megadon_emc@yahoo.com

[2] McGill University and SMBD Jewish General Hospital, Montreal, Canada
b_segal@sympatico.ca

Abstract **–** In indoor propagation, the log-distance path loss model represents the received power as declining with distance from the transmitter according to $1/r^n$, where r is the straight-line distance from the transmitter to the receiver. Previously, the value of the path loss exponent n has been derived from measured received signal strengths at a specific site. In this paper, the value of n is estimated from the geometry of the room and the electrical properties of the walls. Using the Sabine model, these determine the room absorption and hence the received power as a function of distance from the transmitter. Then, a least-square-error curve fit of the log-distance path loss model to the Sabine model determines the value of n. The electric field strength in a typical rectangular room is compared using ray tracing, the Sabine model, and the path loss model. Then the value of the path loss exponent is presented as a function of the power absorption coefficient of the walls, floor and ceiling of the room, for a typical ceiling height. Evaluating n from analytic information rather than from measurement enhances the usefulness of the path loss model in simulations of the coverage of antennas for the design of wireless local area network installations at specific sites.

I. INTRODUTION

In indoor propagation, communication must be established between a transmitter and a receiver located inside a building [1-3]. For a fixed transmitter position and a roaming receiver, the signal strength of the transmitter must be sufficiently large; the delay spread of the multipath components sufficiently small; and the interference from other transmitters operating on the same frequency sufficiently small. Designing the location of access-point antennas for a wireless local area network would benefit from a simple method for an approximate assessment of the field strength of each antenna throughout the whole floor plan.

The "log-distance path loss model" [1, 3, 4, 5] represents the received power in an indoor environment as declining with distance according to,

$$P(r) = \frac{P_0}{r^n} \quad (1)$$

where n is the "path loss exponent" [6] or "slope index" [4], r is the distance between the transmitter and the receiver, and P_O is the received power at a one-meter distance. The value of n depends on the construction of the walls of the room and on other factors. The path loss model is applied to both line-of-sight (LOS) and non-line-of-sight (NLOS) scenarios. If the ray from the source to the observer passes through a wall, the power can be reduced by a "wall attenuation factor", which is often approximated as a fixed number of dB independent of the incidence angle or polarization. This model is empirical, with the value of n determined from measured received powers. Values of the path loss exponent n are cited from the literature for various environments in [1]. Values of n from 1.6 to 2.1 for factory environments were given in [5], where there was a LOS path from the transmitter to the receiver. Where the LOS path is obstructed by partitions or by furnishings, values of n greater than two were used, and the field strength decreased more quickly with distance r than it would in free space. Some authors use free-space propagation ($n=2$) closer to the antenna than a "break point" distance, and the log-distance path loss model for larger distances [6]. The break point distance depends on the size of the first Fresnel zone [4, 5] compared to the position of obstacles in the room that obstruct the direct path from the transmitter to the observer. The log-distance path loss model is site specific in that the power associated with a ray passing through a wall is reduced by a wall attenuation factor, but otherwise the floor plan information is not used.

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Site-specific predictions of the electric field strength throughout a floor plan are often made using ray tracing [4,7]. Ray paths are identified joining the transmitter to the location of the receiver, accounting for specular reflection from walls, and transmission through walls. The "vector sum method" [2] adds the field strengths of the rays accounting for phase and vector direction. The rapid variations of this "local" field strength as the position of the observer changes are called "fast fading". To assess coverage, it is sufficient to estimate the local mean power, obtained by averaging the received power along a path of length 5 to 40 wavelengths [3]. Averaging removes the rapid variations of fast fading and leaves the slow changes due to attenuation with distance and shadowing, called "slow fading". Evaluating the fast fading at closely-spaced points followed by explicit averaging is computationally expensive. However, the local mean power can be estimated by ray tracing by the "power sum method" [3], which combines the field strengths of the rays on an energy basis. Since the local mean power varies slowly with position, much more widely spaced points can be used, and so the computation is much faster.

The Sabine method is less well known, and is based on Sabine's method in acoustics extended to electromagnetics. The Sabine method characterizes the room by its "room absorption", which is calculated from the angle-averaged power absorption coefficient of each surface of the room [8] and the area of the surface. "Live" rooms with low power absorption use Sabine's formula for the room absorption, but when the absorption is high the room is said to be "dead" and Eyring's formula is used [9,10]. To predict the decline in the received field strength as a function of distance from the source, the field is split into the "direct" field strength, which is the field of the source in free space, and the "indirect" field strength or "multipath" field strength, which is the contribution of the room [11, 12]. The local mean power is obtained by adding the power in the direct field and in the indirect field. The calculation of the local mean power by the Sabine method is simple enough to be done with pencil and paper. The Sabine method is readily extended to complex floor plans [11] and because it is computationally inexpensive, it is useful for assessing the field strength of many sources transmitting at various locations throughout a complex floor plan.

To the authors' knowledge, the value of the path loss exponent n has not been explicitly related to the geometry of the room and to the construction of the walls. This paper will derive the value of n in rooms where there is a LOS path between the transmitter and the receiver. The value of n will be determined from the geometry of the room and the average power absorption coefficients of the various room surfaces. The room properties will determine the Sabine room absorption, which in turn will be used to find the local mean power as a function of distance from the transmitter. Then least squares approximation will be used to curve-fit the log-distance path loss model to the local mean strength, to determine the value of n. The method will be illustrated for a small rectangular room. Electric field strengths using the log-distance path loss model will be compared with fields found by ray tracing using the "power sum method", and using the Sabine method. Then the value of n will be graphed for a square room as a function of the power absorption coefficient of the walls for various room areas from small to large.

II. THE SABINE METHOD

The Sabine method [12] divides the electric field strength into the direct field E_d, which is the field of the transmitter in free space, the and the multi-path field, E_m, which is the net field strength due to rays which reflect and re-reflect from the surfaces of the room. The power received by an antenna of effective area equal to unity and operating into a matched load is,

$$P_s(r) = \frac{1}{\eta}\left(E_d^2(r) + E_m^2\right) \quad \text{watts,} \tag{2}$$

where η is the intrinsic impedance of space and r is the separation distance. Assuming that there is an unobstructed path between the transmitter and the receiver, that is, that the first Fresnel zone is clear of furnishing and clutter [4, 13], the direct field is given by,

$$E_d(r) = \sqrt{\frac{\eta D P_t}{4\pi r^2}} \quad \text{volts/meter} \tag{3}$$

where D is the directive gain of the transmitter, and P_t is the transmitted power. The Sabine method gives the local mean value of the multipath field as [12],

$$\widetilde{E}_m = \sqrt{\frac{4\eta P_t}{A_{in}}} \quad \text{volts/meter} \tag{4}$$

where the tilde indicates the local area average. The "indirect" room absorption [12] is,

$$A_{in} = \frac{A S_T}{S_T - A} \tag{5}$$

where S_T is the total area of the surfaces of the room, and where the room is characterized by the Sabine room absorption, given by,

$$A = \sum_{k=1}^{N} S_k \widetilde{\alpha}_k \ . \tag{6}$$

The room has N surfaces, the area of the k^{th} surface is S_k, and $\tilde{\alpha}_k$ is the angle- and polarization-averaged power absorption coefficient [8, 12]. If the walls of the room are modelled as uniform layered structures, the power absorption coefficient is readily evaluated. If the surfaces of the room are highly absorbing, then the room might be classified as "dead" or in the "non-reverberated regime" in the terminology used in [10], and the Eyring formula can be used to calculate the room absorption according to,

$$A_E = S_T \ln\left(\frac{1}{1-\alpha}\right) \tag{7}$$

where $\alpha = A/S_T$. The Eyring indirect absorption is given by,

$$A_{E,in} = \frac{S_T}{1-\alpha}\ln\left(\frac{1}{1-\alpha}\right). \tag{8}$$

Thus the Sabine model consists of either using equations (5) or (8) to calculate the indirect room absorption, then equation (4) for the multipath field strength, which is by definition constant throughout the room, and equation (3) for the direct field, which varies with distance from the transmitter. Then equation (2) is used to find the received power. Note that functional form of the decline in received power with distance is contained in the direct field term and is different from that of the log-distance path loss model of equation (1).

III. EVALUATING THE PATH LOSS EXPONENT

The Sabine model relates the decline in field strength with distance from the transmitter to the geometry of the room, to the room construction, and to the electrical properties of the room surfaces thorough the average power absorption coefficients. Consider a path running radially away from the transmitter from distance r_a to distance r_b. The value of the path loss exponent will be found by minimizing the mean square error in decibels between the path loss model (1) and the Sabine model (2). The square of the mean square error is given by,

$$e^2 = \frac{1}{r_b - r_a}\int_{r_a}^{r_b}\left(\ln\left(\frac{P_0}{r^n}\right) - \ln\left(P_s(r)\right)\right)^2 dr\ . \tag{9}$$

To minimize the error, choose n such that $\frac{\partial}{\partial n}\left(e^2\right) = 0$ to obtain,

$$n = \frac{1}{\int_{r_a}^{r_b}\ln^2(r)dr}\left[\begin{array}{l}\ln(P_0)\int_{r_a}^{r_b}\ln(r)dr \\ -\int_{r_a}^{r_b}\ln(r)\ln(P_s(r))dr\end{array}\right]. \tag{10}$$

Equation (10) is readily evaluated as follows. Parameter P_0 is the power at r=1 m from the antenna in free space and using equation (3), $P_0 = DP_t/(4\pi)$ W. The integrals in equation (10) can be approximated with the rectangular rule. Thus choose a set of evenly-spaced distances $\{r_k : k = 1,...,N\}$ over the interval r_a to r_b with $\Delta = (r_b - r_a)/(N-1)$ and use equation (2) to compute the received power at each distance $\{P_{sk} = P_s(r_k)\}$. Then,

$$n \approx \frac{1}{\Delta\sum_k \ln^2(r_k)}\left[\begin{array}{l}\ln(P_0)\Delta\sum_k \ln(r_k) \\ -\Delta\sum_k \ln(r_k)\ln(P_{Sk})\end{array}\right]. \tag{11}$$

Equation (11) is readily evaluated with a short computer program. Note that the received powers $\{P_{sk} = P_s(r_k)\}$ could also be computed from ray-tracing field strengths using the "power sum method".

In the Sabine model, the received power of equation (2) is always greater than that of the transmitter in free space, because it is enhanced by the multipath field given by equation (4). Hence, the value of the path loss exponent computed with equation (11) will always be less than two. As the average power absorption coefficient α approaches unity, the room absorption approaches the surface area of the room, $A \to S_T$, and the indirect absorption becomes large, $A_{in} \to \infty$. Then the multipath field $\tilde{E}_m$ becomes small, and the signal strength approaches that of free space, from above. The value of n given by equation (11) approaches the free space value of two.

IV. FIELD STRENGTH IN A RECTANGULAR ROOM

This section compares the field strength found using the log-distance path loss model (1) with the field strength from the Sabine model and from the ray-tracing model. The value of the path loss exponent n is obtained using equation (11).

Figure 1 is a plan of a rectangular room, 6.83 m wide by 8.68 m deep, with a ceiling height of $h = 3.75$ m. The transmitter was a vertical, half-wave dipole radiating 100 mW at 2388 MHz, with directivity $D = 1.64$, centered

1.03 m above the floor. The receiver was moved along the path shown in the figure, starting at r_a=1 m from the antenna and ending at r_b = 4.8 m away, and was 1.07 m above the floor. The walls of the rectangular room were modelled as layered structures with 1.5 cm of concrete (ε_r =5.37, σ =149.5 mS/m), 0.8 cm of brick (ε_r =4.38, σ =18.5 mS/m), a center air layer 7.8 cm thick, and symmetric layers of brick and concrete. The angle- and polarization-averaged power absorption coefficient at 2388 MHz was $\tilde{\alpha} = 0.65$. The floor and ceiling were modelled as concrete slabs of thickness 30 cm, and average power absorption coefficient of 0.79. One wall of the room had a row of metal lockers, 3.48 m from the path, as shown in Fig. 1, with a power absorption coefficient of zero. In the ray-tracing simulation, ray paths with up to 32 reflections were calculated. Field strengths were measured in this room, and [12] reports reasonable agreement with the Sabine model using equation (5) and with the ray-tracing model.

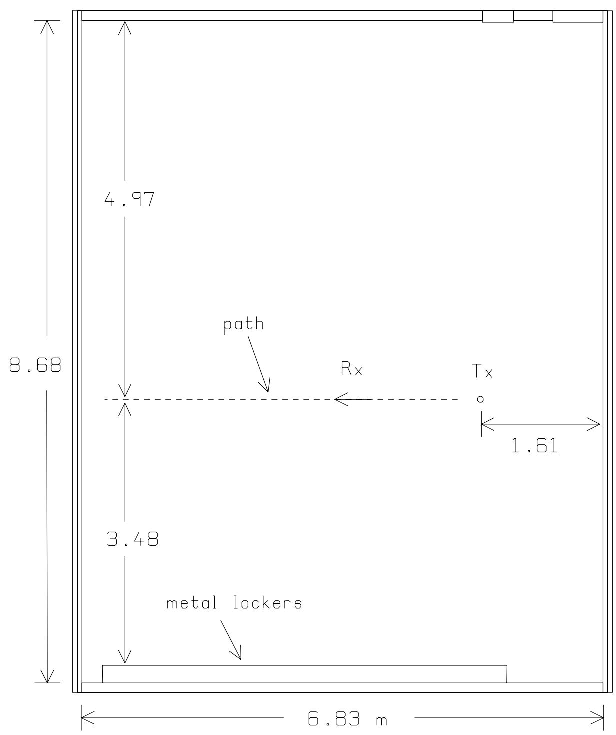

Fig. 1. Plan of the rectangular room.

To use the Sabine model, the area of the room surfaces was calculated to be $S_T = 239$ m^2, and the Sabine room absorption was $A = 166$ m^2, hence the average power absorption coefficient for the room was $\alpha = A/S_T = 0.69$. This was high enough that the room might classified as "dead", in which case the Eyring model for the absorption might be used. The Eyring value for the room absorption was $A_E = 284$ m^2. The Sabine indirect absorption of equation (5) was $A_{in} = 543$ m^2, and the Eyring indirect absorption of equation (8) was $A_{E,in} = 903$ m^2. These values were used with equation (3) to calculate the multipath field strength, which was smaller by a factor of about 0.78 when the Eyring model was used.

Figure 2 shows the electric field strength as a function of separation distance from the transmitter along the path shown in Fig. 1. The ray-tracing method was used to find the local mean field strength (solid curve) by the "power sum method" [3]. The Sabine approximation (dashed curve with crosses) using the Sabine room absorption agreed closely with the ray-tracing curve. When the Eyring room absorption was used (long-dashed curve) the field strength was too small in comparison to the ray-tracing value. Thus, although the room might be classified as "dead", the Sabine absorption led to better agreement with the ray tracing model than did the Eyring absorption.

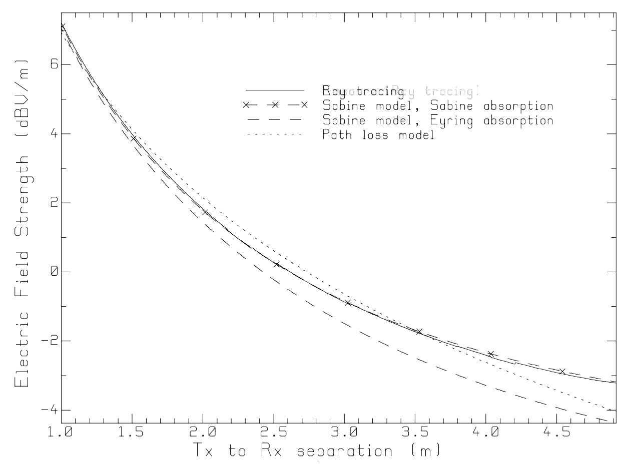

Fig. 2. Electric field strength as a function of distance from the antenna in the rectangular room.

The path loss exponent of $n = 1.58$ was found by evaluating equation (11) using received powers along the path computed with equation (2) using the Sabine absorption. The electric field strength associated with the path loss model is given by,

$$E(r) = \frac{E_0}{r^{n/2}}, \qquad (12)$$

where $E_0 = \sqrt{\eta D P_t / (4\pi)}$ is the field strength at one-meter distance from the transmitter. Figure 2 compares the field strength from the path loss model (dotted curve), from ray tracing (solid curve), and from the Sabine method (dashed curve with crosses). Choosing n with equation (11) leads to a best-fit approximation of the

Sabine model by the path loss model. The field strength of the path loss model was too large between 1 and 3.5 m distance, but the error is small. Towards the end of the path, the field strength of the path loss model decreased too quickly with distance, with a 0.8 dB error at the end of the path.

Figure 2 showed that the Sabine absorption led to a better approximation of the ray-tracing mean value than did the Eyring absorption, so in the following the Sabine absorption will be used to demonstrate the dependence of the path loss exponent on the power absorption coefficient of the room surfaces and the room geometry.

V. PATH LOSS EXPONENT IN A TYPICAL ROOM

Figure 3 shows the variation of the path loss exponent n with the floor area of a square room and with the average power absorption coefficient of the room surfaces, for a square room of ceiling height h=2.75 m. The room had side length w, floor area $S_F = w^2$, and surface area $S_T = 2w^2 + 4wh$. The average power absorption coefficient of the surfaces of the room was α, the Sabine absorption was $A = \alpha S_T$ and the indirect absorption was $A_{in} = \alpha S_T / (1-\alpha)$. To evaluate n, a vertical dipole of directivity $D = 1.64$, radiating P_t watts, was used. The received power was calculated on a path starting at distance $r_a = 1$ m from the dipole and ending at distance $r_b = \sqrt{2}w - 1$ m. Given the value for the average power absorption coefficient α, equation (2) was used to compute a set of received power values at intervals of $\Delta = 1$ cm from r_a to r_b, and then equation (11) was used to compute the path loss exponent. The calculation was repeated as the power absorption coefficient α varied from 0.01 to 0.99, and as floor area varied from $S_F = 10$ m^2 (e.g., an office) to $S_F = 400$ m^2 (e.g. a large auditorium).

These calculations show that at a given absorption, the path loss exponent increases with room size. Thus for absorption α = 0.5, an office of area 10 m^2 would have a path loss exponent of n = 0.69, a mid-sized room of area 50 m^2 would have n = 1.13, and an open-plan office of area 200 m^2 would have n = 1.32. For small rooms and low power absorption coefficients, the multipath field strength of equation (4) dominated the direct field over most of the area of the room, and the received power of equation (2) was almost constant with distance. Thus it was not possible to calculate a path loss exponent. For example, for an office of area 10 m^2 with power absorption coefficient less than 0.3, no path loss exponent could be evaluated. For a power absorption coefficient greater than 0.3, the path loss exponent increased rapidly with absorption. As the room got larger, the minimum absorption for which a path loss exponent could be found decreased. Thus for a room of floor area 50 m^2, the path loss exponent increased from zero starting at absorption $\alpha = 0.1$. For all rooms, as the power absorption coefficient approached unity, corresponding to perfectly-absorbing or "free space" walls, the path loss exponent approached $n = 2$, corresponding to free space propagation.

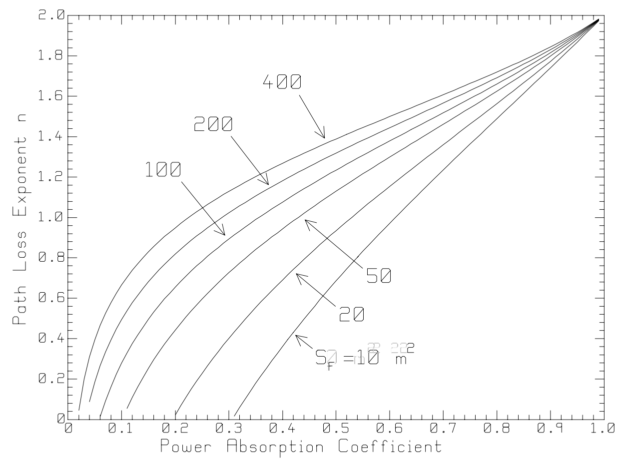

Fig. 3. Path loss exponent as a function of power absorption coefficient and floor area in a square room.

The rectangular room of Fig. 1 had surface area $S_T = 239$ m^2 and average power absorption coefficient α =0.69. Using Fig. 3, the value of n is between 1.5 and 1.55, which is close to the value of 1.58 found above for the rectangular room. Note that the ceiling height of the rectangular room was 3.75 m, considerably higher than the ceiling height of 2.75 m used to draw Fig. 3.

Table 4.6 in [5] gave values of n = 1.6 and 1.8 for a metal-working factory and a paper/cereals factory respectively, both with a LOS path. However, no floor area or ceiling height was given, nor an indication of the wall construction. Assuming a power absorption coefficient of $\alpha = 0.7$, typical of many wall constructions, Fig. 3 shows that for a 200 m^2 area, $n = 1.57$; for 400 m^2, $n = 1.61$, these values being not greatly different from those in [5]. A larger floor area would lead to a larger value of n.

VI. CONCLUSION

The log-distance path loss model (1) is often used to approximate the received power as a function of distance from a transmitter, using values of the path loss exponent n based on measurements in an indoor environment when there is a line-of-sight path from the transmitter to the receiver. This paper showed how to derive the value of n from the geometry of the room, and from the electrical properties and construction of the walls, floor and ceiling.

Figure 2 demonstrated that in a mid-sized rectangular room, field strengths from the Sabine model and from a ray-tracing model agreed closely. The value of n was derived from the Sabine field strengths, and then the field strength of the path loss model were close to the Sabine values.

Figure 3 showed the behavior of the path loss exponent as a function of the floor area of a square room and of the power absorption coefficient of the walls. For very low absorption, the path loss model was not useful. For higher absorption, the path loss exponent increased rapidly and approached $n = 2$ as the absorption coefficient of the walls approached unity, making the walls perfectly absorbing. Figure 3 can be used to estimate the path loss exponent and hence the received power when there is an unobstructed LOS path in an indoor environment, from a knowledge of the wall materials and construction of the room dimensions, and should be useful in the design wireless local area networks.

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Christopher W. Trueman received his Ph.D. from McGill University in 1979. He has applied the methods of computational electromagnetics to problems such as aircraft antenna performance, antenna-to-antenna coupling and EMC on aircraft, aircraft and ship radar cross-section at HF frequencies, suppression of scattering of the signal of a commercial radio station from power lines, dielectric resonators, unconditionally-stable formulations for the finite-difference time-domain method, and the fields of portable radios such as cellular phones held against the head. Recently, his research has investigated the radar cross-section of ships at VHF frequencies, composite materials for aircraft, indoor propagation, and EMC issues between portable radios and medical equipment. Dr. Trueman is currently the Associate Dean for Academic Affairs in the Faculty of Engineering and Computer Science at Concordia University.

Donald P. Davis received the B.Eng in electrical engineering degree from Concordia University in 1991, the MaSc. degree in electrical engineering from Concordia University in 1994 and the PhD. Degree in electrical engineering from McGill University 2004. Since 1995 he has been a sessional instructor teaching both graduate and undergraduate engineering courses at Concordia University. In 2006 he joined the Jewish General Hospital of Montreal as a post-doctoral researcher studying electromagnetic compatibility of medical equipment due to the presence of wireless communication systems. His research interests include indoor radio wave propagation measurement and modeling, electromagnetic interference estimation and safe implementation of wireless communication systems within clinical environments.

Bernard Segal is a highly-inter-disciplinary Research Engineer (Electrical Engineering, with a Master's in Electrical (Biomedical) Engineering and a PhD in Neurophysiology, all from McGill University). He is an Associate Professor in the Ear-Nose-Throat (ENT) Department of McGill University, with appointments in Biomedical Engineering and in Physiology. He is the Director of Research in ENT at McGill, and at the SMBD-Jewish General Hospital. He is the Director of Graduate Studies in ENT at McGill. He has contributed to many medical-EMC standards, EMC-healthcare policies, and EMC-healthcare recommendations. He has organized over thirty national and international conferences, workshops and teaching sessions on EMC in Healthcare. His current research is directed towards clarifying the nature of electromagnetic-interference risk in hospitals, towards minimising this risk, and towards evaluating how to best use wireless informatics in health care safely.